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REPORT OF THE COMMITTEE FOR THE PREPARATION OF MONETARY TABLES

I. 1958 CSO AND CET MORTALITY TABLES ON THE AGE LAST BIRTHDAY BASIS

Introduction

There are indications that several companies will use the age last birthday basis (as distinguished from the age nearest birthday basis traditional for Ordinary life insurance) for premium rates, nonforfeiture values and reserves on Ordinary life insurance policies based on the 1958 CSO and CET mortality tables. The committee concluded that the volumes it was preparing of monetary tables based on these mortality tables should include (in addition to the age nearest birthday material) 1958 CSO and CET mortality tables expressed on the age last birthday basis by an appropriate uniform method.

Recommended Method of Calculation

After the first year of life, it has been found generally acceptable to assume that deaths are uniformly distributed over the year of age. The commonly used exposure formulas (including those used in the basic data underlying these mortality tables), adjustments of annuities for payments more frequently than annually, and adjustments of commutation columns to the continuous basis are all based on this assumption. In this context, this assumption leads to the following formulas, where $\underline{x}\rfloor$ designates age determined on a last birthday, and (x) on a nearest birthday, basis:

$$l_{\underline{x}\rfloor} = \tfrac{1}{2}\,[l_{(x)} + l_{(x+1)}] \,*$$

$$d_{\underline{x}\rfloor} = l_{\underline{x}\rfloor} - l_{\underline{x+1}\rfloor}$$

$$q_{\underline{x}\rfloor} = d_{\underline{x}\rfloor}/l_{\underline{x}\rfloor}.$$

Tests by the Committee

The committee, as a test of this assumption, computed the following for male ages ending in 5, as shown in Exhibit I:

* When the sum in the bracket ends in an odd integer, $l_{\underline{x}\rfloor}$ is taken as the nearest even integer, to avoid biased rounding.

$1{,}000q_{\underline{x}\rfloor}$ derived from the 1958 CSO (nearest birthday) table by the above formula.

$1{,}000q_{\underline{x}\rfloor}$ derived from 1958 CSO Basic $q_{(x)}$ by a third degree polynomial interpolation formula.

The margin per 1,000 between these two rates of mortality, and between the already adopted rates of mortality on these tables for the bracketing nearest birthday ages (that is, x and $x + 1$).

EXHIBIT I

Age x	1,000 $q_{\underline{x}\rfloor}$		Margin per 1,000	
	1958 CSO	1958 CSO Basic	Age Last Birthday	(x) $(x+1)$
5.....	1.33	.52	.81	.80 .81
15.....	1.50	.59	.91	.90 .91
25.....	1.94	.94	1.00	1.00 1.01
35.....	2.57	1.47	1.10	1.10 1.11
45.....	5.59	4.24	1.35	1.32 1.37
55.....	13.60	11.46	2.14	2.07 2.19
65.....	33.22	28.88	4.34	4.14 4.53
75.....	76.16	66.26	9.90	9.57 10.33
85.....	166.47	145.13	21.34	21.02 22.55
95.....	370.64	320.80	49.84	48.21 57.20

NOTE.—

$$1{,}000\,q_{\underline{x}\rfloor}^{\text{58CSO}} = 1{,}000\left[1 - \frac{l_{(x+1)} + l_{(x+2)}}{l_{(x)} + l_{(x+1)}}\right],$$

where $l_{(x)}$ is taken from *TSA* X, p. 697 and $1{,}000q_{\underline{x}\rfloor}$ is shown to two decimal places because the rates with which it is compared are available to only two decimals.

$$1{,}000\,q_{\underline{x}\rfloor}^{\text{58CSO Basic}} = 562.5\,[q_{(x)} + q_{(x+1)}] - 62.5\,[q_{(x-1)} + q_{(x+2)}],$$

where $1{,}000q_{(x)}$ is taken from *TS*.1 X, p. 695.

In each instance, the age last birthday margin so determined is intermediate between the bracketing nearest birthday margins. From the nature of the construction of the nearest birthday tables already adopted, it is clear that similar results would be obtained for the other male ages, the special female tables below age 14, and the CET table.

