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# MORTALITY INVESTIGATION WITH EXPECTED MORTALITY ESTIMATED AT ISSUE BY USE OF PERSISTENCY FACTORS

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THIS note describes a mortality investigation procedure in which the expected mortality applicable to each new policy for a period of about three years after issue is estimated by use of persistency factors and is punched into the individual IBM card. The total expected deaths for comparison with actual deaths in any group are then obtained by simply tabulating the cards. The note goes on to show some results produced by this method and concludes with an opinion of the merits and applicability of the approach.

TABLE 1

TERMINATION RATES (OTHER THAN DEATH) BY POLICY YEAR
STANDARD NONPENSION BUSINESS, LIFE PLANS

| Issue Age | During 1st Year | During 2nd Year | During 3rd Year | During 4th Year | Total, 4 Years |
|---|---|---|---|---|---|
| | Percentages of Original Business | | | | |
| 0–14 | 5% | 10% | 3% | 2% | 20% |
| 15–19 | 7 | 13 | 3 | 2 | 25 |
| 20–24 | 8 | 15 | 3 | 2 | 28 |
| 25–29 | 6 | 12 | 3 | 2 | 23 |
| 30–34 | 4 | 8 | 3 | 2 | 17 |
| 35–39 | 3 | 7 | 3 | 2 | 15 |
| 40 and over | 3 | 5 | 3 | 2 | 13 |

## THE METHOD

It is necessary first to study early termination rates in considerable detail, ascertaining differences by type of business, by plan, by issue age and by policy duration. The rates considered appropriate are then combined with suitable mortality rates to estimate the exposed to risk and the fraction of one expected death attributable to each policy in the block of business being studied. To illustrate this, let us look at "Standard Nonpension Business, Life Plans," which is one of thirteen categories treated separately in our application of this method. Table 1 shows the percent-

ages of original business that our studies showed could be assumed to terminate during each of the first four policy years. (Terminations for nonpayment of an anniversary premium are assigned to the following year.)

Mortality rates used at issue ages between 10 and 64 for the first policy year were 95% of the 1946–1949 Select Basic Table (*TSA* II, 506), and for the second, third and fourth policy years were 85% of that table. Thus for issue age group 35–39, the rates per thousand were 0.92, 1.18, 1.51, and 1.76, respectively.

From the above information the exposed (equal to the business entering each policy year decreased by a proportion, $p$, of the voluntary terminations of that year) is readily determined. In our case $p$ was taken as 2/3 in the first policy year, 5/6 in the later policy years. The expected mortality for each policy was then obtained by applying the mortality factor to this estimated exposed. For Standard Nonpension Business, Life Plans, issue ages 35–39, this produced expected mortality per policy as follows:

| 1st Year | 2nd Year | 3rd Year | 4th Year |
|---|---|---|---|
| .00090 | .00107 | .00132 | .00150 |

However, in these studies it was desired to produce mortality by calendar year rather than policy year, and to reduce machine work by using only even-numbered policies for the expected mortality while using both even-numbered and odd-numbered policies for the deaths. Accordingly, the policy year values obtained as above were converted to a calendar year basis applicable to business grouped by calendar quarter of issue (January–March, April–June, etc.), and were then multiplied by two. Allowance for delayed reporting of deaths was made by deducting one-half a month from the exposure period measured from the middle of each calendar quarter. The results were expressed cumulatively, so that in our illustrated category at issue ages 35–39 the expected mortality per policy became the following:

EXPECTED MORTALITY PER POLICY

| Anniversary Months | First Calendar Year | First Two Calendar Years | First Three Calendar Years | First Four Calendar Years |
|---|---|---|---|---|
| January–March . . . | .0015 | .0036 | .0061 | .0091 |
| April–June . . . . . | .0011 | .0030 | .0055 | .0083 |
| July–September . . . | .0006 | .0025 | .0048 | .0076 |
| October–December | .0002 | .0020 | .0042 | .0068 |

These values were prepared for all the separate categories by type of business, plan, issue age and anniversary quarter, and were gang-punched into cards that contained the information about each policy it was desired to study, *viz.* amount; medical or nonmedical; sex; agency; type of agent; substandard rating; build, impairment and occupational data.

