

II. EXTENSIONS AND MODIFICATION OF CERTAIN BASIC DISABILITY VALUES

This report describes the methods employed in making two extensions and one modification of certain basic disability values published in the 1952 Disability Study of the Society of Actuaries.

The purpose of the extensions and modification was to permit the preparation of monetary tables for disability benefits which would be as complete as possible in giving recognition to the current practices of companies in the issuance of disability benefits. The two extensions and one modification are described below.

- (1) Extension of rates of disablement for Benefit 5, Period 2, at the younger ages, below age 18 down to age 5, and at the older ages, above age 59 to age 64.
- (2) Extension of termination rates for Benefit 5 corresponding to the extension of the rates of disablement for this benefit mentioned in item (1) above.
- (3) Modification of the rates of disablement for Benefit 4, Period 2, from a 4 months to a 6 months waiting period.

1. *Extension of Rates of Disablement for Benefit 5, Period 2*

The graduated rates of disablement for Benefit 5, Period 2, were shown in the 1952 Disability Study for attained ages 18 to 59, inclusive. These rates decreased as the age decreased. At attained age 18, the published rate of disablement was .61 per thousand. For attained age 17, a graduated rate of .52 per thousand was actually obtained but not published. Below this age the data available in the Study were not sufficient to produce reliable results, and there are presently no other statistics at young ages which are deemed acceptable for this purpose. Because of these facts, and also because the rates of disablement at the young ages are relatively low, so that any changes in these rates would have only a minor effect on the net premiums, it was considered desirable to use the graduated age 17 rate of disablement of .52 per thousand as the Benefit 5, Period 2, rate of disablement for attained ages 5 to 17, inclusive. In this connection, it should be noted that the rate of disablement of .52 per thousand was used for ages 10 to 17, inclusive, in Mr. Manuel R. Cueto's paper "Monetary Values for Ordinary Disability Benefits" (TSA VI, 110).

With respect to the extension of the Benefit 5, Period 2, rates of disablement at the older ages 60 to 64, inclusive, it was decided to use a previously prepared extension of the Benefit 1, Period 2, rates of disablement as a guide in determining the Benefit 5 rate of disablement at age 64.

While this extension of the Benefit 1 rates had not been published anywhere, it was prepared by Mr. Cueto to calculate reserves for disability benefits of a type issued many years ago (which provided for disability coverage beyond age 60) and has been approved for valuation purposes by various state insurance departments. This extension of the Benefit 1, Period 2, rates of disablement produced values of $1,000 r'_x$ at attained ages 60 through 85 by applying the following third degree equation:

$$1,000r'_x = 1,000r'_{59} + 3.0018(x - 59) + .0939(x - 59)^2 + .0443(x - 59)^3,$$

where $60 \leq x \leq 85$.

From this equation, the Benefit 1, Period 2, rate of disablement at attained age 64 was determined to be 45.91 per thousand.

TABLE 1
Period 2, $1,000 r'_x$

Attained Age	Benefit 1 (1)	Benefit 5 (2)	(2) ÷ (1) (3)
55.....	13.65	7.12	52.2%
56.....	15.56	8.38	53.9
57.....	17.76	9.33	52.5
58.....	20.24	10.35	51.1
59.....	23.02	11.44	49.7

The next step in the extension of the Benefit 5, Period 2, rates of disablement was to observe the relationship between these rates at attained ages 55 to 59 and the corresponding Benefit 1, Period 2, rates of disablement as indicated in Table 1. Accordingly, it seemed reasonable to assume that the Benefit 5 rate of disablement at age 64 would be equal to 50% of the Benefit 1 rate (45.91 per thousand) at the same age, or 22.95 per thousand.

In order to obtain the Benefit 5 rates of disablement for attained ages 60 to 63, a third degree formula, which reproduced the published Benefit 5 rate of disablement at attained age 59 and the rate of 22.95 per thousand at attained age 64, was developed, as follows:

$$1,000r'_x = 1,000r'_{59} + 1.125(x - 59) + .035(x - 59)^2 + .0401(x - 59)^3,$$

where $60 \leq x \leq 64$.

