Premium Distribution and Market Competitiveness Under Rate Regulation

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AUTHOR Zia Rehman, Ph.D., FCAS **SPONSOR** Society of Actuaries

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CONTENTS

This paper concerns optimal new business written premium mix at the firm and state levels under varying competitive conditions. We address three common problems as well as a new result in linear algebra, shown in an appendix. The first is to determine a company's optimal new business written premium mix for a given geographical area such as a state. The approach we present incorporates the idea of mean-variance optimization, an important attribute of competitive firms. We develop an optimizing function that maximizes return on equity. The second problem concerns predicting statewide product mixes under varying rate regulation scenarios involving profit caps. Knowing the effects of proposed rate regulations will help regulators encourage adequate capacity in particular lines of business in their respective states. The results also help a company know when to exit a line of business in a given state. The third problem, also a regulator's problem, involves measuring and testing a market's competitiveness for a given line of business. Generally speaking, competitive markets require less rate regulation. Our fourth theorem provides necessary and sufficient conditions for market competitiveness. We use hypothetical data to demonstrate the usefulness of our results, but, in practice, one can easily generate them with widely available company level and, where appropriate, industry-level data. The presented solutions link to company reserves and can be updated along with reserve parameters as new data come into play. The methods we present fit well with current annual (or more frequently occurring) reserve reviews and rate filings—an advantage for both companies and regulators. The scope of the paper is international, and we use the United States as a base example to make our points. The model applies to any line of business where there is risk transfer and hence triangulation of data is possible. This includes property liability, individual and group life, individual and group health, disability and accidental insurance.

Section 1: Introduction

The managers of any insurance company operating in any country know about at least some of the complexities presented by state insurance regulation. These include differences in rating laws and regulations and, thus, competitive environments among the different states and across lines of insurance. For example, auto insurance rate regulations in Michigan differ from those in Illinois, and homeowners' rate regulations differ from commercial property owners' rate regulations in Texas.

Rate regulations affect a company's ability to cover losses, pay expenses and sufficiently compensate capital providers for the risk that they assume. Thus, trating regulations should and will influence an insurance company's managerial decisions regarding whether to write a line of business in a given geographic region and, if so, how much to write and at what rate.

When regulators set rate caps too low, companies may reduce their writings in a state or choose to withdraw altogether from a line of business in a state's market. Either way, market disruptions occur, and in some cases, rates may actually go up in the long term—the opposite of what was hoped for with rate suppression (see, for example, Regan, Tennyson and Weiss 2008). Farmers Insurance Group, one of Texas's largest homeowners insurance companies, stopped writing new homeowners insurance in August of 2002 following arguments with state regulators over policy pricing, resuming writing several months later on a limited basis (Associated Press 2003). The short-run effect was fewer markets for homeowners when seeking coverage. More recently, State Farm announced in 2009 that they planned to leave Florida's homeowners insurance market (Simpson 2009), subsequently striking a deal with the state to remain, but shedding 125,000 policies (Patel 2011). Many of State Farm's policyholders shifted to Citizens Property Insurance, Florida's state homeowners insurer of last resort (Patel 2011). Some point to rate suppression as a primary reason that Citizens eventually grew to insure approximately 1.5 million policies in 2005 before a plan for depopulating it to a more reasonable size was adopted (Patel 2011; Vinson 2015). Citizens Property Insurance Corporation presents its own problems because it is largely underfunded (Vinson 2015). The case of Citizens shows what can happen to market structure when the combination of public market capital and rate suppression crowds out private market risk-bearing capital.

In their defense, determining what rates should be charged for a given line of insurance in a particular jurisdiction is not an easy task for regulators. Although they possess the guiding principles that rates should be "adequate, not excessive and not unfairly discriminatory," the challenge of implementing them with the information provided by insurance companies in rate filings can be significant.

Considering what rates seem reasonable for a given line of insurance in a state market presents a vexing challenge, not just for regulators, but for actuaries and insurance company managers too. In this paper, we present some tools for doing just that.

In the section that follows, we introduce the foundation for the rest of the paper, including an equation for profits, key definitions and case study. The third section focuses on our basic result on optimization when perfectly competitive markets exist and expected profits vary freely. There we specify a company's optimizing function and show necessary but not sufficient conditions for market efficiency at the company level. The fourth section concerns regulated markets, and it is here that our major contributions to the literature begin. Employing a constrained optimization problem, we enlist a result by Gotoh (2001) to find optimal product mix weights using eigenvectors, using Theorem 2 to describe the circumstances. The results help in understanding when a company should exit a line of business in a particular state. Next, we turn our attention to the case of optimal premium mix in statewide markets. The third and fourth theorems culminate in the major contributions of this paper. Our third theorem, which shows how the statewide premium mix can be found from company-level information even if firms are not mean-variance optimizers, should prove especially useful to state insurance regulators charged with reviewing company profit filings. The development is made feasible by proving a new result in linear algebra that is shown in Appendix C. Our fourth and last theorem gives regulators a mathematical way to measure competitiveness of insurance markets in their respective states even when at least one firm is not a mean-variance optimizer. This has not been done before in the literature and is a major contribution of this paper. In summary, the mathematical derivations presented in the paper are not an exact or minor variation of mean-variance portfolio theory; rather, we present new mathematical results and an insurance setting that is applicable to regulators. The novel mathematical results and concepts will be made obvious in the paper in bold italicized font.

Although we use hypothetical data to demonstrate the usefulness of our results, in practice, one can easily generate them with widely available company-level and, where appropriate, industry-level data aggregated for a particular jurisdiction. Hence, the model proves very practical in implementation. Also, the presented solutions link to company reserves and can be updated along with reserve parameters as new data come into play, adding another source of usefulness to the model. The methods we present fit well with current annual (or more frequently occurring) reserve reviews and rate filings, an advantage for both companies and regulators.

Before proceeding, understand that our intent is not to investigate the competitive structure of the U.S. insurance industry. Other researchers, including Cummins and Xie (2013), Choi and Weiss (2005), Cummins, Weiss and Zi (1999), Tombs and Hoyt (1994), Mayers and Smith (1988), and King (1975), to name a few, have one already done so using data aggregated by firm across the jurisdictions in which they operate. Critically, their methods do not let them determine which line(s) of business a company should expand (or contract) in a given state or territory. Our methods allow for companies to do just that under different rating environments.

Our paper is not about determining underwriting profit provisions by line, and these are assumed to be known for companies. These underwriting profit provisions are determined as a result of a process that is part science, part art and regulation. See Myers and Read (2001) for one such approach based on capital allocation.

Taylor (1987) found that constant unit expense rates lead to optimal premium rates of substantial negative profitability, and the adjustment to reflect marginal expenses properly can cause very significant changes to these low premium rates. Rothschild and Stiglitz (1992) discussed the equilibrium in competitive insurance markets with imperfect information. They focused on sales offers, which consist of both a price and a quantity, a particular amount of insurance that the individual can buy at that price. What's more, fully revealed information for an individual can make everyone better off. Paul and Haberman (2005) built the optimal control model for general insurance pricing. For two demand functions, an optimal premium strategy is well defined and smooth for certain parameter choices, especially for a linear demand function that these strategies yield the optimal dynamic premium if the market average premium is lognormal distributed. Taylor

(2006) paid attention to what individual insurers were attempting to achieve in following the market and found that optimal strategies do not follow what might be thought the obvious rules. The optimal strategies depend on various factors, including the predict time, price elasticity of demand and rate of return. They also found when the current coverage market rate lies below the break-even rate, return to substantial profitability in the very near future may be possible. Paul (2007) analyzed the pricing problems with two forms of constraint: a bounded premium and a solvency requirement. A lower bound is placed on the premium then an analytic solution can be found, but for solvency constraints, we can get numerical results only using control parameters. Taylor (2008) built the dynamics model for insurance market, whichs includes 11 essential parameters with physical interpretation, some of which can be used as regulatory controls. But these regulatory controls need to be applied with great caution lest they induce preserve effects. Pantelous, Athanasios and Eudokia (2013) considered the volume of business, average market premium, the company's premium, which is a control function, and a linear stochastic disturbance when studying a company expected to drop part of the market. In this model, the optimal premium strategy can be defined analytically and endogenously by maximizing the total expected linear discounted utility of the wealth over a finite time horizon. Pantelous, Athanasios and Passalidou (2015) built a discrete-time stochastic dynamic programming model to connect a company's optimal strategy with market competition, which is available for both negative and positive effects on the volume of business depending on the company's reputation for non-life insurance pricing. When the company has a very great reputation, the company is very flexible to choose any premium it wishes. Pantelous, Athanasios and Eudokia (2017) introduced the quadratic utility function into a discrete-time stochastic nonlinear premium-reserve model to optimize the reserve in a competitive insurance market. Besides the company's reserve, for the very first time, the derived optimal premium in a competitive market environment is also dependent on the break-even premium, the expectation of the market's average premium as it did in the linear models, the income insurance elasticity of demand and other factors.

Our paper concerns the optimal new business written premium mix at the firm and state levels under varying competitive conditions. Our whole discussion is around the mean-variance optimizers, and four theorems are derived. We propose an extension of mean-variance optimization to the case of an insurer or insurers seeking to optimize the mix of premiums across various lines of business.

