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DIVIDEND MODEL FOR NONCONTRIBUTORY DEPOSIT ADMINISTRATION GROUP ANNUITY CONTRACTS

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In recent years, the Deposit Administration Group Annuity contract—chiefly noncontributory—has become a popular funding medium for insured pension plans. There appears to be no actuarial demonstration of the operation of the deposit fund, experience fund and dividend formulation. It is the purpose of this paper to demonstrate how these funds, in theory, develop and enter a dividend formula.

ASSUMPTIONS AND DEFINITIONS

A noncontributory deposit administration group annuity contract will be traced through one contract year by some general relations which can be adapted to handle cases that may be difficult to treat directly. Familiarity with the deposit administration approach, as discussed by Mr. Bronson and others (*TSA* I, 219), and by Mr. Houseman (*TSA* IV, 231), will be assumed. There will be no benefit prior to purchase of an annuity from the insurance company at retirement date. The type of funding method, treatment of cost fluctuations, benefit formula, and special benefits, such as disability payments or widow's benefits, will not enter the analysis. Contingency reserves and pooling devices will be mentioned only briefly, as part of the dividend system. This demonstration assumes no pooling, allowing each case to benefit from its own mortality experience. Purchase rate and interest rate guarantees will remain unchanged during the contract year. Expense loadings will be constant throughout.

Definitions

For the Deposit Fund:

F_0 = Deposit fund at start of current contract year

$1-\lambda$ = Expense factor; $1-\lambda$ may be of the order of 5%

E_0 = Annual contract charge deducted at start of current contract year (usually decreasing with increase in deposit)

i = Deposit fund interest rate

P = Total deposits during current contract year

π = Total gross single premiums used to purchase annuities for retirees, during current contract year

F_1 = Deposit fund at end of current contract year

For the Experience Fund:

V_0 = Retired life reserves at start of current contract year

L_0 = Accumulated annual deficit of prior years, at start of current contract year, carried until a surplus is achieved

E_0' = Expense charges assumed incurred at start of current contract year (which may provide for amortization of acquisition charges), including allowance for overhead and margin

i' = Experience fund interest rate

B = Total payments to retirees during current contract year

V_1 = Retired life reserves at end of current contract year

j = Retired life reserve valuation interest rate

S_1 = Surplus available for distribution, if positive, or surplus deficit, if negative, at end of current contract year. In the latter case, this will become L_0 for the next following contract year.

Superscript i or i' affixed to the upper left-hand corner of P, π or B represents an accumulation of each fractional part of P, π or B during the year with interest at rate i or i' from date of receipt or date of disbursement to end of current contract year.

DEPOSIT FUND

The deposit fund is a repository for contributions placed with the company by the employer. This is a fluid account with ins and outs occurring throughout the contract year. The ins are the deposits, the outs the annual contract charge and single premiums for retirees. It is the operation of this fund, throughout the contract year, combined with the remaining balance from prior years, which probably accounts for the origin of the term "Deposit Administration." (See *TSA* I, 282 for a brief discussion, by Mr. Peterson, of the significance.)

An algebraic statement of the deposit fund balance, at the end of the contract year, follows:

$$F_1 = (F_0 - E_0)(1 + i) + {}^iP - {}^i\pi . \tag{1}$$

The reserve for the deposit fund is λF_1, so that the year's increase in reserve, from (1), is

$$\begin{aligned} \lambda F_1 - \lambda F_0 &= i\lambda F_0 + \lambda{}^iP - \lambda E_0(1 + i) - \lambda{}^i\pi \\ &= i\lambda F_0 + {}^iP - (1 - \lambda){}^iP - \lambda E_0(1 + i) - \lambda{}^i\pi . \end{aligned} \tag{2}$$

Equation (2) shows that the increase in deposit fund reserve consists of interest on the reserve at the start of the current contract year plus accumulated deposits less the sum of a general expense contribution, an annual contract charge (if any) and the net single premiums used to purchase annuities for retirees, during the current contract year.