For policy years beginning within the first six months of life, the nonuniform distribution of deaths within the year of age was recognized in the construction of the 1958 CSO table (see Mr. C. M. Sternhell's paper, *TSA* IX, 6). Therefore, various alternatives to the uniform distribution assumption were tested, as described in Exhibit II. Each of these alterna-

EXHIBIT II

VALUES OF $1{,}000q_{\underline{0}\rfloor}$ RESULTING FROM VARIOUS ASSUMPTIONS
(1958 CSO, MALES)

$1{,}000q_{\underline{0}\rfloor}$	Assumptions
6.50	The intercompany 1,000 $q_{(55/365)}$ of 2.74 and 1,000 $q_{(1)}$ of 1.00 can be given weights of .857 and .143 (approximately reflecting the relationship of the first policy year exposures at issue age 0 in the intercompany study to one-half those for age 1), leading to 1,000 $q_{\underline{55/365}\rfloor} =$ 2.49, which is 90.9% of 1,000 $q_{(55/365)}$. The 1958 CSO Basic 1,000 $q_{\underline{0}\rfloor}$ can thus be taken as 90.9% of the 1958 CSO Basic 1,000 $q_{(0)}$ of 6.33, or 5.75. Addition of the .75 margin in 1958 CSO 1,000 $q_{(0)}$ gives 6.50. (See C. M. Sternhell, *TSA* IX, 7-9.)
6.38	The 1958 CSO 1,000 $q_{(0)}$ of 7.08 and 1,000 $q_{(3/4)}$ of 1.82 (linearly extrapolated from $q_{(1)}$ and $q_{(2)}$) can be given weights of .867 and .133 (reflecting more closely the relationship of the intercompany exposures), leading to 1,000 $q_{\underline{0}\rfloor} = 6.38$.
4.45	Equal weights can be given to the 1958 CSO 1,000 $q_{(0)}$ and 1,000 $q_{(3/4)}$ leading to 1,000 $q_{\underline{0}\rfloor} = 4.45$.
4.43	Deaths as well as exposures can be assumed uniformly distributed, leading to the usual formula 1,000 $q_{\underline{0}\rfloor} = 1{,}000\ [1 - (l_{(1)} + l_{(2)})/(l_{(0)} + l_{(1)})]$. (Shown to two decimal places because the rates with which it is compared are available to only two decimals.)

tives assigns a higher value to $1{,}000q_{\underline{0}\rfloor}$ than the uniform distribution assumption. This assumption, therefore, leads to more conservative (*i.e.*, higher) reserves and cash values (although slightly lower net annual premiums) than the other alternatives. It has the advantage of being obviously consistent with the method indicated above as appropriate at the higher ages.

EXHIBIT III

1958 CSO MORTALITY TABLE

Age Last Birthday Basis

Age Last Birthday x M	F	l_x	d_x	$1{,}000q_x$	Age Last Birthday x M	F	l_x	d_x	$1{,}000q_x$
	0	9,983,614	39,356	3.9421	25	28	9,566,396	18,607	1.9450
	1	9,944,258	15,315	1.5401	26	29	9,547,789	18,857	1.9750
	2	9,928,943	13,703	1.3801	27	30	9,528,932	19,152	2.0099
	3	9,915,240	13,088	1.3200	28	31	9,509,780	19,542	2.0549
	4	9,902,152	12,526	1.2650	29	32	9,490,238	19,976	2.1049
	5	9,889,626	12,016	1.2150	30	33	9,470,262	20,456	2.1600
	6	9,877,610	11,557	1.1700	31	34	9,449,806	20,978	2.2199
	7	9,866,053	11,198	1.1350	32	35	9,428,828	21,545	2.2850
	8	9,854,855	10,989	1.1151	33	36	9,407,283	22,201	2.3600
	9	9,843,866	10,926	1.1099	34	37	9,385,082	23,039	2.4549
	10	9,832,940	10,964	1.1150	35	38	9,362,043	24,107	2.5750
	11	9,821,976	11,100	1.1301	36	39	9,337,936	25,398	2.7199
	12	9,810,876	11,331	1.1549	37	40	9,312,538	27,052	2.9049
	13	9,799,545	11,661	1.1900	38	41	9,285,486	29,061	3.1297
	14	9,787,884	12,088	1.2350	39	42	9,256,425	31,377	3.3898
0		9,964,600	44,138	4.4295	40	43	9,225,048	33,992	3.6848
1		9,920,462	16,270	1.6400	41	44	9,191,056	36,808	4.0048
2		9,904,192	14,758	1.4901	42	45	9,154,248	39,817	4.3496
3		9,889,434	14,142	1.4300	43	46	9,114,431	43,061	4.7245
4		9,875,292	13,578	1.3749	44	47	9,071,370	46,577	5.1345
5		9,861,714	13,067	1.3250	45	48	9,024,793	50,443	5.5894
6		9,848,647	12,607	1.2801	46	49	8,974,350	54,691	6.0941
7		9,836,040	12,246	1.2450	47	50	8,919,659	59,352	6.6541
8		9,823,794	11,984	1.2199	48	51	8,860,307	64,449	7.2739
9		9,811,810	11,872	1.2100	49	52	8,795,858	70,003	7.9586
10		9,799,938	11,956	1.2200	50	53	8,725,855	76,031	8.7133
11		9,787,982	12,186	1.2450	51	54	8,649,824	82,459	9.5330
12	15	9,775,796	12,611	1.2900	52	55	8,567,365	89,295	10.4227
13	16	9,763,185	13,229	1.3550	53	56	8,478,070	96,584	11.3922
14	17	9,749,956	13,894	1.4250	54	57	8,381,486	104,322	12.4467
15	18	9,736,062	14,604	1.5000	55	58	8,277,164	112,578	13.6010
16	19	9,721,458	15,360	1.5800	56	59	8,164,586	121,410	14.8703
17	20	9,706,098	16,063	1.6549	57	60	8,043,176	130,816	16.2642
18	21	9,690,035	16,618	1.7150	58	61	7,912,360	140,747	17.7882
19	22	9,673,417	17,073	1.7649	59	62	7,771,613	151,211	19.4568
20	23	9,656,344	17,478	1.8100	60	63	7,620,402	162,164	21.2802
21	24	9,638,866	17,783	1.8449	61	64	7,458,238	173,504	23.2634
22	25	9,621,083	18,039	1.8749	62	65	7,284,734	185,222	25.4260
23	26	9,603,044	18,246	1.9000	63	66	7,099,512	197,284	27.7884
24	27	9,584,798	18,402	1.9199	64	67	6,902,228	209,656	30.3751