First, we specify a company's optimizing function and show necessary but not sufficient conditions for market efficiency at the company level. Next, employing a constrained optimization problem, we enlist a result by Gotoh (2001) to find optimal product mix weights using eigenvectors, using Theorem 2 to describe the circumstances. Then we show how the statewide premium mix can be found from company-level information even if firms are not mean-variance optimizers. Finally, we give regulators a practical way of determining whether insurance markets remain competitive in their respective states even when at least one firm is not a mean-variance optimizer.

Compared with other research about investigating the competitive structure of the U.S. insurance industry, our methods allow for companies to do just that under different rating environments.

In addition, we offer a new way of estimating the profit covariance in Appendix B, and our model proves very practical in implementation. We also posit two ways in which regulators might impose constraints on insurers in the appendix, which offer an interesting and potentially useful extension to the problems cited.

Section 2: Foundations

To develop the paper systematically, we will first introduce foundational material. For readers familiar with insurance, this material may look elementary, but some statistical enhancements are highlighted.

A random written premium, *P*, for a prospective (brand new) policy year can be broken into its essential components of losses, expenses, profit and investment income offset as follows:

$$P = Loss + Expenses + Profit - (Investment\_Income\_Offset)$$

Similar to the approach taken by Robbin (2004), we add here an investment income offset term to account for the fact that investment income earned on premiums reduces the amount required to transfer risk.

The quantity (Profit — Investment Income Offset) equates to the Underwriting Profit Provision (UPP). Companies may show UPP charges in their rate filings. Hence, we use the equation above. Rearranging the UPP equation to solve for profit yields: Profit = UPP + Investment Income Offset.

Random losses are the undiscounted, ultimate values for a new policy year and include allocated loss adjustment expenses. Expenses, also random, include company overhead, marketing costs and similar items. Some of these expenses, such as sales commissions, depend on the random written premium, *P*. ***We treat the profit as random, and this is a technical enhancement because the literature generally treats profit as a constant.*** Normalizing the above,

$$1 = \frac{Loss}{P} + \frac{Expenses}{P} + \frac{Profit}{P} - \frac{(Investment\_Income\_Offset)}{P}$$

For a new policy year, $\frac{Loss}{P}$, referred to as the permissible loss ratio (PLR)[1], is random. The expense ratio and the investment income offset ratio are assumed fixed[2] and known as they do not change significantly from year to year. Finally, the ratio $\frac{Profit}{P}$ is random. In symbols for a line of business k = 1, 2, …, n,

$$1 = U_k + e_k + R_k - f_k \tag{1}$$

$U_k = \frac{Loss}{P}$ = Random permissible loss ratio

e_k = Fixed expense ratio as a percent of written premium

$R_k = \frac{Profit}{P}$ = Random underwriting profit provision as a percent of written premium

$f_k = \frac{(Investment\_Income\_Offset)}{P}$ = Fixed investment income offset as a percent of written premium.

Taking expectations, we get the following "totality constraint" with $EU_k = u_k$, $ER_k = \pi_k$:

$$1 = u_k + e_k + \pi_k - f_k \tag{2}$$

For a given line, our dataset includes historical risk faced by the company. In Appendix B we briefly describe the estimation of the profit covariance matrix. Equations (3)—(7) are found in this appendix.

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[2] This is the same as target loss ratio. Readers more familiar with the term target loss ratio can replace this when reading the paper.
[3] In actuality, however, these ratios may not be fixed. Various factors may cause these ratios to vary over time. For example, expense ratios may vary because of changes in the commission schedules for insurance agents. Also, the investment income offset ratio may vary because of changes in interest rates or the returns insurers earn on their investments. Fortunately rate filings are usually done annually, and we assume that these changes are small during the one-year period.

Section 3: Case Study

We understand that the information presented above may feel remote to the reader. To make the results seems more tangible and accessible, we present a case study developed using hypothetical data. We begin with a company writing five lines of insurance. Table 1 gives rate filing information for five lines in company X.[3]

Table 1: 2013 Rate Filing Information

	Permissible Loss Ratio	Expense Ratio	Underwriting Profit Provision	Investment Income Offset	Profit
Line 1	72.0%	30.0%	−2.0%	5.1%	3.1%
Line 2	65.0	30.0	5.0	−2.4	2.6
Line 3	62.0	30.0	8.0	−0.1	7.9
Line 4	60.0	30.0	10.0	3.3	13.3
Line 5	70.0	30.0	0.0	8.5	8.5

We now provide an intuitive result (Theorem 1) on perfectly competitive markets. Although the result itself is not startling, it provides a mathematical basis to prove Theorem 4, and this makes it necessary to prove it. Second, Theorem 1 is novel in the sense that it shows that perfectly competitive market assumption leads to maximization of a certain ratio. Third, Theorem 1 sets the tone to think about the problems in this paper.

[3] Figures 1 and 2 are located in Appendix B and discuss data and estimation of profit covariance matrix.

Section 4: Perfectly Competitive Markets

We assume that expected profit ER_k is always given to us by line of business k. Market forces or regulation determine the values of ER_k. Our ultimate goal in this paper is to find the optimal premium mix, w_k, subject to a certain optimizing function and constraints. We specify the optimizing function below.

Section 4.1 Company Optimizing Function

Suppose that the company prospectively writes a total of $p_k \geq 0$ (in U.S. dollars) premium for line k. We can define the company-wide profit as[4]

$$R = \frac{\sum_{k=1}^{n} p_k R_k}{\sum_{k=1}^{n} p_k} = \sum_{k=1}^{n} w_k R_k \tag{8}$$

$$w_k = \frac{p_k}{\sum_{k=1}^{n} p_k} \tag{9}$$

From (8),

$$\text{Expected company profit} = \sum_{k=1}^{n} w_k ER_k \tag{10}$$

Using (8) again, we can measure the total portfolio risk with proportions

$w_k \in [0 \leq w_k \leq 1]$ such that $\sum_{k=1}^{k=n} w_k = 1$,

[4] This is an abstract quantity because profits are charged only by line. Nonetheless it is mathematically correct to define the company-wide profit because we are simply aggregating a quantity across all lines.

$$\sigma^2 = Var(R) = Var\sum_{k=1}^{n} w_k R_k = \sum_{i=1}^{i=n}\sum_{k=1}^{k=n} w_i w_k \operatorname{cov}(R_i, R_k) \tag{11}$$

Inspecting the covariance matrix $\{w_i w_k \operatorname{cov}(R_i, R_k)\}_{k,l}$, the contribution of a line *l* variance to the total portfolio variance (component risk),

$$w_i \sum_{k=1}^{k=n} w_k \operatorname{cov}(R_i, R_k) \tag{12}$$

The above can be verified by summing across l and that will result in $Var(R)$:

$$\sum_{l=1}^{n} w_i \sum_{k=1}^{k=n} w_k \operatorname{cov}(R_i, R_k) = \sum_{l=1}^{n}\sum_{k=1}^{k=n} w_i w_k \operatorname{cov}(R_i, R_k) = VarR$$

Also since $R = \sum_{k=1}^{k=n} w_k R_k$, we have

$$\operatorname{cov}(R_l, R) = \operatorname{cov}\left(R_l, \sum_{k=1}^{k=n} w_k R_k\right) = \sum_{k=1}^{k=n} w_k \operatorname{cov}(R_i, R_k) \tag{13}$$

Therefore (12) can also be written as

$$w_l \operatorname{cov}(R_l, R) \tag{14}$$

The return contribution for line *l* is $w_i E(R_i)$. It can be verified by summing across *l*,

$$\sum_{l=1}^{n} w_l E(R_l) = R$$

Define

Mean-variance ratio of a line = $\dfrac{w_l E(R_l)}{w_l \operatorname{cov}(R_l, R)}$ (15)

Drawing on standard portfolio theory, we make the assumption that companies are mean-variance optimizers and define the optimizing function with respect to line weight w_i:

$$f(w_1,..w_n) := \frac{E(R)}{\sigma^2} = \frac{\sum_{k=1}^{n} w_k ER_k}{\sum_{i=1}^{i=n}\sum_{k=1}^{k=n} w_i w_k \operatorname{cov}(R_i, R_k)} \tag{16}$$

The mean-variance optimization assumption is critical and will be carried through in the first parts of our paper. Later this assumption is relaxed through deviance multipliers (Theorem 3). A practical rationale for companies to adopt equation (16) as the optimizing function is whenever pricing capital (Robbin 2004) is strictly monotonically increasing in σ, then maximizing $f(w_1,\ldots,w_n)$ is equivalent to maximizing the return on equity (ROE).

Section 4.2 Company Behavior

From standard portfolio theory (Markowitz 1952), equations (15) and (16) are set equal under conditions of perfect completion, including in insurance markets:

$$\frac{w_l E(R_l)}{w_l \operatorname{cov}(R_l, R)} = \frac{E(R)}{\sigma^2}$$

The rationale is that profits will "set themselves" to satisfy the above equation. To see this, suppose that the left-hand side of the above equation exceeds the right-hand side. In this case, the company will increase w_l since it is a mean-variance optimizer. If that is not possible due to market forces, the company will lower $E(R_l)$, a possibility that exists if markets are perfectly competitive and profits are allowed to vary freely. In response, other insurers will change their portfolios to let this company increase w_l. The equation represents a kind of equilibrium when markets are perfectly competitive.

