OPTIONS ON ANNUITY RATES

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The work of actuarial science is often to develop theoretical justifications for practical expedients.

ONE of the most rapidly changing branches of insurance has been the field of group pension plans. In addition to intercompany competition in the industry, there has been the competition from what is known popularly as the trusteed method of funding, whereby contributions are turned over to a trustee to handle the investment of the pension fund.

In Canada, Parliament has just passed amendments to the Insurance Acts which will permit the issuing of contracts without guarantee as to investment performance or capital. These so-called variable contracts (whether or not the annuity from the plan is of the fixed dollar type or the variable dollar type) will enable the insurance industry in Canada to offer what is almost a "trusteed" method of funding group pension plans through a segregated asset account. Within this account, individual employer funds or pooled funds will probably be established. The share value of an employer's pension fund in a pooled fund will fluctuate in value with the asset value and investment performance of the pooled fund. The assets of the segregated asset account will be earmarked and entirely separate from the general insurance funds of the company. Broadly speaking, the quantitative restrictions of the Insurance Acts regarding investments will not apply, though the qualitative restrictions will.

At retirement or vesting, conventional fixed dollar annuities may be purchased from the insurance branch of the company or variable annuities purchased within the segregated asset account. In the latter case, only mortality would be guaranteed by the insurance company.

To retain the insurance feature required of an insurance company (and compete more effectively with other organizations in the field of uninsured plans), it will be necessary to attach purchase rates to the variable contracts as is done now in the conventional deposit administration contract used for group pension plans. A major difference in concept from the deposit administration contract arises, however, in that a fixed dollar annuity purchase rate must recognize the fact that the dollars applied will be, in effect, "new money" in the insurance branch of the company at the time received. This is a contrast to the deposit administration case where

the withdrawal of funds from the deposit account and subsequent transfer to the annuity premium account at retirement are simple bookkeeping entries with the same insurance assets behind the dollars in the deposit account and the annuity purchased by them.

Thus, at the retirement of an employee covered under one of these new contract forms, cash is withdrawn from the segregated asset account and used to buy a single premium annuity on then current "new money" rates, or, if more favorable, on the purchase rates guaranteed with the segregated asset account at the time the funds were originally deposited. Methods have been devised to project mortality assumptions, but the problem remains as to the interest rate to be used in the guaranteed purchase rates.

This paper will concentrate on the interest rate problem, but the technique developed could be applied also to possible variations from expected mortality.

The first approach might be to consider the analogy to savings types of insurance or annuity contracts issued to individuals where the annuitant has freedom of choice at maturity to take a lump sum in cash or a life annuity with varying minimum periods of payment. The common principle is to make use of a "settlement option" purchase rate on conservative interest assumptions, with either participation during part or all of the term of the annuity, or advance experience-rating in some fashion, such as through the use of a percentage of the current individual annuity rates of the company in lieu of the settlement option factors.

The proceeds of a policy applied under settlement options are really new money as they are under the complete control of the beneficiary or insured (except in certain special settlements arranged by the insured). To prevent financial selection against the insurance company, settlement option interest rates guaranteed are therefore low.

A deposit administration contract or fully insured nonparticipating group annuity contract is generally underwritten so that selection against the insurance company financially is not possible. This is usually done by eliminating any cash option clause at maturity. At vesting, the annuity is established on interest assumptions of the purchase rates. The *current* new money interest rates do not usually apply and funds are never paid out, even in theory. The interest rate in the purchase rate can be at a higher level than would be the case if the individual or the employer were in a position to select against the company and choose whether or not to use the purchase rate attached to the deposit account or involved in the premium rate assumption.

For group pension plans in competition with other uninsured forms of

funding, the Ordinary contract system will possibly not be sufficient, because of the very conservative nature of the settlement option type of purchase rate. The questions thus facing the insurance company entering the uninsured field with a segregated asset account are:

(1) How high an interest rate can safely be used for a basic purchase rate which will not require subsidy?
(2) What is the cost or option price of a guaranteed purchase rate on a higher interest assumption?
(3) How should this cost be charged to the employer or pension fund?

These questions will be discussed below.

Basic Purchase Rates

The interest rates in settlement options currently in Canada are in the neighborhood of 3%. I shall, therefore, consider this interest rate as the "safe" interest rate at which single premiums for immediate annuities could always be accepted.