EXPERIENCE FUND

The experience fund parallels the deposit fund and represents the actual experience taking place during the contract year. In a sense, an asset share is traced through one contract year, with the valuation reserve then deducted and the balance representing surplus available for distribution or surplus deficit. The net result of these operations at the end of the contract year is shown algebraically below, and then separated into components:

$$S_1 = (\lambda F_0 + V_0 - L_0 - E_0')(1 + i') + {}^{i'}P - {}^{i'}B - \lambda F_1 - V_1 . \quad (3)$$

Substitute F_1, from (1), so that

$$\begin{aligned} S_1 = {} & V_0(1 + j) + \lambda\,{}^{i}\pi - {}^{i'}B - V_1 \\ & + (i' - i)\lambda F_0 + ({}^{i'}P - {}^{i}P) - (i'E_0' - i\lambda E_0) + (i' - j)V_0 \\ & + (1 - \lambda)\,{}^{i}P + \lambda E_0 - E_0' \\ & - L_0(1 + i') . \end{aligned} \quad (4)$$

The final arrangement in (4) can be interpreted as a contribution formula which furnishes a rough breakdown of the total gain into gain from mortality experience on retired lives, gain from interest, and gain from expenses, less any accumulated annual surplus deficit carried over from prior years. No attempt has been made to isolate completely the gain from each source. If $\lambda = 1$ in (4), the expense contribution is reduced to $E_0 - E_0'$, which may be negative and would have to be recovered from gains derived from other sources. Ideally, λ should be chosen so that the gain from expenses, over the long run, is positive. In practice, this may not be possible and gains from interest must then be drawn upon.

DIVIDEND PAYABLE

Consideration of the various gains which develop as part of the experience fund balance makes possible a distinction between stable and unstable gains. (Here, a more or less uniform flow of ins and outs is assumed.) If there have been no retirements, the gain from interest acts to overcome the deficit caused by the excess of the initial expenses over the initial expense contribution and produces a net positive balance, after several years, which gradually increases. The timing and peak of the deficit are a function of all the items entering in equation (3).

Unlike interest and expense, the contribution from mortality—now considering one retired life—is initially a deduction at the end of the year of retirement, when the purchase price is withdrawn and reserves are set up. Since the purchase price reflects the actuarial estimate of emerging mortality, this deduction provides for some future mortality allowance which is subject to correction as experience develops. At the end of the year of death of the retired annuitant, the reserve released creates a large gain and would probably produce a corresponding dividend. As such a gain might very well be preceded and followed by surplus deficits, surplus releases on account of mortality gains should be controlled. Various methods can be employed, depending upon company philosophy, such as intergroup pooling, intragroup pooling, or contingency reserves. Intergroup pooling refers to a method which recognizes immediately some proportion of mortality fluctuations and pools the balance among other similar groups. Intragroup pooling allows a gradual release within a group. Contingency reserves, expressed as a percentage of F_1 (which may be accumulated gradually), generally attempt to smooth out fluctuations in mortality experience, but may be extended to also cover other contingencies, such as possible reductions in interest earnings, increase in future expenses or fluctuations in size of group.

After having made provision for smoothing of large mortality fluctuations, as discussed in the previous paragraph, any positive surplus is available for dividend distribution. A negative surplus is carried forward as a deficit into the calculations for the next contract year. For an alternate verbal statement of dividend declared, see Mr. Dreher's paper in *TSA* XI, 634.

CONCLUSION

There are many complications which could be introduced into this examination of the operation of deposit administration funds. The approach here, admittedly sparse, has been solely an attempt to isolate basic theory. This analysis will be considerably enhanced if some of the technical problems briefly referred to above, such as contingency reserves, pooling devices and modifications of technique, are more fully discussed by those members—and there are many—who are more experienced in this difficult field than the author.

DISCUSSION OF PRECEDING PAPER

RAYMOND W. BENDER:

This discussion is in response to Mr. Berin's invitation to enlarge on some of the points in the paper.

Mr. Berin uses as his "reserve for the deposit fund" λF, where $1 - \lambda$ is the "expense factor." The reserve, for use in dividend work, can also reflect differences between the guaranteed interest and annuity purchase rates applicable to the fund and the dividend valuation basis. If $1 - \lambda$ is the loading factor in the guaranteed annuity purchase rates, Mr. Berin has in effect used the interest and mortality rates of the guarantees as the dividend valuation basis.

The dividend process for a group annuity contract, including those with deposit administration features, involves, in some form, comparison of the "experience fund" (Mr. Berin's term) for the contract with the insurance company's liabilities for future benefits and expenses under the contract.

Once the mortality and interest basis for valuing liabilities for future benefits is chosen, it is readily applied to annuities already purchased for individuals under the contract. Application of the basis to determine the liabilities for any deposit administration fund is less simple.