Using (13), define

$$\beta_l := \frac{\operatorname{cov}(R_l, R)}{\sigma^2} = \frac{\sum_{k=1}^{k=n} w_k \operatorname{cov}(R_l, R_k)}{\sum_{l=1}^{l=n}\sum_{k=1}^{k=n} w_l w_k \operatorname{cov}(R_l, R_k)} \tag{17}$$

Therefore,

$$E(R_l) = \beta_l E(R) \tag{18}$$

The above looks like the capital asset pricing model (CAPM), but the quantities are completely different.

Further from (18),

$$E(R) = \sum_{l=1}^{n} w_l E(R_l) = E(R)\sum_{l=1}^{n} w_l \beta_l$$

$$\sum_{l=1}^{n} w_l \beta_l = 1 \tag{19}$$

At a company level, the above is a necessary (but not sufficient) condition for market efficiency. Note that because of randomness in estimating β_l, it is not possible to draw conclusions about market efficiency based on the data of a single company. Nonetheless equation (19) is a useful theoretical result.

Section 4.3 Company Premium Mix

We calculate optimal weights such that the firm is mean-variance optimized. Specifically, we wish to solve for a unique combination of weights such that $f(w_1, \ldots, w_n)$ is maximized,

$$f(w_1, .. w_n) = \frac{E(R)}{\sigma^2}$$

$$\ln f(w_1, .. w_n) = \ln E(R) - \ln \sigma^2$$

$$\ln f(w_1,..w_n) = \ln \sum_{k=1}^{k=n} w_k E(R_k) - \ln \sum_{l=1}^{l=n}\sum_{k=1}^{k=n} w_k w_l \operatorname{cov}(R_k, R_l)$$

Set $\dfrac{\partial \ln f(w_1,..w_n)}{\partial w_k} = 0; k = 1...n$

$$\frac{E(R_k)}{\sum_{k=1}^{k=n} w_k E(R_k)} = \frac{\sum_{l=1}^{l=n} w_l \operatorname{cov}(R_k, R_l)}{\sum_{l=1}^{l=n}\sum_{k=1}^{k=n} w_k w_l \operatorname{cov}(R_k, R_l)}$$

Therefore,

$$E(R_k) = \frac{\sum_{l=1}^{l=n} w_l \operatorname{cov}(R_k, R_l) \sum_{k=1}^{k=n} w_k E(R_k)}{\sum_{l=1}^{l=n}\sum_{k=1}^{k=n} w_k w_l \operatorname{cov}(R_k, R_l)}$$

Using (18) and (19) we have

$$E(R_k) = \frac{E(R)\sum_{k=1}^{k=n} w_k \beta_k \sum_{l=1}^{l=n} w_l \operatorname{cov}(R_k, R_l)}{\sum_{l=1}^{l=n}\sum_{k=1}^{k=n} w_k w_l \operatorname{cov}(R_k, R_l)} = \frac{E(R)\sum_{l=1}^{l=n} w_l \operatorname{cov}(R_k, R_l)}{\sum_{l=1}^{l=n}\sum_{k=1}^{k=n} w_k w_l \operatorname{cov}(R_k, R_l)}$$

$$\frac{E(R_k)}{E(R)} = \beta_k = \frac{\sum_{l=1}^{l=n} w_l \operatorname{cov}(R_k, R_l)}{\sum_{l=1}^{l=n}\sum_{k=1}^{k=n} w_k w_l \operatorname{cov}(R_k, R_l)}$$

Using (17), the right-hand side is precisely the definition of β_k. Thus, we conclude that the “no-arbitrage argument” results in optimal firm-wide weights. We state the result as a theorem.

THEOREM 1: Under perfectly competitive insurance markets, a necessary (but not sufficient) consequence is that the company is naturally mean-variance optimized, and thus, the business mix of a firm is optimal.

This theorem provides a starting point of our paper and addresses only the situation where perfectly competitive markets exist. Thus, we now turn our attention to regulated markets.

Section 5: Regulated Markets

The mathematical novelty in this section is due to introduction of equal column matrix C under unit sum weight constraint. This reduces our problem to the well-known problem of maximizing a certain ratio $f(w) = \frac{w^{'}Cw}{w^{'}\Sigma w}$ ***and will be made clear to the reader.***

Section 5.1 Company Premium Mix

Under regulated or less-competitive market situations, we cannot appeal to no-arbitrage arguments. Instead, we now have a constrained optimization problem. Suppose that the expected profits for a given line are capped at τ_k because of rate regulation. They are of the form $-\min(ER_k, \tau_k)$. We address the issue of profit caps more completely in Appendix A. At this point, we need not assume an explicit formula for capping and can continue simply by requiring that post-regulation capped loads exist. To avoid new notation, we will not introduce "capped" notation and assume that $E(R_k)$ is capped and known. We wish to maximize the optimizing function and solve for weights:

$$f(w_1, ..w_n) = \frac{\sum_{k=1}^{k=n} w_k E(R_k)}{\sum_{l=1}^{l=n} \sum_{k=1}^{k=n} w_k w_l \operatorname{cov}(R_k, R_l)}$$

subject to the totality constraint $\sum_{k=1}^{k=n} w_k = 1$. This is the constrained optimization problem. Let $E(R_k) = \pi_k$ so that the problem can be written in matrix notation:

$$w' = [w_1, \ldots. w_n]$$
$$\Sigma = \left\{ Cov(R_k, R_l) \right\}_{k,l}$$
$$\pi' = [\pi_1, \pi_2 \ldots \pi_n]$$
$$C = \begin{pmatrix} \pi_1, \pi_1 \ldots\ldots \pi_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \pi_n, \pi_n \ldots\ldots \pi_n \end{pmatrix}_{n*n}$$
$$i' = [1, 1 \ldots\ldots 1]$$

We want to maximize $f(w) = \dfrac{w'\pi}{w'\Sigma w}$ subject to the constraint $w'i = 1$. Then the feasible solutions are found in the set $W = \left\{ w \in R^n \mid w'i = 1, w > 0 \right\}$. Now,

$$w'Cw = (w_1, w_2, \ldots, w_n) \begin{pmatrix} \pi_1, \pi_1, \ldots, w\pi_1 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \pi_n, \pi_n, \ldots, w\pi_n \end{pmatrix} \begin{pmatrix} w_1 \\ \ldots \\ w_n \end{pmatrix} = w'\pi w_1 + w'\pi w_2 + \cdots + w'\pi w_n$$

$$= w'\pi(w_1 + w_2 + \cdots + w_n) = w'\pi$$

The last line follows from $w'i = 1$. Thus, the problem changes to maximizing $f(w) = \dfrac{w'Cw}{w'\Sigma w}$ subject to the constraint $w'i = 1$. Assuming that Σ is a nonsingular, positive definite matrix, the solution is facilitated by Gotoh (2001):[5]

THEOREM 2: Assuming that Σ is nonsingular positive definite, the maximum of $f(w)$ with respect to $w \in R^n \setminus \{0\}$ is given by the largest eigenvalue λ of the matrix $\Sigma^{-1}C$ and is attained by the eigenvector ξ associated with the largest eigenvalue of $\Sigma^{-1}C$.

[5] We have reduced the problem into a form that can be solved using their theorem.

Note that the final solution is given by $w = \frac{\xi}{\xi' i}$, and multiplication by a constant still results in w as an eigenvector for λ but ensures that the linear constraint is satisfied.

Section 5.2 Decision to Exit a Line from a State

Sometimes companies need a formal study[6] to decide the exit of a line from a given state. Since both negative and positive eigenvectors are solutions in Theorem 2, the positive components are taken as the solution since weights are positive. However, if the sign of the components of $\frac{\xi}{\xi' i}$ changes, then some weights are necessarily negative, implying an exit from the state. In this case the line should be removed and weights redetermined until all component signs are the same.

In some cases, more than one line has an opposite sign. In this case, the choice to remove a line could be based on inspecting its ratio $\frac{ER_k}{\sigma_k^2}$ with the lowest ratios removed first.

Section 5.3 Case Study Continued (Company Premium Mix)

Using Table 1 in our case study, in Table 2 we form profit matrix C_s.

Table 2: Profit Matrix C_s

3.13%	3.13%	3.13%	3.13%	3.13%
2.56	2.56	2.56	2.56	2.56
7.85	7.85	7.85	7.85	7.85

[6] Insurers will consider different factors in making their decisions as to whether to exit a line of insurance in a state. Beyond how rates are regulated in that line or state (as well as an insurer's sense of how rates will be regulated in future years), their considerations would include the amount of sunk costs they would lose by exiting a market, how their exit would affect their relationships with insurance agents, and any economies of scope they achieve by writing multiple lines of insurance.

13.29	13.29	13.29	13.29	13.29
8.46	8.46	8.46	8.46	8.46

Next, we show below steps to calculate covariance matrix Σ_s based on five lines and the corresponding triangles. In Table 3 we show calculation of covariance of lines 1 and 2 using triangles. The column "0" is the permissible loss ratio for the policy year times its written premium. It is an "inserted" column, and its rationale is explained in Appendix B.

