



Beyond Actual to Table: Models in Experience Studies





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AUTHOR

Brian D. Holland, FSA, MAAA Society of Actuaries SPONSOR

Individual Life Experience Committee

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Section 1: Purpose of the Study

Traditional life experience studies show actual claims as a ratio of a reference table. Various splits of the experience show familiar relationships, illustrate new insights, and allow benchmarks. Despite traditional studies' advantages, actuaries are familiar with a disadvantage: distortions due to different mixes of business between segments within the study, and between the study and an actuary's own book of business. Ratios to the reference table can be raised or lowered due to the inclusion of another business segment and therefore make for poor adjustment factors.

Despite the issue that mixing heterogeneous segments can distort results, segments are typically aggregated to gain a larger volume of experience. This paper demonstrates an approach to addressing the issue by modeling experience, and then interpreting the model and residual errors instead of actual / table ratios alone. Appendix G of the 2017 ILEC report (select mortality by age, plan, and duration) is taken as an example, modelling on more attributes than the splits shown in the report. Two types of models are presented: generalized linear models (GLM), and a regression tree.

The GLM, in practical terms, determines the 2015 VBT adjustment factors for various splits of business in order to match the total experience corresponding to each factor simultaneously. For example, adjustment factors for each face amount band, underwriting class, etc. are calculated in tandem, so the total adjusted mortality within each specific face band, underwriting class, and other adjusted category is 100%. The adjustment factors are then compared to the actual / table ratios to find anomalies. More formally, the GLM used here uses a log link with an offset, i.e. item that is adjusted, of the expected claims under the 2015 VBT. The splits chosen are those for which actual / table ratios are typically presented. These models are less susceptible to being skewed by different mixes of business. There is a disadvantage: a large adjustment to the underlying expected in a cell could imply an underlying mortality rate over 100%. For examining experience, however, this disadvantage is heavily outweighed by the advantage of communicability of the factors, and comparability to the more familiar actual / table presentation of experience. The adjustment factors are also volatile for thin cells. The experience is quite heterogeneous as is clear from the ranges of issue and study years. The two GLMs fit policy count and face amount on splits of the following characteristics:

- 1. **issue year (iy_band1):** -1989, 1990-1999, 2000-2009, 2010-. There is little experience through 1989, as experience is durations 1-25 and the study period is 2009-13.
- 2. **underwriting class (class_key):** showing whether SM/NS, number of underwriting classes, and which underwriting class: Nonsmoker 2 1, Smoker 2 1, etc.
- 3. insurance_plan: Other, Perm, Term, UL, ULSG, VL, VLSG
- 4. **level term period (ltp):** the anticipated level term period. 20-year term and "N/A (Not Term") are combined in this field as "20 yr or N/A (Not Term)" because the insurance plan field already distinguishes between term and other plans. Without combining the 20-year term indicator and the "not term" indicator we would have two indicators whether the business is term, in the level term period field and in the plan field. Note that much of the source term data has "unknown" level term period.
- 5. issue age band (ia_band1): generally quinquennial bands

- 6. duration band (dur_band1): 01, 02, 03, 04-05, 06-10, 11-15, 16-20, 21-25
- 7. observation_year: 2009, 2010, 2011, 2012, and 2013 individually, each with a factor
- 8. gender: Female, Male
- 9. **face_amount_band:** 1-9999, 10000-24999, etc. The low face business is of questionable quality. It may include insurance which has not been fully underwritten.

Some of these characteristics will be correlated. Issue year ranges in iy_band1 will overlap the study period in different duration bands, for example. Face amounts and products issued as well as underwriting structures have changed over time, and will only be observed in some durations. The correlations between the explanatory variables are an important topic to address when developing a model to be used for prediction, and is intentionally not covered in this description of experience. This paper is not a solution to those problems, but a supplemental way to examine and analyze the experience.

A second type of model was used for describing these results. A tree approach was applied to the observed actual/table ratios, so as to split the data into more homogeneous groups, as measured by their relative actual/table factors. Technically, the rpart routine of R was used, with each record's actual/table ratio as the dependent variable, and the same dependent variables as in the regression model. After the tree was build, the average actual/table from the applicable leaf was assigned to each record as an adjustment factor. Conceptually, this is not that different from a regression: we are essentially assigning an average value of similar records to represent the single value of the observed actual/table. This is not unlike a regression fit where the model value is a best fit by averaging similar data to the specific data point in question.

Note that this paper is not a recommendation of a model of industry experience for prediction. The intent is to supplement current experience reporting. A model intended to fit the experience would be considerably more complex, as it would need to fit various cross combinations of business that are not reflected in the models in this paper. The number of possible cross combinations grows quickly with the number of variables modelled. A model for predicting should also be tested against un-modeled data.

Section 2: Acknowledgements and Resources

2.1 Individual Life Experience Committee

The SOA extends its gratitude to the Individual Life Experience Committee (ILEC). The ILEC designed the project, completed/oversaw the analyses and authored and peer reviewed the report. The ILEC members are:

- Tony R. Phipps (Chair), FSA, MAAA
- Edward Hui (Vice-Chair), FSA
- Mary J. Bahna-Nolan, FSA, MAAA, CERA
- Tatiana Berezin, FSA, MAAA
- Kathryn A. Campbell, FSA, MAAA
- Christopher M. Condon, FSA, MAAA
- Jeffery T. Dukes, FSA, MAAA
- Roland Fawthrop, FSA, MAAA
- Dieter S. Gaubatz, FSA, FCIA, MAAA
- Brian D. Holland, FSA, MAAA
- Douglas A. Ingle, Underwriter
- Kevin P. Larsen, ASA, MAAA
- Hezhong (Mark) Ma, FSA, MAAA
- Stephen J. MacDonald, Underwriter
- Nikolai D. Serykh, FSA, FCIA
- Maureen A. Shaughnessy, FSA, MAAA
- Frans W. Te Groen, FSA, MAAA

2.2 Other Resources

The SOA contracted with MIB's Actuarial and Statistical Research Group, to collect, validate, and compile the data for this report. Yara Rodgers-Silva, Principal Statistical Modeling at LexisNexis Risk Solutions, helped with the review and discussion of the contents of the report. Mervyn Kopinsky, FSA, EA, SOA Experience Studies Actuary, provided assistance with some of the modeling work, and Erika Schulty, SOA Research Associate, helped coordinate and draft the report.

Section 3: Findings

The most striking results from the GLMs are modeling factors which go in a different direction from the actual / table ratio. One such example is the actual / table for Permanent vs Term. By count, Permanent is 121% of 2015VBT vs. 105% for Term, which would seem to imply that Perm has higher mortality. The Term factor, however, is higher than Perm, so the opposite. To tell why, we can compare the other components of the actual / table ratio between these segments, the average factors for the other categories to find the category with the most differentiation in average factors (Table 1). The average factors for the other categories show that the mix of business within the face amount band is driving the appearance that Perm is higher than Term. Perm is skewed to lower bands, which have higher factors (Figure 1). The ILEC understands that the low bands might include business reported inconsistently by different companies for administrative reasons, which could explain both their presence in a fully underwritten study and the higher mortality allocated to the lower bands by the model.

	insurance_plan	Perm	Term
A/VBT15	Observed	1.21	1.05
	Approximated	1.22	1.05
Factor	insurance_plan	0.93	1.00
Avg factors	Overall	0.93	0.93
	class_key	1.24	1.04
	dur_band1	0.85	0.99
	face_amount_band	1.36	0.99
	gender	1.05	1.07
	ia_band1	0.86	0.93
	iy_band1	1.02	1.04
	ltp	1.00	1.01
	observation_year	1.07	1.07

Table 1: Analysis of A/VBT15 for Permanent and Term

Appendix 2 shows a similar exhibit for each characteristic for which the count and the amount models have a separate factor. The example in Table 1 above is one of the more noteworthy examples, because the actual / VBT2015 ratio and the insurance plan factor move so strongly in opposite directions between Term and Perm.



Figure 1: Distribution of 2015 VBT Expecteds (Count) by Face: Term and Perm vs Durational Factor

Model factors alone show potentially misleading relationships in several cases.