### SOME RESULTS OBTAINED BY THE METHOD

Three investigations of consecutively issued blocks of policies have been made since this method was devised. In each case these comprised a particular block of 100,000 policy numbers, with issue dates approximately as follows:

I. Issues of May 1956–July 1957 observed to December 31, 1959.
II. Issues of July 1957–April 1958 observed to December 31, 1960.
III. Issues of April 1958–July 1959 observed to December 31, 1960.
(This third block will in due course be observed to December 31, 1961.)

The use of blocks of 100,000 policy numbers is not only a convenience, but also a helpful control to make sure that deaths will not be omitted or identified with the wrong exposure-groups. The fact that issue periods do not coincide exactly with calendar quarters introduces an error, but a small one. For example, in Group II, inclusion of April 1958 business creates a one-month understatement in exposure on about 10% of the issues, *i.e.*, an average understatement of 3 days on the entire block.

Table 2 summarizes the results obtained on standard issues, all amounts combined. A striking feature of these results is their general consistency with each other. Even though the numbers of deaths are small, each of the eight sections shows mortality ratios that are uniformly either above or below the 100% level. Detailed figures not shown in this paper break these results down in illuminating fashion by age groups, amount groups and other characteristics.

Since this study is by policies, suitable allowance must be made for multiple policies on individual lives among the deaths.

### USEFULNESS OF THE METHOD

It is obvious that this approach offers the ultimate in flexibility, since the cards when once prepared can be tabulated in as many different ways as there are items of information thereon. It is equally obvious that the method is only as reliable as the persistency factors that are employed. (There is, of course, an additional source of error due to using only even-numbered policies to determine the exposed, but this short cut need not be appraised here as it is not a necessary part of the procedure.) The question is: How large an error is likely to arise due to variation of actual persistency from that assumed?

The practical limit of this error can be visualized by comparing the expected mortality values per policy in the two categories which generally were the farthest apart in persistency factors among standard issues, *viz.* (*a*) Pension Business, Life Plans and (*b*) Pension Business, Term Plans. Such a comparison is shown in Table 3.

These figures tell us that, for example at ages 40–44, an error in

TABLE 2

RESULTS OBTAINED IN THREE INVESTIGATIONS

| | Group I | | | Group II | | | Group III | | |
|---|---|---|---|---|---|---|---|---|---|
| | Expected Deaths | Actual Deaths | Ratio Act. to Exp. | Expected Deaths | Actual Deaths | Ratio Act. to Exp. | Expected Deaths | Actual Deaths | Ratio Act. to Exp. |
| *Nonpension, Males* | | | | | | | | | |
| Medical | 106.98 | 131 | 122% | 63.98 | 69 | 108% | 61.05 | 82 | 134% |
| Nonmedical | 28.87 | 31 | 107 | 21.26 | 22 | 103 | 22.67 | 24 | 106 |
| *Nonpension, Females* | | | | | | | | | |
| Medical | 13.01 | 7 | 54% | 8.18 | 4 | 49% | 6.84 | 1 | 15% |
| Nonmedical | 12.24 | 8 | 65 | 9.06 | 4 | 44 | 8.04 | 4 | 50 |
| *Pension, Males* | | | | | | | | | |
| Medical | 30.72 | 26 | 85% | 16.56 | 9 | 54% | 13.09 | 7 | 53% |
| Nonmedical* | 26.16 | 29 | 111 | 16.12 | 21 | 130 | 10.70 | 16 | 150 |
| *Pension, Females* | | | | | | | | | |
| Medical | 7.07 | 5 | 71% | 4.71 | 0 | 0% | 4.19 | 2 | 48% |
| Nonmedical* | 6.99 | 1 | 14 | 4.42 | 1 | 23 | 3.28 | 1 | 30 |

* Does not include Automatic Issue.

TABLE 3

COMPARISON OF TOTAL EXPECTED DEATH VALUES TO END OF THIRD COMPLETE CALENDAR YEAR AFTER ISSUE ON POLICIES WITH JANUARY–MARCH ANNIVERSARIES

| Issue Ages | Pension Business, Life Plans | | Pension Business, Term Plans | | Ratio (2)÷(4) % |
|---|---|---|---|---|---|
| | Assumed Termination Rate, 4 Years (1) | Expected Death Value $\times 10^2$ (2) | Assumed Termination Rate, 4 Years (3) | Expected Death Value $\times 10^2$ (4) | (5) |
| 25–29 | 26% | .49 | 44% | .42 | 117% |
| 30–34 | 21 | .57 | 44 | .48 | 119 |
| 35–39 | 18 | .90 | 44 | .73 | 123 |
| 40–44 | 14 | 1.37 | 44 | 1.08 | 127 |
| 45–49 | 11 | 2.21 | 35 | 1.83 | 121 |
| 50–54 | 11 | 3.19 | 26 | 2.84 | 112 |
| 55–59 | 11 | 4.88 | 18 | 4.64 | 105 |