In developing the above formula the conditions were imposed that at attained age 59 the values of the first and second derivatives be equal to the corresponding values produced by the second degree equation:

$$1,000r'_x = 1,000r'_{56} + .915(x - 56) + .035(x - 56)^2.$$

This latter equation reproduced the published Benefit 5 rates of disablement at ages 56 through 59.

The resulting extended Benefit 5, Period 2, rates of disablement at the older ages are shown in Table 2.

2. Extension of Termination Rates for Benefit 5

The extension of the rates of disablement for Benefit 5 at the younger and older ages required a corresponding extension of the termination rates.

As background information, it should be noted that the 1952 Disability Study produced only select termination rates for Benefit 5. No ultimate termination rates were produced for Benefit 5 because of the lack of data,

TABLE 2
EXTENSION OF 1,000 r'_2 FOR BENEFIT 5,
PERIOD 2

Attained Age	1,000 r'_2
60.....	12.64
61.....	14.15
62.....	16.21
63.....	19.07
64.....	22.95

but the graduated ultimate termination rates for Benefits 2 and 3 combined were used for the purpose of calculating disability annuities for Benefit 5.

The select termination rates produced for Benefit 5 in the 1952 Disability Study included select annual termination rates for the first 15 years of disability and select monthly termination rates for the 7th through the 24th month of disability. The select termination rates were published for quinary age groups (15-19, 20-24, . . . 55-59) according to the insurance age at the policy anniversary preceding the date of disablement.

The ultimate termination rates produced for Benefits 2 and 3 combined in the 1952 Disability Study included ultimate termination rates for disability years 16 and over at individual attained ages $32\frac{1}{2}$, $33\frac{1}{2}$, . . . $99\frac{1}{2}$.

In Mr. Manuel Cueto's paper "Monetary Values for Ordinary Disability Benefits" (*TSA* VI, 108), the author assumed that the 1952 Disability Study Benefit 5 select termination rates for quinary age groups were applicable to the central age of each quinary age group and calculated select termination rates for intervening individual ages by means of the Karup-King third difference interpolation formula (*TSA* VI, 111). This procedure

produced select termination rates for insurance ages 17 to 57, inclusive, at the policy anniversary preceding the date of disablement. In addition, Mr. Cueto extended these select termination rates down to insurance age 10 and up to insurance age 59 at the policy anniversary preceding the date of disablement and also extended the 1952 Disability Study ultimate termination rates for Benefits 2 and 3 combined from age $32\frac{1}{2}$ down to age $25\frac{1}{2}$.

As the extension of the Benefit 5, Period 2, rates of disablement down to age 5 and up to age 64 required a further extension of termination rates beyond the ages covered by Mr. Cueto in the paper referred to in the preceding paragraph, the Committee investigated the possibility of using Mr. Cueto's extensions as a basis for their further extensions. The Committee decided that it could use Mr. Cueto's extension at the older ages as a basis for the further extension to age 64, but that it could not use Mr. Cueto's extension at the younger ages because, while his extension produced reasonable values down to age 10, it did not produce reasonable values when it was carried through down to age 5.

The Committee, therefore, started with Mr. Cueto's individual age select termination rates for insurance ages 17 to 59, inclusive, at the policy anniversary preceding the date of disablement and by graphical methods produced new select termination rates for insurance ages 5 to 16 and for insurance ages 60 to 64. The Committee also used graphical methods to extend the 1952 Disability Study ultimate termination rates for Benefits 2 and 3 combined from attained age $32\frac{1}{2}$ down to attained age $20\frac{1}{2}$.

In applying these graphical methods, the Committee analyzed separately the rates of termination by death and recovery as well as the total termination rates. The Committee also examined the grading by age and duration as well as the grading of the select termination rates into the ultimate termination rates. Monthly termination rates for the 7th through the 24th month of disability were also obtained by graphical methods and were adjusted so that they would compound into the corresponding annual termination rates for the first and second year of disability.