- **Insurance plans:** the factors show the opposite relationship from the actual/table ratios between permanent and term plans.
- Issue ages: the factors show a declining adjustment to 2015VBT, while the actual / VBT ratio is more constant.
- **Durations:** the factors show a falling relationship versus 2015VBT, while the actual / VBT ratio turns upward after the duration 6-10 band.
- Face amounts: lower face amount bands have a higher adjustment factor than the actual / VBT ratio alone would indicate.
- **Issue years:** the actual/2015VBT ratio is high for the latest issue cohort. The model attributes that elevated mortality to the low duration band, which has a higher factor.

Exhibits for all of the factors follow below in Appendix 1, showing A/2015VBT compared to the factors, showing the corresponding approximation of the A/2015VBT ratio as the product of the average factors.

Note that the low face amount bands heavily skew results by count due data quality concerns already noted.

The low face amount bands also have different durational relationships. These features could be reflected in the model by including adjustments for cross-combinations of different business characteristics. The tree model shows these cross combinations immediately. The full tree is shown in the appendix, and includes the top-level splits shown immediately following. At each node the A/2015VBT is shown, and the criterion by which the tree is split to go to the next level down. A "yes" or "no" goes left or right as indicated at the top-level node. The first three splits in the tree peel off low band, early duration business (face band minimum < 17500, duration band minimum < 3.5) as having a 315.2% actual / table. Such an exhibit quickly indicates to the subject

matter expert that there could be an irregularity, such as the inclusion of so much business with face below 10,000 that is purportedly fully underwritten. The full model is shown in Figure 22.

Figure 2: Tree model by count: top level splits



As a reminder, 2015VBT as adjusted with any of these models will match the experience in total for any subset for which there is a specific factor. Such a subset is, for example, a face amount band or policy duration band for which there is a factor; or a leaf node of a tree model, which uses the actual / table ratio in that node as a factor. It will not, however, necessarily match the experience at a more granular level within that cell.

3.1 Possible Future Directions

Topics which could be covered in future reports are comparisons of further models and modelling techniques:

- Analysis of additional exhibits besides Appendix G from the ILEC report to compare actual / table to adjustment factors
- Calibrating complexity of the model by tuning it against holdback data
- Calibrating the model for prediction of the next year of claims data
- Calibrating the model for variable selection using familiar significance tests
- Use of dimension reduction methods to find and rank interactions of variables

The topics to be explored will depend in part on community interest. Please feel free to express your interest to the ILEC, to the SOA research department, or to me directly.

Appendix 1: 2017 ILEC Report Exhibit G-Format Model Results

This section shows Exhibit G and also Exhibit G redone using each model instead of 2015VBT.

The GLMs fit 100% of the experience in total for the edges (plan total, duration band total, face band total) for the metric, face or count, for which they are run. They will not fit within cross combinations of characteristic without the introduction of more complicated factors.

	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG	All
face_min	1	87%	135%	140%	130%	167%	182%	140%	135%
	10000	144%	127%	205%	126%	141%	134%	129%	128%
	25000	149%	116%	194%	130%	122%	119%	122%	125%
	50000	140%	104%	147%	124%	113%	112%	115%	118%
	100000	187%	91%	104%	110%	99%	100%	103%	102%
	250000	199%	86%	89%	105%	95%	97%	96%	92%
	500000	264%	83%	83%	103%	95%	101%	93%	88%
	1000000	331%	87%	79%	104%	88%	105%	102%	87%
	2500000	0%	86%	87%	96%	97%	103%	132%	93%
	500000	108%	65%	90%	88%	86%	140%	134%	88%
	1000000	97%	111%	68%	89%	79%	100%	104%	84%
	All	158%	120%	104%	121%	102%	107%	111%	115%
dur_band1	01	117%	249%	139%	177%	125%	108%	126%	157%
	02	181%	203%	126%	174%	120%	73%	107%	140%
	03	129%	174%	115%	159%	104%	132%	90%	126%
	04-05	273%	155%	104%	133%	99%	102%	111%	113%
	06-10	464%	150%	98%	117%	98%	97%	100%	108%
	11-15	393%	134%	96%	118%	102%	103%	108%	113%
	16-20	183%	118%	124%	119%	104%	108%	125%	118%
	21-25	133%	114%	127%	121%	124%	114%	125%	116%
	All	158%	120%	104%	121%	102%	107%	111%	115%

Table 2a: Exhibit G Actual / qx2015vbt by Count; by Plan and Duration

Table 2b: Exhibit G Actual / qx2015vbt by Amount; by Plan and Duration

	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG	All
face_min	1	352%	133%	178%	134%	187%	190%	140%	134%
	10000	265%	124%	191%	126%	146%	133%	130%	126%
	25000	147%	114%	177%	131%	123%	120%	122%	124%
	50000	125%	102%	140%	123%	113%	112%	115%	116%
	100000	240%	90%	101%	109%	97%	99%	102%	100%
	250000	200%	86%	88%	105%	94%	96%	96%	91%
	500000	235%	83%	82%	102%	94%	101%	94%	87%
	1000000	290%	87%	79%	102%	88%	106%	105%	87%
	2500000	0%	87%	86%	94%	98%	101%	129%	93%
	500000	168%	64%	89%	87%	85%	145%	138%	88%
	1000000	92%	100%	73%	86%	78%	96%	100%	82%
	All	142%	94%	87%	103%	90%	104%	105%	93%
dur_band1	01	10%	110%	96%	136%	106%	71%	60%	100%
	02	259%	93%	92%	108%	93%	57%	111%	94%
	03	55%	89%	87%	90%	88%	133%	83%	88%
	04-05	17%	93%	87%	95%	74%	90%	92%	86%
	06-10	417%	93%	85%	90%	91%	106%	94%	88%
	11-15	1,392%	97%	83%	98%	95%	100%	118%	92%
	16-20	861%	94%	100%	108%	98%	106%	103%	101%
	21-25	72%	94%	116%	120%	121%	111%	131%	106%
	All	142%	94%	87%	103%	90%	104%	105%	93%

	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG	All
face_min	1	45%	101%	99%	85%	118%	136%	105%	100%
	10000	79%	102%	124%	91%	103%	106%	99%	100%
	25000	90%	98%	129%	99%	93%	98%	100%	100%
	50000	92%	95%	113%	101%	95%	98%	93%	100%
	100000		97%	98%	104%	98%	99%	97%	100%
	250000	142%	103%	94%	114%	107%	103%	102%	99%
	500000	180%	104%	91%	119%	113%	113%	104%	99%
	1000000	243%	110%	87%	124%	108%	117%	114%	99%
	2500000	0%	99%	87%	105%	108%	110%	138%	100%
	500000	78%	79%	94%	100%	100%	157%	147%	100%
	1000000	76%	145%	75%	107%	95%	120%	120%	100%
	All	100%	100%	99%	100%	100%	100%	100%	100%
dur_band1	01	71%	140%	91%	98%	87%	78%	98%	99%
	02	104%	127%	92%	106%	93%	57%	89%	99%
	03	79%	120%	95%	109%	90%	114%	79%	100%
	04-05	188%	116%	96%	103%	96%	96%	104%	100%
	06-10	294%	110%	98%	96%	101%	90%	95%	100%
	11-15	196%	102%	98%	97%	106%	96%	102%	100%
	16-20	102%	98%	111%	100%	101%	102%	101%	100%
	21-25	88%	98%	117%	101%	111%	106%	104%	100%
	All	100%	100%	99%	100%	100%	100%	100%	100%

Table 3a: Exhibit G Actual / g_count by Count; by Plan and Duration

Table 3b: Exhibit G Actual / g_count by Amount; by Plan and Duration

	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG	All
face_min	1	182%	101%	120%	89%	133%	142%	105%	101%
	10000	146%	100%	115%	90%	106%	105%	99%	98%
	25000	89%	97%	118%	100%	93%	99%	100%	99%
	50000	83%	94%	107%	100%	95%	98%	93%	98%
	100000	177%	96%	95%	104%	96%	98%	97%	98%
	250000	142%	103%	92%	114%	107%	103%	103%	99%
	500000	161%	104%	90%	118%	113%	112%	105%	99%
	1000000	213%	109%	87%	123%	107%	119%	117%	100%
	2500000	0%	100%	86%	103%	109%	108%	135%	100%
	500000	121%	79%	94%	99%	99%	163%	151%	100%
	1000000	72%	129%	80%	104%	95%	115%	115%	98%
	All	105%	100%	91%	107%	103%	108%	110%	99%
dur_band1	01	6%	81%	72%	96%	87%	57%	51%	76%
	02	164%	76%	77%	85%	85%	49%	102%	79%
	03	38%	82%	81%	82%	89%	126%	82%	83%
	04-05	14%	93%	89%	99%	83%	93%	97%	89%
	06-10	313%	91%	92%	100%	107%	110%	100%	98%
	11-15	924%	99%	94%	107%	119%	106%	125%	102%
	16-20	659%	102%	101%	110%	114%	110%	108%	106%
	21-25	61%	102%	119%	114%	125%	111%	116%	108%
	All	105%	100%	91%	107%	103%	108%	110%	99%

