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## THE CASE FOR REFINEMENT IN METHODS OF ALLOCATING INVESTMENT INCOME

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THE techniques of fund accounting have an important place in actuarial analysis. As applied in preparing the Gain and Loss Exhibit, they give management and supervisory authorities a broad picture of the relative financial standing of the several major lines of business. Applied to classes and subclasses within the major lines in the preparation of historical asset shares, they provide management with a measure of the profit margins in each such class and serve as a guidepost in the development of dividend formulas. They even extend to the direct calculation of dividends on individual group insurance and group annuity cases which are of sufficient size to warrant substantially complete experience rating.

It is the purpose of this paper to examine the questions of equity and practicality of refinements in the methods of allocation of investment income in these and allied areas.

Fund accounting is the historical accumulation of premium income, investment income and asset gains less expenses, benefit payments and asset losses for the particular line, class or subclass under study. Some items for the fund account can be obtained directly from the company's books and others must be allocated by formula. In general, the smaller the class under study, the more the allocation must be made by formula. Since there is no direct way to allocate investment income and expenses and asset gains and losses for jointly held assets (i.e., not segregated by line or class) to each of the separate classifications of business which have contributed to those assets, such items must be allocated by formula.

There is a history of continuing refinement in the methods of allocating expense and mortality and morbidity charges and in the subdivision of these items into smaller classes both in setting premium and cash value levels and in evaluating financial results. Long-term swings in the level of interest rates have been reflected to a limited extent in establishing the level of gross premiums and reserve and cash value structures, especially for nonparticipating contracts and single premium annuities. The investment earnings assumption in these rate structures has been, at best, a compromise between yields available at the time the policy is issued and expected long-term ones. Beyond this, relatively little recogni-
tion has been given to variations between the different lines, classes and subclasses in allocating investment income and asset gains and losses. Within the last few years, considerable interest has developed in methods of allocation of investment income which recognize the fact that funds received at different times are invested and reinvested at different rates of interest. A number of companies have introduced, and others are considering introducing, this principle into the methods by which the financial experience of individual group annuity cases are evaluated and group annuity dividends are determined. At least one company has used it in the allocation of investment income by lines. However, in general, the current company average rate of return, independent of when monies may have been received and invested, has been used in preparing the Gain and Loss Exhibit and developing historical asset shares and other studies of profit margins by class.

## EQUITY AND RISK SHARING

There are interesting discussions of what constitutes equity in J. B. Maclean's and Edward W. Marshall's Actuarial Study No. 6 on "Distribution of Surplus," Robert T. Jackson's paper, "Some Observations on Ordinary Dividends" (TSA XI, 764), and Harlow B. Staley's discussion of Mr. Jackson's paper. In general, equity seems to require that any allocation made to the fund of any one class of policyholder should be in proportion to the contribution of that class to the item being allocated and that the classification system be subdivided sufficiently to reflect any major differences in characteristics which affect financial results.

Both Actuarial Study No. 6 and Mr. Jackson's paper refer to a possible conflict between the equity principle and the sharing of risk principle if the subdivision of classifications is too fine. However, Mr. Staley has cautioned that fundamental differences between one class and another should not be ignored just because a class is small. It seems to me that these points of view can be shown to be compatible by the philosophy set forth in Mr. Rosenthal's paper, "Limits of Retention for Ordinary Life Insurance" (RAIA XXXVI, 6), in which he treats the stabilization of mortality experience in terms of an insurance company's expected mortality as a unit rather than any breakdown of classes in this mortality into age groups, substandard classifications or the like. Even though classes and subclasses or even lines of business may be combined for risksharing and stabilization of experience in the mortality, morbidity, investment or expense elements, the fund account for each class can and should reflect major differences in characteristics of the class which can
reasonably be shown to affect the financial results of the pool of experience of which it is a part. Thus, mortality rates broken down by age, sex, duration, plan of insurance, and occupation and state of health at time of issue and expense factors reflecting such differences as size of policy, method of premium payment, type of coverage, degree of claim activity and commission rates contribute to equity by recognizing characteristics which affect financial results without doing violence to the fundamental insurance concept.

Turning to the interest factor, historically interest rates have followed both short-term and long-term cycles. These cycles on high-grade corporate bonds for the period 1850-1952 were shown graphically by Dr. Raymond W. Goldsmith at the Society's Forum on Economic Trends and Life Insurance (TSA V, 82). More recent data are included in the annual reports of Dr. James J. O'Leary to the membership of the Life Insurance Association of America. My economist friends tell me there is no reason to believe that interest cycles will not be present in the future regardless of the possible dampening effect of government controls. Since interest rates move in cycles, the characteristic which most affects the relative amount that the fund of any line, class or subclass contributes to the investment return of the entire portfolio is the relationship of the incidence of increments to and decrements from the fund to those of the total portfolio.

Where this incidence has been approximately parallel for all classes, the allocation of investment income by applying the current average rate of return on the entire portfolio to the current mean funds of each line, class and subclass produces substantial equity. However, where there is any marked variation in the pattern of the growth of the different funds, a further refinement in the method of allocation appears to be necessary to fulfill the previously mentioned definition of equity. Such a refinement would recognize that amounts made available for investment at different times are invested at different rates and, hence, contribute differently to the investment return of the entire portfolio.

## PRINCIPLES OF AN INVESTMENT-GENERATION ALLOCATION FORMULA

For purposes of discussion, let us define an investment-generation method of allocating interest as one under which the allocation to an individual class of its pro-rata share of the interest earned on the total portfolio is determined by calendar year of investment. To illustrate: If class A provided $10 \%$ of the investments for the year 1950, it would receive credit in 1961 for $10 \%$ of the 1961 return on such 1950 investments and their successor reinvestments. If the same class provided $12 \%$
of the investments for the year 1951, it would receive credit for $12 \%$ of the 1961 return on such investments and their reinvestments. A class which had been in existence from 1950 on would receive credit in 1961 for its pro-rata share of the 1961 return on the investments of each of the years 1950-61 and their successor reinvestments. There is no reason why a time period other than the calendar year could not be used except for convenience and the fact that the calendar year normally would be a long enough period to provide a base of sufficient size and short enough to reflect important variations.

Many allocation formulas varying as to details and differing in complexity of application and precision of results can be developed within the basic principles of the investment-generation method. For instance, the rate of reinvestment can be a precise year-by-year figure or it can be graduated over a period of years. Calendar years back of a time when the force of reinvestment has turned over most of the original portfolio can be lumped together. The reinvestment factor can be introduced in the form of a year-by-year adjustment to the original calendar year fund growth to bring reinvested funds into the year of reinvestment. Alternatively, the original calendar year fund growth can be held constant and the original calendar year interest rate adjusted annually to reflect changes in the original rate of return resulting from reinvestment. Cash and short-term investments in the nature of cash can be included in determining each year's rate of return or they can be excluded and introduced as part of a final adjustment to make the total allocations equal the total receipts.

Whatever formula is used, it seems to me that there are two basic principles which can be used to test its equity and its consistency with the broad insurance principle of risk sharing.

In the first place, each class should get the advantage of a rise in a company's over-all interest rate to the extent to which it contributed to such rise and should not suffer in a decline of a company's over-all interest rate to the extent that other classes contributed to the decline. This means that the formula must recognize that the investments of a particular calendar year are dynamic rather than static, that maturities, calls, sales and repayments require continuous reinvestment of part of the original investments in the then current market. It also means that the formula must recognize the incidence of all increments to and decrements from the fund-that is, investment income, asset gains, benefit payments, expenses and asset losses as well as premium income. A formula which recognizes these facts will maintain the dollar-averaging principle to the full extent that it is appropriate in each class.

Secondly, the formula should maintain the stability and diversity of pooled investments which are inherent in the design and offer to the buying public of any particular type of contract. This means that the formula would use as a base the return on the total portfolio rather than on any specifically segregated items of the portfolio. The total return would be broken down by a series of parameters categorized by the time at which funds were received and the rate of return earned by the common pool on investments being made at that time. Thus, the refinements in the formula in making allocations to the different classes would reflect the characteristics of each class which influenced the pattern of flow of funds. They would not be influenced by changes in the portfolio resulting from the exercise of managerial judgment under various market conditions except as they showed up in the rate of return. The principle of maintaining the stability and diversity of pooled investments does not preclude the recognition in the allocation formula of the effect on return of specific classes of investments which are unique to specific lines or classes of business. Examples would include such things as policy loans in the ordinary line, investments made in a foreign currency for policies issued in such currency, or equities in a special account for fundtype group annuities with limited guarantees.

## CLASSIFICATION

The degree of refinement used in classification is a major contributing factor to the equity of an allocation method. As stated earlier, equity seems to require that the classification system be subdivided sufficiently to reflect any major differences in characteristics which affect financial results. Practical considerations such as availability of data and the expense of processing data put some limits on the degree of refinement. Furthermore, the broad uses to which fund accounting techniques are put, ranging all the way from preparation of the Gain and Loss Exhibit by line down to the calculation of certain group dividends, present a variety of problems in classification.

Since an investment-generation method of allocating investment income is designed to give recognition to the effect of the pattern of the flow of funds of a particular class on the total return of the investment portfolio, classification criteria for its use should be based on characteristics which affect such flow. These would include such things as the pattern of premium payment, the time interval between receipt of premiums and payment of benefits, the proportionate investment element in the coverage, the degree of sophistication of the buyer, the extent to which the policyholder can vary the size and timing of his premium payments and
the presence of alternative methods of providing the benefits. Classes of business which are more or less homogeneous with respect to these characteristics could well be combined in the original allocation and the average of the combined classes used in making allocations to subclasses. For instance, life and endowment policies with level premiums payable over a long period of years could constitute such a class. On the other hand, single premium life insurance policies, paid-up policies, single premium immediate annuities, single premium deferred annuities and group annuities each seem to possess characteristics calling for separate classification. The only justification for combining classes with different characteristics would be a demonstration that the actual fund flow of the classes had been parallel in spite of the difference in characteristics.

Incidentally, there has been some question as to the equity of using an investment-generation method of allocating the investment income assigned to the group annuity line to individual contracts if a similar method is not used in allocating total income to the group annuity line. Such a procedure does recognize the relative contribution of the separate group annuity contracts to whatever amount was allocated to the line. Of course, if, by coincidence, the flow of funds in the group annuity line had paralleled that of the total portfolio, it would also recognize the relative contribution of the separate contracts to the earnings on the total portfolio. However, in recognizing homogeneity of characteristics affecting the pattern of flow of funds, it would seem that, in general, any grouping of classifications should be among subclasses rather than among primary classes such as lines of business.

## PERIOD OF USE

Once an investment-generation method of allocation is introduced, it should be maintained over a long period of time if equity is to be maintained. Such a formula would be advantageous to existing policyholders in securing higher yields over the life of their policies by attracting money to the pool of investments in which they are participating when market yield rates are high and reducing the flow of money from those seeking higher than market rates when market yield rates are low. However, if the formula were used only during a period of high rates and abandoned when rates were low, financial antiselection could be expected which would take away the advantages. If the allocation method is equitable during one phase of the interest cycle, it would be equitable in all phases of the cycle and should be continued over the entire cycle. This, of course, would not preclude improvement and refinement in the formula from time to time.

## LEVELS OF INTRODUCTION

There are three levels at which an investment-generation method of allocation can be introduced into fund accounting. The first recognizes only the flow of funds and new investment rates for funds received after the date of introduction. The second makes all allocations after the date of introduction in accordance with the past history of flow of funds. The third rebuilds the fund from the beginning assuming the formula has been in use over the entire time of the existence of the fund. The second level can be modified to recognize the effect on current allocations of using the formula over the past duration of the fund without making the retroactive adjustment involved in the rebuilding of the fund.