The resulting termination rates for ages at policy anniversary preceding disablement, 5 through 17, are shown in Table 3 for the 7th through the 24th month of disability, and in Table 4 for the 1st through 15th year of disability. Table 4 also shows the extension of the ultimate termination rates from individual attained age $32\frac{1}{2}$ down to individual attained age $20\frac{1}{2}$. The values for age 17 in Tables 3 and 4 are the same as those published in the 1952 Disability Study and are included to illustrate the grading of the new values into those previously published.

TABLE 3
 EXTENSION OF 1930-1950 GRADUATED TERMINATION RATES—BENEFIT 5
 MONTHLY RATES PER 1,000 FOR DISABILITY YEARS 1 AND 2
 (Based on Amount of Premium Waived)

DISABILITY MONTH	AGE AT POLICY ANNIVERSARY PRECEDING DISABLEMENT												
	5	6	7	8	9	10	11	12	13	14	15	16	17
7.....	28.0	28.4	28.9	29.5	30.1	30.9	31.7	32.7	33.8	34.9	36.1	37.3	38.5
8.....	24.0	24.3	24.8	25.3	26.1	26.8	27.8	28.7	29.8	31.2	32.5	33.8	34.9
9.....	20.1	20.6	21.1	21.7	22.4	23.1	24.2	25.2	26.6	27.8	29.2	30.5	31.9
10.....	17.2	17.8	18.2	18.9	19.6	20.6	21.4	22.6	23.9	25.2	26.7	28.1	29.6
11.....	15.8	16.1	16.7	17.3	18.1	19.0	20.0	21.1	22.3	23.9	25.4	26.9	28.3
12.....	14.9	15.4	16.0	16.7	17.4	18.4	19.4	20.7	22.0	23.5	25.0	26.6	28.2
13.....	17.8	18.2	18.9	19.5	20.3	21.2	22.2	23.4	24.8	26.3	27.8	29.4	31.0
14.....	20.9	21.4	21.9	22.6	23.3	24.1	25.1	26.3	27.5	28.9	30.3	31.9	33.3
15.....	24.8	25.2	25.7	26.3	27.0	27.8	28.7	29.7	31.0	32.2	33.7	35.0	36.4
16.....	28.8	29.1	29.6	30.2	31.0	31.6	32.5	33.5	34.6	35.8	37.1	38.4	39.7
17.....	32.4	32.6	33.2	33.6	34.1	34.9	35.8	36.7	37.7	38.8	40.0	41.3	42.5
18.....	35.1	35.4	35.7	36.2	36.8	37.5	38.2	38.9	39.9	41.0	42.0	43.1	44.3
19.....	36.8	37.1	37.5	38.0	38.6	39.1	39.7	40.5	41.3	42.1	43.1	44.0	44.8
20.....	37.9	38.1	38.4	38.7	39.0	39.4	40.1	40.6	41.2	41.8	42.5	43.2	43.9
21.....	38.5	38.7	38.9	39.1	39.3	39.6	39.8	40.3	40.5	40.9	41.2	41.6	42.1
22.....	38.7	38.9	39.0	39.1	39.2	39.4	39.5	39.5	39.6	39.7	39.8	39.9	40.0
23.....	38.7	38.7	38.8	38.9	39.0	39.0	38.9	38.8	38.7	38.6	38.4	38.2	38.0
24.....	38.1	38.1	38.2	38.2	38.4	38.7	38.7	38.6	38.4	38.0	37.6	37.0	36.5

TABLE 4

EXTENSION OF 1930-1950 GRADUATED TERMINATION RATES—BENEFIT 5
 SELECT ANNUAL RATES PER 1,000 FOR DISABILITY YEARS 1 TO 15
 (Based on Amount of Premium Waived)