	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG	All
face_min	1	37%	100%	104%	81%	115%	121%	88%	99%
	10000	64%	105%	141%	86%	107%	96%	88%	101%
	25000	72%	102%	148%	96%	98%	88%	88%	101%
	50000	75%	100%	126%	99%	99%	91%	88%	101%
	100000	116%	99%	108%	100%	100%	93%	91%	102%
	250000	133%	99%	101%	103%	103%	96%	92%	101%
	500000	183%	98%	99%	104%	107%	104%	92%	101%
	1000000	235%	104%	95%	108%	100%	108%	102%	100%
	2500000	0%	97%	100%	97%	104%	102%	127%	101%
	500000	79%	79%	110%	96%	98%	148%	136%	101%
	1000000	72%	146%	88%	105%	96%	113%	115%	101%
	All	84%	102%	109%	96%	101%	93%	90%	101%
dur_band1	01	78%	218%	124%	137%	111%	95%	110%	137%
	02	117%	191%	122%	144%	115%	68%	97%	132%
	03	88%	176%	121%	142%	107%	132%	85%	128%
	04-05	195%	159%	114%	124%	105%	105%	107%	119%
	06-10	306%	137%	105%	103%	99%	91%	89%	108%
	11-15	190%	115%	103%	99%	100%	92%	92%	105%
	16-20	87%	100%	116%	95%	93%	93%	86%	99%
	21-25	68%	95%	118%	93%	104%	94%	88%	95%
	All	84%	102%	109%	96%	101%	93%	90%	101%

Table 4a: Exhibit G Actual / g_amount by Count; by Plan and Duration

Table 4b: Exhibit G Actual / g_amount by Amount; by Plan and Duration

	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG	All
face_min	1	149%	100%	130%	84%	130%	126%	88%	100%
	10000	118%	103%	132%	86%	111%	95%	88%	100%
	25000	72%	101%	135%	96%	98%	89%	88%	100%
	50000	67%	98%	120%	99%	99%	91%	88%	100%
	100000	149%	98%	104%	100%	99%	93%	91%	100%
	250000	134%	99%	100%	103%	103%	96%	93%	100%
	500000	163%	98%	98%	104%	106%	103%	93%	100%
	1000000	205%	103%	95%	107%	99%	109%	105%	100%
	2500000	0%	98%	99%	95%	105%	99%	124%	100%
	5000000	124%	78%	110%	95%	97%	154%	140%	100%
	1000000	69%	131%	94%	101%	95%	108%	110%	100%
	All	100%	100%	100%	100%	100%	100%	100%	100%
dur_band1	01	7%	120%	96%	122%	106%	68%	57%	100%
	02	185%	109%	99%	102%	100%	57%	110%	100%
	03	42%	113%	101%	93%	100%	142%	87%	101%
	04-05	13%	120%	104%	104%	86%	100%	98%	100%
	06-10	305%	109%	99%	97%	102%	109%	92%	100%
	11-15	874%	106%	97%	99%	106%	98%	110%	100%
	16-20	530%	99%	104%	99%	98%	99%	89%	100%
	21-25	47%	96%	118%	103%	113%	96%	97%	100%
	All	100%	100%	100%	100%	100%	100%	100%	100%

	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG	All
face_min	1	68%	104%	99%	105%	130%	141%	119%	104%
	10000	109%	98%	107%	102%	108%	105%	104%	99%
	25000	118%	91%	112%	107%	94%	97%	99%	99%
	50000	116%	88%	103%	105%	91%	95%	93%	98%
	100000	188%	91%	103%	110%	98%	100%	102%	102%
	250000	224%	93%	97%	113%	97%	103%	101%	99%
	500000	296%	90%	93%	110%	98%	107%	98%	96%
	1000000	356%	94%	90%	110%	91%	110%	108%	95%
	2500000	0%	93%	99%	98%	99%	108%	139%	99%
	500000	113%	70%	101%	88%	86%	147%	140%	91%
	1000000	102%	123%	77%	90%	80%	106%	108%	87%
	All	137%	97%	100%	106%	96%	99%	102%	100%
dur_band1	01	120%	125%	116%	115%	102%	102%	128%	116%
	02	177%	108%	109%	114%	99%	69%	105%	107%
	03	124%	91%	101%	104%	86%	123%	85%	97%
	04-05	260%	122%	102%	118%	95%	101%	110%	106%
	06-10	439%	104%	98%	103%	93%	93%	97%	99%
	11-15	272%	97%	95%	103%	96%	97%	102%	98%
	16-20	144%	98%	110%	107%	99%	100%	108%	102%
	21-25	116%	95%	114%	107%	110%	101%	99%	99%
	All	137%	97%	100%	106%	96%	99%	102%	100%

Table 5a: Exhibit G Actual / g_tree_count by Count; by Plan and Duration

Table 5b: Exhibit G Actual / g_tree_count by Amount; by Plan and Duration

	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG	All
face_min	1	276%	104%	118%	108%	148%	149%	119%	104%
	10000	201%	96%	99%	102%	112%	105%	105%	98%
	25000	117%	90%	102%	107%	94%	98%	100%	98%
	50000	104%	87%	98%	104%	91%	95%	93%	96%
	100000	241%	90%	100%	109%	97%	100%	101%	100%
	250000		93%	95%	113%	97%	102%	101%	98%
	500000	264%	90%	92%	109%	97%	107%	99%	95%
	1000000	310%	94%	90%	108%	90%	111%	111%	95%
	2500000	0%	94%	98%	97%	100%	106%	136%	99%
	500000	176%	69%	101%	87%	85%	153%	144%	91%
	1000000	97%	110%	82%	87%	80%	102%	104%	86%
	All	148%	91%	94%	103%	91%	105%	108%	96%
dur_band1	01	10%	102%	101%	128%	109%	76%	64%	104%
	02	274%	88%	98%	101%	96%	60%	116%	97%
	03	58%	84%	93%	85%	90%	138%	86%	92%
	04-05	17%	95%	94%	93%	75%	95%	96%	90%
	06-10	428%	92%	93%	89%	91%	109%	97%	92%
	11-15	1,329%	92%	91%	99%	98%	102%	121%	96%
	16-20	846%	92%	100%	109%	102%	107%	104%	101%
	21-25	76%	90%	117%	116%	119%	105%	110%	102%
	All	148%	91%	94%	103%	91%	105%	108%	96%

Appendix 2: Comparisons of Factors and A/2015VBT

This section shows the total actual / 2015VBT compared to the model factor as calculated from the multivariate fit.

Count is used for the count model, **g_count**, and amount for the amount model, **g_amount**.