EXHIBIT III—*Continued*

Age Last Birthday x		l_x	d_x	$1{,}000q_x$	Age Last Birthday x		l_x	d_x	$1{,}000q_x$
M	F				M	F			
65	68	6,692,572	222,332	33.2207	85	88	1,205,692	200,709	166.4679
66	69	6,470,240	235,264	36.3609	86	89	1,004,983	179,281	178.3921
67	70	6,234,976	248,306	39.8247	87	90	825,702	157,726	191.0205
68	71	5,986,670	261,038	43.6032	88	91	667,976	136,650	204.5732
69	72	5,725,632	272,833	47.6512	89	92	531,326	116,556	219.3681
70	73	5,452,799	283,079	51.9144	90	93	414,770	97,812	235.8223
71	74	5,169,720	291,248	56.3373	91	94	316,958	80,646	254.4375
72	75	4,878,472	297,028	60.8855	92	95	236,312	65,180	275.8218
73	76	4,581,444	300,591	65.6105	93	96	171,132	51,454	300.6685
74	77	4,280,853	302,453	70.6525	94	97	119,678	39,577	330.6957
75	78	3,978,400	303,012	76.1643	95	98	80,101	29,689	370.6446
76	79	3,675,388	302,506	82.3059	96	99	50,412	21,853	433.4881
77	80	3,372,882	300,912	89.2151	97	100	28,559	15,686	549.2489
78	81	3,071,970	297,756	96.9267	98	101	12,873	9,665	750.7962
79	82	2,774,214	292,266	105.3509	99	102	3,208	3,208	1000.0000
80	83	2,481,948	283,916	114.3924					
81	84	2,198,032	272,442	123.9481					
82	85	1,925,590	257,880	133.9226					
83	86	1,667,710	240,646	144.2973					
84	87	1,427,064	221,372	155.1241					

Conclusion

Exhibits III and IV present the 1958 CSO and CET mortality tables on the age last birthday basis, calculated by the method recommended above. ($1{,}000q_x$, age last birthday basis, is shown to four decimal places, the smallest number needed to satisfy the relationship $d_{\underline{x}\rfloor} = l_{\underline{x}\rfloor} \cdot q_{\underline{x}\rfloor}$.) The committee considers these tables appropriate for use in all calculations of nonforfeiture values and reserves involving the 1958 CSO and CET mortality tables on the age last birthday basis.