DISABILITY YEAR	AGE AT POLICY ANNIVERSARY PRECEDING DISABLEMENT												
	5	6	7	8	9	10	11	12	13	14	15	16	17
1.....	114.2	116.5	119.4	122.7	126.6	131.1	136.2	141.9	148.4	155.4	162.7	169.8	176.7
2.....	326.5	328.7	331.4	334.7	338.6	342.8	347.4	352.5	358.1	364.1	370.3	376.4	382.5
3.....	304.6	306.6	308.7	310.9	313.2	315.8	318.5	321.3	324.2	327.3	330.6	333.6	336.4
4.....	255.1	257.5	259.9	262.3	264.7	267.2	269.7	272.2	274.7	277.3	279.9	282.5	285.1
5.....	208.4	210.9	213.2	215.4	217.6	219.8	221.9	224.0	226.0	227.9	229.6	231.1	232.3
6.....	169.9	172.4	174.5	176.3	178.0	179.4	180.7	181.8	182.6	183.2	183.4	183.4	183.2
7.....	132.9	137.2	140.8	143.7	145.9	147.3	148.0	148.2	148.0	147.4	146.4	144.9	142.8
8.....	108.6	111.1	112.8	113.8	114.3	114.5	114.5	114.4	114.3	114.0	113.4	112.5	111.3
9.....	90.2	91.1	91.9	92.6	93.2	93.8	94.2	94.5	94.6	94.2	93.4	92.3	91.2
10.....	86.1	86.2	86.3	86.4	86.4	86.3	86.2	86.0	85.8	85.4	84.3	82.7	80.7
11.....	82.4	82.6	82.8	82.9	82.9	82.9	82.9	82.6	82.2	81.5	80.2	78.4	76.7
12.....	78.8	79.3	79.6	79.9	80.0	80.1	79.9	79.4	78.7	77.8	76.6	75.3	73.8
13.....	75.5	76.4	76.9	77.2	77.3	77.4	77.3	76.9	76.3	75.3	74.1	72.9	71.2
14.....	73.9	74.3	74.6	74.8	74.9	74.9	74.8	74.5	73.9	73.0	71.8	70.6	68.8
15.....	72.7	72.9	73.1	73.1	73.1	73.0	72.8	72.3	71.6	70.7	69.6	68.3	66.7
ULTIMATE ANNUAL RATES PER 1,000 (BY ATTAINED AGE)													
ATTAINED AGE													
	20½	21½	22½	23½	24½	25½	26½	27½	28½	29½	30½	31½	32½
16 and over	71.6	71.7	71.7	71.7	71.5	71.3	70.9	70.2	69.4	68.5	67.6	66.5	65.4

The resulting termination rates for ages at policy anniversary preceding disablement, 57 through 64, are shown in Table 5 for the 7th through the 24th month of disability, and in Table 6 for the 1st through 15th year of disability. The select termination rates for age 57 in Tables 5 and 6 and the ultimate termination rates in Table 6 are the same as those published in the 1952 Disability Study and are included to illustrate the grading of the new values into those previously published.

TABLE 5
EXTENSION OF 1930-1950 GRADUATED TERMINATION RATES—BENEFIT 5
MONTHLY RATES PER 1,000 FOR DISABILITY YEARS 1 AND 2
(Based on Amount of Premium Waived)