Graphical comparisons

Model g_count: Graphical Comparisons of A/2015VBT and Model Factor



Figure 4: Model g_count, Factors by iy_band1 vs Actual/VBT2015

	iy_band1	-1989	1990-1999	2000-2009	2010-
A/VBT15	Observed	1.16	1.17	1.11	1.39
	Approximated	1.17	1.19	1.12	1.39
Factor	iy_band1	1.00	1.03	1.05	1.02
Avg factors	Overall	0.93	0.93	0.93	0.93
	class_key	1.23	1.21	1.09	1.06
	dur_band1	0.81	0.86	1.00	1.31
	face_amount_band	1.30	1.25	1.05	1.03
	gender	1.06	1.05	1.06	1.07
	ia_band1	0.89	0.87	0.87	0.88
	insurance_plan	0.97	0.97	1.00	1.00
	ltp	1.00	1.00	1.01	1.01
	observation_year	1.06	1.07	1.07	1.08





	class_key	NS	SM	SM	SM	Unk									
		21	22	31	32	33	41	42	43	44	nan	21	22	nan	nan
A/VBT15	Observed	0.89	1.38	0.72	0.87	1.17	0.71	0.88	1.04	1.25	1.16	0.90	1.21	1.21	1.50
	Approximated	0.90	1.39	0.73	0.87	1.18	0.71	0.89	1.04	1.26	1.18	0.90	1.22	1.23	1.50
Factor	class_key	1.00	1.44	0.76	0.92	1.22	0.71	0.89	1.03	1.22	1.21	0.87	1.13	1.19	1.44
Avg	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
factors	dur_band1	0.91	0.91	1.00	1.00	1.03	1.03	1.02	1.04	1.06	0.85	0.99	0.99	0.85	0.85
	face_amount_ban	1.00	1.14	0.94	0.96	1.00	0.93	0.95	0.94	0.98	1.26	1.02	1.10	1.31	1.50
	d														
	gender	1.07	1.06	1.07	1.07	1.07	1.07	1.07	1.08	1.07	1.06	1.07	1.07	1.05	1.03
	ia_band1	0.90	0.87	0.92	0.88	0.85	0.93	0.89	0.90	0.88	0.87	0.93	0.90	0.90	0.83
	insurance_plan	1.00	0.99	1.00	1.01	1.01	1.00	1.00	1.00	1.00	0.97	1.00	1.00	0.96	0.94
	iy_band1	1.04	1.03	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.02	1.04	1.04	1.02	1.02
	ltp	1.00	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.00	1.01	1.01	1.00	1.00
	observation_year	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.06	1.07	1.07	1.06	1.08





	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG
A/VBT15	Observed	1.59	1.21	1.05	1.21	1.02	1.07	1.11
	Approximated	1.62	1.22	1.05	1.22	1.03	1.08	1.13
Factor	insurance_plan	1.31	0.93	1.00	1.06	1.01	0.97	0.96
Avg	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93
factors	class_key	1.22	1.24	1.04	1.20	1.11	1.19	1.17
	dur_band1	0.89	0.85	0.99	0.86	1.01	0.88	0.94
	face_amount_band	1.20	1.36	0.99	1.18	1.03	1.07	1.12
	gender	1.07	1.05	1.07	1.06	1.06	1.06	1.06
	ia_band1	0.88	0.86	0.93	0.87	0.81	0.91	0.88
	iy_band1	1.02	1.02	1.04	1.03	1.05	1.03	1.04
	ltp	1.00	1.00	1.01	1.00	1.00	1.00	1.00
	observation_year	1.07	1.07	1.07	1.07	1.07	1.07	1.07

Figure 7: Model g_count, Factors by ltp vs Actual/VBT2015



	ltp	5 yr	10 yr	15 yr	20 yr or N/A	25 yr	30 yr	Not Level	Unknown
					(Not			Term	
	Approximated	1.91	1.07	1.05	Term) 1.19	1.64	1.01	1.05	1.07
Factor	ltp	1.18	1.03	1.10	1.00	1.26	1.03	0.89	1.00
Avg	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
factors	class_key	1.16	1.03	1.00	1.21	1.25	0.95	1.15	1.05
	dur_band1	1.23	1.06	0.99	0.87	0.95	1.01	0.90	0.99
	face_amount_band	1.10	0.98	0.99	1.24	1.00	0.94	1.08	0.99
	gender	1.06	1.08	1.07	1.05	1.06	1.06	1.07	1.07
	ia_band1	0.96	0.88	0.89	0.87	1.00	1.00	0.96	0.93
	insurance_plan	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00
	iy_band1	1.04	1.05	1.05	1.03	1.04	1.04	1.03	1.04
	observation_year	1.05	1.06	1.06	1.07	1.07	1.07	1.06	1.07



Figure 8: Model g_count, Factors by ia_band1 vs Actual/VBT2015

	ia_band1	18-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95+
A/VBT15	Observed	1.36	1.29	1.17	1.18	1.18	1.18	1.17	1.15	1.16	1.14	1.13	1.10	1.06	0.90	0.98	1.93
	Approximated	1.37	1.30	1.18	1.19	1.19	1.20	1.19	1.17	1.17	1.15	1.14	1.11	1.06	0.90	0.97	1.95
Factor	ia_band1	0.99	1.07	1.02	1.03	1.00	0.96	0.92	0.87	0.83	0.79	0.77	0.74	0.71	0.63	0.66	0.98
Avg	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
factors	class_key	1.20	1.16	1.15	1.14	1.14	1.15	1.16	1.18	1.20	1.22	1.23	1.24	1.24	1.23	1.20	1.27
	dur_band1	0.92	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.87	0.88	0.90	0.94	0.98	1.01	1.05	1.36
	face_amount_band	1.18	1.10	1.07	1.07	1.10	1.14	1.18	1.22	1.29	1.31	1.29	1.23	1.15	1.05	1.08	1.12
	gender	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.06	1.05	1.05	1.04	1.04	1.03	1.03	1.04	1.06
	insurance_plan	0.98	0.98	0.99	0.99	0.99	0.98	0.98	0.98	0.97	0.97	0.98	0.99	1.00	1.01	1.01	0.96
	iy_band1	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.04	1.05	1.05	1.04	1.03
	ltp	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	observation_year	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07





	dur_band1	01	02	03	04-05	06-10	11-15	16-20	21-25
A/VBT15	Observed	1.59	1.42	1.26	1.14	1.08	1.13	1.18	1.17
	Approximated	1.59	1.42	1.26	1.14	1.09	1.14	1.19	1.18
Factor	dur_band1	1.48	1.34	1.20	1.07	1.00	0.92	0.85	0.81
Avg	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
factors	class_key	1.07	1.06	1.06	1.06	1.08	1.15	1.22	1.23
	face_amount_band	1.03	1.03	1.02	1.02	1.04	1.14	1.27	1.30
	gender	1.07	1.07	1.06	1.06	1.06	1.06	1.05	1.06
	ia_band1	0.89	0.89	0.88	0.88	0.87	0.88	0.87	0.89
	insurance_plan	1.00	1.00	1.00	1.00	1.00	0.98	0.97	0.97
	iy_band1	1.03	1.03	1.04	1.05	1.05	1.04	1.03	1.01
	ltp	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00
	observation_year	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07



Figure 10: Model g_count, Factors by observation_year vs Actual/VBT2015

	observation_year	2009	2010	2011	2012	2013
A/VBT15	Observed	1.10	1.13	1.21	1.15	1.17
	Approximated	1.12	1.15	1.23	1.17	1.19
Factor	observation_year	1.00	1.04	1.09	1.06	1.09
Avg factors	Overall	0.93	0.93	0.93	0.93	0.93
	class_key	1.18	1.19	1.20	1.18	1.17
	dur_band1	0.90	0.88	0.89	0.89	0.90
	face_amount_band	1.21	1.22	1.23	1.20	1.19
	gender	1.06	1.06	1.06	1.06	1.06
	ia_band1	0.89	0.89	0.88	0.88	0.88
	insurance_plan	0.98	0.97	0.98	0.98	0.98
	iy_band1	1.02	1.02	1.03	1.03	1.04
	ltp	1.00	1.00	1.00	1.00	1.01



Figure 11: Model g_count, Factors by gender vs Actual/VBT2015

	gender	Female	Male
A/VBT15	Observed	1.15	1.17
	Approximated	1.17	1.19
Factor	gender	1.00	1.10
Avg factors	Overall	0.93	0.93
	class_key	1.21	1.17
	dur_band1	0.88	0.90
	face_amount_band	1.28	1.15
	ia_band1	0.86	0.89
	insurance_plan	0.97	0.98
	iy_band1	1.03	1.03
	ltp	1.00	1.00
	observation_year	1.07	1.07



