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SALARY SCALES

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INTRODUCTION

THIS paper surveys the actuarial techniques connected with salary scale mathematics and presents and examines some new data in the hope that others may be stimulated to follow with further statistical surveys and that the implications of the statistics may be readily available to all.

The use of a salary scale was proposed in the early 1900's to solve the technical problem of estimating the cost of a future payment related to the future salary of an employee. The actuarial present value of such a payment is the product of:

(i) the interest factor,
(ii) the probability of the event occasioning the payment, and
(iii) the amount of the payment.

The interest rate is an initial assumption, the probability of the event is determined from tables derived from relevant statistics, and it remains to complete the solution of the problem by estimating the amount of the future payment. A figure may be obtained for this from the amount of the employee's current salary and information as to salary increases in the future. The procedure is to develop a table of expected increases from this information, and when the increases are applied to a radix to give a sequence of absolute values the result is known as a "salary scale."

To proceed from the particular to the general, it was necessary to take the step of incorporating this salary scale into commutation columns. This step was, I believe, first taken by H. W. Manly and the technique was subsequently elaborated by him and by George King in their classic papers on pension funds.[1]

The salary scale is of prime importance in the calculation of liabilities for pension plans provided for salaried employees. In these plans the

[1] Manly, *JIA* 36, 209; 37, 193; 38, 101. King, *JIA* 39, 129.

pension is usually related to the salary of the employee in some manner such as a stated fraction of the "average salary during the period of employment" or of the "average salary of the n years preceding retirement" and so on. Since the compensation of these employees is customarily increased year by year to match increasing responsibilities, it follows that the pension liabilities are similarly increased, and, in fact, that a proper estimate of the liabilities for the pensions cannot be made without an attempt to forecast the salary increments. In the larger employments, for consistent and equitable control of salary increments, it has been the practice to establish grades within which progression from the minimum to the maximum salary for the grade occurs in stipulated increments related to the period of service within the grade. The effect on the salary history of the individual employee is a steady increase in compensation with age, with occasional "flats" while a move from one grade to the next is awaited. In the bulk statistics of employees, grouped by age or by duration of service, all these individual salary movements are combined to exhibit a steady rate of increase of compensation calendar year by calendar year, age by age, or service duration by service duration. There is a mass of information available on these phenomena which requires only examination and development.

THE SALARY SCALE TECHNIQUE

In making cost estimates the salary scale is combined with the total actual compensation of a group in such a way as to apply the proportionate increases age by age to the total compensation of the group and thus to estimate the future compensation of the group.

In symbols:

let $(TAS)_x$ be the present total compensation of a group of

n_x employees of mean age x,

and let s_x denote the salary scale value at age x.

Then the application of the method increases the value of compensation of the n_x employees by the end of t years to

$$(TAS)_x \cdot s_{x+t}/s_x \,.$$

The application of the probabilities of survival in service from the service table adopted reduces the volume of compensation to correspond with the expected survivors. In the calculation the final expression for the expected future compensation of the survivors of the n_x original employees would be:

$$(TAS)_x \cdot (s_{x+t}/s_x) \cdot {}_tp'_x \,,$$

where ${}_tp'_x$ is the probability of survival in service derived from a service table which may include deaths, withdrawals and disability retirements or other causes of decrement.

Ignoring decrements, the expected group increment in the volume of salaries in one year is:

$$(TAS)_x(s_{x+1}/s_x - 1),$$

and the assumption of the method will be satisfied so long as the sum of the compensation increments to the individual members of the group do not exceed the total group expected increment. The individual increments are thus not of importance, though specially large increments introduce their own problems. Within the method one individual may take up a large portion of the expected increment, and provided the other members satisfactorily refrain from acquiring compensation increases in the aggregate more than the balance of the expected increment, no strain will ensue. Manly, *JIA* 36, 227, in developing the method points out that the application of the salary scale is relative to the total salaries of the group and does not mean that each individual employee's salary is expected to rise year by year in exact proportion to the scale values.

In practice the effect of actual separations from active service is eliminated by considering only those persons who survived the year or exposure. Thus the expected salaries may be taken as

(total of salaries at age x of those who survive to $x + 1$) $\times$ s_{x+1}/s_x and the corresponding actual salaries would then be

(total of salaries at age $x + 1$ of those who survive to age $x + 1$).

METHODS OF DERIVING SALARY SCALES

Any method of deriving a salary scale can be relied upon to produce a satisfactory result only if it is applied to a homogeneous group of employees. It has been assumed that the procedure which follows will be applied to such a group.

There are two main methods of deriving salary scales from any available data. The first method may be referred to as the Current Average Method and the second as the Increase Ratio Method.