Table 3: Line 1 Data and Error Triangle

LINE 1: DATA AND ERROR TRIANGLE											
pol_yr	_name_	0	12	24	36	48	60	72	84	96	108
2000	InsRisk	46626088	48962669	49330414	49160522	49161441	49060192	49008980	48998750	48993364	49017743
2001	InsRisk	54401695	56455375	56539405	56577908	56481454	56379874	56316201	56302856	56369719	0
2002	InsRisk	51737143	50138872	49772205	49722349	49664590	49577476	49549720	49551433	0	0
2003	InsRisk	55136470	53019269	52558149	52329009	52294807	52308224	52290406	0	0	0
2004	InsRisk	58883995	57905328	57304304	57228641	56989183	56967681	0	0	0	0
2005	InsRisk	67767614	66627556	66772152	66919289	66732202	0	0	0	0	0
2006	InsRisk	57902520	57596782	57295053	57111064	0	0	0	0	0	0
2007	InsRisk	64464615	64046763	62294141	0	0	0	0	0	0	0
2008	InsRisk	88926002	88314038	0	0	0	0	0	0	0	0
2009	InsRisk	79038753	0	0	0	0	0	0	0	0	0
	Log Ratio	0 - 12	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 -
1	2000	0.04890	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
2	2001	0.03706	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
3	2002	-0.03138	-0.01	0.00	0.00	0.00	0.00	0.00			
4	2003	-0.03916	-0.01	0.00	0.00	0.00	0.00				
5	2004	-0.01676	-0.01	0.00	0.00	0.00					
6	2005	-0.01697	0.00	0.00	0.00						
7	2006	-0.00529	-0.01	0.00							
8	2007	-0.00650	-0.03								
9	2008	-0.00691									
10	2009										

LINE 2: DATA AND ERROR TRIANGLE

pol_yr	_name_	0	12	24	36	48	60	72	84	96	108
2000	InsRisk	106899696	107350642	107594689	107725044	107735964	107829442	107876347	107908361	108007128	108002516
2001	InsRisk	111601714	111065524	110824320	110877586	111151159	111084670	111025941	111003944	111026514	0
2002	InsRisk	118155250	116693238	116120634	116186452	116056376	115908761	115849317	115799339	0	0
2003	InsRisk	120253173	116453383	115003098	114579948	114279658	114200316	114078314	0	0	0
2004	InsRisk	121428756	116942681	115243327	114561218	114156392	113967227	0	0	0	0
2005	InsRisk	123305650	119168530	117872337	117333081	116907745	0	0	0	0	0
2006	InsRisk	122456951	121024652	120195487	119443446	0	0	0	0	0	0
2007	InsRisk	127734926	127202366	126394911	0	0	0	0	0	0	0
2008	InsRisk	128204161	127825667	0	0	0	0	0	0	0	0
2009	InsRisk	135788483	0	0	0	0	0	0	0	0	0

	Log Ratio	0 - 12	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 -
1	2000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
2	2001	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
3	2002	-0.01	0.00	0.00	0.00	0.00	0.00	0.00			
4	2003	-0.03	-0.01	0.00	0.00	0.00	0.00				
5	2004	-0.04	-0.01	-0.01	0.00	0.00					
6	2005	-0.03	-0.01	0.00	0.00						
7	2006	-0.01	-0.01	-0.01							
8	2007	0.00	-0.01								
9	2008	0.00									
10	2009										

The covariance by age (using errors) is calculated using two error triangles in Table 4.

Table 4: Covariance Matrix by Age

	0 - 12	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96
0 - 12	0.03046%	0.01390%	0.00588%	0.00659%	0.00286%	0.00220%	0.00134%	0.00042%
12 - 24	0.00128%	0.00231%	0.00115%	0.00086%	0.00066%	0.00042%	0.00026%	0.00021%
24 - 36	-0.00075%	-0.00011%	0.00004%	0.00001%	-0.00008%	0.00000%	-0.00006%	-0.00015%
36 - 48	0.00185%	0.00069%	0.00036%	0.00018%	0.00012%	0.00002%	0.00003%	0.00006%
48 - 60	-0.00168%	-0.00066%	-0.00029%	-0.00019%	-0.00005%	-0.00005%	-0.00001%	-0.00001%
60 - 72	-0.00053%	-0.00021%	-0.00007%	-0.00008%	-0.00002%	-0.00002%	-0.00001%	0.00000%
72 - 84	-0.00010%	-0.00004%	0.00000%	-0.00002%	-0.00001%	0.00000%	0.00000%	0.00000%
84 - 96	-0.00059%	-0.00029%	-0.00005%	0.00015%	-0.00010%	-0.00006%	-0.00003%	-0.00005%

sum 0.07%

The covariance matrix by line is then assembled similarly for all lines in Table 5. We show a single entry pertaining to lines 1 and 2, presented as case study in our paper, and other "introduced" lines will have similar calculations.

Table 5: Covariance Matrix by Line

	Line 1	Line 2	Line 3	Line 4	Line 5
Line 1	0.12%	**0.07%** (sum, as above)	0.24%	0.38%	0.26%
Line 2	0.07	0.08	0.20	0.37	0.21
Line 3	0.24	0.20	0.62	1.02	0.67
Line 4	0.38	0.37	1.02	1.91	1.08
Line 5	0.26	0.21	0.67	1.08	0.72

The error covariance matrix can be converted to profit covariance matrix Σ_s by multiplying by the permissible loss ratio (PLR) for each line. We skip this step. We show in Table 6 the eigenvalues and eigenvectors corresponding to $\Sigma_s^{-1} C_s$.

Table 6: Eigenvalues and Eigenvectors for the Matrix $\Sigma_s^{-1} C_s$

Eigenvalues	15.8145411	−5.60E-16	−1.13E-17	−1.13E-17	1.91E-17
Eigenvectors					
	1	2	3	4	5
1	0.48306088	0.279984683	−0.3210583	−0.3210583	−0.329742014
2	0.35912127	−0.647776436	−0.430533952	−0.43053395	−0.423579758
3	0.712650327	0.651957714	0.808127965	0.808127965	0.818208988
4	0.172843455	−0.277305496	−0.195912974	−0.19591297	−0.174357789
5	0.316130901	−0.006860464	0.13937726	0.13937726	0.109470574

Note that four out of five eigenvalues are zero in Table 6. To understand this mathematically, refer to Appendix C.

Next, we find the "normalized" eigenvector corresponding to the largest eigenvalue of the matrix $\Sigma_s^{-1} C_s$ shown as "Mix," short for product mix, in Table 7. This result corresponds with Theorem 2.

Table 7: Normalized Eigenvector and Derived Product Mix

Line	Eigenvalue	Derived Mix
1	0.483060881	23.64%
2	0.359121275	17.57
3	0.712650327	34.87
4	0.172843455	8.46
5	0.316130901	15.47
Total	**2.043806839**	**100.00**

Section 5.4 Statewide Premium Mix

The theorem proved in this section is novel for two reasons. First, it should prove especially useful to state insurance regulators charged with reviewing rate filings and considering the effects of different profits on premium mix statewide. Second, for mathematicians, this analysis leads to a new result in linear algebra formalized in Appendix C.

We now determine the premium mix at a statewide level. Assume that each company is mean-variance optimized and there are *s = 1, 2, …, m* companies in the state. Each company has a current known total premium in dollars p_s and known eigenvector $\frac{\xi_s}{\xi_s^{'} i}$ of the type discussed in the previous section. The statewide premium for all companies combined would be $\sum_{s=1}^{s=m} p_s \frac{\xi_s}{\xi_s^{'} i}$. The statewide premium mix is therefore

$$\eta = \frac{\sum_{s=1}^{s=m} p_s \frac{\xi_s}{\xi_s^{'} i}}{\sum_{s=1}^{s=m} p_s} = \sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s^{'} i} \qquad (20)$$

$$r_s = \frac{p_s}{\sum_{s=1}^{s=m} p_s}$$

The above shows that statewide premium mix is a weighted average of company premium mix (eigenvectors). Now using the fact that $\frac{\xi_s}{\xi_s^{'} i}$ is the eigenvector of

$\Sigma_s^{-1} C_s$,

$$\Sigma_s^{-1} C_s \frac{\xi_s}{\xi_s^{'} i} = \lambda_s \frac{\xi_s}{\xi_s^{'} i}$$

$$\frac{\Sigma_s^{-1}C_s}{\lambda_s}\xi_s = \xi_s$$

$$\left(\sum_{s=1}^{s=m}\frac{r_s\Sigma_s^{-1}C_s}{\sum_{j=1}^{j=m}r_j\lambda_j}\right)\left(\sum_{k=1}^{k=m}r_k\frac{\xi_k}{\xi_k^{'}i}\right)=\sum_{s=1}^{s=m}\sum_{k=1}^{k=m}\frac{r_s\Sigma_s^{-1}C_s}{\sum_{j=1}^{j=m}r_j\lambda_j}r_k\frac{\xi_k}{\xi_k^{'}i}$$