In view of the low levels of interest rates during World War II and the early postwar period, this assumption could possibly be challenged. On the other hand, the recent past might be considered sufficiently abnormal and thus the possibility of recurrence judged sufficiently small to warrant even a higher basic rate, such as $3\frac{1}{2}\%$.

Costs of Option Rates

If a guaranteed purchase rate is attached to a deposit into the segregated asset account on an interest assumption greater than our 3% maximum rate, there is a possible future cost to the insurance company for such rate.

For example, if 3% is a safe interest assumption, what price should be charged for an option on a 4% annuity purchase rate to be attached to a contribution received today? It must be kept in mind that because of either purchase elsewhere or advance experience rating, the insurance company will only be held to the option purchase rate at times when the new money interest assumption for annuity single premiums is below 4% (aside from the effects of mortality).

The question resolves to a certain extent into an economic and political guessing game as to the probabilities of interest rate levels in the future. However, estimates of the cost can be made.

First, I shall define the following terms.

(1) S is the net dollar amount of contribution into a fund in a certain year (at time $t = 0$).

(2) s_t is the portion of S withdrawn at time t together with interest to purchase an annuity for a retiring participant in the fund:

$$\int_0^\infty s_t dt = S .$$

(3) $^m a_y$ is the annuity purchase rate at normal retirement age y at any interest rate i_m and represents the single premium for a life annuity of \$1.00 per annum.

(4) i_r is the option interest rate in the annuity purchase rate.

(5) j is the effective compound annual interest rate earned by S in the fund to time t (note that j is a function of t and may be positive or negative).

(6) i is the valuation interest rate of the cost of the option, or the interest rate on the insurance funds of the Company.

(7) $^m\Delta^r a_y = {}^m a_y - {}^r a_y =$ the excess (deficiency) of the annuity single premium at interest rate i_m over that at interest rate i_r.

(8) p_t^m is the probability of an interest rate i_m being applicable to new single premium annuity rates at time t.

On the above assumptions, formula (1) below represents the cost at time t of applying s_t dollars to the purchase of an annuity at interest rate i_r when the new money interest rate assumption in annuity single premiums is i_m:

$$\frac{(1+j)^t s_t}{{}^r a_y} \int_{.03}^\infty p_t^m \cdot {}^m\Delta^r a_y \cdot d(i_m) . \tag{1}$$

If we place the limitation on the integral that $i_m \leq i_r$ as the option rates will not be used otherwise, and sum all purchases over the full application of the original contribution of S, we obtain the total cost of the option with respect to the contribution of S at time $t = 0$ as:

$$\int_0^\infty v_i^t \cdot (1+j)^t \cdot \frac{s_t}{{}^r a_y} \int_{.03}^{i_r} p_t^m \cdot {}^m\Delta^r a_y \cdot d(i_m) \, dt . \tag{2}$$

I do not propose to apply any further mathematical manipulation to the above theoretical formula. It is intended only as a guide and the assigning of values to p_t^m or the devising of formulas for s_t would be difficult. For example, I do not believe anyone has yet successfully developed any law for p_t^m, especially since the advent of Lord Keynes. However, some simplifications may be made in the formula on an approximate basis.

(1) Let $j = i$ on the grounds that the interest rate in the segregated asset account will not be significantly different from the insurance company's rate in the case of fixed income securities, or that reserves can

be carried in the segregated asset account in similar investments to those in which S has been applied. (This latter method, through the company ownership of investment units, may be necessary in the case of pooled equity funds where j could be considerably in excess of i because of capital gains.)

(2) Assume that p_t^m is independent of t and may be replaced by p^m. The level of interest rates one year ahead often appears to be almost as much of a question as the level thirty years ahead.

(3) Because of the previous assumptions, the incidence of s_t is now of little importance and the integral with respect to t may be eliminated and replaced by S.

(4) For practical use, values of p^m can be estimated for whatever values of i_m may be desired for handling the summation within the second integral on a discrete basis.

Note, however, that in any application to a new plan some select period of t might require recognition, affecting p_t^m and s_t.

Our option price now reduces down to the following simplified formula:

$$\text{Price} = \frac{S}{{}^r a_y} \sum_{i_m = .03}^{i_m - i_r} p^m \cdot {}^m \Delta^r a_y . \tag{3}$$

Returning now to our original question as an example, where $i_r = .04$, if $p^{.035} = .20$, $p^{.0375} = .35$ and $p^{.04 \to \infty} = .45$, and on a certain projected mortality basis and retirement age ${}^{.035}\Delta^{.04} a_y = 0.50$, ${}^{.0375}\Delta^{.04} a_y = 0.30$ and ${}^{.04} a_y = 11.0$, then:

$$\text{Price} = \frac{S}{11} [.20 \, (.50) + .35 \, (.30) + .45 \, (0)] = .019 S .$$

This discussion has ignored the effects of other features of the pension plan which should probably be mentioned.