predicting persistency all the way from a 14% termination rate to a 44% termination rate—surely a much larger misestimate than should arise if reasonable care is used—would be needed to affect the mortality ratio by as much as 27%. From this and other observations it is concluded that the method is sufficiently reliable to give a general picture of mortality levels in the highly important early years. Clearly we should not attempt to use it over such a lengthy period of policy years that business actually remaining in force has a chance to diverge too far from the estimate.

To offset its admitted lesser degree of precision the method offers advantages that seem well worth having—in particular, promptness of results and flexibility. Expected deaths can be calculated when each block of policies is issued, so that mortality ratios can be observed even as the death claims emerge. The opportunity to appraise underwriting results while they are still fresh can be a stimulus to analysis of underwriting rules and procedures. The method can be used to locate possible problem areas which can then be analyzed further, either by the customary mortality investigation method or simply by testing a sample to verify that actual persistency was close to that assumed.

It is even possible that the method could be used in intercompany studies of special types of business. For example, companies interested in getting a rapid, approximate idea of early mortality under policies issued by exercise of the guaranteed purchase option might use this mortality investigation method to pool their experience while awaiting the time when the usual formal type of study could be undertaken.

Some actuaries might prefer a variation of the procedure described in this note that would produce mortality ratios by amounts rather than by policies. This has not been tried, but appears to be feasible. It would require use of a multiplying-punch instead of a gang-punch to record the expected mortality values on the individual cards. On the other hand, the method we have used does yield mortality results separately for any desired amount-groups.

## DISCUSSION OF PRECEDING PAPER

LOUIS LEVINSON:

The method developed by Mr. Moorhead in this paper provides actuaries with a means of conducting a rapid and practical investigation of mortality experience for the first few years following issue. In this modern age when new forms of coverage are being adopted as fast as they can be fashioned, the ability to check early on the closeness of the mortality assumptions to actual experience is much to be desired. This product of Mr. Moorhead's ingenuity should serve actuaries well in this important area.

Mr. Moorhead's method was applied to the insurances issued in the Massachusetts Mutual and dated 1956. The data were those included in our reports to the continuous mortality investigation of the Society with respect to medically examined male lives. Expected deaths were determined by the 1946–1949 Select Basic Tables and persistency by the Massachusetts Mutual's experience for the period. The nature of the data cast some question on the validity of first year ratios; accordingly, figures only for the second, third and fourth policy years are shown in the table on the following page.

In obtaining expected deaths Mr. Moorhead's basic method, but not his procedure (involving punching cards, etc.), was followed. To obtain the results shown, appropriate mortality and persistency factors were applied to the amounts of new business dated 1956 in the age categories indicated. The results, of course, are essentially those which would have been obtained by Mr. Moorhead's procedure.

It is interesting to note that the ratios derived by the conventional method are, in practically every instance, extremely close to those obtained by Mr. Moorhead's method. This suggests that the procedure may also present a simple and direct way for testing the reasonableness of the results arising from the conventional methods of investigating mortality experience.

JOHN M. BRAGG:

Mr. Moorhead is to be congratulated for this valuable paper outlining a method by which early mortality may be investigated with promptness and flexibility.

The method requires rather precise knowledge of expected persistency, broken down by age group and class of business, on a policy year basis.

MASSACHUSETTS MUTUAL EXPERIENCE
ISSUES DATED 1956
EXPECTED DEATHS BASED ON 1946–1949 SELECT BASIC TABLE
PERSISTENCY BASED ON MASSACHUSETTS MUTUAL EXPERIENCE