With a soundly constructed formula the third level gives the most accurate picture of the extent to which each class of business has contributed to past earnings and current surplus. It could show some surprising results. For instance, a block of single premium deferred annuities issued in the 1920's near the peak of the interest cycle might not have caused as much of a drain on surplus as management had been led to believe by the traditional method of allocating interest. Mr. McVity in his paper, "Some Reflections on Fund Accounts" (TSA I, 106), considers the treatment of irrecoverable losses. A rebuilding of the various funds using the investment-generation method of allocating investment income might give a different picture of irrecoverable losses in certain classes or subclasses and suggest equitable methods of write-off.

Introducing the investment-generation method of allocation at the third level requires adequate records of the flow of funds and new investment rates over past years which may not be readily available. Furthermore, it produces a discontinuity which management might consider undesirable unless means are found to spread the adjustment over a period of time. The first level requires no past records and maintains the maximum of continuity. The second level requires past records but does not involve the possible sharp break of a retroactive adjustment.

## PRACTICAL OBJECTIVES

The practical objective of the investment-generation allocation method is twofold. The first is to provide lower costs to the general body of policyholders through the higher yield on investments resulting from the removal of the incentive for investment antiselection. Obviously, under the portfolio average method of allocation the pension buyer and the purchaser of the single premium contract are going to be more interested in the insurance company product at a time when the average rate of
return on portfolios built up over prior years is higher than the return available in the open market than at a time when the converse is the case. To a lesser extent the individual buyer of level premium insurance will be influenced in his choice of type of policy and time of purchase by the return available on competing investment media.

In the discussion of Mr. McDiarmid's paper, "Inflation and Life Insurance," the writer pointed out that the percentage growth of capital flow arising from voluntary systematic personal security plans has kept pace with that of personal income-that the decline in the over-all rate of asset growth of life insurance companies has been offset by a corresponding rise in the rate of growth of corporate pension plans. This has occurred during a period of rising interest rates, a time when the mass buyer, using the different criteria mentioned in that discussion, has been able to secure a significantly higher return on his accumulating pension funds by investing directly in the market than by buying into a portfolio average rate of return under a group annuity contract. It is worth noting that premium payments for insured pension plans in the United States have almost doubled in the past decade, while the flow of funds into the capital market through corporate pension funds has increased more than three and one-half times. During the same period, the total flow of funds into the capital market through life insurance companies has increased about one and three-eighths times.

To the extent that the results of the investment-generation allocation are reflected in net costs, it can be expected, in comparison with the company average method, to encourage the flow of funds to the insurance company when market rates of return are high and discourage financial antiselection when the rates are low. It is practical to introduce the in-vestment-generation allocation directly into the calculation of dividends or experience rate credits for group annuity contracts since fund accounting is usually the basis of the calculation for each contract. There are complications in using the method directly in an ordinary dividend formula somewhat similar to those arising in connection with the use of a generation mortality table for annuity rates. It would involve a separate dividend scale each year for policies issued in each calendar year if this period were assumed to represent a generation. However, the variation from year to year would be relatively small for the large block of life and endowment policies with level premiums payable over a long period of years which constitute a large part of the ordinary business. It would seem practical to have a single scale based on the combined fund accounts as a guide for these policies, while still introducing variations for single pre-
mium and paid-up contracts. If terminal dividends are paid, they could reflect variations in the accumulated surplus of the different subclasses arising from variations in the pattern of receipt of funds.

The second part of the practical objective of the investment-generation allocation method is to enable the insurance company to offer to the public a wider scope of coverages and a greater degree of flexibility within coverages. From time to time when management has been concerned with declining yields on new investments, it has attempted to discourage policyholders from taking policies with substantial investment elements through underwriting restrictions, such as imposing limits on the amount of single premium business it would accept from any one source. One large company withdrew entirely from writing new group annuities during a period of low new money rates. Maximum and minimum limits have been placed on yearly premiums payable under group annuity contracts, thereby limiting the extent to which the contract-holder can adapt his payments to the financial condition of his business. All of these devices to slow down investment antiselection could be relaxed under the in-vestment-generation allocation method. Just as refinement of mortality factors to recognize substandard and superstandard classifications resulted in a wider availability of protection, refinement of interest factors would tend to extend the services of insurance companies.

## AN ILLUSTRATION

Table 1, based on the experience of the John Hancock Mutual Life Insurance Company, shows the wide variation in the pattern of flow of funds of certain classes of business over the past twenty years. The growth of funds in each class for each year is expressed as a percentage of the Company's total growth of funds for that year, exclusive of policy loans which are attributable to the ordinary line only. Classes A-E are individual group annuity cases and Class F is the group annuity line. With patterns varying as widely as these, there is no question but that the investment-generation method of allocation measures more accurately than the portfolio average method the actual contribution of each class to the current earnings on the total portfolio. The final two lines of the table show the proportion of the 1960 earnings on the total portfolio, exclusive of policy loan interest, which would be allocated to each class under the portfolio average and an investment-generation method respectively, assuming the methods had been in operation for the last twenty years.

The premium payments under Class B have remained more or less constant in absolute amount. Those under Class A have been growing
steadily. In 1956 those under Class C dropped substantially with a shift to split-funding and those under Class $D$ were discontinued entirely. Class $\mathbf{E}$ was a single lump-sum purchase made in 1942.

It can be noted that there is very little difference between the results of the investment-generation method and the portfolio average method in Classes B and C whose patterns of fund growth are similar to that of many individual policy classes. The largest relative difference occurs in Class E , which is more nearly akin to individual policy single premium classes except that in the Class $E$ example the fund account began to decline immediately with an excess of benefit payments over investment income.

TABLE 1

| Year | Pegcentage of Total Growth in Funds Attributable to Class |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | D | E | F |
| 1941 | . 640 | 271 | 758 | . 597 |  | 20.460 |
| 1942 | 422 | 194 | . 954 | 1. 244 | . 952 | 16.147 |
| 1943 | . 994 | 197 | 857 | . 708 | $-.042$ | 14.288 |
| 1944 | . 987 | 285 | . 745 | . 845 | $-.032$ | 16.846 |
| 1945. | . 954 | . 246 | . 732 | . 376 | $-.027$ | 15.497 |
| 1946. | . 302 | 417 | 1.066 | . 590 | $-.022$ | 18.021 |
| 1947 | . 301 | 180 | . 593 | . 620 | $-.034$ | 22.134 |
| 1948. | . 618 | . 206 | . 664 | . 478 | $-.035$ | 24.200 |
| 1949. | . 831 | 218 | . 786 | . 610 | $-.033$ | 27.704 |
| 1950. | 1.075 | . 185 | . 682 | . 638 | $-.038$ | 28.255 |
| 1951 | 1.617 | . 168 | 1.125 | 1.006 | $-.027$ | 34.080 |
| 1952 | 1.887 | . 187 | . 913 | . 754 | $-.023$ | 33.020 |
| 1953. | 2.315 | . 118 | . 752 | . 375 | $-.019$ | 34.686 |
| 1954. | 2.302 | . 109 | 1.165 | . 675 | $-.018$ | 35.534 |
| 1955. | 2.377 | . 140 | 1.038 | . 534 | $-.016$ | 35.380 |
| 1956. | 2.917 | . 148 | . 536 | - . 707 | $-.015$ | 32.699 |
| 1957 | 3.500 | . 172 | . 482 | - . 256 | $-.018$ | 36.988 |
| 1958. | 3.720 | . 188 | . 532 | $-.031$ | $-.018$ | 38.622 |
| 1959. | 4.356 | . 218 | . 611 | $-.055$ | $-.017$ | 39.466 |
| 1960. | 4.654 | . 221 | . 489 | -. 054 | $-.014$ | 40.594 |
|  | Percentage of 1960 Interest Earnangs Allocated to Class |  |  |  |  |  |
| By Portfolio Average Method. | 1.774 | . 167 | . 659 | . 332 | . 002 | 26.280 |
| By Investment-Genera tion Method | 1.944 | . 168 | . 655 | . 301 | . 001 | 27.392 |

## COROLLARY PROBLEMS

There are two allocation problems closely allied to that of investment income, namely, that of asset gains and losses and investment expenses. Each is worthy of a paper in itself. I hope they will be forthcoming, as well as additional more precise and detailed papers on the subject of the refinement of the interest factor.

Suffice it to say at this point that much of the reasoning applicable to the allocation of investment income is equally applicable to asset gains and losses, since the two together constitute the total return on the investment portfolio. An investment-generation type of formula for allocation of investment gains and losses has many of the same practical advantages and equity traits as it does for allocation of investment income.

Investment expenses can, of course, be separated into initial and maintenance expenses just as insurance expenses are separated into first year and renewal through a variety of formulas. However, the maintenance, through investment and reinvestment, of a portfolio of investments to which there is a steady flow of new and renewal premiums and income from investments of all sorts, and from which there is a steady flow of benefit payments and expenses, involves a different expense pattern than does the selling and servicing of business. Considerable justification can be found for spreading the investment expenses of any year other than those specifically and directly related to new investments evenly over the entire portfolio of the year.

In any consideration of methods of allocating investment income and asset gains and losses, the existence of guaranteed cash values and statutory limitations thereon cannot be ignored. The investment-generation method of allocation approximates the results that would be achieved with the portfolio average method if the total portfolio were valued at current market price and the various fund accounts were continually adjusted for the change in market values of the assets behind them. Such adjustments might readily bring the fund of a particular subclass at certain times below the aggregate of its guaranteed cash values. This situation could be particularly acute in the case of single premium life or endowment policies where the minimum cash surrender value under the standard nonforfeiture law is the full present value of future benefits with a $3 \frac{3}{2} \%$ ceiling on the interest rate.

Certainly any dividend distribution based on an investment-generation method of allocation should take into account the level of guaranteed cash values. It might be desirable to review the existing nonforfeiture laws, especially in the area of single premium contracts, to see if any revision is needed in fairness to the general body of policyholders. The
guaranteeing of transfer values in group annuity contracts which could result in a substantial profit to the contract-holder at the expense of other policyholders under certain market conditions has been a matter that has received considerable attention. There may be a similar problem with certain kinds of contracts in the individual policy field. The portfolio average method of allocating investment returns, the valuation of bonds on an amortized basis, the guaranteeing of liberal cash surrender values, are particularly appropriate for life and endowment policies with level premiums payable over a long period of years. Some revisions in these practices may be needed to recognize the extension of insurance company contracts into other areas, the increasing tendency of the in-surance-buying public to distinguish between the protection and investment elements of an insurance contract and its increasing interest in other media for investment.

At the time of writing this paper there are certain complications involved in introducing a refined method of allocating investment income. There is question whether such a method is permissible in preparing the annual statement under Regulation 33 of the New York State Insurance Department or the instructions in the National Association of Insurance Commissioners' Annual Statement Blank. The Department is currently considering revisions in Regulation 33 which would clearly make permissible a method recognizing year-by-year variations in the yield on new investments. Beyond this, the Department is considering the imposing of certain detailed restrictions on the method and its use in other than accounting procedures which it may feel necessary to meet its interpretation of the antidiscrimination law. The desirability of a revision in the instructions accompanying the Annual Statement Blank has been brought to the attention of the NAIC Blanks Committee but the subject is not yet on their formal agenda. It is hoped that these matters will be brought to a conclusion which will permit refinements in methods of allocating investment income which are both practical and equitable.