The question may be raised as to whether or not a cash value of the option price would be requested on funds released in contributory plans through terminations of employment and the refund of employee contributions. However, the option price would be charged only on net new dollars coming into the pension fund of the employer.

The formula above determines the option price on the basis of a certain normal retirement age, but this will not apply for annuities purchased on early or deferred retirement. For any i_m, ${}^m \Delta^r a_y$ increases with decrease in the age y and hence the option price would be insufficient on early retirements. However, mortality profits might well cover such loss, as the mortality amongst early retirements is usually high. On deferred retire-

ments, the option price charge would be excessive, though it might be offset to a certain extent by exceptionally good mortality.

To prevent selection against the company, it will be necessary to restrict in the contract the right of the employer to purchase annuities for retiring employees with funds outside of those under the contract. Otherwise an employer would be tempted to use current contributions for annuity purchases from some other company except when i_m was less than i_r.

Method of Payment for the Option

The simplest method of covering the option charge is by a simple deduction of the necessary percentage of each deposit as part of the expense provision. This is the method developed by my formulas.

The administrators of uninsured pension funds usually charge for their investment management services through a deduction from the yield of the fund (possibly expressed as a fractional percentage of the assets in the fund). The common percentages applied by trust companies in Canada to the employer's share of the assets under administration in a pooled fund to determine the management fee are: $\frac{1}{2}\%$ on the first $500,000 in the fund, $\frac{1}{4}\%$ on the next $1,500,000, $\frac{1}{8}\%$ on the next $3,000,000 and $\frac{1}{16}\%$ on the excess over $5,000,000. Insurance companies establishing segregated asset accounts will probably make use of an investment management fee of a similar type. Provision, therefore, could be made for the option cost through this charge. The amount collected in the first few years would be insufficient but, after a reasonably short period of years after a deposit was made, the necessary funds could be built up to cover the cost of the purchase rate option. A group with relatively few retirements for some years after issue would contribute a relatively large amount to the option fund, except to the extent that the combined investment–option charge might be experience-rated.

An advantage of the first system of a flat percentage deduction from the consideration is that it provides immediate cash to pay commissions, enabling full "cash-out" clauses to be incorporated into the variable contracts without restriction or possible loss to the insurance company. It also enables the insurance company to offer two types of variable contracts—one corresponding to a contract used by a trust company or a bank, with only investment charges and conservative or "safe" purchase rates, and alternatively, if the employer wishes to pay the price, a contract with more favorable purchase rates. This may help emphasize to the public the risks undertaken in an uninsured plan.

The system using the investment management fee to also collect the

option charge is difficult to operate equitably; however, it might be possible to provide for the cost of a relatively favorable purchase rate for the employer within an investment management fee that is still competitive with that used by the trust companies or banks.

A third system would, of course, be additional loading in the purchase rates themselves; this would give the same effect as if a purchase rate were used on an interest assumption in between the interest assumption of the option rate and the interest assumption considered safe. The contingency loading method is always popular, and through the methods of this paper a theoretical justification for the amount of loading can now be made.

Through the option approach of this paper, I have attempted to develop expressions for determining the price of an option on an annuity rate at a certain interest assumption. Using this method the actuary can estimate the charges or the contingency loadings required for such an annuity rate attached to funds in a segregated asset account. It is even conceivable that the trustees of an uninsured plan might wish to purchase options for certain funds or employees.

There will be many new and interesting problems for the portion of the Canadian insurance industry entering this new field. This particular problem of guaranteed purchase rates applicable to future new money has not received much attention, though some companies have developed special provisions for ordinary or group policies used in split-funded retirement plans. However, these plans have not had the contingent liabilities now possible with large pension funds under the new contract forms.

DISCUSSION OF PRECEDING PAPER

CONRAD M. SIEGEL:

Mr. Rudd's paper adds to the recent work of American and Canadian actuaries in the difficult field of finding the cost of various types of options.

