TRANSACTIONS OF SOCIETY OF ACTUARIES
1968 VOL. 20 PT. 1 NO. 56 AB

BOOK REVIEWS AND NOTICES*

Henry Aaron, "The Social Insurance Paradox," *Canadian Journal of Economics and Political Science*, XXXII, No. 3 (August, 1966), 371–74.

In this paper the author attempted to demonstrate that, under a social insurance program, larger benefits could be payable in the aggregate when reserves are not accumulated than when they are. In my opinion the underlying reasoning is fallacious, because the simplified model used in the demonstration and the underlying assumptions are not on a reasonable basis, as is necessary in the consideration of a long-range social insurance program. This is especially true with respect to the economic and demographic situation in a well-developed country that is operating on a stable basis and will continue to operate thereon. Moreover, the type of social insurance pension system that is postulated is only one of several different types, and other results would have been obtained on other bases. In other words, there is by no means any universal standard plan that can be used as a basis for "proving" the existence of a paradox of general nature.

In this review I shall first set forth certain factual errors that Aaron made (not all of which affect his reasoning and conclusions), and then I shall take up his major argument.

FACTUAL POINTS

Aaron states that under private insurance a reserve is accumulated for each person. Although this is generally and customarily so, it need not be the case in every instance. A reserve accumulates only if the benefit payments and administrative expenses in the early years of operations are less than the premium income and the investment income (the converse would be the case later on). It is possible to set up a scheme so that their present values would be equal yet with no reserve accumulation (in fact, a deficit could occur at some time during the period of protection but would be wiped out by the end of the period).

Aaron states that he presents a *rigorous* demonstration of the paradox, but it seems to me that the model is oversimplified to such an extent that this adjective hardly applies. For example, he assumes that every individual has the same wage in a particular year, without any variation by age or sex. Also, he assumes that all contribution payments and all pension payments are made annually at the beginning of each year. Further, he makes the very unrealistic assumption that all persons live to exactly age $A + n$ (actually to slightly before this age, because of the foregoing oversimplification about persons' being paid annually).

In defining k (the year in which the situation is to be considered, with the first year in which a stable situation prevails with regard to the annual number of

* Books and other publications noted with an asterisk (*) may be borrowed from the library of the Society of Actuaries under the rules stated in the *Year Book*.

new entrants at age A and the annual rate of increase in the number of such new entrants being designated as year 0), it is stated that, for equation (1) to be applicable, k must be less than n (the length of life of all individuals); actually, k must be equal to or greater than n, so that the population at all ages from A to $A + n$ will be stable (being made up entirely of entrants who have entered at least since time 0, when entry conditions were assumed to have been stabilized). Also in regard to equation (1), P_0 is defined as the population at time zero, whereas actually it is the number of new entrants at age A in year zero. Finally, in regard to equation (1), under the foregoing conditions the equation is in error since the factor i^k should be i^{k-m}.

In equation (5) the second equality sign should be a multiplication sign.

EXAMINATION OF THE REASONING INVOLVED

The entire so-called paradox rests on the assumption that the combination of the rate of growth of population and the rate of increase in earnings will continuously and forever exceed the rate of interest. This assumption seems to me to be untenable. For one thing, we cannot have a positive rate of growth of the population for the long-range future since we will eventually run out of standing room on this planet.

In the economic area it seems most unlikely to me that the rate of increase of earnings alone will, over the long-range future, exceed the interest rate. Furthermore, even if it does or even if it is close, there is a question of whether the social insurance system that is postulated should have benefits that bear a constant relationship to the earnings level rather than to the price level. If the latter type of social insurance system had been considered, the story would have been considerably different. Certainly, over any future period in an economically stable country it would be impossible for the price level to increase at a more rapid rate than the interest rate. Under these circumstances—since we must discard the assumption of a perpetual increase in population—the paradox falls. In popular terms, it might be said that the stated paradox is merely the well-known one—proved fallacious by time—of "borrowing from Peter to pay Paul" or the "Ponzi routine."