Figure 12: Model g_count, Factors by face_amount_band vs Actual/VBT2015

	face_amount_band	1-9999	10000-	25000-	50000-	100000-	250000-	50000-	100000-	2500000-	500000-	1000000+
			24999	49999	99999	249999	499999	999999	2499999	4999999	9999999	
A/VBT15	Observed	1.36	1.29	1.26	1.19	1.02	0.92	0.88	0.87	0.93	0.88	0.84
	Approximated	1.36	1.29	1.26	1.19	1.03	0.93	0.89	0.88	0.95	0.89	0.85
Factor	face_amount_band	1.55	1.43	1.30	1.17	1.00	0.90	0.87	0.87	0.95	0.91	0.88
Avg	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
factors	class_key	1.29	1.23	1.22	1.20	1.13	1.06	1.04	1.03	1.06	1.09	1.07
	dur_band1	0.85	0.85	0.86	0.87	0.92	0.96	0.98	0.99	1.00	1.01	1.02
	gender	1.03	1.04	1.06	1.06	1.07	1.07	1.07	1.08	1.07	1.07	1.06
	ia_band1	0.82	0.86	0.88	0.90	0.91	0.91	0.91	0.89	0.83	0.78	0.78
	insurance_plan	0.93	0.95	0.99	1.00	1.00	1.00	1.00	1.00	1.01	1.02	1.02
	iy_band1	1.02	1.02	1.02	1.03	1.04	1.04	1.04	1.04	1.05	1.05	1.05
	ltp	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.00	1.00	1.00
	observation_year	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07	1.07

Model g_amount: Graphical Comparisons of A/2015VBT and Model Factor



Figure 13: Model g_amount, Factors by iy_band1 vs Actual/VBT2015

	iy_band1	-1989	1990-1999	2000-2009	2010-
A/VBT15	Observed	1.07	0.99	0.89	0.88
	Approximated	1.07	0.99	0.89	0.88
Factor	iy_band1	1.00	1.02	0.98	0.88
Avg factors	Overall	0.93	0.93	0.93	0.93
	class_key	1.13	1.10	1.08	1.05
	dur_band1	1.03	1.00	1.00	1.12
	face_amount_band	1.05	0.99	0.94	0.94
	gender	1.00	1.00	1.00	1.00
	ia_band1	0.97	0.96	0.94	0.96
	insurance_plan	0.96	0.99	1.01	1.00
	ltp	1.00	1.00	1.00	1.01
	observation_year	1.02	1.02	1.02	1.01





	class_key	NS	SM	SM	SM	Unk									
		21	22	31	32	33	41	42	43	44	nan	21	22	nan	nan
A/VBT15	Observed	0.85	1.21	0.69	0.79	1.04	0.68	0.83	0.96	1.19	0.99	0.81	0.97	1.07	1.32
	Approximated	0.85	1.21	0.69	0.79	1.04	0.68	0.83	0.96	1.19	0.99	0.81	0.97	1.07	1.33
Factor	class_key	1.00	1.42	0.82	0.96	1.28	0.80	0.99	1.14	1.42	1.10	0.93	1.11	1.14	1.31
Avg factors	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
	dur_band1	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.02	1.01	1.00	1.01	1.01	1.02
	face_amount_band	0.95	0.97	0.94	0.93	0.93	0.94	0.94	0.94	0.94	1.01	0.96	0.97	1.05	1.11
	gender	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00
	ia_band1	0.95	0.94	0.96	0.94	0.92	0.96	0.96	0.96	0.95	0.96	0.96	0.96	0.96	0.96
	insurance_plan	1.00	1.00	1.01	1.02	1.02	1.00	1.00	1.00	1.01	0.98	1.00	1.01	0.97	0.98
	iy_band1	0.99	0.99	0.98	0.98	0.97	0.97	0.97	0.96	0.96	1.00	0.98	0.98	1.00	1.00
	ltp	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	0.99	1.00	1.00	1.00	0.99
	observation_year	1.02	1.02	1.02	1.02	1.02	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.02



Figure 15: Model g_amount, Factors by insurance_plan vs Actual/VBT2015

	insurance_plan	Other	Perm	Term	UL	ULSG	VL	VLSG
A/VBT15	Observed	1.42	0.94	0.87	1.03	0.90	1.04	1.05
	Approximated	1.42	0.94	0.87	1.03	0.90	1.03	1.05
Factor	insurance_plan	1.57	0.88	1.00	1.05	1.01	1.04	1.11
Avg factors	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93
	class_key	1.10	1.13	1.03	1.13	1.12	1.13	1.12
	dur_band1	1.05	1.01	1.00	1.00	1.00	1.00	1.00
	face_amount_band	0.92	1.03	0.95	0.98	0.93	0.97	0.96
	gender	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	ia_band1	0.96	0.96	0.96	0.94	0.92	0.96	0.96
	iy_band1	0.95	1.00	0.98	0.99	0.97	1.00	0.98
	ltp	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	observation_year	1.01	1.02	1.02	1.02	1.02	1.02	1.02

Figure 16: Model g_amount, Factors by ltp vs Actual/VBT2015



	ltp	5 yr	10 yr	15 yr	20 yr	25 yr	30 yr	Not	Unknown
					or N/A			Level	
					(Not			Term	
					Term)				
A/VBT15	Observed	1.39	0.90	0.87	0.96	1.03	0.89	0.83	0.88
	Approximated	1.38	0.90	0.87	0.96	1.02	0.89	0.83	0.88
Factor	ltp	1.47	1.04	1.03	1.00	1.02	1.09	0.87	0.99
Avg factors	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
	class_key	1.07	1.05	1.02	1.11	1.17	0.97	1.08	1.03
	dur_band1	1.04	1.01	1.00	1.00	1.00	1.00	1.00	1.01
	face_amount_band	0.99	0.94	0.95	0.97	0.96	0.94	0.96	0.95
	gender	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00
	ia_band1	0.97	0.96	0.96	0.95	0.98	0.98	0.97	0.96
	insurance_plan	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	iy_band1	0.94	0.96	0.98	0.99	0.98	0.97	0.99	0.98
	observation_year	1.01	1.01	1.01	1.02	1.01	1.01	1.02	1.03



Figure 17: Model g_amount, Factors by ia_band1 vs Actual/VBT2015

	ia_band1	18-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84	85-89	90-94	95+
A/VBT15	Observed	1.09	1.02	0.91	0.91	0.94	0.91	0.91	0.91	0.98	1.03	0.98	0.90	0.94	0.73	0.61	1.21
	Approximated	1.09	1.02	0.91	0.91	0.94	0.90	0.91	0.91	0.98	1.03	0.98	0.90	0.94	0.73	0.61	1.22
Factor	ia_band1	1.04	1.03	0.96	0.98	1.00	0.95	0.94	0.93	0.97	1.00	0.96	0.90	0.90	0.68	0.62	1.13
Avg	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
factors	class_key	1.09	1.07	1.05	1.05	1.05	1.06	1.06	1.07	1.09	1.11	1.12	1.14	1.18	1.22	1.09	1.20
	dur_band1	1.03	1.01	1.01	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.15
	face_amount_band	1.02	0.99	0.97	0.96	0.96	0.97	0.97	0.98	0.99	0.99	0.96	0.93	0.93	0.92	0.95	0.96
	gender	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	insurance_plan	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.01	1.02	1.03	1.03	1.03	0.98
	iy_band1	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.97	0.89
	ltp	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
	observation_year	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.01



Figure 18: Model g_amount, Factors by dur_band1 vs Actual/VBT2015

	dur_band1	01	02	03	04-05	06-10	11-15	16-20	21-25
A/VBT15	Observed	1.01	0.94	0.88	0.85	0.89	0.92	1.01	1.06
	Approximated	1.00	0.94	0.88	0.86	0.89	0.92	1.01	1.06
Factor	dur_band1	1.23	1.13	1.03	0.97	1.00	0.98	1.00	1.03
Avg	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
factors	class_key	1.05	1.05	1.06	1.07	1.08	1.08	1.12	1.13
	face_amount_band	0.94	0.94	0.94	0.94	0.94	0.96	1.00	1.04
	gender	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	ia_band1	0.96	0.96	0.95	0.94	0.94	0.95	0.96	0.97
	insurance_plan	1.00	1.00	1.00	1.01	1.01	1.01	0.98	0.96
	iy_band1	0.90	0.91	0.94	0.97	0.98	1.00	1.02	1.01
	ltp	1.01	1.01	1.01	1.01	1.00	1.00	0.99	1.00
	observation_year	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02