Dis- ABILITY MONTH	AGE AT POLICY ANNIVERSARY PRECEDING DISABLEMENT							
	57	58	59	60	61	62	63	64
7.	33.4	29.7	25.7	24.3	23.4	22.9	22.6	22.8
8.	32.2	29.0	25.6	24.2	23.3	22.9	22.5	22.7
9.	30.9	28.3	25.4	23.9	23.1	22.6	22.3	22.5
10.	29.3	26.9	24.3	22.9	22.0	21.6	21.2	21.4
11.	27.5	25.3	23.0	21.5	20.7	20.3	19.9	20.1
12.	25.6	23.4	21.2	19.8	18.9	18.5	18.1	18.3
13.	22.3	20.5	18.8	18.0	17.4	17.0	16.8	16.8
14.	20.5	18.6	16.7	16.0	15.3	14.9	14.7	14.7
15.	18.9	17.0	15.0	14.2	13.6	13.2	13.0	13.0
16.	17.5	15.5	13.5	12.8	12.1	11.8	11.5	11.5
17.	16.3	14.3	12.4	11.6	11.0	10.7	10.4	10.4
18.	15.4	13.5	11.7	10.9	10.3	9.9	9.7	9.7
19.	14.7	13.0	11.3	10.6	9.9	9.6	9.3	9.3
20.	14.3	12.8	11.4	10.6	10.0	9.6	9.4	9.4
21.	13.9	12.7	11.4	10.7	10.0	9.7	9.4	9.4
22.	13.6	12.6	11.7	10.8	10.1	9.8	9.5	9.5
23.	13.2	12.4	11.5	10.9	10.3	9.9	9.6	9.7
24.	12.9	12.3	11.7	11.0	10.3	10.0	9.7	9.7

In order to illustrate the effect on disabled life annuities of the difference between the Committee's extension of termination rates at the younger ages and Mr. Cueto's extension, Table 7 compares disabled life annuities computed on both bases for representative ages at policy anniversary preceding disablement.

3. Modification of Rates of Disablement for Benefit 4

Benefit 4 provided for a 120-day waiting period clause, and accordingly the rates of disablement developed in the 1952 Disability Study for this Benefit reflect a 4 months waiting period. However, because current policies providing disability income benefits are generally based on a 6 months

waiting period, it was felt that Benefit 4 values should be based on a 6 months waiting period, thereby eliminating the need for subsequent adjustments in net premiums and other values where such waiting period is used. Accordingly, rates of disablement for this Benefit were modified to a 6 months waiting period. This was done by applying the necessary select monthly termination rates as follows:

$$r'_x(6 \text{ months}) = r'_x(4 \text{ months}) \cdot (1 - {}_{1/12}q^i_{[x+1/2]+4/12}) (1 - {}_{1/12}q^i_{[x+1/2]+5/12})$$

$$= r'_x(4 \text{ months}) \cdot \frac{l^i_{[x+1/2]+6/12}}{l^i_{[x+1/2]+4/12}},$$

TABLE 6

EXTENSION OF 1930-1950 GRADUATED TERMINATION RATES—BENEFIT 5
SELECT ANNUAL RATES PER 1,000 FOR DISABILITY YEARS 1 TO 15

(Based on Amount of Premium Waived)

DISABILITY YEAR	AGE AT POLICY ANNIVERSARY PRECEDING DISABLEMENT							
	57	58	59	60	61	62	63	64
1.....	166.1	152.0	136.7	129.0	124.5	122.0	120.1	121.0
2.....	177.0	161.8	146.3	138.5	132.0	128.0	125.0	125.2
3.....	144.4	138.1	132.2	128.5	125.3	121.5	120.1	119.8
4.....	122.7	121.9	121.6	117.8	115.7	114.2	114.1	114.5
5.....	107.0	108.2	109.9	108.6	108.2	108.0	109.1	110.2
6.....	95.8	97.1	98.7	99.5	101.0	101.9	104.1	105.8
7.....	88.1	88.9	89.7	91.6	94.1	95.8	99.1	101.2
8.....	82.9	83.3	83.7	86.1	88.4	90.3	94.1	96.6
9.....	79.4	80.1	80.8	83.6	85.6	87.4	90.6	94.1
10.....	77.5	79.1	81.1	83.8	86.5	88.0	91.3	95.6
11.....	76.7	79.4	82.6	85.5	89.0	91.2	95.4	100.4
12.....	76.9	80.2	84.1	87.9	92.0	96.0	101.0	106.8
13.....	78.1	81.8	86.0	90.6	95.2	101.0	106.7	113.4
14.....	79.8	83.2	87.1	93.4	98.9	106.2	112.6	120.2
15.....	82.0	85.2	89.8	97.2	103.8	111.6	119.0	127.5
	ULTIMATE ANNUAL RATES PER 1,000 (BY ATTAINED AGE)							
	ATTAINED AGE							
	72½	73½	74½	75½	76½	77½	78½	79½
16 and over.....	85.3	89.7	95.6	102.4	109.7	117.6	126.1	135.3

where

r'_x (4 months) = Benefit 4, Period 2, rates of disablement based on a 4 months waiting period

$[x + \frac{1}{2}]$ = age at disablement .