The paradox would fall even harder if nations could become economically mature enough in their thinking and actions that earnings would stay relatively level; increases in productivity would then be reflected in lower prices (and thus higher real wages would result). This stable situation would equitably benefit all sectors of the population—the retired as well as the active workers and their dependents. Under these circumstances the level of interest rates would be lower, but it would still exist, so that the paradox of Aaron would fall.

It may be noted that, if benefits are automatically adjusted to changes in the wage level, it could be argued that every other item in the economic area (such as consumer debt and bonds) should be similarly adjusted. Under these circumstances we would in essence have a new monetary unit, which would be completely stable in nature.

I might point out that the situation discussed by Aaron can exist—at least

for a temporary period—in countries that have inflationary problems. Under these circumstances there is obviously no advantage in accumulating reserves under social insurance systems (or, in fact, even under private pension plans). This fact has been recognized by actuaries for a number of years (for example, see my paper "Actuarial Analysis of Pension Plans under Inflationary Conditions," *Transactions of the Sixteenth International Congress of Actuaries*, Vol. I [June, 1960]).

ROBERT J. MYERS

Tony Lynes, *French Pensions*, pp. 163, "Occasional Papers on Social Administration," G. Bell & Sons, Ltd., London, England, 1967.

During the last two years there have been quite a few articles on the social and financial aspects of pensions. Interest on the part of actuaries has probably been sparked by Bernstein's book, the report of the President's Committee on Private Pensions, and legislation in a few Canadian provinces. It does not seem, however, that much work has been done thus far to discover how other countries have attempted to solve the same problems. This book will give English-speaking actuaries the opportunity to learn what the French have done.

The financial security of the aged in France is insured by a four-tiered system —the *Régime général*, the *Régimes complémentaires*, and two other income-maintenance programs providing benefits on a means-test basis. The *Régime général* was set up after the last war and is the universal social security program of France; it does not, however, cover the whole population since certain major occupational groups have been allowed to have their own schemes. The *Régimes complémentaires* are the French supplementary or "private" pension plans, although the term "private" can hardly be applied to them, for reasons to be discussed later.

The *cadres* (staff employees), who in the 1930's had been excluded from the social insurance scheme established at the time, set up their own private pension plans. At the end of the war the pensions that they had been promised were now inadequate, as a result of the severe inflation of the war years, and the investments in their funds were by no means sufficient to bear the cost of increasing the benefits to an adequate level. On the other hand, the radical thinking prevalent in the postwar years toyed with the idea of a universal social security scheme. The *cadres* were opposed to being forced into the *Régime général* and agreed only to promises of being allowed a complementary pension plan. This was to be the *Régime complémentaire*.

Because of their previous experience the *cadres* did not contemplate anything but a pay-as-you-go pension plan. Different funds were established in 1947 to cater for one or many firms in one or more industries; they have been co-ordinated by the *Association Générale des Institutions de Retraite des Cadres*. One of the responsibilities of this body is to make financial transfers among different member funds, according to their demographic characteristics.

All existing *cadre* pensioners (if they could be found) were blanketed into the scheme and were paid an earnings-related pension. All pension credits earned in

a given year are converted into pension points, which are revalued yearly. The value of the point follows a salary index, enabling retired *cadres* to share in any increase in the prosperity of the active members, since pensions in payment are also adjusted yearly. One pension point, once earned, is fully preserved in the fund in which it was earned. Naturally, the contributions paid also increase in proportion with each change in the value of the pension point.

The scheme has been successful so far, partly because of the good demographic balance of the *cadre*-scheme membership and the impossibility of tracing all existing pensioners. This success led the unions to press for the same plans for their membership, and in 1956 the first *Régime complémentaire* was established for manual workers. These have now been extended to a very large proportion of the population covered by the *Régime général*. Co-ordination of a large proportion of these funds was made compulsory in 1961. The extraordinary coverage achieved by the *Régimes complémentaires* is partly due to the fact that the French minister of labor has power to extend by decree the collective agreement of employers' and workers' representatives to the whole industry concerned. The French trade unions have taken a keen interest in negotiating pension agreements (for the reason given above) and in administering the funds themselves (under another regulation of the Ministry of Labor, management and workers have to be equally represented in the management of each fund).