| AGE GROUP | ACTUAL DEATHS | EXPECTED DEATHS | | RATIO ACTUAL TO EXPECTED | |
|---|---|---|---|---|---|
| | | Moorhead | Conventional | Moorhead | Conventional |
| | POLICY YEAR 2 | | | | |
| 10–14 | $ 0 | $ 426 | $ 452 | 0.00% | 0.00% |
| 15–19 | 0 | 1,829 | 1,942 | 0.00 | 0.00 |
| 20–24 | 13,300 | 9,543 | 9,517 | 139.37 | 139.75 |
| 25–29 | 15,000 | 37,330 | 37,612 | 40.18 | 39.88 |
| 30–34 | 50,000 | 61,732 | 63,463 | 81.00 | 78.79 |
| 35–39 | 89,800 | 115,616 | 119,143 | 77.67 | 75.37 |
| 40–44 | 156,700 | 125,767 | 130,758 | 124.60 | 119.84 |
| 45–49 | 60,100 | 127,980 | 131,081 | 46.96 | 45.85 |
| 50–54 | 47,800 | 88,797 | 92,967 | 53.83 | 51.42 |
| 55–59 | 25,000 | 62,953 | 66,849 | 39.71 | 37.40 |
| 60–64 | 2,100 | 35,971 | 38,318 | 5.84 | 5.48 |
| 65 and over | 0 | 8,876 | 9,460 | 0.00 | 0.00 |
| Total | $459,800 | $676,820 | $701,562 | 67.94% | 65.54% |
| | POLICY YEAR 3 | | | | |
| 10–14 | $ 0 | $ 490 | $ 536 | 0.00% | 0.00% |
| 15–19 | 0 | 2,067 | 2,149 | 0.00 | 0.00 |
| 20–24 | 0 | 10,753 | 10,412 | 0.00 | 0.00 |
| 25–29 | 49,000 | 42,733 | 42,539 | 114.67 | 115.19 |
| 30–34 | 62,500 | 75,696 | 87,718 | 82.57 | 71.25 |
| 35–39 | 87,500 | 144,520 | 146,509 | 60.55 | 59.72 |
| 40–44 | 254,600 | 168,828 | 172,205 | 150.80 | 147.85 |
| 45–49 | 128,000 | 168,740 | 168,065 | 75.86 | 76.16 |
| 50–54 | 217,200 | 122,500 | 123,176 | 177.31 | 176.33 |
| 55–59 | 90,000 | 85,600 | 89,725 | 105.14 | 100.31 |
| 60–64 | 17,100 | 47,813 | 50,293 | 35.76 | 34.00 |
| 65 and over | 22,000 | 11,023 | 11,490 | 199.58 | 191.47 |
| Total | $927,900 | $880,763 | $904,817 | 105.35% | 102.55% |
| | POLICY YEAR 4 | | | | |
| 10–14 | $ 0 | $ 527 | $ 587 | 0.00% | 0.00% |
| 15–19 | 0 | 2,114 | 2,246 | 0.00 | 0.00 |
| 20–24 | 0 | 11,156 | 10,589 | 0.00 | 0.00 |
| 25–29 | 97,000 | 45,189 | 43,553 | 214.65 | 222.72 |
| 30–34 | 95,800 | 83,780 | 94,367 | 114.35 | 101.52 |
| 35–39 | 100,000 | 164,392 | 162,688 | 60.83 | 61.47 |
| 40–44 | 103,900 | 198,219 | 198,851 | 52.42 | 52.25 |
| 45–49 | 98,700 | 194,599 | 190,845 | 50.72 | 51.72 |
| 50–54 | 135,600 | 147,346 | 142,329 | 92.03 | 95.27 |
| 55–59 | 122,800 | 102,062 | 102,996 | 120.32 | 119.23 |
| 60–64 | 53,000 | 55,103 | 57,775 | 96.18 | 91.74 |
| 65 and over | 6,000 | 12,252 | 12,563 | 48.97 | 47.76 |
| Total | $812,800 | $1,016,739 | $1,019,389 | 79.94% | 79.73% |

Many smaller companies do not have records suitable for investigating persistency on a policy year basis and might therefore have trouble applying the method. This discussion will suggest a procedure under which Mr. Moorhead's method could be modified and adopted by such companies.

From recent paid business and valuation records, it is possible to determine the portion of a given calendar year's paid business which remains in force (1) at the end of such calendar year, (2) at the end of the second calendar year, etc.

Let us define

$B_x$ = business paid for in calendar year $x$
$I_x^y$ = business paid for in calendar year $x$, still in force at end of calendar year $y$.

We are interested in obtaining factors to apply to $B_x$ to estimate "exposed to risk" in calendar year $y$.

For practical purposes we can obtain such factors as follows. Specific calendar years are shown to illustrate the method.