# DISCUSSION OF PRECEDING PAPER 

## J. EDWIN MATZ:

Mr. Green's paper, I am sure, will be thought-provoking to everyone. I would also describe it as a very valuable paper, for the customary reason that I am largely in agreement with the things he has said so well in it.

It seems to me that the paper focuses our attention on the fact that the two predominant principles of mutual life insurance, first, pooling of risk and, second, individual equity, are essentially opposites. If we gave complete recognition to the pooling principle, we would have absolute uniformity of cost for all classes of policyholders. If, on the other hand, we gave complete recognition to the individual equity principle, our operation would degenerate into an individual trust operation and could no longer even be called insurance. As Mr. Green points out, we have long since taken a position intermediate between the two extremes of complete pooling and complete equity so far as the mortality and expense elements of insurance are concerned. With regard to the investment element, however, we have scarcely moved at all from the extreme position of complete pooling. The reasons which suggest themselves quickly for this somewhat anomalous situation do not seem sufficiently convincing:

1. The complete pooling of investment experience may seem to relate to the genuine need for a diversified portfolio of securities. However, diversification can still be achieved even though recognition is given to time of investment.
2. Pooling of investment experience may seem to be required by the existence of guaranteed returns. The unique guarantees that insurance offers must indeed be retained, but there does not seem to be any reason why this cannot be done while at the same time some recognition is given to the particular contribution a class of policyholders has made to investment returns. We have, in fact, accomplished this with respect to mortality and expenses.
3. Anything short of complete pooling of investment experience may have been felt to be impracticable. This may have been a perfectly legitimate assumption in the past, and it is still true that a shift from one extreme to the other on the pooling-equity scale would be not only undesirable but impracticable. However, with modern office equipment some recognition of the equity principle seems clearly to be feasible.

It is interesting to consider the factors which have determined the exact points at which we have stood on the scale between pooling and equity and which have from time to time caused our business to shift its position. Considering the period of the last 30 years, we could set down a number of facts which, when considered together from the point of view of a life insurance company, might lead to a conclusion. It is more interesting to look at the facts from the point of view of the life insurance buyer, the typical American family. The disposable personal income of the family is $3 \frac{1}{2}$ times what it was 30 years ago. The insurance owned by the family is $3 \frac{1}{4}$ times as large also, but more of it is in the form of term insurance or other lower cost forms. The cost of living is up just $75 \%$ over 1930, so that a greater proportion of the disposable personal income represents discretionary spending power. The family has not only some group insurance coverage but also a substantial amount of social security coverage. The members of the family are indeed aware of insurance needs. They are also aware of recent inflation trends and predictions of more to come, the rapid rise in the stock market in recent years, the existence and great growth of mutual funds. The family has been conditioned to do much of its buying in supermarkets and discount houses.

There are other factors, of course, which have affected buyer attitudes. It might be said that the insurance companies themselves have a good bit to do with formulating them. Whatever the reasons for their existence may be, the buyer attitudes are the market conditions we must meet if the institution of insurance is to be as viable in the future as it has been in the past. We have met the need for greater equity in the past with such innovations as preferred risk underwriting and quantity discounts. It does seem to be the time now to recognize the need for either rewarding or penalizing the policyholder, equitably, for putting his money into an insurance policy instead of into some other investment medium at a given time.

## JOHN L. STEARNS:

Mr. Green's paper has given a very illuminating exhibit regarding the effect on participation in the group annuity area when account is taken of the calendar year in which the invested money was received. This naturally opens the question as to what the situation would be in the Ordinary department. The purpose of this discussion is not to give final answers on the many problems involved but to point out the numerous phases of the subject that should be considered in coming to a conclusion.

We have therefore made some approximate calculations to at least explore the ground rules underlying such a system. We will first construct a
model table. Rather than make theoretical assumptions we will go at this task the other way by taking the actual gain shown in the Gain and Loss Exhibit and spreading it back through the various sources of gains and loss in the Ordinary department including the recent years of issue. In such an analysis as this, whether the distribution between years of issue be right or wrong the total is the exact total, and therefore any comment can only result in a reallocation in one year at the expense of another. Such an exhibit would appear as shown in Table 1, first, on the year-byyear basis and then on the accumulative basis.

TABLE 1

|  | Balance Gain | $\begin{gathered} \text { Accumulated } \\ \text { Gain } \end{gathered}$ |
| :---: | :---: | :---: |
| New Business 1960. | -20,863,000 | -20,863,000 |
| Renewal 1959 | + 1,075,000 | -19,788,000 |
| 1958 | + 1,355,000 | -18,433,000 |
| 1957. | + 1,454,000 | -16,979,000 |
| 1956 | + 2,318,000 | -14,661,000 |
| 1955. | + 1,323,000 | -13,338,000 |
| 1954. | + 855,000 | -12,483,000 |
| 1953. | + 953,000 | -11,530,000 |
| 1952. | + 973,000 | -10,557,000 |
| 1951. | + 763,000 | - 9,794,000 |
| 1950. | + 1,004,000 | - 8,790,000 |
| 1949 | + 1,078,000 | - 7,712,000 |
| 1948. | + 1,471,000 | - 6,241,000 |
| Prior Issues and Paid-up. | +10,695,000 |  |
| Individual Annuities. | + 1,350,000 |  |
| Supplementary Contracts. | + 1,270,000 |  |
| Total Gain, Ordinary Dept. | + 7,074,000 |  |

This seems a very pessimistic picture in regard to the length of time it takes to repay the first year deficit. One of the causes is that the business issued seven years ago shows a gain but represents a much smaller volume of business than that just issued. Therefore this table could be reconstructed on a constant volume of new business basis appearing as in Table 2.

Even these modified deficits are so large and persist for so long that one's attention may be diverted from the interest rate question to wonder whether any dividend at all is earned in the early policy years and whether customary scales of surrender values are defensible. The answer, in our opinion, is that much first year expense properly chargeable as such in the aggregate is not necessarily so chargeable in determining the level of early benefits to individual policyholders. Many items of so-called first year
expense can be viewed equally well either as expenditures for the general welfare of the company or as expenses incurred for the purpose of collecting renewal premiums and providing service in renewal years.

We now go back to the interest problem. In the Distribution of Surplus textbook by Maclean and Marshall they state on page 30 that a typical way of paying excess interest in the dividend is a computation based on the initial reserve. Examining Schedule M of some of the leading companies indicates that this practice is still being followed. In obtaining the interest element in the dividend a high rate basis on new investments might be assumed, or an average rate of the company; but it is equally

TABLE 2

|  | Balance Gain | $\begin{gathered} \text { Accumulated } \\ \text { Gain } \end{gathered}$ |
| :---: | :---: | :---: |
| Premium-paying New Business 1960. | -20,863,000 | -20,863,000 |
| Renewal 1959. | + 1,076,000 | -19,787,000 |
| 1958. | + 1,443,000 | -18,344,000 |
| 1957. | + 1,649,000 | -16,695,000 |
| 1956. | + 2,786,000 | -13,909 000 |
| 1955. | + 2,000,000 | -11,909,000 |
| 1954. | + 1,655,000 | -10,254,000 |
| 1953. | + 1,872,000 | - 8,382,000 |
| 1952. | + 2,132,000 | - 6,250,000 |
| 1951. | + 1,856,000 | - 4,394,000 |
| 1950. | + 2,774,000 | - 1,620,000 |
| 1949. | + 3,341,000 | + 1,721,000 |
| 1948. | + 4,031,000 | + 5,752,000 |

important to determine the basis for the calculation, whether it be the full initial reserve or some lower figure representing the funds actually on hand. We must therefore study further the import of our assumptions in making Table 2.

From the viewpoint of a life insurance company, the group of individuals who buy insurance in any particular year are a perfectly good financial risk. Therefore the company is justified in withdrawing money from surplus to set up the reserve liability. Another way of looking at this situation is to assume that the initial expenses are taken care of by monies withdrawn from the surplus to be repaid later by the particular group of policyholders. In the second and subsequent years this money is paid back to the surplus. Eventually, of course, interest on this fund should be returned to the surplus. The repayment to the surplus can be scheduled on such a basis as to leave available the initial reserve on which to calculate the dividend interest. If the schedule of repayment is more exacting, the full
initial reserve would not be available. If the higher new money rate were applied to a figure which is lower than the initial reserve, it might well produce less of an interest credit than the average rate on the initial reserve.

We now come to consideration of the relative weight of this interest factor in the dividends. Table 3 shows a typical set of dividends together with the amount of interest determined under the three-factor method by applying an average interest rate to the initial reserve.

TABLE 3

| Year | Dividend | Interest <br> Element | Ratio <br> Interest <br> to Total |
| :---: | :---: | :---: | :---: |
| 1. | \$1.45 | \$ . 21 | $14.5 \%$ |
| 2. | 1.76 | . 37 | 21.0 |
| 3. | 2.08 | . 54 | 26.0 |
| 4 | 2.39 | . 71 | 29.7 |
| 5. | 2.72 | . 88 | 32.4 |
| 6. | 3.04 | 1.05 | 34.5 |
| 7. | 3.40 | 1.23 | 36.2 |
| 8. | 3.81 | 1.41 | 37.0 |
| 9. | 4.27 | 1.58 | 37.0 |
| 10. | 4.76 | 1.77 | 37.2 |
| 11. | 5.27 | 1.95 | 37.0 |
| 12. | 5.79 | 2.13 | 36.8 |
| 13. | 6.30 | 2.32 | 36.8 |
| 14. | 6.79 | 2.51 | 37.0 |
| 15. | 7.24 | 2.69 | 37.2 |
| 16. | 7.66 | 2.88 | 37.6 |
| 17. | 8.07 | 3.07 | 38.0 |
| 18. | 8.47 | 3.26 | 38.5 |
| 19. | 8.84 | 3.45 | 39.0 |
| 20. | 9.15 | 3.64 | 39.8 |

There is another phase of this problem that should be taken into account. Interest is earned on the assets of the company. Roughly $10 \%$ of these assets could be considered to cover the surplus items. Although part of the surplus has been inherited from previous generations, nevertheless the contribution to surplus by the present body of policyholders obviously came from those years of issue which have liquidated the initial expense. The net result of this reasoning would be that the interest earnings of this earned surplus should be returned to these longer-duration policyholders. If returned in the form of an increased interest dividend factor, it could produce a higher interest allocation factor for old policies than new policies even if the generation method is used with a higher current rate.

If the new money approach is applied to the Ordinary department, a
careful consideration should be given to the many different facets of this problem as pointed out above. The conclusion to this demonstration is, in my opinion, that for Ordinary premium-paying policies the major determination of the dividend scale arises from the mortality return factor, the return of the expense loading, and the fundamental assumption as to the liquidation of the capital invested in the particular block of business. In the early policy years the choice of an interest rate is of very little moment. In the later policy years the interest rate assumed is, of course, very important, but the greater the number of calendar years involved in the accumulation of the reserve the more different investment years are taken into consideration, with the result that the average interest rate of the company itself becomes a very close approximation to such a generation interest calculation.

## WILLLAM M. RAE:

The development of investment generation theories and practices represents a major break-through in actuarial science. Mr. Green's paper is outstanding, well worthy of the magnitude and importance of the subject with which he deals. My comments will be in the nature of expansion on a few items.