Most actuaries will be concerned at this point about the astronomical cost of running pension schemes on a pay-as-you-go basis. But, as shown in Chapter vii of the book, if the rate of interest obtained under pension schemes is lower than the rate at which liabilities increase, then a pay-as-you-go scheme is cheaper than a fully funded scheme as long as that situation persists. Under the French schemes, with all benefits indexed, such a situation is certainly more likely than it is under a typical North American career average pension plan. Tony Lynes is not the first to expose what H. Aaron has called the "social insurance paradox."[1] Others who come to mind are D. C. Bronson,[2] Paul Nowlin,[3] Robert Myers,[4] and Hans Ammeter.[5]

After reading all this literature, one wonders why the *Study Notes* for Part 10E still say that a pension fund is a "growing fund which theoretically reaches stable size" and that, except for termination, the fund's basic pupose is "to provide investment income so that, together with contributions, benefit payments can be made without touching the principal of the funds." Certainly the experience of most final salary plans since the war would indicate that a large propor-

[1] "The Social Insurance Paradox," *Canadian Journal of Economics and Political Science*, Vol. XXXII, No. 3 (August, 1966).

[2] *Concepts of Actuarial Soundness in Pension Plans*, Homewood, Ill.: Richard D. Irwin, Inc., 1957.

[3] "Insufficient Premiums," *TSA*, XI, 100.

[4] "Actuarial Analysis of Pension Plans under Inflationary Conditions," *Transactions of the 16th International Congress of Actuaries*, I, 301.

[5] "Funding of Retirement Income," *Transactions of the 17th International Congress of Actuaries*, III, 25.

tion of the investment income has been used not to provide benefits but to pay for the increase in liabilities caused by changes in salary due to inflation or productivity gains. It seems to me that students ought to be exposed to those considerations as well.

Those interested in the book need not worry about the large number of French technical words in both the review and the book, for there is a three-page glossary appended to the book.

CLAUDE GARCIA

SELECT CURRENT BIBLIOGRAPHY

In compiling this list, the Committee on Review has digested only those papers which appear to be of direct interest to members of the Society of Actuaries; in doing so, the Committee offers no opinion on the views which the various articles express. The digested articles will be listed under the following subject-matter classifications: 1—"Actuarial and Other Mathematics, Statistics, Graduation"; 2—"Life Insurance and Annuities"; 3—"Health Insurance"; 4—"Social Security"; 5—"Other Topics."

The review section of the *Journal of the Institute of Actuaries* contains digests in English of articles appearing in foreign actuarial journals.

ACTUARIAL AND OTHER MATHEMATICS, STATISTICS, GRADUATION

Karl Borch, "The Theory of Risk," *Journal of the Royal Statistical Society*, Series B, XXIX (1967), 432–67.

Borch, a Norwegian economist and actuary, has been a prolific writer in the field of risk theory and insurance economics. This paper provides an excellent review of risk theory. Classical, or individual, risk theory and collective risk theory are succinctly summarized. The section titled "Modern Risk Theory" is devoted to a risk model that provides for dividend payments when funds reach a reflecting or dividend barrier. One key problem in this model, which Borch and others have discussed elsewhere, is the determination of an optimal dividend policy that will seek to maximize the present expected value of future dividends subject to the consideration of maintaining a suitably long, expected future lifetime of the insurance company. The modern risk theory section is largely devoted to specific elementary examples rather than to general results. The discussion of the paper is particularly valuable because it includes comments by several European actuaries.

Lawrence Fisher, "Computer Algorithms for Finding Exact Rates of Return," *Journal of Business*, XXXIX (1966), 111–18.