It seems that the Current Average Method was chronologically first in the field. The first step in it is the derivation of the actual average salaries of employees, probably in quinary age groups but possibly in single age groups if the volume of the data permits. For the dual purpose of reducing the amount of the calculations and producing as smooth a sequence of figures as possible it has always seemed most satisfactory to use the quinary averages. A series of figures representing the current average salary of present employees is thus obtained at five-year age intervals. No figure in this series has any relation to any other figure, but practically the assumption is made that the figures do give an indication of the progression. These averages may be supplemented by other relevant information. For instance, a similar series may be available from statistics of the year before, or at a date five years earlier or ten years earlier. Current salary policies may be in the course of change. Their effect may require an adjustment to past average salaries. It may be noted that the originator of the Current Salary Method did not divide his statistics according to employment grade or class. He obtained his average salaries from the statistics of all persons, separating only men from women. All persons of the same sex in the salaried group of employees were thus included, from office boy to president. The next stage in the proceeding was to graduate the crude values, usually by the graphic method. A tendency for average salaries to exhibit lower figures in the age groups over 60 was remarked and smoothed out of the final curve. Individual values were then read off the final curve and the individual first differences smoothed by eye.

The Increase Ratio Method was suggested by Mr. James Bacon in 1907.[2] He proposed that the card records of persons at the valuation date should be marked with their current salary rate. The new entrants in the preceding five years would then be taken out and, for the remainder, the salaries at the point five years earlier would be recorded. A similar procedure would record the salaries ten years earlier for those who survived to the five years earlier date. Grouping the cards in quinary groups of ages, salary increase ratios over five-year periods were obtained. After recording these at each fifth age, a salary scale was obtained by successive multiplication. The quinquennial point values were smoothed graphically and the individual values were read off. There was controversy over the relative merits of the two methods. A protagonist of the Current Average Method commented as follows:

> Which was the most likely to give a correct estimate of those future salary scales:—a scale based on the salary actually being paid at the moment of observation, or one based on an event that happened five or ten years previously? A scale based on the experience of all existing members? Or one from which a large proportion of members was omitted? A scale giving full weight to the actual salaries at each age? Or one which was built up by a series of ratios upon the foundation of a deduced salary at one particular age?

The latter reference is, of course, to the fact that under the Increase Ratio Method a radix at the earliest age is necessary. It does not seem to

[2] *JIA* 42, 46.

have been recognized that the scale figures themselves do not need to bear a resemblance to salaries as long as they are in proper relationship to each other. Equally, in the discussion on the technique, no thought is apparent as to the effect of inflation on the results of the two methods.

In the discussion, however, emphasis is laid upon the application of experience, wisdom and skill to effect an "intelligent graduation" of the results; it was pointed out that "past experience is only an indication of what is going to happen in the future and that it is necessary to take into consideration many events which affect the scale in years to come."

DERIVATION OF SALARY SCALES FROM CURRENT DATA

In order to exhibit the effect of the two approaches to the salary scale problem I have had recourse to statistics of two large pension plans, kindly supplied to me by their actuaries. The plans are referred to as Plan A and Plan B. Plan A statistics derive from a wholly clerical operation, while Plan B includes a substantial number of technicians and manual operators. Both plans cover men and women employees and the relevant statistics for each sex were provided.

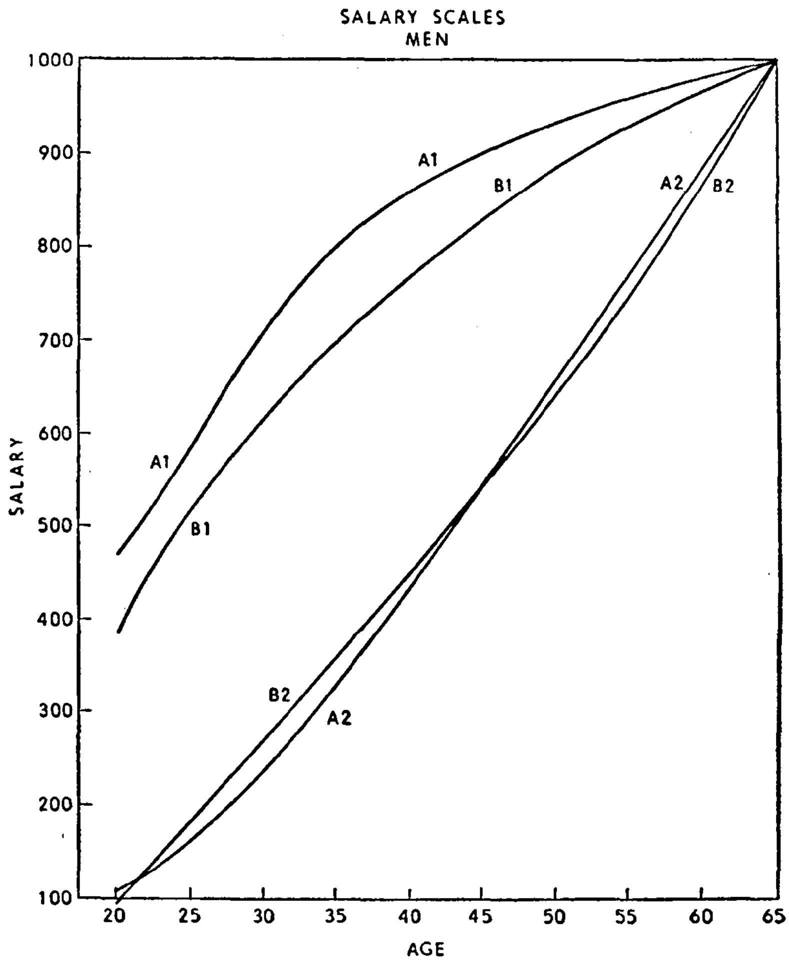
In the statistics of Plan A it was possible to examine the average salaries in 5-year age groups of both male and female employees at two points of time five years apart, 1955 and 1960. For each of the intervening five years, the data supplied included the actual total salaries of those participants, grouped five consecutive ages together, who were employed both at the beginning of the year and at the end of the year. Thus it was possible to determine the annual increase rate exhibited in each of the five years at the central age of the group. In these statistics the age group is constant; employees enter on attaining the earliest age and leave on attaining the entry age of the next group.