Figure 19: Model g_amount, Factors by observation_year vs Actual/VBT2015

	observation_year	2009	2010	2011	2012	2013
A/VBT15	Observed	0.94	0.96	0.96	0.94	0.90
	Approximated	0.94	0.96	0.96	0.94	0.90
Factor	observation_year	1.00	1.03	1.04	1.03	0.99
Avg	Overall	0.93	0.93	0.93	0.93	0.93
factors	class_key	1.09	1.10	1.09	1.09	1.08
	dur_band1	1.01	1.01	1.00	1.00	1.00
	face_amount_band	0.98	0.98	0.97	0.96	0.96
	gender	1.00	1.00	1.00	1.00	1.00
	ia_band1	0.96	0.95	0.95	0.95	0.95
	insurance_plan	0.99	0.99	1.00	1.00	1.00
	iy_band1	0.99	0.99	0.99	0.98	0.98
	ltp	1.00	0.99	0.99	1.01	1.01



Figure 20: Model g_amount, Factors by gender vs Actual/VBT2015

	gender	Female	Male
A/VBT15	Observed	0.95	0.93
	Approximated	0.95	0.93
Factor	gender	1.00	0.99
Avg factors	Overall	0.93	0.93
	class_key	1.10	1.08
	dur_band1	1.00	1.01
	face_amount_band	0.98	0.96
	ia_band1	0.94	0.95
	insurance_plan	1.00	1.00
	iy_band1	0.99	0.99
	ltp	1.00	1.00
	observation_year	1.02	1.02



Figure 21: Model g_amount, Factors by face_amount_band vs Actual/VBT2015

face_amount_band.

	face_amount_band	1-	10000-	25000-	50000-	100000-	250000-	50000-	100000-	250000-	500000-	1000000+
		9999	24999	49999	99999	249999	499999	999999	2499999	4999999	9999999	
A/VBT15	Observed	1.34	1.27	1.24	1.17	1.00	0.91	0.88	0.87	0.93	0.88	0.83
	Approximated	1.35	1.27	1.24	1.17	1.00	0.92	0.88	0.87	0.93	0.88	0.83
Factor	face_amount_band	1.40	1.30	1.22	1.12	1.00	0.95	0.93	0.93	0.98	0.91	0.87
Avg	Overall	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
factors	class_key	1.16	1.14	1.14	1.13	1.11	1.08	1.06	1.06	1.09	1.12	1.10
	dur_band1	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00
	gender	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	ia_band1	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.95	0.93	0.92	0.92
	insurance_plan	0.89	0.92	0.96	0.99	0.99	1.00	1.00	1.00	1.01	1.02	1.02
	iy_band1	1.01	1.01	1.00	1.00	0.99	0.99	0.98	0.98	0.98	0.98	0.97
	ltp	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	observation_year	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02

Appendix 3: Model Formulae

The model formulae are expressed in *patsy*, an R-style formula specification language. While the syntax is different in other statistical software packages, the logic in the formulae below can be interpreted and translated to other languages.

The dependent variable is specified first, followed by the independent variables. Each independent variable is specified in this case with a C() because they are categorical variables. Each category has one type which is the default against which others are measured, specified as the "treatment" below.

The only difference between the formulae is the dependent variable.

The GLM would be executed as follows, using the *statsmodels* Python package:

```
import statsmodels.formula.api as smf
```

```
fit = smf.glm( data = data
```

- , formula = "(the formula below)"
- , offset = np.log(offset column)
- , family = sm.families.Poisson(link=sm.families.links.log)).fit()

Formula for model *g_count*, using offset column *expected_death_qx2015vbt_by_policy*:

number_of_deaths ~

- + C(class_key)
- + C(dur_band1, Treatment(reference='06-10'))
- + C(face_amount_band, Treatment(reference=' 100000-249999'))
- + C(ia_band1, Treatment(reference='40-44'))
- + C(gender)
- + C(observation_year)
- + C(insurance_plan, Treatment(reference='Term'))
- + C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))
- + C(iy_band1)

Formula for model *g_amount*, using offset column *expected_death_qx2015vbt_by_amount*:

death_claim_amount ~

- + C(class_key)
- + C(dur_band1, Treatment(reference='06-10'))
- + C(face_amount_band, Treatment(reference=' 100000-249999'))
- + C(ia_band1, Treatment(reference='40-44'))
- + C(gender)
- + C(observation_year)
- + C(insurance_plan, Treatment(reference='Term'))
- + C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))
- + C(iy_band1)

The tree model uses the following formula, and was run in R using the *rpart* package. The maximum depth and complexity parameter cp were chosen for presentation purposes. A model used for prediction should be tuned against holdback data and random forest techniques should be considered. In this case, the tree branches when the variance within a node is sufficiently reduced by some split within that node among the variables given. The variance is the weighted squared error of the observations from the mean when split, compared to the mean when not split. Note that the data are somewhat aggregated before receipt by the ILEC, so variance within the individual records in the source data gets lost.

tree_count <- rpart(ae_count ~ preferred_class</pre>

- +number_of_preferred_classes +smoker_status +dur_band1_min +face_min +ia_band1_min +gender +observation_year +ltp +iy_band1_min
- , data=data g
- , maxdepth=8
- , cp=0.0001
- , weights=expected_death_qx2015vbt_by_policy)

Appendix 4: Statistical Model Output

The model factors as output by the Python package *statsmods* include statistical fit metrics as shown below. The columns are:

- The variable for which a factor is calculated. The variable "C(face_amount_band, Treatment(reference=' 100000-2499999'))[T. 250000-4999999]" is an indicator variable of 1 if the face amount band is 250000-499999, and zero otherwise.
- The natural logarithm of the adjustment factor to be applied to the adjusted metric (expecteds using VBT15)
- The standard error of the coefficient
- The z score: normalizing the coefficient as if it is a normally distributed variant with mean zero and standard deviation of the standard error
- P>|z|, the probability that a standard normal would be at least as large as the coefficient
- The 95% confidence interval of the estimate of the coefficient, calculated as the middle 95% of a normal distribution with mean of the coefficient and standard error shown.

Model factors are applied by adding up all applicable factors for a given cell of data, and exponentiating the result. Each category has one value which has no adjustment factor and serves as the default category, such as duration band 06-10 for the duration category.

Note that or fits of amount, the statistical descriptors have no meaning.

The model factors fit that category in total. The statistical values such as confidence intervals do not reflect the correlation between amounts. The model effectively counts each dollar of benefit as a separate trial.

This issue is the same issue as having multiple policies on one life, but much more extreme. It is clear from the z-scores of the amount model that the correlations between individual dollars are not considered, as they are the equivalent of standard normal variates that come out in the thousands in many cases.