Seymour Kaplan, "Computer Algorithms for Finding Exact Rates of Return," *Journal of Business*, XL (1967), 389–92.

The first paper suggests using the Newton-Raphson iteration technique for finding the force of interest associated with a financial project that involves multiple outputs and inputs. The article is accompanied by a FORTRAN program. The only check for nonconvergence or for multiple rates of return is to stop the program after a hundred trials if a solution has not been found. It is suggested that the determination of the internal rate of return on a pension fund would be an application of the program.

The second paper discusses modifications in the program proposed in the first paper. These modifications are to prevent the nonconvergence of the algorithm. The possible causes of the failure of the Newton-Raphson algorithm are well known in numerical

analysis. The problem of multiple yields rates is not discussed in detail. Figures 1 and 3 in the paper have been interchanged and this may cause some minor confusion to readers.

Robert L. Winkler, "The Assessment of Prior Distributions in Bayesian Analysis," *Journal of the American Statistical Association*, LXXII (1967), 776–800.

Actuaries have been introduced to applications of Bayesian statistics in their science in recent papers (Jones, *TSA*, Vol. XVII; Jones-Kimeldorf, *TSA*, Vol. XIX; and Mayerson, *PCAS*, Vol. LI). However, the feasibility of actually formulating prior distributions from vague knowledge and the questioning techniques for drawing out such information have not been extensively discussed in the actuarial journals.

In this paper four techniques for formulating prior distributions are tested. One of these techniques involves formulating a probability density function, and another requires the specification of a cumulative distribution function.

A sample of University of Chicago students was asked to formulate prior distributions by each of the four methods for the six probabilities of occurrence associated with six characteristics of a hypothetical, randomly selected University of Chicago student. For example, one such characteristic involved the wearing of glasses. The author concludes that it is feasible to question about subjective prior probabilities but that certain methods lead to more consistent results than others do. Training in statistics also improves the success of a person in making coherent estimates of prior distributions.

This paper should be of interest to actuaries who are concerned with the necessary but perplexing task of actually assigning prior probabilities.

Robert L. Winkler, "The Quantification of Judgment: Some Methodological Suggestions," *Journal of the American Statistical Association*, LXII (1967), 1105–20.

In this essay a discussion of the problems involved in eliciting personal probability assessments, which was started in the author's earlier paper ("The Assessment of Prior Distribution in Bayesian Analysis," *JASA*, LXII [1967], 776–800), is continued. The use of penalties and side bets to compel serious probability assessment and to force coherence is discussed. The ideal attributes of a hypothetical probability assessor are explicitly listed. The problems inherent in finding or training a probability assessor with these attributes and in devising a probability-assessment situation that will force these characteristics are discussed. Actuaries who are responsible for the selecting and the rating of insurance risks may find this paper of interest, since they are concerned with the announced subject of the paper.

Health Insurance

Helen Hershfield Avnet, *Physician Service Patterns and Illness Rates*, pp. xxvi, 452, Group Health Insurance, Inc., New York, 1967.

This study was undertaken "to demonstrate the wealth of medically relevant data which can be produced as a by-product of medical insurance administration."

"This volume is the third original research report to be published by GHI-GHDI in five years." The prior two were "Psychiatric Insurance," by Mrs. Avnet, and "Insured Dental Care," by Mrs. Avnet and Mata Kouvari Nikias.

U.S. National Center for Health Statistics, *Utilization of Short-Stay Hospitals by Characteristics of Discharged Patients, United States, 1965*, pp. 39, Public Health Service Publication No. 1,000, Series 13, No. 3, Washington, December, 1967.

Hospital discharges, days of care, average length of stay, discharge rates, daily hospital-bed usage rates, and rates of days of care are analyzed by age, sex, and marital status.