Realizing fully the limitations attached thereto, let us examine the additional loading that he develops, 1.9% of the segregated account contribution. This is roughly equivalent to a reduction of $\frac{1}{8}$% in investment income. In a large case this additional charge might be viewed as an additional expense equal to 75% or 100% of the basic investment management expense. An employer, faced with these costs, may have second thoughts as to the desirability of the option, or, in fact, of insurance company funding. Effective experience rating of the option charge seems impractical in view of its basic purpose.

I agree with Mr. Rudd's approach in the case of split-funded plans where the basic, fixed-dollar insurance contract is written on the individual policy, group permanent, deferred group annuity or terminal funded deposit administration forms. In each of these cases, preretirement reserves are usually allocated to specific employees. At retirement, assuming the total annuity to be provided on a fixed dollar basis, funds must be transferred from the segregated fund and applied, as new money, to buy fixed dollar annuities.

However, I do not believe Mr. Rudd's analysis is normally applicable where the basic contract is either of the Standard or the Immediate Participation Guarantee Deposit Administration forms. Here preretirement reserves are not allocated to specific employees. At retirement it is quite feasible to purchase the entire annuity from the active life fund of the D.A. contract. The segregated fund would be used simply to form a portion of the preretirement reserves of succeeding generations of active employees. If the investment philosophy of the employer was based on the maintenance of a constant percentage split in total reserves between fixed dollar and common stock investments, there would be no transfers for many years. (See my discussion on page D285.) As the plan approached maturity, a portion, and later all, of the investment earnings of the segregated fund would be transferred to the D.A. contract each year. These transfers, unrelated to retirements, would simply be treated as new money deposited in the active life fund and would be subject to the same guarantees as any other source of new money deposited in the active life fund. If the contract is written to provide for the handling of all transfers in this

way and if, as is usual, withdrawals from the active life fund are made on a first-in, first-out basis, there would be no need for a guarantee of annuity rates to apply to funds transferred at retirement. In addition, if interest is credited on the "new money" or "investment year" basis, the possibility of investment antiselection is negligible.

The approach outlined above has several additional advantages to the insurance company. The average number of years a given dollar of new money remains invested in the active life fund is reduced, thus providing a hedge against overly-liberal interest guarantees. The reduction can also have a beneficial effect on postretirement mortality guarantees, especially where a single table of annuity rates is used, rather than the battery of rates produced by the full projection technique.

Several advantages accrue to the policyholder. By achieving the desired split in investment reserves primarily through gradual adjustments in the contribution split, unnecessary "churning" of the common stock fund is avoided. The sale of securities or "units" in the segregated fund will be determined solely on investment merit and will not be "forced"on the occasion of a retirement. Finally, the approach does not require an option charge.

In order to give some semblance of the "insurance" connotation to what is virtually a trust arrangement, a guarantee related to then current "new money" rates might be provided.

J. B. MACDONALD:

There is one possibly important point which is not brought out in the paper. The guaranteed annuity purchase rates attached to a segregated fund may, or may not, contain an expense loading factor, depending upon the method in which a particular company operates its segregated fund. In any event the loading in the guaranteed purchase rates will almost certainly be lower than the loading in any annuity rate available in the open market. Thus we may expect the option rate to be exercised even at a time when the new money rate is slightly higher than the guaranteed option rate, and this to some extent should offset the times when the option rate exceeds the new money rate.

(AUTHOR'S REVIEW OF DISCUSSION)

D'ALTON S. RUDD:

I am indebted to Mr. MacDonald for bringing out the effect of loading differences between the annuity purchase rates and individual single premiums. Most companies in this field are offering as the alternative to the annuity purchase rates a percentage of individual annuity rates approxi-

mately eliminating the commission element in the loading. This has the effect of generally giving a very small difference between the loading in the annuity purchase rates and the residual loading in the individual annuity rates.

Mr. Siegel examined the effect of the charge for a preferred annuity purchase rate through the equivalent reduction in investment income. If one considers the active life fund, the reduction in investment income is affected by the average duration that funds remain in the active life fund before application to purchase annuities on a first-in, first-out basis. The shorter the duration the greater the charge when measured as a reduction in investment income.

Mr. Siegel then examined two approaches to split-funded plans. I would agree with his analysis that when the uninsured fund is used solely as a side fund there would be no advantage to the employer in purchasing the contract form with the preferred annuity purchase rates. An insurance company adopting the preferred rate system of the paper would, however, usually have available also a contract form without special charges and only the "safe" annuity purchase rates.

I greatly appreciate the contributions to my paper brought out by the discussions of Mr. MacDonald and Mr. Siegel.