Model fit statistics: model g_count

	coef	std err	Z	P> z	[0.025	0.975]
Intercept	-0.0763	0.014	-5.519	0.000	-0.103	-0.049
C(class_key)[T.Nonsmoker 2 2]	0.3665	0.008	47.431	0.000	0.351	0.382
C(class_key)[T.Nonsmoker 3 1]	-0.2786	0.012	-22.405	0.000	-0.303	-0.254
C(class_key)[T.Nonsmoker 3 2]	-0.0788	0.011	-6.908	0.000	-0.101	-0.056
C(class_key)[T.Nonsmoker 3 3]	0.1978	0.009	20.880	0.000	0.179	0.216
C(class_key)[T.Nonsmoker 4 1]	-0.3444	0.013	-25.987	0.000	-0.370	-0.318
C(class_key)[T.Nonsmoker 4 2]	-0.1136	0.013	-8.562	0.000	-0.140	-0.088
C(class_key)[T.Nonsmoker 4 3]	0.0278	0.016	1.789	0.074	-0.003	0.058
C(class_key)[T.Nonsmoker 4 4]	0.1978	0.015	13.536	0.000	0.169	0.226
C(class_key)[T.Nonsmoker nan]	0.1892	0.007	26.930	0.000	0.175	0.203
C(class_key)[T.Smoker 2 1]	-0.1375	0.013	-10.387	0.000	-0.163	-0.112
C(class_key)[T.Smoker 2 2]	0.1255	0.012	10.296	0.000	0.102	0.149
C(class_key)[T.Smoker nan]	0.1699	0.008	22.436	0.000	0.155	0.185
C(class_key)[T.Unknown nan]	0.3669	0.009	42.502	0.000	0.350	0.384
C(dur_band1, Treatment(reference='06-10'))[T.01]	0.3910	0.017	22.390	0.000	0.357	0.425
C(dur_band1, Treatment(reference='06-10'))[T.02]	0.2945	0.015	19.832	0.000	0.265	0.324
C(dur_band1, Treatment(reference='06-10'))[T.03]	0.1791	0.013	14.191	0.000	0.154	0.204
C(dur_band1, Treatment(reference='06-10'))[T.04-05]	0.0685	0.008	8.393	0.000	0.052	0.084
C(dur_band1, Treatment(reference='06-10'))[T.11-15]	-0.0787	0.007	-11.871	0.000	-0.092	-0.066
C(dur_band1, Treatment(reference='06-10'))[T.16-20]	-0.1613	0.008	-19.528	0.000	-0.177	-0.145
C(dur_band1, Treatment(reference='06-10'))[T.21-25]	-0.2089	0.009	-23.577	0.000	-0.226	-0.192
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 1-9999]	0.4380	0.006	73.809	0.000	0.426	0.450
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 10000-249999]	0.3590	0.005	70.633	0.000	0.349	0.369
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 25000-49999]	0.2614	0.005	52.435	0.000	0.252	0.271
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 50000-99999]	0.1557	0.005	33.302	0.000	0.147	0.165
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 250000-499999]	-0.1052	0.006	-17.533	0.000	-0.117	-0.093
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 500000-999999]	-0.1446	0.008	-18.493	0.000	-0.160	-0.129
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 1000000-2499999]	-0.1443	0.010	-14.675	0.000	-0.164	-0.125
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 2500000-4999999]	-0.0485	0.025	-1.970	0.049	-0.097	-0.000
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 5000000-9999999]	-0.0968	0.032	-3.030	0.002	-0.159	-0.034
C(face_amount_band, Treatment(reference=' 100000-249999'))[T.1000000+]	-0.1330	0.052	-2.551	0.011	-0.235	-0.031
C(ia_band1, Treatment(reference='40-44'))[T.18-24]	-0.0111	0.012 0.010	-0.927	0.354	-0.035	0.012
C(ia_band1, Treatment(reference='40-44'))[T.25-29]	0.0717 0.0197	0.010	7.324 2.419	0.000	0.052	0.091
C(ia_band1, Treatment(reference='40-44'))[T.30-34] C(ia_band1, Treatment(reference='40-44'))[T.35-39]	0.0197	0.007	3.625	0.000	0.012	0.036
C(ia_band1, Treatment(reference='40-44'))[T.45-49]	-0.0377	0.006	-6.020	0.000	-0.050	-0.025
C(ia_band1, Treatment(reference='40-44'))[T.50-54]	-0.0864	0.006	-14.194	0.000	-0.098	-0.025
C(ia_band1, Treatment(reference='40-44'))[T.55-59]	-0.1411	0.006	-23.737	0.000	-0.153	-0.129
C(ia_band1, Treatment(reference='40-44'))[T.60-64]	-0.1872	0.006	-31.913	0.000	-0.199	-0.176
C(ia_band1, Treatment(reference='40-44'))[T.65-69]	-0.2344	0.006	-38.262	0.000	-0.246	-0.222
C(ia_band1, Treatment(reference='40-44'))[T.70-74]	-0.2672	0.007	-38.745	0.000	-0.281	-0.254
C(ia_band1, Treatment(reference='40-44'))[T.75-79]	-0.3063	0.009	-35.634	0.000	-0.323	-0.289
C(ia_band1, Treatment(reference='40-44'))[T.80-84]	-0.3428	0.013	-27.249	0.000	-0.367	-0.318
C(ia_band1, Treatment(reference='40-44'))[T.85-89]	-0.4646	0.027	-17.281	0.000	-0.517	-0.412
C(ia_band1, Treatment(reference='40-44'))[T.90-94]	-0.4088	0.097	-4.201	0.000	-0.600	-0.218
C(ia_band1, Treatment(reference='40-44'))[T.95+]	-0.0249	0.129	-0.193	0.847	-0.278	0.228
C(gender)[T.Male]	0.0974	0.003	35.214	0.000	0.092	0.103
C(observation_year)[T.2010]	0.0397	0.006	7.106	0.000	0.029	0.051
C(observation_year)[T.2011]	0.0890	0.005	16.629	0.000	0.079	0.100
C(observation_year)[T.2012]	0.0610	0.006	11.060	0.000	0.050	0.072
C(observation_year)[T.2013]	0.0818	0.006	14.524	0.000	0.071	0.093
C(insurance_plan, Treatment(reference='Term'))[T.Other]	0.2729	0.089	3.052	0.002	0.098	0.448
C(insurance_plan, Treatment(reference='Term'))[T.Perm]	-0.0770	0.008	-9.649	0.000	-0.093	-0.061
C(insurance_plan, Treatment(reference='Term'))[T.UL]	0.0553	0.008	6.973	0.000	0.040	0.071
C(insurance_plan, Treatment(reference='Term'))[T.ULSG]	0.0148	0.009	1.567	0.117	-0.004	0.033
C(insurance_plan, Treatment(reference='Term'))[T.VL]	-0.0354	0.009	-3.842	0.000	-0.054	-0.017
C(insurance_plan, Treatment(reference='Term'))[T.VLSG]	-0.0423	0.013	-3.143	0.002	-0.069	-0.016
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T. 5 yr]	0.1661	0.035	4.736	0.000	0.097	0.235



C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.10 yr]	0.0304	0.011	2.744	0.006	0.009	0.052
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.15 yr]	0.0922	0.011	8.373	0.000	0.071	0.114
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.25 yr]	0.2289	0.027	8.390	0.000	0.175	0.282
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.30 yr]	0.0317	0.017	1.844	0.065	-0.002	0.065
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.Not Level Term]	-0.1116	0.011	-9.737	0.000	-0.134	-0.089
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.Unknown]	0.0033	0.009	0.381	0.703	-0.014	0.020
C(iy_bandl)[T.1990-1999]	0.0329	0.005	7.253	0.000	0.024	0.042
C(iy_band1)[T.2000-2009]	0.0529	0.008	6.357	0.000	0.037	0.069
C(iy_band1)[T.2010-]	0.0190	0.016	1.179	0.239	-0.013	0.051