In order to eliminate minor anomalies in results, slight adjustments were made in the figures produced by the aforementioned formula.

The rates of disablement for Benefit 4, Period 2, on the basis of a 6 months waiting period are given in Table 8, together with the corresponding rates based on a 4 months waiting period for purposes of comparison.

TABLE 7
DISABLED LIFE ANNUITIES FOR BENEFIT 5 BASED
ON 1952 DISABILITY STUDY 1930-1950
TERMINATION RATES WITH $2\frac{1}{2}\%$ INTEREST

$$\bar{a}_{[x+1/2]+1/2:\overline{u-x-1}}^i$$

Age at Policy Anniversary Preceding Disablement x	Based on Extension of Termination Rates in Mr. Cueto's Paper (TSA VI, 165)	Based on Proposed Extension of Termination Rates in Committee Report
$u = x + 10$		
5.....	—	2.949
10.....	2.845	2.782
15.....	2.544	2.554
20.....	2.356	2.356
25.....	2.234	2.234
$u = x + 20$		
5.....	—	3.624
10.....	3.440	3.373
15.....	3.067	3.079
20.....	2.851	2.851
25.....	2.731	2.731
$u = 100$		
5.....	—	4.048
10.....	3.821	3.758
15.....	3.425	3.439
20.....	3.204	3.204
25.....	3.090	3.090

TABLE 8
 BENEFIT 4, PERIOD 2
 GRADUATED RATES OF DISABLEMENT PER 1,000
 1,000 r'_x

Attained Age x	4 Months Waiting Period	6 Months Waiting Period
15.....	2.44	2.09
16.....	2.44	2.09
17.....	2.44	2.09
18.....	2.44	2.09
19.....	2.44	2.09
20.....	2.44	2.09
21.....	2.44	2.09
22.....	2.44	2.09
23.....	2.44	2.09
24.....	2.44	2.09
25.....	2.44	2.09
26.....	2.44	2.09
27.....	2.44	2.09
28.....	2.44	2.09
29.....	2.55	2.18
30.....	2.68	2.29
31.....	2.82	2.41
32.....	2.99	2.54
33.....	3.16	2.68
34.....	3.33	2.83
35.....	3.52	2.99
36.....	3.71	3.16
37.....	3.92	3.34
38.....	4.16	3.54
39.....	4.44	3.79
40.....	4.76	4.06
41.....	5.13	4.39
42.....	5.56	4.76
43.....	6.04	5.19
44.....	6.56	5.64
45.....	7.13	6.15
46.....	7.73	6.69
47.....	8.37	7.27
48.....	9.03	7.88
49.....	9.75	8.55
50.....	10.53	9.28
51.....	11.60	10.27
52.....	12.81	11.41
53.....	14.23	12.76
54.....	15.83	14.30
55.....	17.60	16.01
56.....	19.51	17.90
57.....	21.64	20.02
58.....	24.02	22.43
59.....	26.70	25.18

Conclusion

The above extensions and modifications are being incorporated in the forthcoming publication of *Monetary Tables for Disability Benefits* based on the 1952 Disability Study combined with the 1958 CSO Table. The Committee considers them appropriate for use in all calculations involving the use of the affected disability functions.

SUPPLEMENT

In presenting these reports the Committee wishes to express its indebtedness for the special contribution of Bert A. Winter to the Age-Last-Birthday report and of Manuel Cueto to the Disability Extensions and Modification report.