Other Topics

U.S. National Center for Health Statistics, *The Influence of Interviewer and Respondent Psychological and Behavioral Variables on the Reporting in Household Interviews*, pp. 65, Public Health Service Publications No. 1,000, Series 2, No. 26, Washington, March, 1968.

The study "was designed to identify some of the major variables relating to the level of the reporting of health information in a household interview. . . . The findings of this study indicate that behaviors are the most important variables in determining the course of the interview, with psychological and demographic characteristics having minimal predictive value."

U.S. National Center for Health Statistics, *Marriage Statistics Analysis, United States, 1962*, pp. 30, Public Health Service Publication No. 1,000, Series 21, No. 10, Washington, January, 1967.

Marriage data are presented by geographic area, sex, age, color, former marital status, and month of marriage. Trends for annual totals and for rates of marriages are described.

U.S. National Center for Health Statistics, *Natality Statistics Analysis, United States, 1964*, pp. 38, Public Health Service Publication No. 1,000, Series 21, No. 11, Washington, February, 1967.

The report presents data on the characteristics of births, including analyses of the sex ratio, plural births, birth weight and period of gestation, season of birth, and illegitimacy. Fertility statistics by color, by state and geographic area, and by metropolitan residence are presented. Recent trends in cohort fertility rates, period fertility rates, timing of births, and the influence of contraceptive pills are described.

U.S. National Center for Health Statistics, *Fertility and Educational Attainment, Puerto Rico, 1962*, pp. 20, Public Health Service Publication No. 1,000, Series 21, No. 12, Washington, September, 1967.

Fertility rates of mothers and characteristics of newborn infants are related to the educational attainment of the parents.

U.S. National Center for Health Statistics, *Divorce Statistics Analysis, United States, 1963*, pp. 57, Public Health Service Publication No. 1,000, Series 21, No. 13, Washington, October, 1967.

Divorce trends, international comparisons, and annulments are discussed. Detailed divorce statistics are presented by age, geographic area, race, duration of marriage, children involved, and legal grounds for divorce.

U.S. National Center for Health Statistics, *Multiple Births, United States, 1964*, pp. 50, Public Health Service Publication No. 1,000, Series 21, No. 14, Washington, October, 1967.

Data based on live births and fetal deaths occurring in multiple deliveries are presented by race, live-birth status, sex, and state.

U.S. National Center for Health Statistics, *Trends in Illegitimacy, United States, 1940–1965*, pp. 90, Public Health Service Publication No. 1,000, Series 21, No. 15, Washington, February, 1968.

This report describes trends in the illegitimacy rate and illegitimacy ratio, numbers of illegitimate births, characteristics of unwed mothers, factors accounting for color differences, factors accounting for the increasing rate, and health characteristics.

U.S. National Center for Health Statistics, *United States Life Tables by Causes of Death, 1959–61*, I, No. 6, pp. 63, Public Health Service, Washington, May, 1968.

This is the last report on the official United States life tables program for 1959–61. Mortality in these reports is derived from the census of population taken April 1, 1960, and from recorded deaths for the period 1959–61. The program was a joint effort of the National Center for Health Statistics, Public Health Service, and the Division of the Actuary, Social Security Administration. This report, prepared by Mr. Francisco Bayo, Deputy Chief Actuary, Social Security Administration, includes life tables eliminating the leading causes of death for the total population, for white males, for white females, for nonwhite males, and for nonwhite females. Multiple decrement tables by causes of death are presented along with the probability of eventually dying from these causes and the gain in expectation due to their elimination as causes of death.

This is the first report of this nature, but comparisons are made in it with the results of similar unpublished calculations associated with previous censuses. The methodology used in the preparation of the tables is described in detail, and the development of the most important approximations is given.

*Karl-H. Wolff, *Methoden der Unternehmensforschung im Versicherungswesen* ("Methods of Operations Research in the Life Insurance Business"), pp. vi, 266, Springer-Verlag, Berlin, 1966.

This book, written in German, is the fourth volume in a series entitled "Econometrics and Operations Research."