One approach would be to make the best possible estimate of how the fund will be used. Based on this estimate, the value of the benefits to be generated by the fund can be found by application of the dividend valuation basis. For example, assume a fund of $1,000 has a guaranteed interest rate of 3% and a gross guaranteed life annuity purchase rate of $150 for $1 per month for a male at age 65. Assume the dividend valuation basis is $3\frac{1}{2}\%$ and a mortality table which produces a life annuity value of $140 for $1 per month for a male at age 65. If the fund is to be applied two years from the valuation date, the liability on the dividend valuation basis for the benefits to be provided by the $1,000 fund is:

$$\$1{,}000 \times \frac{(1.03)^2}{(1.035)^2} \times \frac{140}{150} = \$924\,.$$

This approach described above is, of course, troublesome to apply in practice, since it requires a prediction of the application of the fund. Also, use of generation mortality in the dividend valuation basis, and the presence of several funds under a contract with varying guarantees, increase the complications.

A simpler approach is to carry dividend liabilities for a deposit administration fund as some arbitrary percentage of the fund—100% or 95% as Mr. Berin suggests. This percentage should probably be higher than any that would be developed by a more complicated approach. As the fund is allocated to annuity purchases, the excess of

(1) the product of the arbitrary percentage and the amount of the fund applied over
(2) the resulting dividend reserve for the purchased annuities

emerges as earnings. The earnings will also reflect the difference between the "experience fund" interest rate and the guaranteed deposit administration fund interest rate applied to the fund balance. If either of these sources of earnings yields negative amounts, some recognition of this is probably desirable, perhaps by setting aside some portion of the current year's earnings.

It is possible for the guarantees applicable to older deposit administration funds to be more liberal than the current dividend valuation basis. If this is so, it seems necessary to carry more than 100% of such funds as a liability for dividend purposes to avoid paying apparent earnings as a dividend on a faulty premise as to the contract's dividend surplus position.

Some deposit administration funds are subject to a contractual right of transfer to a substitute funding medium. When a transfer occurs, the "experience fund" is reduced by the amount transferred. It may also be reduced by a charge which reflects the difference between the interest rate at which funds are currently being invested and the average interest rate at which funds under the contract are invested. The dividend liability for a deposit administration fund subject to transfer should not be less than the amount that could be transferred and any additional charge that would be made against the "experience fund."

(AUTHOR'S REVIEW OF DISCUSSION)

BARNET N. BERIN:

I would like to thank Mr. Bender for his interesting comments which will be covered in order of presentation.

By his use of quotation marks and parenthetic insert, Mr. Bender implies that the author has defined terms which are not understood outside of the context of the paper. It is important to dwell upon this point because our *language* is constantly being expanded, particularly in the Group field, and it appears that no one body exists that is both willing and able to assume the responsibility of defining terms. It has reached the point where meaningful exchanges require careful elaboration of what should be basic material. Mr. Bender does well to emphasize this, although the terms so identified should be familiar to all concerned with employee retirement plans.

Language difficulties can work in both directions. If "a dividend valuation basis" refers to the method by which interest beyond the guaranteed rate is credited and the retired lives reserves valued, then, as defined in the paper, the interest and mortality rates of the guarantees need not be the same as the dividend valuation basis. Note that i, i', j are not related nor is the method by which π or V is determined necessary to the development. If V is stronger than π, a contingency reserve is established whenever a retired life reserve is set up.

I agree with Mr. Bender that the first approach he describes is troublesome and complicated. While an argument could be made for determining λ so that other contingencies beyond expenses are provided for, it seems to me that this is best handled independently by specific earmarking of either a mortality contingency reserve (implicit in use of a stronger V than π) or a percentage of deposits. The dividend formula should be reasonable, fair and justifiable to the group which has no retirees owing to (say) staggered retirements as well as to more mature groups.

The percentage λ should not be arbitrary, although it clearly will be influenced by competitive considerations. If $\lambda < 1$, a definite contribution for expenses beyond the contract charge is available. If $\lambda = 1$, this element disappears and the contribution from expenses is almost certainly negative with an interesting effect upon the size and pay-out of the dividend. From formula (3), it can be shown that if $\lambda = 1$ and the funds are growing rapidly the dividend will decrease, and where funds are decreasing the dividend will increase compared to a choice of $\lambda < 1$. If $\lambda > 1$, a contingency reserve is, in effect, established. If λ is considered as generating some provision for expenses, λ should probably increase with increase in deposit and increase, to a higher level, in renewal years as compared with year of issue.

I believe that Mr. Bender's second approach is a dividend developed by a fund method which is simply formula (3).

Mr. Bender's next to last paragraph states that where older funds have more liberal guarantees a contingency reserve is necessary. This is certainly true. The last paragraph refers to contractual terminations and dividend formula rules should cover this possibility.