We have the opportunity to choose any matrices on the left-hand side. The equality is due to the additivity property of matrices. Now, $\Sigma_s^{-1}C_s$ is a matrix with identical columns. Note that each column is also the eigenvector of the respective $\left\{\Sigma_s^{-1}C_s\right\}_{s=1,\ldots,m}$. Therefore,

$$\Sigma_s^{-1}C_s r_k\frac{\xi_k}{\xi_k^{'}i}=\begin{Bmatrix}\lambda_s r_s\dfrac{\xi_s}{\xi_s^{'}i};k=s\\ \lambda_k r_k\dfrac{\xi_s}{\xi_s^{'}i};k\neq s\end{Bmatrix}=\lambda_k r_k\frac{\xi_s}{\xi_s^{'}i} \tag{21}$$

The eigenvalue λ_k is the sum of components of $\frac{\xi_k}{\xi_k^{'}i}$. Hence from equation (21),

$$\sum_{s=1}^{s=m}\sum_{k=1}^{k=m}\frac{r_s\Sigma_s^{-1}C_s}{\sum_{j=1}^{j=m}r_j\lambda_j}r_k\frac{\xi_k}{\xi_k^{'}i}=\sum_{s=1}^{s=m}\sum_{k=1}^{k=m}\frac{\lambda_k r_k r_s\xi_s}{\xi_s^{'}i\sum_{j=1}^{j=m}r_j\lambda_j}=\sum_{s=1}^{s=m}r_s\frac{\xi_s}{\xi_s^{'}i}\sum_{k=1}^{k=n}\frac{\lambda_k r_k}{\sum_{j=1}^{j=m}r_j\lambda_j}=\sum_{s=1}^{s=m}r_s\frac{\xi_s}{\xi_s^{'}i}$$

Thus $\sum_{s=1}^{s=m}r_s\frac{\xi_s}{\xi_s^{'}i}$ is recognized as the eigenvector of $M=\sum_{s=1}^{s=m}\frac{r_s\Sigma_s^{-1}C_s}{\sum_{j=1}^{j=m}r_j\lambda_j}$ associated with an eigenvalue of 1. The result is stated in the theorem. To prove that remaining eigenvalues of M are zero, note that M is an equal column matrix with the column as the eigenvector. With a single eigenvector and $2\le j\le m$, $M\xi=\lambda_j\xi$ and $M\xi=\xi$, so that $\lambda_j\xi=\xi$ and $\left(\lambda_j-1\right)\xi=0$. Thus, the largest eigenvalue

$\left\{\lambda^{\max} : \lambda^{\max} = 1, \xi \neq 0\right\}$. Since we seek nonzero eigenvectors, remaining eigenvalues $\left\{\lambda_j : \lambda_j = 0; 2 \leq j \leq n\right\}$.

THEOREM 3: Assuming that each company is mean-variance optimized, the statewide premium mix $\eta = \left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s' i}\right)$ can be determined from the eigenvector of the matrix $\sum_{s=1}^{s=m} \frac{r_s \Sigma_s^{-1} C_s}{\sum_{j=1}^{j=m} r_j \lambda_j}$ associated with eigenvalue of 1. Alternatively, the state premium can be obtained directly from $\eta = \left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s' i}\right)$ since company premium mixes are known.

Section 5.5 Case Study Continued (Pre-Regulation)

We present pre- and post-regulation results separately. Only one scenario will be presented where expected profits are capped for lines 2 and 3 under post-regulation. Observed written premium weight r_s by company for a state Y is given in Table 8.

Table 8: Observed Written Premium Weight

	Written Premium	Weight
Company 1	\$10,400,000	16.4%
Company 2	20,000,000	31.4
Company 3	5,200,000	8.2
Company 4	28,000,000	44.0
Total	**63,600,000**	**100**

Using the same methods as above with company X, next find eigenvectors and company premium mix. The details will be skipped here. In Table 9 we present the expected profits by line and company.

Table 9: Expected Profits by Line and by Company

	Company 1	Company 2	Company 3	Company 4
Line 1	4.87%	4.96%	4.95%	5.50%
Line 2	5.74	5.11	5.86	5.69
Line 3	4.94	5.20	5.18	4.59
Line 4	5.96	5.05	4.83	5.50
Line 5	5.95	4.83	4.09	4.30

Using the weights r_s above we can combine the calculated company mix to obtain statewide premium mix $\left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s^{'} i}\right)$ in Table 10. This result connects to Theorem 3.

Table 10: Statewide Premium Mix

	Company 1	Company 2	Company 3	Company 4	Computed Statewide
Line 1	11.44%	23.39%	19.64%	29.62%	23.87%
Line 2	13.04	14.88	14.53	31.38	21.82
Line 3	30.89	22.42	19.64	7.41	16.97
Line 4	21.57	12.08	34.76	21.98	19.84
Line 5	23.07	27.23	11.43	9.60	17.50
Total	**100**	**100**	**100**	**100**	**100**

Section 5.6 Predicting Statewide Regulatory Impact on Premium Mix

Suppose the state regulator wants to predict the impact of profit caps on statewide premium mix. For example, lines with rate caps are expected to have a less dedicated capacity, post-regulation. The regulator knows the statewide premium mix of each company $s = 1, 2 \ldots, m$ before capping. They want to know the revised premium mix of the line after capping. We solve this problem below.

DEVIANCE MATRIX: Pre-regulation, the regulator can use Theorem 3 and obtain the eigenvector $\eta = \left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s^{'} i} \right)$. However, Theorem 3 assumes that each company in the state is mean-variance optimized. To the extent that this is true, our theoretical premium mix will match the current market premium mix. However, such a mean-variance optimization assumption is unlikely to hold true in practice. If Theorem 3 was applied, we would get a premium mix that would differ from actual. Let us call this phenomenon "market deviance." We use a "deviance matrix" with real elements to measure this phenomenon:

$$D = \begin{pmatrix} d_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & d_n \end{pmatrix}$$

After application of Theorem 3 we will have η. Suppose we know the current observed statewide premium mix $\left\{ \mu_k : \sum_{k=1}^{n} \mu_k = 1 \right\}$. Then $\{d_k\}_{k=1 \ldots n}$ is defined as (writing the vectors η and μ in component form)

$$d_k := \frac{\mu_k}{\eta_k} \tag{22}$$

The usefulness of defining $\{d_k\}_{k=1 \ldots n}$ in this way will be explained later in Theorem 4.

PREDICTED MATRIX (POST-REGULATION): Revise the appropriate rows of the profit matrix C_s for each company to get a capped matrix $\Sigma_s^{-1} C_s$. To avoid unnecessary notation, we will not introduce any new symbols and assume that in this section the matrix is capped through C_s. Next, use Theorem 3 to recalculate the eigenvector $\left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s^{'} i} \right)$. The predicted post regulation premium mix is given by

$$\eta = \frac{D\left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s^{'} i}\right)}{D\left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s^{'} i}\right)^{'} i} \qquad (23)$$

The above adjusts for market deviance as long as *D* remains unchanged between pre- and post-regulation periods. The deviance matrix corrects for this violation once at the pre-regulation time. If the violation itself changes, then *D* will also change. Note also that the covariance matrices Σ_s remain unchanged post-regulation as the regulator is interested in capping C_s with components of the form $r_{ks} = E\left(R_{ks}\right)$ rather than the random variable R_{ks} with $k = 1, \ldots, n$.

The use of deviance matrix *D* in equation (22) is an ad hoc adjustment to reflect market deviance, because factors driving the deviance come from outside the model. Hence, the matrix $D\left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s^{'} i}\right)$ will no longer sum to 1 and "normalization" is necessary, leading to $\eta = \frac{D\left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s^{'} i}\right)}{D\left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s^{'} i}\right)^{'} i}$.

Section 5.7 Case Study Continued (Post-Regulation)

Now we cap expected profits at 5% for lines 2 and 3 (see Table 11).

Table 11: Profit Caps

	Company 1	Company 2	Company 3	Company 4
Line 1	4.87%	4.96%	4.95%	5.50%
Line 2	5.00	5.00	5.00	5.00
Line 3	4.94	5.00	5.00	4.59

Line 4	5.96	5.05	4.83	5.50
Line 5	5.95	4.83	4.09	4.30

Using the weights r_s above we can combine the calculated (capped) company mix to obtain in Table 12 the capped statewide premium mix $\left(\sum_{s=1}^{s=m} r_s \frac{\xi_s}{\xi_s' i} \right)$.

Table 12: Capped Statewide Premium Mix

	Company 1	Company 2	Company 3	Company 4	Computed Statewide Mix
Line 1	11.52%	23.34%	21.38%	29.94%	24.15%
Line 2	11.24	14.45	11.27	29.18	20.15
Line 3	31.74	22.04	19.86	8.04	17.29
Line 4	21.94	12.64	35.76	22.41	20.35
Line 5	23.56	27.54	11.73	10.43	18.06
Total	**100**	**100**	**100**	**100**	**100**

Now we calculate the final market deviance-adjusted “predicted” statewide premium mix (see Table 13). This mix is what should exist if a 5% cap is enacted for lines 2 and 3.

Table 13: Predicted Statewide Premium Mix

	Computed Statewide Mix	Market Deviance	Predicted Mix
Line 1	24.15%	1.05	25.42%
Line 2	20.15	0.89	17.91
Line 3	17.29	1.08	18.66
Line 4	20.35	1.2	24.34
Line 5	18.06	0.76	13.67
Total	**100**		**100**

Section 5.8 Market Competitiveness

The primary purpose of this section is to provide a mathematically formal way to measure market competitiveness because this is lacking in the literature. To do so, we need our fundamental result in Theorem 1, which will be used to prove Theorem 4.