Undue refinement in applying the theory can result in less rather than more equity. For example, assume that the yield on new long-term investments was $5 \%$ during both 1960 and 1961. Assume, also, that Insurer A's uninvested cash increased by $\$ 5$ million during 1960, because of investment decisions and other chance events, but decreased by $\$ 5$ million in 1961 to its more normal level. Insurer A invests about $\$ 100$ million each year. If changes in cash position are taken into account in determining investment generation new money rates, Insurer A will have a $\$ 5$ million investment at zero interest in 1960, resulting in a new money rate distinctly less than $5 \%$. On the other hand, the next year it could be said to have a negative $\$ 5$ million investment, at zero interest, resulting in a new money rate distinctly more than $5 \%$. This fluctuation in investment generation new money rates, caused by fluctuations in uninvested cash, appears to be unrealistic and undesirable. Other ways of including cash in determining new money rates can produce equally ludicrous situations over the years.

Another example has to do with maturities, calls and sales. Suppose Employer B deposits a large amount in a group pension contract in 1951 when new money rates are, say, $4 \%$, but deposits little in 1952 when they are also $4 \%$. Employer C does just the opposite. He deposits little in 1951 but a large amount in 1952. Comes 1961. A significant portion of the 1951
investments mature or are called, or are sold, by reason of management decision. By chance little of the 1952 investments come back in 1961 for reinvestment. Should Employer B have a significant portion of his fund reinvested in 1961 at the then new money rate of, say, $5 \%$, while Employer $C$ has little reinvested? I doubt it. A smoothing of the rate of reinvestment among the years seems indicated, particularly by a small or middle-sized insurer where the fluctuations can be very significant.

These examples also serve to illustrate that no one investment generation method is necessarily the best for all insurers. Wide differences in method may be needed to meet the different situations of different companies.

The examples also point up the fact that the objective should be to reasonably reflect the yield on new investments, and not to go overboard on pseudo-precision. In this connection I am reminded of the twenties when we worked disability rates to three decimals, only to find later that our general level of rates was off the mark by several hundred percent.

There may be an inference in Mr. Green's paper that the records which insurers keep on each group pension contract for the purpose of computing dividends are mere junior editions of the fund accounts by line of business, and that consequently the interest credited to such a contract-holder is merely a suballocation of the total allocated for annual statement purposes to the pension line. This is not the case in my company. I do not believe it to be the case in many companies, and I do not believe Mr . Green intended the inference. There are fundamental differences between the two. For example, (1) the records for dividend purposes usually reflect a contribution to surplus from interest earnings, (2) they reflect an averaging of mortality experience in varying degrees, (3) they usually spread capital gains and losses over a period of years, etc. The fund accounts by line, on the other hand, are simply historical records of receipts and disbursements.

Mr. Green wisely cautions about using an investment generation method for dividends on any ordinary, industrial or group contract which may be surrendered for a lump sum. This caution deserves to be underscored. The situation is materially different because of the paying out of amounts in full settlement which may be substantially in excess of the market values of the supporting securities. This complication is in addition to the mechanical problems and other considerations with respect to such contracts.

Investment generation methods are in their infancy. Improvements will undoubtedly be forthcoming in future years, provided there is sufficient freedom to encourage their development.

Again let me compliment Mr. Green most highly on his fine paper.

## WILLIAM J. NOVEMBER:

The author has performed a valuable service in presenting his thoughtful discussion of this important topic to the Society. The subject is bound to require more and more of our attention and it is well to have before us the fundamentals that have been brought out by the author.

The paper presupposes the existence of funds or historical asset shares for the various line subdivisions discussed. The nonexistence of identifiable funds for subdivisions of some lines of business brings into question what form an allocation to these subdivisions could take. It strikes me that the application of the method when separate funds do not exist, as is frequently the case, does not represent an allocation of investment income in an accounting sense so much as a recognition of the sources of earnings for dividend distribution purposes.

I regret that the author has not included among his classification criteria for the investment-generation method the absence of a right to surrender a contract for a substantial cash value. Toward the end of the paper he recognizes that the existence of guaranteed cash values presents problems, but I wonder whether he has gone far enough. It seems to me rather fundamental that the investment-generation method will not operate well if the policyholder has the option of cashing out a policy which has been purchased at a time of low returns so that he may replace it with a new contract when the rate of return on new investments is substantially higher. Such a surrender and repurchase would not really bring new money to the company for investment at current rates. The situation may be likened to, and is perhaps dramatized by, the practice in the savings bank field. If a bank were to attempt to recognize the investment return available at the time of receipt of new deposits, it would be in an impossible position in respect to the retention of old deposits after the investment rate on new money has risen substantially. The right of the depositor to withdraw his old funds is the key to this situation. Of necessity banks have to work with an average rate in their interest credits.

A similar problem exists in respect to single premium life insurance and single premium deferred annuities which have cash value privileges. When a high return is obtainable on new purchases, the only deterrent to the surrender of old contracts with the intent of repurchasing new ones is the absorption of additional acquisition costs, assuming in the case of a life insurance policy that the policyholder is still insurable. As an increase of return to the policyholder can hardly be justified in these circumstances, I would not be as ready as the author to include single premium life insurance policies and single premium deferred annuities among the contracts that "possess characteristics calling for separate classification."

Another comment I have in a critical vein relates to the part of the paper that deals with "levels of introduction." The first level, which is entirely prospective, seemed the most natural one to us, but I can understand how a case can be made for the second level, which recognizes the way in which funds were received in the past for the purpose of making future allocations. The third level, which involves an adjustment of past allocations, troubles me. This ex post facto imposition of an adjusted amount on the policyholder seems to me to run counter to principles of equity and fairness that have guided us in the past. I hope this level was included for the sake of completeness and not because it was seriously considered as a proper basis of operation.

## MARK W. HILL:

From my reading of Mr. Green's paper, I get the impression that he is primarily concerned with more equitably measuring the investment income contribution of each class of business to divisible participating surplus. To a company writing nonparticipating business exclusively, principal objectives in using the more refined methods would be an improvement in the allocation of investment income to the major lines as set forth in the annual statement and an improvement in the interest factor used in the calculation of premiums, particularly single premiums. The difference in the results from using the "portfolio average" method and the "invest-ment-generation" method, as illustrated in the author's Table 1, seems rather unimportant for annual statement purposes alone. The calculation of premiums on a nonparticipating basis should ideally recognize the expected course of interest rates far into the future and the suggested refinements would probably be of value to the extent that they result in a more realistic picture of the present and the past. In the absence of the ideal situation a company can reflect the new money rates in its premium calculations provided it compensates by assuming a declining rate over the contract period.

Mr. Green defines an investment-generation formula as one that allocates the investment income of the calendar year (or other appropriate period) according to the timing of, and amounts of, contributions by the various lines to invested funds. He points out that "the formula should maintain the stability and diversity of pooled investments. . .." Thus, there is no intention that particular lines should have assigned to them particular investments of the portfolio. This is an important point, since it would be intolerable to have competition among the lines for the more desirable investments. It would not be acceptable to have one line subsidized, in effect, at the expense of the others, even to improve the position of the company in competition.

An important objective of the suggested refinements would be that the application of the investment-generation formula more equitably reflect any investment antiselection against the company. This could operate in such areas as the incidence of single premium business and the use of settlement options and would be of importance with respect to both participating and nonparticipating business.

As Mr. Green suggests, the effect of capital gains and losses should not be ignored. Indeed, reflection of investment gains and losses follows naturally and consistently if the investment-generation formula described by him is used. The present format of the annual statement, however, precludes reflecting such gains in the Summary of Operations.

The author discusses expenses, but a problem not definitely covered in the paper is that of complications in allocation of that portion of the federal income tax which a company chooses to treat as investment expense. If, for example, a company considers the Phase 1 tax to be investment expense and allocates to line on the basis of the tax law formula, there will be the complication of different "earnings rates" for each line. This is not an insurmountable obstacle but adds one more complication to an already difficult problem.

JOHN H. TUROFF:
Mr. Green is to be congratulated for his very timely and thoughtprovoking paper. He suggests therein that "many allocation formulas varying as to details and differing in complexity of application and precision of results can be developed within the basic principles of the invest-ment-generation method . . .," and I thought it would be of interest to the membership to have one such family of formulas demonstrate how items of an investment nature may be allocated by lines of business using an investment-generation approach.

The particular method of allocation to be discussed here ensures that all funds initially arising from insurance operations and becoming available for acquisition of new investments will carry forward into succeeding years their relative weights by lines of business, augmented by investment income and by asset changes resulting from reinvestments.

The method is predicated on two basic suppositions:
A. That a line of business distribution for new investments is established for each year during which new investments are acquired.
B. That any monies generated by subsequent activity arising from previously acquired investments would be apportioned with respect to each such year of acquisition in accordance with the line of business distributions derived in supposition A.

The fundamental problem, of course, is how to determine the line of business distributions required by supposition A.

If we could follow the flow of funds into investments for a typical life insurance company, we would find income tending to expand working capital and outgo tending to deflate working capital. Since competent management would have decided on the desirable level at which the cash float should be held, any excess of income over outgo not required to maintain that level would be channeled into new investments.

The various incremental and decremental items involved may be combined to show the existence of three main sources of funds becoming available for new investments during a particular year of operation.
Source 1. The net result of insurance operations and policy loan activities. In essence, this item consists of increments arising from premium deposits (includes considerations for supplementary contracts and surplus distributions left on deposit), policy loan repayments, interest on policy loans, miscellaneous income, etc., and decrements arising from payments to policyholders and beneficiaries, policy loans granted, insurance expenses (includes field and home office compensation), insurance taxes, and policy loan expenses and taxes.
Source 2. Interest, dividends, rents and miscellaneous investment income received less associated investment expenses and taxes disbursed.
Source 3. Considerations received for the sale of assets, repayment of mortgages, etc.

The foundation for establishing line of business distributions for new investments as required for supposition A rests on the allocation of source 1 funds by lines of business by direct or other acceptable methods. Source 2 and source 3 funds arise from all previously acquired investments, must be broken down by years of acquisition and then apportioned by lines of business in accordance with the requirement of supposition B . All three sources may then be aggregated to produce the line of business distribution for the current year's new investments as required for supposition $A$.

Assuming a new company commences business at the beginning of calendar year $z$, then the line of business distribution for new investments acquired during calendar year $z+m$ may be developed by tracing the sources of funds becoming available for new investments in that year. Symbolically this may be expressed as follows:

$$
\begin{equation*}
{ }^{2+m} V^{0}={ }^{2+m} P^{s}+{ }^{2+m} I^{v}+{ }^{2+m} C^{*}, \tag{1}
\end{equation*}
$$

where

```
z+m}\mp@subsup{N}{}{s}=\mathrm{ New investments acquired during calendar year z+m
z+m}\mp@subsup{P}{s}{s}=\mathrm{ Source 1 funds during calendar year z+m
z+m}\mp@subsup{I}{}{*}=\mathrm{ Source 2 funds during calendar year z+m
z+m}\mp@subsup{}{}{*
```

and the superscript " $s$ " on the right hand side signifies all lines of business combined.