EXHIBIT IV

1958 CET MORTALITY TABLE

Age Last Birthday Basis

Age Last Birthday x M	F	l_x	d_x	$1{,}000q_x$	Age Last Birthday x M	F	l_x	d_x	$1{,}000q_x$
	0	10,002,356	46,940	4.6929	25	28	9,384,800	25,292	2.6950
	1	9,955,416	22,799	2.2901	26	29	9,359,508	25,504	2.7249
	2	9,932,617	21,157	2.1301	27	30	9,334,004	25,762	2.7600
	3	9,911,460	20,517	2.0700	28	31	9,308,242	26,109	2.8049
	4	9,890,943	19,931	2.0151	29	32	9,282,133	26,501	2.8551
	5	9,871,012	19,396	1.9649	30	33	9,255,632	26,933	2.9099
	6	9,851,616	18,916	1.9201	31	34	9,228,699	27,409	2.9700
	7	9,832,700	18,535	1.8850	32	35	9,201,290	27,925	3.0349
	8	9,814,165	18,303	1.8650	33	36	9,173,365	28,529	3.1100
	9	9,795,862	18,220	1.8600	34	37	9,144,836	29,308	3.2049
	10	9,777,642	18,236	1.8651	35	38	9,115,528	30,490	3.3448
	11	9,759,406	18,347	1.8799	36	39	9,085,038	32,114	3.5348
	12	9,741,059	18,557	1.9050	37	40	9,052,924	34,173	3.7748
	13	9,722,502	18,861	1.9399	38	41	9,018,751	36,703	4.0696
	14	9,703,641	19,261	1.9849	39	42	8,982,048	39,608	4.4097
0		9,960,850	51,602	5.1805	40	43	8,942,440	42,830	4.7895
1		9,909,248	23,684	2.3901	41	44	8,899,610	46,318	5.2045
2		9,885,564	22,144	2.2400	42	45	8,853,292	50,059	5.6543
3		9,863,420	21,502	2.1800	43	46	8,803,233	54,089	6.1442
4		9,841,918	20,915	2.1251	44	47	8,749,144	58,436	6.6791
5		9,821,003	20,379	2.0750	45	48	8,690,708	63,173	7.2690
6		9,800,624	19,896	2.0301	46	49	8,627,535	68,362	7.9237
7		9,780,728	19,512	1.9949	47	50	8,559,173	74,066	8.6534
8		9,761,216	19,230	1.9700	48	51	8,485,107	80,253	9.4581
9		9,741,986	19,094	1.9600	49	52	8,404,854	86,970	10.3476
10		9,722,892	19,154	1.9700	50	53	8,317,884	94,219	11.3273
11		9,703,738	19,358	1.9949	51	54	8,223,665	101,905	12.3917
					52	55	8,121,760	110,058	13.5510
12	15	9,684,380	19,756	2.0400	53	56	8,011,702	118,656	14.8103
13	16	9,664,624	20,344	2.1050	54	57	7,893,046	127,706	16.1796
14	17	9,644,280	20,976	2.1750					
					55	58	7,765,340	137,278	17.6783
15	18	9,623,304	21,652	2.2500	56	59	7,628,062	147,428	19.3271
16	19	9,601,652	22,371	2.3299	57	60	7,480,634	158,142	21.1402
17	20	9,579,281	23,038	2.4050	58	61	7,322,492	169,322	23.1235
18	21	9,556,243	23,556	2.4650	59	62	7,153,170	180,911	25.2910
19	22	9,532,687	23,975	2.5150					
					60	63	6,972,259	192,842	27.6585
20	23	9,508,712	24,342	2.5600	61	64	6,779,417	204,977	30.2352
21	24	9,484,370	24,611	2.5949	62	65	6,574,440	217,262	33.0465
22	25	9,459,759	24,831	2.6249	63	66	6,357,178	229,601	36.1168
23	26	9,434,928	25,003	2.6500	64	67	6,127,577	241,923	39.4810
24	27	9,409,925	25,125	2.6701					

EXHIBIT IV—*Continued*

Age Last Birthday x		l_x	d_x	$1{,}000q_x$	Age Last Birthday x		l_x	d_x	$1{,}000q_x$
M	F				M	F			
65	68	5,885,654	254,137	43.1791	85	88	611,723	132,247	216.1877
66	69	5,631,517	266,119	47.2553	86	89	479,476	111,074	231.6571
67	70	5,365,398	277,687	51.7552	87	90	368,402	91,376	248.0334
68	71	5,087,711	288,289	56.6638	88	91	277,026	73,578	265.5996
69	72	4,799,422	297,214	61.9270	89	92	203,448	57,934	284.7607
70	73	4,502,208	303,764	67.4700	90	93	145,514	44,534	306.0462
71	74	4,198,444	307,403	73.2183	91	94	100,980	33,334	330.1050
72	75	3,891,041	307,884	79.1264	92	95	67,646	24,199	357.7299
73	76	3,583,157	305,516	85.2645	93	96	43,447	16,933	389.7392
74	77	3,277,641	300,927	91.8121	94	97	26,514	11,356	428.3020
75	78	2,976,714	294,593	98.9658	95	98	15,158	7,264	479.2189
76	79	2,682,121	286,826	106.9400	96	99	7,894	4,403	557.7654
77	80	2,395,295	277,645	115.9127	97	100	3,491	2,435	697.5079
78	81	2,117,650	266,664	125.9245	98	101	1,056	934	884.4697
79	82	1,850,986	253,322	136.8579	99	102	122	122	1000.0000
80	83	1,597,664	237,406	148.5957					
81	84	1,360,258	219,007	161.0040					
82	85	1,141,251	198,517	173.9468					
83	86	942,734	176,679	187.4113					
84	87	766,055	154,332	201.4633					