THEOREM 4: Suppose that there is at least one company *s* in the industry *s* > 0 that is not mean-variance optimizing its portfolio for line(s) $\{k_i\}_{0<i\leq n}$. Then a necessary and sufficient condition of perfect market competitiveness for lines $\{k_i\}_{0<i\leq n}$ is the existence of $\left\{d_{k_i} = \frac{\mu_{k_i}}{\eta_{k_i}} = 1\right\}_{0<i\leq n}$.

NECESSITY: We are given that markets are perfectly competitive. Hence, from Theorem 1, the subset of lines $\{k_i\}_{0<i\leq n}$ for each individual company are naturally mean-variance optimized. Thus, $\{\eta_{k_i} = \mu_{k_i}\}_{0<i\leq n}$ since the necessary conditions to calculate η_{k_i} are identical to actual market conditions.

SUFFICIENCY: We are given that $\{\mu_{k_i} = \eta_{k_i}\}_{0<i\leq n}$ as well as the fact that there is at least one company *s* in the industry *s > 0* that is not mean-variance optimizing its portfolio for line(s) $\{k_i\}_{0<i\leq n}$. We need to show that markets are perfectly competitive for lines $\{k_i\}_{0<i\leq n}$.

We claim that the theoretical industry premium mix is given by component set $\left\{\eta_{k_i} = \sum_{s=1}^{s=m} r_{sk} \left.\frac{\xi_{sk}}{\xi'_{sk} i}\right|_{k=k_i}\right\}$ iff each company *s* is mean-variance optimized. The sufficiency of this statement is obvious from our discussion that each mean-variance optimized company will lead to component set $\left\{\frac{\xi_{sk}}{\xi'_{sk} i}\right\}_{k=k_i}$ as the optimal solution for a company. For necessity note that from equation (20) the component set $\left\{ \eta_{k_i} = \sum_{s=1}^{s=m} r_{sk} \left. \frac{\xi_{sk}}{\xi_{sk}^{'} i} \right|_{k=k_i} \right\}$ implies that each company has a premium mix given by the component set $\left\{ \frac{\xi_{sk}}{\xi_{sk}^{'} i} \right\}_{k=k_i}$. But this set maximizes the objective function for the combination of company and line.

Hence, if the component set $\left\{ \eta_{k_i} = \mu_{k_i} = \sum_{s=1}^{s=m} r_{sk} \left. \frac{\xi_{sk}}{\xi_{sk}^{'} i} \right|_{k=k_i} \right\}$ was *observed*, then each company is also *actually* mean-variance optimized for lines $\{k_i\}_{0<i\le n}$. But such mean-variance optimization cannot be due to a company's own efforts since at least one company is not mean-variance optimized. Thus, such optimization is due to market conditions.

To recap briefly, we have "some" market conditions that lead to mean-variance optimized portfolios for all companies in the industry. From Theorem 1, we recognize this to be "perfectly competitive" market conditions. We now turn our attention to a case study to demonstrate how the model might be deployed.

Section 5.9 Case Study Continued (Pre-Regulation Market Competitiveness)

Next, we calculate the deviance matrix using actual observed statewide "line premiums" and the computed statewide mix above (see Table 14). To calculate the observed statewide mix we would require an expanded Table 8 with written premiums by both line and company. The expanded figure is not shown here.

Table 14: Market Deviance

	Computed Statewide Mix	Observed Statewide Mix	Market Deviance
Line 1	23.87%	25.17%	1.054

Line 2	21.82	19.44	0.891
Line 3	16.97	18.35	1.082
Line 4	19.84	23.77	1.198
Line 5	17.50	13.26	0.758
Total	**100**	**100**	

Employing Theorem 4, we see that lines 1 and 3 have a market deviance close to 1, making them reasonably competitive.[7]

Section 5.10 Case Study Discussion

The observed statewide premium mix (pre-regulation) can be compared to the predicted mix (post-regulation) to try to understand the effects of rate caps. As a result of rate caps, the predicted rate mix for line 2 suggests a more than 1.5% decline in writings in line 2, going from an observed statewide mix of 19.44% to a predicted share of 17.91%. The difference for line 3 was much smaller and in a different direction, with an observed share of 18.35% adjusting slightly upward with the predicted mix of 18.66% to reflect a small shift from line to 2 to line 3 writings. Likewise, lines 1, 4 and 5 each picked up a small share of the offset from line 2 shifts. Further, upon inspecting the market deviances, we note that lines 1 and 3 are reasonably competitive, with values close to 1.

[7] Some subjectivity is involved because market deviances are not exactly equal to one.

Section 6: Model Limitations and Discussion

Since the model is applicable to regulators, we offer observations to place the significance of rate regulation in the broader context of the full set of regulatory policies and practices. Specifically, we provide discussion on "file and use" versus "prior approval" rate regulation laws. First, the empirical literature indicates that insurance markets are structurally competitive at the national and state levels. This is the case for any market of any significance, such as auto insurance, home insurance, workers compensation insurance and the like. It is generally the conclusion of academic researchers that strict rate regulation does not improve market performance but can create significant market distortions. It is possible that insurers in a given state or line may be more "aggressive" in competing with each other; that is, competition in a given state or line may exceed the standards for workable competition, at least for a limited period of time. Hence, the degree of competitiveness could still vary across lines of business in a state with the qualification stated above.

Second, in the United States, the type of rate filing system (e.g., prior approval, file and use) in a given state and for a given line of business, by itself, is not necessarily a good indicator of how insurers' rates are regulated. In some prior approval states, for a given line of insurance, regulators may attempt to constrain insurers' rates, whereas in other prior approval states, for a given line of insurance, regulators do not attempt to constrain insurers' rates. By the same token, in some file and use states, regulators do not attempt to constrain insurers' rates, whereas in others they do attempt to constrain insurers' rates. In our model this is not a problem because rate regulation is reflected only in terms of the extent to which regulators might attempt to constrain insurers' rates or profits, which could vary by state and line of insurance.

Third, other aspects of state regulatory policies and practices may also influence insurers' operations and their decisions regarding what they consider to be their optimal mix of business across different lines of insurance. These aspects include the regulation of solvency, underwriting and pricing, policy design and claims settlement practices, among others.

The paper provides an extension of mean-variance optimization to the case of an insurer or insurers seeking to optimize the mix of premiums across various lines of business. However, in practice, an insurer could choose to exit a market or reduce its premiums in a given line of insurance for reasons other than achieving its optimal set of weights.

Regarding applicability of the paper from a regulator standpoint, we make some comments. Generally, what regulators consider to be most important are the "affordability" and "availability" of coverage for a given line of insurance. When an insurer files for a large rate increase, especially in a market where rates are already high, regulators can become concerned that if they approve (or do not disapprove) such a rate increase, it would have a significant and negative financial impact on consumers. At the same time, regulators are generally aware that if they place severe constraints on insurers' rates, this could negatively affect the supply of insurance. Hence, regulators tend to balance considerations with respect to both the affordability and availability of insurance. That said, as suggested in the Introduction, regulators in a given situation may not be able to predict how their decisions on rate filings will affect insurers' decisions regarding how much coverage they will offer, although, in some cases, insurers may inform regulators on the consequences of their decisions. The work presented in this paper could serve as a foundation for further work on models that could help regulators to predict the effects of their decisions.

Appendix A posits two ways in which regulators might impose constraints on insurers: (1) placing a cap on their expected profits and (2) setting a cap on the maximum allowable rate increase. The first way seems more likely to happen in practice. The second way (i.e., a uniform cap on rate increases that would apply to all insurers) may be less consistent with reality. There may be some situations where regulators do impose a uniform cap, but the more common scenario is for regulators to impose caps on insurers' rate increases that vary by insurers.

Section 7: Conclusion

The manager's choice of what products to offer in what quantities in a given state, district or territory is an important and highly pragmatic decision. Mean-variance optimization provides one way of considering how allocations such as product choices ought to be made under risky circumstances. Most would agree that underwriting insurance is an inherently risky business.

Finance theory suggests that returns from firm activities should be sufficient to compensate capital providers for their risk. Insurers create returns through two primary activities: underwriting and investments. These returns can be incorporated into insurance pricing models used by actuaries and regulators alike, commonly through a profit term and an investment income offset term.

We use hypothetical data in showing some practical results of four theorems developed in this paper. We begin with the unregulated markets case. Our first theorem states that under perfectly competitive markets, a necessary but not sufficient consequence is that the company is naturally mean-variance optimized, and, thus, the business mix of the firm is optimal. Companies using the optimizing function will maximize ROE. Next, we introduce rate-regulated markets. Our second theorem enlists a result by Gotoh (2001) to find optimal product mix weights by firms using eigenvectors in a constrained optimization problem. The results provide guidance in determining whether a company should exit from a line of business in a particular state. Next, we turn our attention to the case of an optimal premium mix in statewide markets. Our third theorem, which shows how the statewide premium mix can be found from company-level information even if firms are not mean-variance optimizers, should prove especially useful to state insurance regulators charged with reviewing company profit filings. Our fourth and last theorem gives regulators a practical way of determining whether insurance markets remain competitive in their state, even if at least one firm is not a mean-variance optimizer.