Certain other relationships may also be expressed in this form. Thus if
${ }^{z+m} D^{z}=$ Original cost of assets disposed of during calendar year $z+m$
${ }^{x+m} A^{*}=$ Assets owned at end of calendar year $z+m$, cost basis
${ }^{2+m} M^{s}=$ Assets owned at end of calendar year $z+m$, market basis
${ }^{z+m} R^{s}=$ Realized asset gains during calendar year $z+m$
${ }^{2+m} U^{s}=$ Unrealized asset gains during calendar year $z+m$,
then

$$
\begin{align*}
{ }^{2+m} A^{s} & ={ }^{2+m-1} A^{s}+{ }^{2+m} N^{s}-{ }^{z+m} D^{s} \\
& ={ }^{2+m-1} A^{s}+{ }^{2+m} P^{s}+{ }^{s+m} I^{s}+{ }^{2+m} R^{s}  \tag{2}\\
{ }^{2+m} R^{s} & ={ }^{2+m} C^{s}-{ }^{2+m} D^{s}  \tag{3}\\
{ }^{2+m} U^{s} & ={ }^{2+m} M^{s}-{ }^{2+m} A^{s}-{ }^{2+m-1} M^{s}+{ }^{x+m-1} A^{s}  \tag{4}\\
{ }^{2+m} M^{s} & ={ }^{2+m-1} M^{s}-{ }^{z+m-1} A^{s}+{ }^{2+m} A^{s}+{ }^{s+m} U^{s} \\
& ={ }^{2+m-1} M^{s}+{ }^{2+m} N^{s}-{ }^{2+m} D^{s}+{ }^{z+m} U^{s} \\
& ={ }^{2+m-1} M^{s}+{ }^{2+m} P^{s}+{ }^{2+m} I^{s}+{ }^{2+m} R^{s}+{ }^{2+m} U^{s} . \tag{5}
\end{align*}
$$

The $C, D, I, R$ and $U$ functions may in turn be broken down with respect to the calendar years in which the investments were originally acquired. Thus:

$$
\begin{aligned}
& z+m C^{s}=\sum_{t=0}^{t=m}{ }_{z+m} C_{z+t}^{s} \\
& { }^{2+m} D^{s}=\sum_{t=0}^{t-m}{ }_{z+m} D_{z+t}^{s} \\
& z+m I^{s}=\sum_{i=0}^{t=m}{ }_{z+m} I_{z+t}^{s} \\
& { }^{t+m} R^{s}=\sum_{t=0}^{t=m}\left(z+m C_{z+t}^{z}-{ }^{z+m} D_{z+t}^{s}\right) \\
& z+m U^{*}=\sum_{t=0}^{t=m}{ }_{z+m} U_{z+t}^{s},
\end{aligned}
$$

where the subscripts on the right-hand side refer to the calendar year in which new investments are acquired.

Formula (1) may consequently be re-expressed as follows:

$$
\begin{equation*}
{ }^{z+m} N^{s}={ }^{z+m} P^{s}+\sum_{t=0}^{t=m}\left({ }^{z+m} I_{z+t}^{s}+{ }^{z+m} C_{z+t}^{s}\right) . \tag{6}
\end{equation*}
$$

Also the assets owned at the end of calendar year $z+m$ (cost basis) may be expressed in terms of all new investments ever acquired less all investments ever disposed of (cost basis) thusly:

$$
\begin{align*}
{ }^{2+m} A^{s} & =\sum_{t=0}^{t=m}\left({ }^{z+t} N^{s}-{ }^{z+t} D^{s}\right) \\
& =\sum_{t=0}^{t=m}{ }_{z+t} N^{s}-\sum_{t=0}^{t=m} \sum_{r=0}^{r=t} z+t D_{z+r}^{s}, \tag{7}
\end{align*}
$$

and, substituting formula (6) in formula (7),

$$
\begin{align*}
z+m
\end{align*} A^{s}=\sum_{t=0}^{t=m}{ }^{z+t} P^{s}+\sum_{t=0}^{t=m} \sum_{r=0}^{r=t}\left({ }^{z+t} I_{z+r}^{s}+{ }^{z+t} C_{z+r}^{s}-{ }^{z+t} D_{z+r}^{s}\right) .
$$

## Line Distributions

As previously indicated, source 1 funds as designated by the $P$ function are the basic source for line distribution as determined by direct and allocation methods. For calendar year $z+m$ source 1 funds may therefore be broken down symbolically as follows:

$$
\begin{equation*}
{ }^{2+m} P^{s}={ }^{2+m} P^{1}+{ }^{2+m} P^{2}+{ }^{2+m} P^{3}+\ldots+{ }^{2+m} P^{k}+\ldots, \tag{9}
\end{equation*}
$$

and the factor for apportionment to line $k$ would be determined from

$$
\begin{equation*}
{ }^{z+m} f_{p}^{k}={ }^{z+m} P^{k}+{ }^{z+m} P^{s}, \tag{10}
\end{equation*}
$$

where

$$
1={ }_{p}^{x+m} f^{1}+{ }^{2+m}{ }_{p} f^{2}+{ }^{z+m} f^{3}+\ldots+{ }^{z+m} f^{\prime}+\ldots
$$

The distribution we are seeking, however, is that of the $N$ function, which may be broken down by lines of business symbolically as follows:

$$
\begin{equation*}
{ }^{2+m} N^{s}={ }^{2+m} N^{1}+{ }^{2+m} N^{2}+{ }^{2+m} N^{3}+\ldots+{ }^{2+m} N^{k}+\ldots, \tag{11}
\end{equation*}
$$

and the factor for apportionment to line $k$ would be determined from

$$
\begin{equation*}
{ }^{2+m}{ }_{n} f^{k}={ }^{2+m} N^{k}+{ }^{2+m} N^{s}, \tag{12}
\end{equation*}
$$

where

$$
1={ }^{2+m}{ }_{n} f^{1}+{ }^{2+m f_{n}^{2}}+{ }^{z+m}{ }_{n} f^{3}+\ldots+{ }^{\beta+m}{ }_{n} f^{k}+\ldots
$$

Introducing the apportionment factors from formulas (10) and (12) into formula (6), the following expression with respect to line of business $k$ results:

$$
\left.\begin{array}{rl}
{ }^{x+m} N^{k} & ={ }_{n}^{a+m} f^{k} \cdot z+m \\
& =\underset{p}{s+m} f^{k} \cdot z+m P^{s}+\sum_{i=0}^{t=m}{ }_{n}^{z+t} f^{k}(\varepsilon+m  \tag{6k}\\
i+t \\
s
\end{array}{ }^{z+m} C_{z+t}^{s}\right), ~ l
$$

from which the apportionment factor ${ }^{a+m} f_{n}$ may be determined, since from ( $6 k$ ), bringing all items involving ${ }^{2+m}{ }_{m} k$ to one side,

$$
\begin{align*}
{ }_{n}+m & f^{k}\left({ }^{z+m} N^{s}-{ }^{z+m} I_{z+m}^{s}-{ }^{z+m} C_{z+m}^{s}\right)= \\
& +\sum_{t=0}^{i=m}{ }_{p}{ }_{p} f_{n}, z+m P^{s}  \tag{13}\\
&
\end{align*}
$$

Formula (13) is the key to the practical application of the method.
With respect to the first year of operation, calendar year $z$, and putting $m=0$ in formula (13),

$$
{ }^{2+m}{ }_{n}^{k}={ }^{z+m} f_{p} f^{\prime}(m=0),
$$

and the line distribution of the $P$ and $N$ functions are identical. This can be easily rationalized since the source 2 and source 3 funds for the first year of operation could only have arisen from source 1 funds of the same year.

With respect to the second year of operation ( $m=1$ ), and substituting back in formula (13) for known values of ${ }^{x+m_{n} f^{k}}(m=0)$ and ${ }^{2+m} f_{p}^{k}(m=$ 1) for all lines of business, the values for ${ }^{+\pi m} f^{k}(m=1)$ for all lines of business may be determined.

Again, with respect to the third year of operation ( $m=2$ ), values of ${ }^{z+m}{ }_{n} f^{f}(m=0, m=1)$ for all lines of business are known, values of ${ }^{x+m}{ }_{p} f^{k}(m=2)$ for all lines of business are known and values for ${ }^{2+m_{n}} f^{k}$ ( $m=2$ ) for all lines of business can be determined from formula (13).

This process is continued for successive years of operation.
By use of these apportionment factors certain relationships shown earlier with respect to all lines of business combined may now be reexpressed with respect to line of business $k$ thus:

$$
\begin{align*}
& { }^{2+m} U^{k}={ }^{z+m} M^{k}-{ }^{2+m} A^{k}-{ }^{2+m-1} M^{k}+{ }^{2+m-1} A^{k}  \tag{4k}\\
& { }^{z+m} M^{k}={ }^{z+m-1} M^{k}+{ }^{z+m}{ }_{p} f^{k} \cdot z+m P^{s}+\sum_{t=0}^{t=m}{ }_{n}{ }_{n}^{t} f^{k}  \tag{5k}\\
& \times\left({ }^{2+m} I_{z+t}^{z}+{ }^{2+m} R_{z+1}^{z}+{ }^{z+m} U_{z+t}^{z}\right) \\
& { }^{z+m} A^{t}=\sum_{t=0}^{t=m}{ }_{z+i} f^{k} \cdot z+t N^{s}-\sum_{i=0}^{t=m} \sum_{r=0}^{r=m}{ }_{z}{ }_{n} f^{k} . z+t D_{z+r}^{s} . \tag{7k}
\end{align*}
$$

Since the current year's line of business distribution of newly acquired investments relies on all previous years' distributions, the ideal starting point for using this method should be the very first year of operation. If a company has been in business for a significant number of years, the impracticalities of reconstructing any but its more recent years of operations may preclude the possibility of the ideal situation being achieved. However, if a distribution of assets by lines of business is available at a convenient point of time in the past, it can be assumed that all investments effected prior to that time were allocated in accordance with the line distribution of such assets and all future activities pertaining to those same investments would also be reflected by lines of business using the same relationship.

While the basic premises for the method of allocation discussed here are relatively simple, the tremendous increasing burden of analysis involved with each succeeding year can present a formidable and costly process unless the several investment accounting operations are modified to produce the necessary information as a routine by-product.

HAROLD R. LAWSON:
Under the heading of "Corollary Problems" the author refers to the matter of asset gains and losses which, he says, is worthy of a paper in itself, and adds, "Suffice it to say at this point that much of the reasoning applicable to the allocation of investment income is equally applicable to asset gains and losses. . . "There is also another problem that the paper brushes over, if indeed it mentions it at all, and that is how to arrive at an equitable cash withdrawal value. The device of imposing a surrender
charge of, say, $5 \%$ is arbitrary, possibly inequitable, and not too ac ceptable to the prospective policyholder.

It seems to me that both of these problems can be neatly solved, at least in theory, by a "unit" valuation approach. For this purpose the entire ledger assets of the life insurance company would be considered as one big mutual fund, if you will excuse this expression. The Ordinary policyholders do not participate in this fund in their own right, as they have contractual dollar guarantees from the company respecting death benefits, nonforfeiture values, and so forth. It is the company, itself, which invests the net amount of money received from these policyholders, in consideration of the contractual dollar guarantees, in its fund of assets. Certain special policyholders, however-say, those investing pension funds with the company-will be given the right, by contract, to participate fully in the experience of the fund, whether it be favorable or otherwise.

By way of a simple illustration, assume that a certain company has exactly $\$ 100,000,000$ of ledger assets taken at market value (for this purpose the market value of mortgages and real estate would be considered equal to the book value) on January 1, and that the value of 1 unit on this date is $\$ 1$. There are, therefore, $100,000,000$ units in the fund. In the month of January the total net cash income, excluding investment income, might be $\$ 1,000,000$, which would increase the number of units in the fund to $101,000,000$. The market value of the assets on February 1 might be $\$ 102,000,000$, due to investment income including net capital gains, so the unit would acquire a value slightly over $\$ 1$. The net cash income for the month of February would be used to buy units which would be revalued on March 1, and so on.

As I have said, the Ordinary policyholders of this illustrative company would not participate on a unit basis, but would have their contractual dollar guarantees, including, in the case of participating insurance, the right to receive dividends computed in the traditional manner. However, if a pension fund client brought in $\$ 5,000,000$ of new money on, say, February 1, it would be easy to calculate the number of units to which he would be entitled. Thereafter it would be equally easy to determine his share of the earnings from month to month, including capital gains and losses, and the equitable cash withdrawal value to which he would be entitled at any time.