Model fit statistics: model g_amount

					==============	
	coef	std err	Z	P> z	[0.025	0.975]
Intercept	-0.0683	3.36e-05	-2032.213	0.000	-0.068	-0.068
C(class_key)[T.Nonsmoker 2 2]	0.3515	1.51e-05	2.32e+04	0.000	0.351	0.352
C(class key)[T.Nonsmoker 3 1]	-0.1941		-9827.611	0.000	-0.194	-0.194
C(class_key)[T.Nonsmoker 3 2]	-0.0395	1.77e-05	-2234.296	0.000	-0.040	-0.039
C(class_key)[T.Nonsmoker 3 3]	0.2489	1.6e-05	1.56e+04	0.000	0.249	0.249
C(class_key)[T.Nonsmoker 4 1]	-0.2183	2.11e-05	-1.04e+04	0.000	-0.218	-0.218
C(class_key)[T.Nonsmoker 4 2]	-0.0052	2.21e-05	-236.867	0.000	-0.005	-0.005
C(class_key)[T.Nonsmoker 4 3]	0.1312	2.56e-05	5124.140	0.000	0.131	0.131
C(class_key)[T.Nonsmoker 4 4]	0.3509	2.53e-05	1.39e+04	0.000	0.351	0.351
C(class_key)[T.Nonsmoker nan]	0.0935	1.4e-05	6690.558	0.000	0.093	0.093
C(class_key)[T.Smoker 2 1]	-0.0714	2.84e-05	-2518.049	0.000	-0.071	-0.071
C(class key)[T.Smoker 2 2]	0.1030	2.91e-05	3539.230	0.000	0.103	0.103
C(class_key)[T.Smoker nan]	0.1301	1.9e-05	6830.527	0.000	0.130	0.130
C(class_key)[T.Unknown nan]	0.2693	4.05e-05	6654.020	0.000	0.269	0.269
C(dur_band1, Treatment(reference='06-10'))[T.01]	0.2104	3.54e-05	5942.458	0.000	0.210	0.210
C(dur_band1, Treatment(reference='06-10'))[T.02]	0.1259	2.9e-05	4338.172	0.000	0.126	0.126
C(dur_band1, Treatment(reference='06-10'))[T.03]	0.0266	2.34e-05	1137.950	0.000	0.027	0.027
C(dur_band1, Treatment(reference='06-10'))[T.04-05]	-0.0353	1.41e-05	-2505.406	0.000	-0.035	-0.035
C(dur_band1, Treatment(reference='06-10'))[T.11-15]	-0.0182	1.38e-05	-1315.605	0.000	-0.018	-0.018
C(dur_band1, Treatment(reference='06-10'))[T.16-20]	-0.0015	2.05e-05	-75.570	0.000	-0.002	-0.002
C(dur_band1, Treatment(reference='06-10'))[T.21-25]	0.0279	2.41e-05	1158.147	0.000	0.028	0.028
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 1-9999]	0.3329	4.73e-05	7039.162	0.000	0.333	0.333
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 10000-24999]	0.2608	2.75e-05	9498.876	0.000	0.261	0.261
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 25000-49999]	0.2028	2.15e-05	9439.285	0.000	0.203	0.203
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 50000-99999]	0.1162	1.61e-05	7211.599	0.000	0.116	0.116
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 250000-499999]	-0.0528	1.27e-05	-4155.019	0.000	-0.053	-0.053
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 500000-999999]	-0.0762	1.29e-05	-5916.013	0.000	-0.076	-0.076
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 1000000-2499999]	-0.0743	1.23e-05	-6048.979	0.000	-0.074	-0.074
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 2500000-4999999]	-0.0227		-1352.306	0.000	-0.023	-0.023
C(face_amount_band, Treatment(reference=' 100000-249999'))[T. 5000000-9999999]	-0.0959	1.73e-05	-5557.401	0.000	-0.096	-0.096
C(face_amount_band, Treatment(reference=' 100000-249999'))[T.10000000+]	-0.1445	1.84e-05	-7870.527	0.000	-0.145	-0.144
C(ia_band1, Treatment(reference='40-44'))[T.18-24]	0.0378	4.29e-05	880.643	0.000	0.038	0.038
C(ia_band1, Treatment(reference='40-44'))[T.25-29]	0.0324	2.78e-05	1164.636	0.000	0.032	0.032
C(ia band1, Treatment(reference='40-44'))[T.30-34]	-0.0429	2.04e-05	-2104.240	0.000	-0.043	-0.043
C(ia_band1, Treatment(reference='40-44'))[T.35-39]	-0.0245	1.73e-05	-1422.481	0.000	-0.025	-0.025
C(ia_band1, Treatment(reference='40-44'))[T.45-49]	-0.0523	1.59e-05	-3284.498	0.000	-0.052	-0.052
C(ia_band1, Treatment(reference='40-44'))[T.50-54]	-0.0611	1.59e-05	-3847.242	0.000	-0.061	-0.061
C(ia_band1, Treatment(reference='40-44'))[T.55-59]	-0.0766		-4801.886	0.000	-0.077	-0.077
C(ia_band1, Treatment(reference='40-44'))[T.60-64]	-0.0332	1.63e-05	-2033.567	0.000	-0.033	-0.033
C(ia_band1, Treatment(reference='40-44'))[T.65-69]	0.0024	1.72e-05	141.817	0.000	0.002	0.002
C(ia_band1, Treatment(reference='40-44'))[T.70-74]	-0.0429	1.85e-05	-2326.576	0.000	-0.043	-0.043
C(ia_band1, Treatment(reference='40-44'))[T.75-79]	-0.1108	1.96e-05	-5641.219	0.000	-0.111	-0.111
C(ia_band1, Treatment(reference='40-44'))[T.80-84]	-0.1029	2.18e-05	-4717.252	0.000	-0.103	-0.103
C(ia_band1, Treatment(reference='40-44'))[T.85-89]	-0.3833	3.48e-05	-1.1e+04	0.000	-0.383	-0.383
C(ia_band1, Treatment(reference='40-44'))[T.90-94]	-0.4800	0.000	-2885.744	0.000	-0.480	-0.480
C(ia_band1, Treatment(reference='40-44'))[T.95+]	0.1228	0.000	389.104	0.000	0.122	0.123
C(gender)[T.Male]	-0.0062	8.01e-06	-780.339	0.000	-0.006	-0.006
C(observation_year)[T.2010]	0.0296	1.61e-05	1841.690	0.000	0.030	0.030
C(observation_year)[T.2011]	0.0401	1.53e-05	2616.114	0.000	0.040	0.040
C(observation_year)[T.2012]	0.0249	1.55e-05	1607.572	0.000	0.025	0.025
C(observation_year)[T.2013]	-0.0119	1.58e-05	-749.458	0.000	-0.012	-0.012
C(insurance_plan, Treatment(reference='Term'))[T.Other]	0.4504	0.000	2974.392	0.000	0.450	0.451
C(insurance_plan, Treatment(reference='Term'))[T.Perm]	-0.1313	1.83e-05	-7184.902	0.000	-0.131	-0.131
C(insurance_plan, Treatment(reference='Term'))[T.UL]	0.0533	1.7e-05	3128.422	0.000	0.053	0.053
C(insurance_plan, Treatment(reference='Term'))[T.ULSG]	0.0094	1.8e-05	525.518	0.000	0.009	0.009
C(insurance_plan, Treatment(reference='Term'))[T.VL]	0.0396	1.95e-05	2030.704	0.000	0.040	0.040
C(insurance_plan, Treatment(reference='Term'))[T.VLSG]	0.1007	2.82e-05	3568.473	0.000	0.101	0.101

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C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.10 yr]	0.0428	2.04e-05	2097.696	0.000	0.043	0.043
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.15 yr]	0.0340	2.16e-05	1573.938	0.000	0.034	0.034
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.25 yr]	0.0220	6.58e-05	334.878	0.000	0.022	0.022
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.30 yr]	0.0847	2.98e-05	2842.270	0.000	0.085	0.085
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.Not Level Term]	-0.1402	2.61e-05	-5372.097	0.000	-0.140	-0.140
C(ltp, Treatment(reference='20 yr or N/A (Not Term)'))[T.Unknown]	-0.0085	1.69e-05	-503.242	0.000	-0.009	-0.008
C(iy_band1)[T.1990-1999]	0.0154	1.8e-05	855.897	0.000	0.015	0.015
C(iy_band1)[T.2000-2009]	-0.0243	2.45e-05	-989.059	0.000	-0.024	-0.024
C(iy_band1)[T.2010-]	-0.1314	3.75e-05	-3500.907	0.000	-0.132	-0.131

Figure 22: Tree model by count

Displaying a larger tree can be difficult and less helpful than a smaller tree. One of the challenges of trees is tracing where one variable causes a branch, and in relationship with which other variables. An advantage is that the trees can describe reality more closely than a model with a less flexible structure.



The tree shows the A/2015VBT by count at each branch. For example, the top node shows the overall 1.155 A/2015VBT without branching.

Each node in the tree shows a criterion on which the next branch is made, with "yes" to the left and "no" to the right. The top node's criterion is whether the lower bound of the face amount band, face_min, is above or below 75,000. If not, the tree branches to the right, with a higher (darker) value than for the higher face amount bands to the left.

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The Society of Actuaries (SOA), formed in 1949, is one of the largest actuarial professional organizations in the world dedicated to serving 24,000 actuarial members and the public in the United States, Canada and worldwide. In line with the SOA Vision Statement, actuaries act as business leaders who develop and use mathematical models to measure and manage risk in support of financial security for individuals, organizations and the public.

The SOA supports actuaries and advances knowledge through research and education. As part of its work, the SOA seeks to inform public policy development and public understanding through research. The SOA aspires to be a trusted source of objective, data-driven research and analysis with an actuarial perspective for its members, industry, policymakers and the public. This distinct perspective comes from the SOA as an association of actuaries, who have a rigorous formal education and direct experience as practitioners as they perform applied research. The SOA also welcomes the opportunity to partner with other organizations in our work where appropriate.

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Society of Actuaries 475 N. Martingale Road, Suite 600 Schaumburg, Illinois 60173 www.SOA.org