Appendix A: Determining Capped "Expected Profits"

Our approach requires the regulator to know capped profits. However, statewide rate regulation for any line can take different forms, and not all of them will directly provide capped profits. We discuss the two most common types of rate regulations:

1. Caps on expected profit in rate filings and

2. Maximum allowable rate increase.

In the first case, the regulator makes the assumption that absent the regulatory capping, profits in the prospective period would have remained unchanged because all companies would have obtained the required rate changes. Thus, in the prospective period, the profits are impacted only by regulatory capping. This case is handled using the formula given in the paper, and we explicitly consider the impact on companies when the current (i.e., prospective) expected profits are capped at a certain level.

The second case requires a discussion. As part of finding the appropriate "allowable" rate increase, the regulator conducts statewide rate-level indication and determines a statewide permissible loss ratio (PLR). With this information, a hypothetical "maximum allowable rate increase" is set for further review. The impact of this "rate increase cap" varies by company, with different companies affected differently. We show here how the rate increase cap converts to expected profit cap for each individual company. To illustrate, suppose that a hypothetical rate increase cap for line 1 is set at +5%, based on a statewide rate-level indication of +10% and a PLR = 70%. Consider two companies with these profiles:

COMPANY "A," PRE-REGULATION PROFILE: PLR = 65%, expense ratio (ER) 30%, Underwriting Profit Provision (UPP) = 5%

Suppose that company "A" files for an 8% rate increase and receives only 5% because of the cap. The resulting shortfall is 3%. The new UPP equals 3%:

PLR = 65%/0.97 = 67% (losses are the same but written premium is deficient by 3%)

ER = 30% (expense ratio is percentage of written premium and thus remains unchanged)

UPP = 1−67%−30% = 3% (satisfies the totality constraint)

COMPANY “A” PREDICTED PROFILE: PLR = 67%, expense ratio = 30%, and UPP = 3%.

Note that if the “filed” rate increase is less than 5%, then no impact will be seen (post regulation) in the company’s UPP. In all cases Profit = UPP + Investment Income Offset.

Appendix B: Estimation of Profit Covariance Matrix

For notational convenience we shall drop the line subscript k for now and will alert the reader when this is added later.

Data Triangle

In Figure 1, a data triangle is available for M (fixed) policy years i = 1, 2, …, M. Row entries in the table can be obtained by varying i. However, for any fixed M the quantity U_{M-i+2} provides only the last (diagonal) column entry for row i. The value on the adjacent diagonal would be U_{M-i+1}. We do not introduce a variable for the realized part of the rectangle because our interest lies in the unknown (missing) cells of the new policy year, $i = M + 1$. We will be content with just naming the values by their specific symbols in the realized part. The unrealized part will be referenced by adding a subscript, q = 1, 2, …, $i-1$ such that $U_{M-i+1+q}$ will now reference future cells for row i. Define

$U_{i,M-i+2}$ = Ultimate loss for policy year i, estimated at development age $M-i+2$

$\varepsilon_{i,M-i+1}$ = Error for policy year i and development interval ($M-i+1$, $M-i+2$)

$$\varepsilon_{i,M-i+1} = \ln\left(\frac{U_{M-i+2}}{U_{M-i+1}}\right) \qquad (3)$$

Figure 1 consists of triangle of ultimate losses (Schedule P, Part 2–Annual Statement).[8] Before continuing, we note that the model as presented applies to both catastrophic and noncatastrophic lines as long as "triangulation of data is feasible.

Figure 1: Data Triangle: [$M \times M + 1$]

	1	2	3	…	$M-i+1$	$M-i+2$	$M-1$	M	$M+1$
1	$U_{1,1}$	$U_{1,2}$	$U_{1,3}$	…	$U_{1,M-i+1}$	$U_{1,M-i+2}$	$U_{1,M-1}$	$U_{1,M}$	$U_{1,M+1}$
2	$U_{2,1}$	$U_{2,2}$	$U_{2,3}$	…	$U_{2,M-i+1}$	$U_{2,M-i+2}$	$U_{2,M-1}$	$U_{2,M}$	
3	$U_{3,1}$	$U_{3,2}$	$U_{3,3}$	…	$U_{3,M-i+1}$	$U_{3,M-i+2}$	$U_{3,M-1}$		

[8] These are on an accident year basis instead of a policy year basis. To that extent, they are an approximation.

…	…	…	…	…	…	…			
i	$U_{i,1}$	$U_{i,2}$	$U_{i,3}$		$U_{i,M-i+1}$	$U_{i,M-i+2}$			
$i+1$	$U_{i+1,1}$	$U_{i+1,2}$	$U_{i+1,3}$		$U_{i+1,M-i+1}$				
…									
M	$U_{M,1}$	$U_{M,2}$							
$M+1$	$U_{M+1,1}=$								

Figure 2: Error Triangle: [$M \times M$]

	1	2	…	$M-i$	$M-i+1$	…	$M-i$	M
1	$\varepsilon_{1,1}$	$\varepsilon_{1,2}$	…	$\varepsilon_{1,M-i}$	$\varepsilon_{1,M-i+1}$	…	$\varepsilon_{1,M-1}$	$\varepsilon_{1,M}$
2	$\varepsilon_{2,1}$	$\varepsilon_{2,2}$	…	$\varepsilon_{2,M-i}$	$\varepsilon_{2,M-i+1}$	…	$\varepsilon_{2,M-1}$	
…	…	…	…	…	…			
i	$\varepsilon_{i,1}$	$\varepsilon_{i,2}$	…	$\varepsilon_{i,M-i}$	$\varepsilon_{i,M-i+1}$			
$i+1$	$\varepsilon_{i+1,1}$	$\varepsilon_{i+1,2}$	…	$\varepsilon_{i+1,M-i}$				
…	…	…	…					
M	$\varepsilon_{M,1}$							
$M+1$								

Model Postulates

In Figure 2, half the rectangle is observed, and the remaining is unobserved. Since our randomness will be due to unobserved values, we will ignore the first row because it is complete.[9] We make no distributional assumptions about the unobserved errors $\{\varepsilon_{i,M-i+1}\}_{2\le i\le M}\ i=2\ldots M$. More importantly, for estimation purposes, we make the following two assumptions (Figure 2):

(i) For any given column, the errors have the same marginal variance. This permits us to use the sample variance to estimate the population variance.

(ii) Any two columns of errors have the same covariance. This permits us to use the sample covariance as an estimate of the population covariance.

The last two postulates are necessary for calibration of the parameters.

Note the following caveats. First, as new data come each year, the samples estimates are updated. Second, the old "completed rows" (Figure 1) greatly enhance accuracy in estimation and should be retained. All estimation completed subsequently will use these postulates. For the sake of simplicity, we will not refer to these postulates henceforth.

Estimation of Profit Covariance Matrix

We can normalize the data triangle by dividing each row by its respective written premium. This provides a triangle of "loss ratios." Note that this has no impact on Figure 2 because we are dealing with log ratios (across rows). *By construction, the first column in Figure 1 is always the permissible loss ratio of the respective policy years (this is why it's labeled as "0" column in our illustrations).* For year $M+1$, the company charges an average permissible loss ratio in its rate filings: $U_{M+1,1}=u_{M+1}$. We assume that u_{M+1} is available. Suppose that we have the data for M years and wish to determine the permissible loss ratio distribution of year $M=1$ (Figures 1 and 2). Adding subscript $q\in 1,2,\ldots,i-1$ for the future development period in policy year $i\in 1,2\ldots,M+1$ (Figure 2) we get

[9] Adding extra rows will later help in estimation (especially for late development periods with few entries in columns), as we will estimate quantities using columns in Figure 2.

$$\varepsilon_{iq} = \ln\left(\frac{U_{i,M-i+2+q}}{U_{i,M-i+1+q}}\right) \tag{4}$$

Now set $i = M + 1$ and, for convenience, drop the subscript $i = M + 1$ because it is understood that we are working with future policy years. *Instead, we reintroduce a subscript for line of business k.* We now turn to the estimation of $\Sigma = \{C\text{ov}(R_k, R_l)\}_{kl}$. From equation (1), we have

$$\varepsilon_i = \sum_{q=1}^{i-1} \varepsilon_{iq} = \sum_{q=1}^{i-1} \ln\left(\frac{U_{i,M-i+2+q}}{U_{i,M-i+1+q}}\right) = \ln\left(\frac{U_{i,M+1}}{U_{i,M-i+2}}\right) \tag{5}$$

$$C\text{ov}(R_k, R_l) = C\text{ov}(1 - e_k - U_k - f_k, 1 - e_l - U_l - f_l)$$

where e_k is a fixed expense ratio, U_k is a random permissible loss ratio and f_k is a fixed investment income offset for line k. Therefore,

$$C\text{ov}(R_k, R_l) = C\text{ov}(U_k, U_l)$$

$$VarR_k = VarU_k$$

We wish to compute Cov(U_k,U_l) for two given data sets. Each data set is a triangle of not necessarily the same size. Using equation (5) with $i = M + 1$,

$$\varepsilon_k = \ln\left(\frac{U_k}{u_k}\right)$$

$$u_k \exp \varepsilon_k = U_k$$
$$C\text{ov}(U_k, U_l) = C\text{ov}\left(u_k \exp \varepsilon_k, u_l \exp \varepsilon_l\right)$$
$$C\text{ov}(U_k, U_l) \approx C\text{ov}\left(u_k + u_k\varepsilon_k, u_l + u_l\varepsilon_l\right) = u_k u_l C\text{ov}\left(\varepsilon_k, \varepsilon_l\right)$$