As a practical matter certain problems might arise if unrealized capital gains required a write-up in book values, or in the interplay of participating and nonparticipating accounts, but these could probably be solved. I have not attempted to develop the plan fully as it does not seem to have any value for Canadian companies, which now have the statutory right
to segregate funds and value the liabilities in these funds on a unit basis. Nevertheless, I thought that this approach might have in it the germ of an idea for other companies that are differently situated.

This paper describes an entirely new philosophy in investment income accounting and pioneers in a development that promises to become increasingly important as the years go by. Its author is to be highly commended.

JAMES E. HOSKINS:
Mr. Green's only comment regarding the application of the method discussed to individual life and endowment policies with level premiums is that "the variation [in dividend scale] from year to year would be relatively small. . . ."

Even though the change in dividend for a given combination of plan, age, and duration from one year to the next might be small, is it not likely that the average interest rate earned on investments arising from policies issued near the bottom or top of the interest cycle would differ materially from that on all investments, so that over the greater part of the life of a year's issues, the net cost of those issued near the bottom would be too low, under the author's concept of equity, at the expense of those issued near the top?

## J. P. STANLEY:

I believe that the case for the so-called "investment year" method of allocating investment income, as so ably presented by Mr. Green, is a very strong one-so much so that it is likely to be only a matter of time before all of the major life companies in the group annuity field have adopted some variation of the method. I should like to touch briefly two problems which this new method gives rise to, neither of which seems to have received much attention to date.

The "investment year" method makes the operation of a pension plan funded through a Deposit Administration or Immediate Participation contract much more nearly comparable to a Trust Fund invested primarily in fixed-income securities than has formerly been the case, at least with respect to direct investment earnings. This is not true as yet, however, with respect to capital transactions: while variations in new money interest rates will produce market value changes in a trust fund's bond portfolio which can be predicted almost with mathematical precision, depending on the maturity dates of the individual securities, the "market value" for transfer purposes of an insured deposit fund continues to be determined by most insurers by reference to the old $5 \%$ surrender charge arrangement. It would seem that an insurer, having once adopted the
"investment year" method, could with some accuracy impute a "market value" for transfer purposes to each of the deposit funds held under its D.A. and I.P.G. contracts, using a fairly straightforward formula involving the relationship of the average formula interest return on the entire particular fund to the return available on new investments at any given time. Such an approach would seem to make D.A. and I.P.G. contracts more competitive vehicles for corporate pension plans (particularly in "split funded" situations) as they would eliminate the possible fear on the part of the employer that he might be locked into the insured part of the plan with no possible recovery of assets except after payment of a significant surrender charge, and would permit the insurer to hold $100 \%$ of the fixed-income portion of the pension fund without impairing the investment flexibility of the over-all funding medium by eliminating the possibility of capital appreciation on the fixed-income side of the portfolio.

The other problem to which I wish to call attention arises in connection with the conversion to other funding media of pension plans which are funded through individual insurance or annuity contracts. The question then arises as to whether the availability of guaranteed cash values in the individual contracts entitle such cash values to be treated as "new money" if transferred to the group annuity department of the same insurer. If the answer is "no," the fact that other insurers will be willing to treat such cash values as new money will often be a compelling reason for using a different insurer for the revised plan, since for larger groups the advantage of "new money" interest rates applicable to the entire existing cash values will generally outweigh the slightly higher acquisition expenses which may be involved. Nevertheless, to date, many insurance companies, including Mr. Green's, for reasons which are perhaps understandable, have been unwilling to treat as "new money" the cash values of pension business transferred from their Ordinary to their Group departments. To my knowledge, Mr. Green's company has already lost business for this reason -but no doubt it has gained some, too. The problem is one which is difficult to resolve in an equitable manner; yet it seems to be worthy of careful deliberation.

## DORRANCE C. BRONSON:

Since a John Hancock man followed a John Hancock man, I am adopting his precedent by following Dr. Stanley here for just a couple of comments which I haven't heard mentioned this morning on this subject.

Perhaps I don't understand the paper and perhaps you have sufficient ineluctable arguments that could be made to demonstrate this new method and to justify it. One question, however, that it seems to
me should be borne in mind is this: where group annuity contracts in the past have been sold on the guarantee of averages, now if things work around in such a way that those guarantees and the understandings at the time of the sale are not met, aren't the group annuity companies going against their word with their old policyholders? Maybe they can be shown that they are getting just as much out of it as they would have on the basis of averages, but it seems to me that that is a very important thing.

Secondly, without having seen all the new proposals of the new money method on a generation basis-this, that and the other-I have seen enough of them and have been puzzled with enough of them to wonder whether the group annuity companies are going to be able to write new business or give up writing new business and spend their time analyzing, discussing and debating with one another on their respective new money methods. I think that is a possibility.

## IRVING ROSENTHAL:

Mr. Green has constructed an impressive philosophical underpinning for what most of us call the "new money" or "select interest" method. He has provided enough abstract components to satisfy rather divergent views as to what is practical and necessary and this, paradoxically, is what troubles me. I feel that he has opened up a line of reasoning which could conceivably result in serious damage to the entire institution of life insurance in the United States. I would have been happier if his arguments had been cruder, with more reliance on directly practical considerations.

Basing yourself on Mr. Green's approach, you can take the position that in determining annual dividends for Ordinary annual premium life insurance, equity requires that each plan of insurance for each year of issue be regarded as a separate generation or fund whose pattern of growth by calendar year of investment ought to be reflected in the allocation of investment income. This seems reasonable enough and it leads to a dividend formula quite similar in structure to what the "new money" advocates want to use in the group annuity field.

But consider the practical implications of all this. The Ordinary insurance dividend scale for issues of 1962 published in 1961 would need to use the new investment yield of 1961 , say $5 \frac{1}{2} \%$, and assume, furthermore, for illustrative purposes, the continuation of that new investment yield rate indefinitely into the future. A twenty year projection on this $5 \frac{1}{2} \%$ basis would produce illustrative dividends which are very substantially higher than the dividends being paid on old policies. The destructive effect that this would have on the persistency of old business would make the dep-
redation of today's twisters and replacers seem almost beneficent by comparison.

I am quite sure that Mr. Green understands all this and so he suggests that all life and endowment plans for all years of issue could properly, and without violence to his theory, be combined in one fund and that only the growth pattern of this combined fund need be considered in the allocation of investment income to the fund. Since this combined fund would represent a large part of total assets, its growth pattern, in the case of most companies, would be so similar to the growth pattern of total assets as to parallel at a somewhat lower level the traditional total portfolio yield basis for Ordinary dividends. And so we would be pretty much right back where we started from.

The practical reasons for combining all Ordinary life annual premium policies into one fund seem overwhelming. However, when these practical needs are expressed as philosophical generalities they lose their force of conviction. This leads me to my main criticism of Mr. Green's paper.

A radical change, such as the use of the select interest method, does not arise out of abstract theory but out of practical needs. These are rooted in the competitive forces generated in our national system of freely competing insurance entrepreneurs, which include also bank trust departments associated with consulting actuarial firms. These varied entrepreneurs are vying vigorously for the favor of individual prospects who have a wide range of choice based on their own judgment as to what will best serve their own self-interest. The actual structure of these competitive forces makes it quite practical and desirable to use the new money method for group annuity dividends and impractical and undesirable to use the method for Ordinary insurance dividends.

In support of this conviction I would raise the question of what it is that justifies you in placing certain individuals into a class which will differ from other classes in the calculation of the premium or net cost of insurance charges. Mr. Green's theory of classification is based on considerations of equity. In our business, being for equity is like being against sin, but when we consider the problem of classification in the context of the forces generated in our competitive free enterprise-free choice system, it seems to me that the main reason for putting Mr. Sam Jones into a certain premium or net cost class is that you can keep him there by surrounding him with an invisible field of force which will convince him that where he is is just where he wants to be. The classification is useless, or worse than useless, if he can escape your classification and obtain a better one from his point of view by joining another insurance company or entering into some other type of benefit arrangement.

Suppose you try to apply the select interest theory to Mr. Jones' Ordinary insurance policy and that you tell him that "equity" demands that he be placed in a class where $10 \%$ of his reserves will be credited with $2 \frac{1}{2} \%$ interest, $12 \%$ at $3 \%$ interest, and so on up, ending with $9 \%$ of his reserve at the 1961 new money rate of $5 \frac{1}{2} \%$. He is not likely to be very much impressed with your equity argument if he can withdraw his reserve altogether and place it in some other company which will hang a "new money" label on all his reserve money and credit it with $5 \frac{1}{2} \%$. If he is in good health and has no intentions of committing suicide in the next two years, and some agent of your company or some other "new money" company gets to him to explain the situation, he would be a fool not to follow the agent's advice and transfer his reserve into the "new money" category. For example, he would be much better off with a combination of new single premium and new annual premium insurance in a "new money" company.

The situation would be quite different if the only way he could withdraw his reserve would be to take it in instalments over, say, a ten year period, or if the withdrawable reserve were to be calculated prospectively on a variable interest basis which could be adjusted to conform with the market value of the company's assets or, what amounts to the same thing, the interest rate on new investments. Indeed Mr. Green points out that if a company's portfolio interest rate were calculated on the market value of its assets, the portfolio rate would be substantially the same as the new money investment rate. On a variable liability fund which is correlated with the market value of assets the difference between the select interest system and the portfolio rate interest system would vanish altogether.

The reason why the select interest system is practical in the group annuity field is that it is a very troublesome business for an employer to transfer from one carrier to another. There might be no financial advantage in his doing so at all if, as is usually the case, he can only obtain his deposit account funds in small monthly instalments which would drag out the process of fund withdrawal over a fairly long period.

It would, in my opinion, be quite "equitable" to apply the new money system to the allocation of investment income in the Annual Statement and then confine the system insofar as it applies to dividends solely to the group annuity line. Using Mr. Green's Table 1 as an indication of a typical situation, the percentage of investment income allocated to the group annuity line would increase from $26.3 \%$ to $27.4 \%$ as you switch from portfolio basis to new money basis of allocation. This would not necessarily result in any reduction in dividends to the policyholders in the Ordinary line. All that would happen is that the Ordinary line's contribution to
company surplus would be reduced by a minor amount. Nor would it be likely to reduce the dividends on most old group annuity policies, because they are still characterized by a rapid growth of funds. All it would do is increase group annuity dividends substantially for the most rapidly growing policy funds, i.e., those which are most vulnerable to the blandishments of the advocates of self-administered pension plans. It would, correspondingly, strengthen the competitive position of the company in the new pension market compared with self-administered plans.

## HOWARD H. HENNINGTON:

Mr. Green has presented thoughtfully and logically the important points of the investment-generation or investment year method of allocating investment income. I believe that this new system introduces a higher degree of equity than that present in the traditional aggregate or average system. A higher degree of equity as between contracts is particularly important for group annuity contracts with substantial flexibility in the timing of payment of considerations. The new system will remove artificial incentives in such contracts to refrain from making contributions at times of high investment yield and to make substantial contributions in times of low investment yield. As between lines of business, a high degree of equity is important for rapidly growing lines as well as for stable lines. The new system will make investment results appropriate for the rate of growth of the line and will make the results independent of the situation with respect to other lines of business. No line will get an undeserved advantage nor will any line be adversely affected by another line of business.