1) The first factor represents the portion of $B_x$ which is "exposed to risk" in year $x$ and can be taken as

$$\frac{\frac{1}{2}\{B_{60} + I_{60}^{60}\}}{B_{60}}$$

2) The second factor represents the portion of $B_x$ which is "exposed to risk" in year $x + 1$ and can be taken as factor (1) multiplied by

$$\frac{I_{59}^{59} + I_{59}^{60}}{B_{59} + I_{59}^{59}}$$

3) The third factor represents the portion of $B_x$ which is "exposed to risk" in year $x + 2$ and can be taken as factor (2) multiplied by

$$\frac{I_{58}^{59} + I_{58}^{60}}{I_{58}^{58} + I_{58}^{59}}$$

4) The fourth factor represents the portion of $B_x$ which is "exposed to risk" in year $x + 3$ and can be taken as factor (3) multiplied by

$$\frac{I_{57}^{59} + I_{57}^{60}}{I_{57}^{58} + I_{57}^{59}}.$$

By use of two years' paid business records and three years' valuation records, it is therefore possible to determine all necessary factors for esti-

mating "exposed to risk." These calculations can be made on either a "number of policies" or "amount" basis.

It is suggested that this procedure be followed for broad age at issue and class of business groups. Some smoothing of results might prove necessary. If valuation and paid business records are available by mode of premium payment, as well as age, factors based on such information might be particularly helpful, because of the high degree of connection between these items and persistency.

This procedure could be made more accurate by incorporating actual death claims in these formulas, but this refinement might be thought unnecessary in view of the generally approximate nature of the results. Furthermore, factor (1) above could be arbitrarily refined by further study of early lapses and typical distribution of new business by month of issue.

When suitable "exposed to risk" factors have been obtained as above, the expected mortality for any particular policy paid for can be determined by multiplying the appropriate factor by the appropriate select mortality rate. In arriving at the latter it would be borne in mind (1) that business paid for in year $x$ is exposed only for a part of year $x$ (on average, one-half of such year, if new business is evenly distributed) and (2) that select mortality rates require a "half-year of age" adjustment to be suitable.

Mr. Moorhead's method employs different factors for each calendar quarter of issue. This would not be possible under the proposed modification, and some loss of accuracy might result. However, the proposed modification does assure the use of persistency factors determined from recent experience.

Mr. Moorhead deserves thanks for this fine paper, which permits the study of early mortality in the required detail, without delay.

HARRY M. SARASON:

In these days, persistency is almost as important as mortality. Mr. Moorhead's general method could be used to predict terminations as well as deaths. A comparison of actual terminations to predicted terminations would not only help in an analysis of persistency but would also permit a refinement in the calculation of the approximate expected mortality when this seemed advisable. Many companies in the United States have much higher termination rates than those of Mr. Moorhead's company, and have correspondingly greater fluctuation in persistency.

(AUTHOR'S REVIEW OF DISCUSSION)

ERNEST J. MOORHEAD:

Mr. Levinson has inspired confidence in the method by showing that it works satisfactorily in a test situation. Mr. Bragg and Mr. Sarason have enhanced the value of the basic idea by offering in the one case an alternate and in the other case a parallel application of it.

The closeness of the approximation exhibited by Mr. Levinson for the fourth policy year points to the possibility that we may have been unnecessarily timid in restricting our use of the method to as short a period as that described in the paper. Inspection of his results for individual age groups shows, however, that the near identity of the aggregate result was fortuitous. Mr. Bragg's method appears to be feasible, subject only to the inherent problem in a small company of obtaining sufficient early policy year exposure for meaningful results.

Mr. Levinson stresses, I believe, the greatest potential usefulness of the procedure when he mentions its applicability to investigating mortality under new forms of coverage. As an illustration we show below in the same form as Table 2 of the paper the mortality results that we have already obtained for automatic issue business.

| | Group I | | | Group II | | | Group III | | |
|---|---|---|---|---|---|---|---|---|---|
| | Expected Deaths | Actual Deaths | Ratio Act. to Exp. | Expected Deaths | Actual Deaths | Ratio Act. to Exp. | Expected Deaths | Actual Deaths | Ratio Act. to Exp. |
| *Automatic Issue* | | | | | | | | | |
| Males..... | 70.71 | 143 | 202% | 54.81 | 105 | 192% | 45.40 | 103 | 222% |
| Females..... | 14.45 | 15 | 104% | 13.66 | 9 | 66% | 12.85 | 8 | 78% |

These rather startling statistics do not necessarily indicate that automatic issue business on male lives has been unprofitable. They do give incentive for prompt consideration of the question whether that much extra mortality is being covered by savings in acquisition and administrative expense and by reduction in dividends.