We used a Taylor first order approximation of the exponential function, and it is feasible as long as the total error ε_k by policy year is small. To add future errors, we now add the development interval subscript:

$$\varepsilon_k = \sum_{q=1}^{M} \varepsilon_{kq}$$

$$\varepsilon_l = \sum_{p=1}^{N} \varepsilon_{lp}$$

$$Cov(U_k, U_l) \approx u_k u_l Cov\left(\sum_{q=1}^{M} \varepsilon_{kq}, \sum_{p=1}^{N} \varepsilon_{lp} \right) = u_k u_l \sum_{p=1}^{N} \sum_{q=1}^{M} Cov\left(\varepsilon_{kq}, \varepsilon_{lp}\right)$$

Hence,

$$Cov(R_k, R_l) = u_k u_l \sum_{p=1}^{N} \sum_{q=1}^{M} Cov\left(\varepsilon_{kq}, \varepsilon_{lp}\right) \tag{6}$$

Thus, as long as the expected permissible loss ratio $\{u_k\}_{k=1,\ldots,n}$ is available by line, $Cov(\varepsilon_{kq}, \varepsilon_{lp})$ can be estimated from the columns of Figure 2. Note that

$$VarR_k = VarU_k = u_k^2 \sum_{p=1}^{N} \sum_{q=1}^{M} Cov\left(\varepsilon_{kq}, \varepsilon_{kp}\right) \tag{7}$$

Appendix C: Linear Algebra Results on Average Matrices

Definition (Average Matrices): The set of average matrices Λ has square members with at least one nonzero eigenvalue and eigenvector such that there exist matrices and eigenvalues $\{M_i, \lambda_i\}$ $M \in \Lambda$,

$$M = \sum_{i=1}^{i=n} \frac{r_i M_i}{\sum_{j=1}^{j=n} r_j \lambda_j}$$

$$\sum_{i=1}^{i=n} r_i = 1$$

Theorem 5 (equal columns): Let $M \in \Lambda$ be a linear combination of $n \in \mathbb{N}$ matrices $\{M_i\}_{i=1..n}$ having identical columns and those columns are taken as the respective eigenvectors $\{\xi_i, \lambda_i : \lambda_i \neq 0\}_{i=1,\dots,n}$. Then for any valid choice of r_i, the set of average matrices

$$\left\{ M : M = \sum_{i=1}^{i=n} \frac{r_i M_i}{\sum_{j=1}^{j=n} r_j \lambda_j}, \sum_{i=1}^{n} r_i = 1 \right\}$$

has a single nonzero eigenvector ξ with an associated eigenvalue of 1 (the remaining are identically 0) and satisfies $\xi = \sum_{i=1}^{n} r_i \xi_i$.

Corollary 1: Given that $\xi = \sum_{k=1}^{k=n} r_k \xi_k$, then the total matrix $\sum_{i=1}^{i=n} M_i$ has eigenvector and eigenvalue $\left\{ \xi, \sum_{j=1}^{j=n} \lambda_j \right\}$.

Corollary 2: A set of average matrices with a common eigenvector has an average matrix with eigenvalue 1 and the same associated eigenvector.

Proof (theorem):

Use the fact that ξ_i is the eigenvector of M_i,

$$\frac{M_i \xi_i}{\lambda_i} = \xi_i$$

Now,

$$\left(\sum_{i=1}^{i=n} \frac{r_i M_i}{\sum_{j=1}^{j=n} r_j \lambda_j} \right) \left(\sum_{k=1}^{k=n} r_k \xi_k \right) = \sum_{i=1}^{i=n} \sum_{k=1}^{k=n} \frac{r_i M_i}{\sum_{j=1}^{j=n} r_j \lambda_j} r_k \xi_k$$

We have the opportunity to choose any matrices on the left-hand side. The equality is due to the additivity property of matrices. Now pick M_i as a matrix with identical columns. Note that each column is also the eigenvector of respective $\{M_i\}_{i=1..n}$. Therefore,

$$M_i r_k \xi_k = \begin{Bmatrix} \lambda_i r_i \xi_i ; k = i \\ \lambda_k r_k \xi_i ; k \neq i \end{Bmatrix} = \lambda_k r_k \xi_i \tag{1}$$

The eigenvalue λ_k is the sum of components of ξ_k. Hence from this equation,

$$\sum_{i=1}^{i=n} \sum_{k=1}^{k=n} \frac{r_i M_i}{\sum_{j=1}^{j=n} r_j \lambda_j} r_k \xi_k = \sum_{i=1}^{i=n} \sum_{k=1}^{k=n} \frac{\lambda_k r_k r_i \xi_i}{\sum_{j=1}^{j=n} r_j \lambda_j} = \sum_{i=1}^{i=n} r_i \xi_i \sum_{k=1}^{k=n} \left(\frac{\lambda_k r_k}{\sum_{j=1}^{j=n} r_j \lambda_j} \right) = \sum_{i=1}^{i=n} r_i \xi_i$$

Thus $\left(\sum_{i=1}^{i=n} r_i \xi_i \right)$ is recognized as the eigenvector of $M = \sum_{i=1}^{i=n} \frac{r_i M_i}{\sum_{j=1}^{j=n} r_j \lambda_j}$ associated with an eigenvalue of 1. The result is stated in the theorem. To prove that remaining eigenvalues of *M* are zero, note that *M* is an equal column matrix with the column as the eigenvector. With a single eigenvector and $2 \le j \le n$, $M\xi = \lambda_j \xi$, and $M\xi = \xi$. So $\lambda_j \xi = \xi$ and $(\lambda_j - 1)\xi = 0$, and the largest eigenvalue $\{\lambda^{\max} : \lambda^{\max} = 1, \xi \neq 0\}$. Since we seek nonzero eigenvectors, we have the remaining eigenvalues $\{\lambda_j : \lambda_j = 0; 2 \le j \le n\}$.

Proof (corollary 1):

Since $M \in \Lambda$ and the fact that $\xi = \sum_{k=1}^{k=n} r_k \xi_k$,

$$M\xi = \sum_{k=1}^{k=n} r_k M \xi_k = \sum_{k=1}^{k=n}\sum_{i=1}^{i=n} \frac{r_k M_i}{\sum_{j=1}^{j=n} \lambda_j} \xi_k$$

$$\xi \sum_{j=1}^{j=n} \lambda_j = \sum_{k=1}^{k=n}\sum_{i=1}^{i=n} r_k M_i \xi_k = \sum_{i=1}^{i=n} M_i \left(\sum_{k=1}^{k=n} r_k \xi_k \right)$$

Again, using $\xi = \sum_{k=1}^{k=n} r_k \xi_k$,

$$\left(\sum_{i=1}^{i=n} M_i \right) \xi = \left(\sum_{j=1}^{j=n} \lambda_j \right) \xi$$

Hence $\left(\sum_{i=1}^{i=n} M_i \right)$ has eigenvector and eigenvalue $\left\{ \xi, \sum_{j=1}^{j=n} \lambda_j \right\}$.

Proof (corollary 2): We show that a set of average matrices with a common eigenvector has an average matrix with eigenvalue 1 and the same associated eigenvector. The proof is identical to the main theorem with the observation that

$$M_i r_k \xi_k = M_i r_k \xi = \left\{ \begin{matrix} \lambda_i r_i \xi; k = i \\ \lambda_i r_k \xi; k \neq i \end{matrix} \right\}$$

As in Theorem 1,

$$\sum_{i=1}^{i=n}\sum_{k=1}^{k=n} \frac{r_i M_i}{\sum_{j=1}^{j=n} r_j \lambda_j} r_k \xi = \sum_{i=1}^{i=n} \frac{r_i \lambda_i r_i \xi}{\sum_{j=1}^{j=n} r_j \lambda_j} + \sum_{i \neq k}\sum_{k=1}^{k=n} \frac{r_i \lambda_i r_k \xi}{\sum_{j=1}^{j=n} r_j \lambda_j} = \sum_{i=1}^{i=n}\sum_{k=1}^{k=n} \frac{r_i \lambda_i r_k \xi}{\sum_{j=1}^{j=n} r_j \lambda_j} = \sum_{i=1}^{i=n} \left(\frac{r_i \lambda_i}{\sum_{j=1}^{j=n} r_j \lambda_j} \right) \sum_{k=1}^{k=n} r_k \xi = \sum_{k=1}^{k=n} r_k \xi = \xi$$

Thus $\left(\sum_{i=1}^{i=n} r_i \xi_i\right) = \xi$ is recognized as the eigenvector of $\left(\sum_{i=1}^{i=n} \frac{r_i M_i}{\sum_{j=1}^{j=n} r_j \lambda_j}\right)$ associated with an eigenvalue of 1.

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Society of Actuaries
475 N. Martingale Road, Suite 600
Schaumburg, Illinois 60173
www.SOA.org