The paper notes that the investment year method has the same advantages for allocation of investment gains and losses as it has for the allocation of investment income. In the Equitable we expect to apply the system to realized investment gains and losses. With some exceptions, we expect to identify each item of investment income with its corresponding year of investment. The distribution of the investment income by year of investment and the distribution of the outstanding investments will give us the necessary basis for operating the system. It is equally easy to analyze realized capital gains and losses by year of investment and apply the system to such gains and losses in the same manner.

In connection with cash payments it is vital to avoid an inconsistency between the investment income allocation system and the method for determining values on cash transfer. One can introduce refinements in the investment income allocation only if there are sufficient safeguards in the handling of cash payments. I believe we have sufficient protection in the
determination of the timing and amount of cash payments under Equitable group annuity contracts. Apart from the right of employees to withdraw their own contributions under certain circumstances, there is generally no right under deferred annuity contracts for an employer to withdraw funds. Such a withdrawal right exists under a relatively small number of deposit administration contracts, but the withdrawal is associated with a deferment provision whereby the Equitable may elect to spread large cash payments over a period of years, so that the payments will generally be made under differing investment circumstances. Under such an arrangement the dividend formula can then introduce proper equity in establishing the level of a charge which is appropriate to the times at which payments are made. I am assuming a continuing contract with respect to liabilities other than those which have been cashed out.

It has always been a difficult problem to determine the proper charge on account of the investment effects of a substantial cash payment under a group annuity contract. I believe this problem is not as difficult under the investment year system because cash paid out will be treated negatively in the operation of the investment year system. The investment year system will probably produce the right result without the need for a special charge in many instances where a special charge would have been necessary under the old system. In connection with extreme cases a special charge may still be necessary, but the investment year system will probably lead to a sounder basis for determining the appropriate charge.

## StUART J. KIngston:

My remarks are inspired by the remarks of Mr. Matz of the John Hancock, pointing out that there are two extremes: one of complete equity, which has no averaging in it, and the other of complete averaging or pooling, which has no equity in it.

I believe that this analysis of the two extremes is correct. It applies to other factors besides investments. In the case of mortality, for example, in group insurance, when the mortality gain is to be allocated to various group insurance policyholders, a credibility factor is used. This credibility factor really means, aside from all the long formulas, that you really calculate two dividends. The first dividend you calculate is one the case would get if all mortality were pooled. Then you calculate another dividend which is the dividend that the case would get if it were 100 percent experience-rated. Then you take a weighted average of the two, based upon the credibility factor, and that is the weight. It could be 50 percent; or 0 percent for a very small case, i.e., no credibility; or 100 percent for a very large case.

Thus we see that there has been a mathematical technique devised for arriving at which point between the two extremes you wish to be at. The mortality credibility factor is usually based on exposure.

The idea that I got from listening to Mr. Matz is that a similar technique could be derived for the investment factor, so it would not be necessary to go to the full extreme of the investment year method. Instead, you would calculate two allocations of the interest: one on the aggregate method formerly in use, i.e., the traditional method, and the other by the complete investment year method. Then by means of the credibility factor, assign a blend of the two, and then you would probably have to make one more calculation to make sure the total amount you allocate is equal to the total amount available.

In deciding on the credibility factor, one of the things that would be important, like exposure in mortality calculations, would be the amount of assets of the particular group. There is also the question of how much the public prizes the averaging process of the life insurance company. They prize it very much for mortality because there is no other place they can go to get mortality averages. Therefore, you can have a mortality credibility factor of zero in all your individual insurance and a lot of your group life insurance, and the public will still buy it from you.

But in the case of investments, the public, at least at the present moment, does not prize the averaging function of the life insurance company as to investments to the same degree that they prize mortality pooling. This is shown by the great rapidity with which they have deserted insurance companies in order to get a higher immediate investment return on their new money going into pension plans.

Mr. Green's statistics showed that when they could get significantly more interest elsewhere they reduced their deposits with the insurance company. Therefore, a zero credibility factor wouldn't be realistic at this time.

There is no reason why it has to be 100 percent. Perhaps it should be 50 percent for the smallest cases, working up to 100 percent for the largest cases.

Also, by use of this mathematical technique, not only would the ex-posure-i.e., the assets-affect the credibility factor, but, for example, there is the question of whether or not there is a cash value. The more guarantee that is associated with the cash value, the lower the credibility factor could be.

In other words, you can take into consideration various factors which determine which place between the two extremes you should be for the credibility factor for a given case.

There is going to be a time when new money rates will go down, and it is desirable for the insurance company to attract business then, too.

It is to the interest of all present policyholders that the company be a going concern. What I am really saying is that, by departing slightly from 100 percent credibility, you are taking an iota of interest away from the large group annuity cases and using it to get new business when current new money rates are low, by saying, "We are going to give you more interest than you can actually earn elsewhere." That is what you do in Ordinary insurance when you take some of the surplus the previous generation of stockholders has earned and use it to support the sales and other expenses necessary to bring in new policyholders so the company can go on.

This would be exactly the same thing. You have the mathematical formula already worked out in group life insurance. You apply exactly the same technique to this investment contingency that you would to mortality.

That is what I got from Mr. Matz talking about the two extremes. I said to myself that there must be a way to go somewhere in between. Then I started thinking about it, and I thought about the credibility factor.

## HARRY M. SARASON:

When we talk of insurance company "yield," we usually think of a denominator based on admitted assets or on ledger assets.

The true yield based on true asset values is precisely the same for new investments as for current portfolios. That is a truism.

The yield based on market values is almost the same for new investments as for current portfolios-as nearly the same as the two sets of market values are to being the same percentages of true values.

Real yields include capital gains and losses.
Yields based on book values or on admitted values are deceptive in somewhat the same manner as are comparisons of actual mortality to "tabular" mortality, though, usually, to a much lesser degree. More properly, comparisons of yields based on book values to yields based on market values are similar to comparisons between actual mortality and the mortality assumed in old premium and dividend calculations. Perhaps yields based on book values or on admitted values should be called "tabular yields."

In England, the premium rate for a large single premium annuity may reflect "today's yield," i.e., today's market price of investments.
(AUTHOR'S REVIEW OF DISCUSSION)
EDWARD A. GREEN:
I certainly appreciate the extensive discussion of my paper and want to thank those who have taken part in it. Before commenting on some of the points raised in the discussion, I should point out that since the paper was written the New York State Insurance Department has issued a revised Regulation 33, which specifically makes permissible methods recognizing year-by-year variations in the yield on new investments beginning with allocations made in the year 1962, but with certain detailed restrictions on such methods and their use. The revised regulation includes as points of reference Sections 204, 209, 216, 221 and 223 of the New York Insurance Law as well as Section 226 which was the only one referred to in the prior regulation. While the meaning and effect of some of the restrictions and extensions of references remain to be worked out, it is definitely encouraging to have the principle of refinement in methods of allocating investment income recognized favorably.

The question has been raised as to whether the introduction of an in-vestment-generation method at either the second or third levels referred to in the paper can in any way constitute a breach of faith with existing policyholders. These levels have sometimes been called a retrospective, i.e., looking back, application of the formula. For the following reasons, I believe that, with a soundly constructed formula, the answer is, "no."

1. Whether the formula is introduced at the first, second, or third level has no effect on new policyholders entering the fold subsequent to the adoption of the method. Therefore, any possible breach of faith can only be as it affects an existing policyholder of one class as compared to an existing policyholder of another class.
2. Since the adoption of the method can in no way alter the obligation of the insurer to deliver the benefits promised for the tabular premium as set forth in the contract, its only effect can be on the determination of divisible surplus allowed as dividends or retroactive rate credits. While the distribution of surplus clause used in John Hancock policy contracts has varied in detailed wording from time to time and from class to class, it has, in essence, provided that any divisible surplus accruing under a policy or class of contract shall be ascertained and apportioned annually by the Company.
The law prohibits making any estimate as to the dividends or share of surplus to be received in the future or making any false or misleading statement as to the dividends or share of surplus previously paid.

Whenever existing dividend scales or practices have been illustrated to a prospective buyer, it has been a Company requirement that he be informed that they are not guaranteed and are subject to change annually by the Board of Directors. To the best of my knowledge, other insurers have followed similar practices. It would seem that no promise, implied or direct, had been made to continue existing dividend scales or practices without adjustment or refinement. In fact, there are many instances where companies have changed dividend formulas in the interest of greater equity under changing conditions. Just to mention one: many companies have changed dividend practices to take into account the impact of the size of policy on expenses regardless of when the policy was written.
3. The choice as to which of the three levels is used in introducing an in-vestment-generation method has a minimal effect on life and endowment contracts with level premiums payable over a period of years. Its maximum effect is in the group annuity line where there is a wide variation in the pattern of flow of funds from contract to contract. Therefore, if there is any breach of faith with existing policyholders in using the second or third level, it would be largely concentrated within the group annuity line.
In order to show the pattern of flow of funds under contracts affected both favorably and adversely by the use of an investment-generation formula introduced at the second level, an asset share type of analysis has been made of all of the group annuity contracts issued by the John Hancock before January 1, 1950 and which were still on the books of the Company on December 31, 1959. The analysis was limited to contracts issued before 1950 in order to show the relative effect of the method on different members of a group of contract-holders who entered into their contracts under approximately the same expectations. Parenthetically, contracts issued subsequent to January 1, 1950 have contributed only $8.3 \%$ of the Company's total group annuity funds.

The contracts issued prior to 1950 were divided into three categories:
(a) those contracts where the rate of return was substantially unaffected by the adoption of the new method;
(b) those contracts where the rate of return was affected favorably; and
(c) those contracts where the rate of return was affected unfavorably.

A change of as much as $0.1 \%$ in the rate of return was considered to be significant. The history of the growth of funds in each category is set forth in Table A.

Briefly, a majority of plans, representing about $60 \%$ of the total funds,
fell in class ( $a$ ); that is, the investment-generation method resulted in no material change. The plans which benefited from the new apportionment procedure had contributed prior to 1950 only $8.7 \%$ of all the group annuity funds, but came to contribute, in the period of $1955-59,27.3 \%$ of the total group annuity funds arising during that period from contracts issued prior to 1950. The annual increase in funds from this group of contracts grew from $\$ 10.5$ million in 1951 to $\$ 29.2$ million in 1959.

The reverse situation is true with respect to the plans which were affected unfavorably under the new procedure. These plans which contributed $35.8 \%$ of the group annuity funds in the period prior to 1950, a

TABLE A
Grour annuity Contracts Issued Prior to 1950 (1937-49)

| Year | Annual Increase in Fund in Category (000 omitted) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (a) |  | (b) |  | (c) |  | Total |
| 1937. |  |  |  |  | \$ 350 | 100.0\% | \$ 350 |
| 1938 | \$ 5,973 | 93.7\% |  |  | 403 | 6.3 | 6,376 |
| 1939. | 3,609 | 98.8 |  |  | 43 | 1.2 | 3,652 |
| 1940. | 4,295 | 71.8 |  |  | 1,690 | 28.2 | 5,985 |
| 1941. | 11,457 | 48.7 | \$ 1,224 | 5.2\% | 10,822 | 46.1 | 23,503 |
| 1942 | 9,732 | 48.0 | 948 | 4.6 | 9,615 | 47.4 | 20,295 |
| 1943. | 11,065 | 50.8 | 2,218 | 10.2 | 8,485 | 39.0 | 21,768 |
| 1944. | 18,247 | 58.4 | 2,902 | 9.2 | 10,116 | 32.4 | 31,265 |
| 1945. | 15,868 | 51.7 | 3,539 | 11.4 | 11,292 | 36.9 | 30,699 |
| 1946 | 19,841 | 55.6 | 3,159 | 8.9 | 12,669 | 35.5 | 35,669 |
| 1947 | 26,521 | 59.0 | 3,466 | 7.7 | 15,003 | 33.3 | 44,990 |
| 1948. | 29,066 | 55.8 | 5,353 | 10.3 | 17,672 | 33.9 | 52,091 |
| 1949. | 31,811 | 52.0 | 6,682 | 11.0 | 22,633 | 37.0 | 61,126 |
| 1950. | 34,019 | 54.7 | 8,195 | 13.2 | 19,914 | 32.1 | 62,128 |
| 1951 | 44,691 | 55.5 | 10,502 | 13.0 | 25,381 | 31.5 | 80,574 |
| 1952. | 48,404 | 58.1 | 14,048 | 16.9 | 20,803 | 25.0 | 83,255 |
| 1953. | 54,464 | 60.7 | 16,297 | 18.2 | 18,955 | 21.1 | 89,716 |
| 1954. | 58,743 | 61.5 | 18,471 | 19.4 | 18,232 | 19.1 | 95,446 |
| 1955. | 60,974 | 62.6 | 20,167 | 20.7 | 16,226 | 16.7 | 97,367 |
| 1956. | 55,289 | 64.2 | 22,260 | 25.8 | 8,635 | 10.0 | 86,184 |
| 1957. | 59,119 | 65.8 | 25,165 | 28.0 | 5,618 | 6.2 | 89,902 |
| 1958. | 58,081 | 64.6 | 26,793 | 29.8 | 5,018 | 5.6 | 89,892 |
| 1959. | 56,636 | 63.3 | 29,164 | 32.6 | 3,680 | 4.1 | 89,480 |
|  | \$717,905 | 59.7\% | \$220,553 | 18.4\% | \$263,255 | $21.9 \%$ | \$1,201,713 |

(a) Rate of return relatively unaffected by investment-generation method.
(b) Rate of return increased by investment generation method.
(c) Rate of return reduced by investment-generation method.
period when the average new money rate was just under $3 \%$, dropped their contributions to only $8.6 \%$ of the total funds arising from pre-1950 contracts during the period of 1955-59 when the average new money rate had risen to $4.5 \%$. The annual increase in funds from this group of contracts dropped from its peak of $\$ 25.4$ million in 1951 to $\$ 3.7$ million in 1959.

Further analysis showed that, in general, those contract-holders to whose contracts less investment income is allocated under the invest-ment-generation method used were those who purchased group annuity contracts in the decade of the ' 40 's and have since exercised their unilateral right under the contract to discontinue or substantially reduce premium payments. In practically every instance, these contract-holders have shifted to a noninsured trust plan or to a terminal or split-funding plan with all or part of the accumulations during active service made through a noninsured trust fund. To have continued apportioning dividends to such plans on the traditional average portfolio method would have been to allow these discontinuing plans to secure a double favor. Not only would they secure higher rates on their new money in the noninsured plan, but they would also receive-more gradually, but still significantly-higher rates on their old money in the insured plan, higher rates made possible by funds contributed to the insurance company by the continuing plans.

Presumably any breach of faith would be with these latter contractholders, to whose contracts less investment income is allocated under the second or third level than under the first. It seems to me that there is no more breach of faith in the insurer's exercising his right to adjust and refine the method of allocating investment income in determining the dividend of a group annuity contract-holder than in the contract-holder's exercising his right to discontinue or reduce premium payments, and that it is nil in both instances.

Mr. Bronson has referred to guarantees of portfolio average. If any such guarantees have been made they would seem to fall in the same area as other contractual obligations. Incidentally, Mr. Bronson's use of the word "average" without modification has given me an opportunity to point out that an investment generation method following the principles outlined in the paper is an averaging method.

I believe there may be some difference in understanding between Mr . November and myself concerning the third level of introduction of an investment-generation method. He refers to it as involving an adjustment of past allocations. I looked upon it as a method of analysis to give a picture of the extent to which each class of business had contributed to
past earnings and current surplus or, using his words, "recognition of the sources of profit for dividend distribution purposes." In this connection, I pointed out that the second level could be modified to recognize the effect on current allocations of using the formula over the past duration of the fund without making any retroactive adjustment. Used in this way to measure contributions to surplus, I have trouble seeing how it can run counter to principles of equity and fairness. The variations shown by the analysis may very well fall within the bounds of surplus margins maintained for purposes such as fluctuation in investment income.

Irrespective of what level formula may be used in surplus distribution a third level calculation should be informative as to the actual contributions to past earnings and current surplus of various classes of business.

I have been joshed a bit about using the degree of sophistication of the buyer as a classification criterion, as having the possible implication that anything goes if the buyer doesn't know about it. Of course, I did not mean it that way. Coverage designs or rate or dividend practices which permit financial selection by individuals against the body of policyholders still will have little or no effect on financial results if such selection is not exercised. However, a previously equitable structure may become inequitable if buyers are educated to and do exercise such selection.

Dr. Stanley and Mr. Hennington discuss the problem of lump-sum transfer values under fund type group annuity contracts. I agree with them that the investment-generation method makes the solution of this problem simpler, especially if it is used in connection with asset gains and losses as well as investment income. There can be difficulty in putting the definition of the transfer value in contractual language which will be readily understood. The basic principle involved is, of course, to avoid financial selection against other policyholders in the withdrawal of funds as well as in the placing of funds.

Mr. Turoff's development of a family of formulas for allocation of items of an investment nature by line of business using an investmentgeneration approach is a valuable adjunct to the paper. Mr. Lawson and Mr. Kingston have suggested other interesting ways of introducing the investment-generation concept, one involving unit values and the other credibility factors. While Mr. Lawson in his discussion would limit the unit value approach to group annuities, Mr. McDiarmid in his paper, "Inflation and Life Insurance," suggested that it might work in connection with cash values under Ordinary policies as well. The credibility concept may well have merit in some situations if properly applied, but the standard of equity presented in the paper would rule out any usage which was designed to take something away permanently from a class of policy-
holders in order to attract new policyholders during a different phase of the investment cycle. I do not believe that Mr. Kingston intended to imply that any such usage would be analogous to the amortization of acquisition expenses. As Mr. Rae points out, wide differences in methods of introducing the investment-generation principle may be needed to meet different situations and a variety of sound approaches may be forthcoming provided there is sufficient freedom to encourage their development.

I am grateful to Mr. Rae and Mr. November for making clear the distinction between the historical records of receipts and disbursements which constitute fund accounts and the recognition of sources of profit for dividend distribution purposes. While accounting and dividending utilize many of the same techniques-the techniques of fund accounting as I call them in the paper-they are separate procedures legally, equitably and practically. For instance, Mr. Stearns has presented figures showing the size and persistency of first year deficits in the Ordinary department. According to these figures the accounts would indicate that no dividend would be earned for the first dozen years. However, as Mr. Stearns points out, the payment of early dividends can be justified by the principle of amortization of first year expense. Also guaranteed nonforfeiture values rather than statement liabilities are used in testing the effects of Ordinary dividend scales. Mr. Sarason's comments allude to some of the idiosyncrasies of life insurance accounting.

Mr. Hennington has pointed out the appropriateness of the method for making allocations as between lines of business. The use of the invest-ment-generation principle in determining divisible surplus has been further developed, is simpler, and is more necessary for equity and practicality within the group annuity line than within the Ordinary line. Mr. Stearms's analysis sets forth many of the factors that need to be considered in this latter field and seems to back up my statement that the variation between an investment-generation and an average method would be slight within that large block of life and endowment policies with level premiums payable over a long period of years. Several speakers have questioned the feasibility of making immediate and extensive use of the investment-generation principle within the Ordinary line. Mr. Hoskins' suggestion that the investment-generation need not be a year but can be some longer period for individual life and endowment policies may very well point the direction that any application of the investmentgeneration principle to the Ordinary field may take. Incidentally, Mr. Hoskins' discussion leads me to the conclusion that I would have been well advised to have used a broader definition of an investment-generation method and called such a method where the calendar year represents a
generation an investment-year method. Mr. Hill outlines the differences in handling the interest factor between participating and nonparticipating business. I believe that nonparticipating premium and cash value structures have followed the longer swings of the interest cycle more than many other areas of our business.

Several speakers emphasized my statement that in any consideration of methods of allocating investment income and asset gains and losses the existence of guaranteed cash values and statutory limitations thereon cannot be ignored. All I can say is that I agree with them and repeat my suggestion that any dividend distribution based on an investment-generation method of allocation should take into account the level of guaranteed cash values. Though admittedly involving considerable research and education over a period of time, it might be desirable to review existing nonforfeiture laws, especially in the area of single premium contracts, to see what, if any, revision may be needed in fairness to the general body of policyholders.

Mr. Matz has done an excellent job of summarizing the need to reconcile the equity principle and the sharing of risk principle in such a way as to meet the wishes of the buyer. I gather that he thinks there may be need to extend the investment-generation method to some extent beyond the group annuity line as methods of overcoming practical difficulties can be devised. This was brought home to me recently when I was hard put to advance satisfactory reasons to a friend of mine, who had a lot of figures before him, as to why he should not surrender an educational endowment policy taken out a number of years ago but with several years to run and place the proceeds with a high-grade investment company.

Mr. Rosenthal has used the undefined terms "new money method" or "select interest method" as synonymous with the terms used in the paper. I avoided both of these terms because of the possibility that they might imply a fixed rate of return associated permanently with monies invested at a particular point of time and therefore be inconsistent with the basic principles developed in the paper. Actually "new money" becomes "old money" with great rapidity and the rate of return will vary as it ages.

I am surprised at the reaction of Mr. Rosenthal to the line of reasoning set forth in the paper. As far as I can see, his main objections concern the effect on the Ordinary line in two areas-net cost illustrations and cash values guaranteed at a level higher than market values can support over a period of time-neither of which should be insurmountable. Net cost illustrations could be published on a historical basis or on the basis of dividends and interest currently payable on similar contracts whose funds have grown over a spread of the interest cycle. Obviously net costs based
on the high period of the interest cycle would not be representative of net costs which would be experienced over the entire cycle on Ordinary policies with level premiums payable over a period of years. The illustration set forth in the paper shows very little difference between the results of the investment generation method and the portfolio average method in Classes B and C whose patterns of fund growth are similar to that of many individual policy classes. An important consideration in the development of any practical application of the investment generation principle to any class of business is the need to avoid aggravation of the replacement problem. High cash values, to the extent that they are not covered by higher gross premiums, can be backed up by lower surplus distribution which may bring us at least part way back to where we started, as Mr. Rosenthal suggests. However, I fail to see how we have lost anything by going through a rational analysis which leads to this conclusion.

I agree with Mr. Rosenthal that an acceptable level of equity can be attained if the investment-generation principle is used in allocating investment income to lines of business but with its use in determining dividends limited to the group annuity line. There is very little business issued in the Ordinary line which has the important characteristics affecting flow of funds of the group annuity line. Individual single premium insurance, in addition to being what the lawyers call "de minimis," must have a high minimum cash surrender value under the standard nonforfeiture law, which, as Mr. November has pointed out, is an important classification criterion.

The substantially greater importance of investment income to the earnings of the group annuity line than to those of the Ordinary line can be shown by considering its effect on gains from operations before dividends as shown in the annual statement. For instance, the excess of interest earned over interest required was $77 \%$ of our group annuity gains for 1960. This compares with $41 \%$ in the Ordinary line.

In closing, may I again thank all those who took part in this interesting and valuable discussion and express the hope that papers will be submitted by various members of the Society on the many ramifications of refining methods of allocating investment income.