From the statistics supplied in respect of Plan B, the average salaries at a date in 1954 and at the same date in 1959 were obtainable and were produced in the same 5-year age groups. The 1959 data also included the number and salaries of the survivors of the employees who had constituted the groups taken in the 1954 data. Thus average salaries of employees in an age group in 1954 could be compared with the average salaries of the survivors of the same employees five years later.

In developing the salary scales, the methods described earlier were employed. For the current salary method, the absolute salary values at the fifth age points were plotted, a smooth curve was drawn, and the values at each age were read off and "polished" by adjusting first differences. Substantially the same method was employed for the salary increase method except that the relative increase ratios were graduated graphically and a salary scale developed by continuous multiplication.

The unadjusted average salaries and ratios derived from the data are shown in Appendix 1, while the salary scales developed are tabulated in Appendix 2. In developing the salary scales by the Increase Ratio Method, the increases in the cost of living have been subtracted from the smoothed ratios of increase developed. The cost of living increases were determined from the values of the Consumers Price Index published by the Bureau of Labor Statistics. In the calculations on the Plan A data the

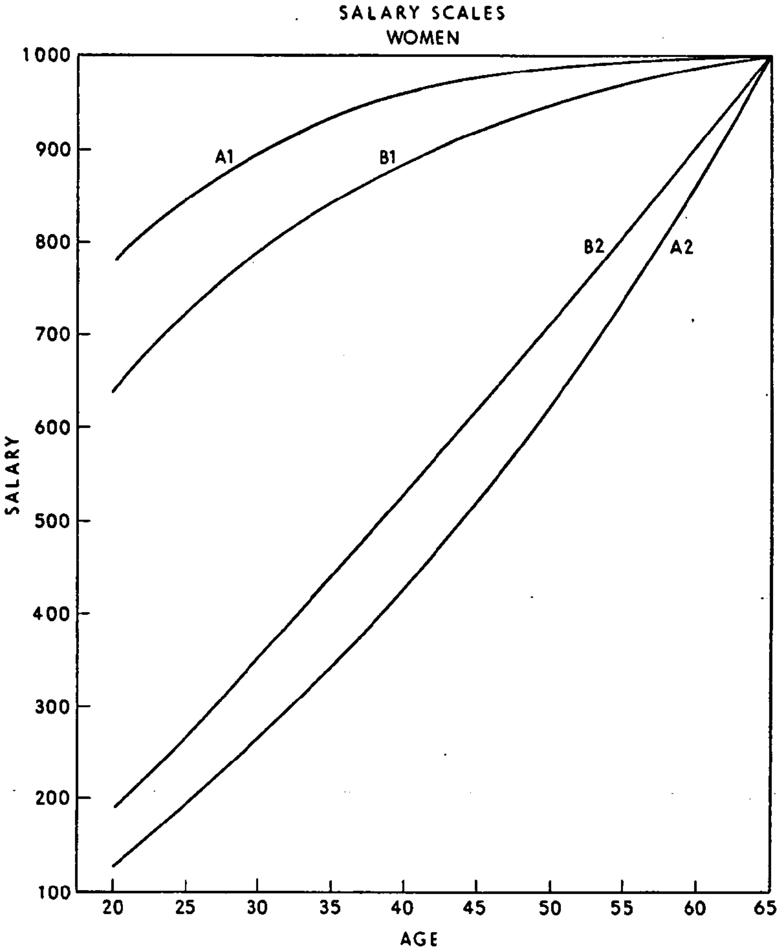
CHART 1



deduction from each age ratio was .02 and for Plan B the deduction was .096 from the five-year rate.

The scale of values derived as current averages at the beginning and end of the five-year period exhibit a characteristic commonly found on salary scale data. The current average scale at the end of the period is found to be slightly less steep than its counterpart at the beginning of the period. This occurs in each of the four sets of average salaries—*i.e.*, Plan A, Men; Plan A, Women: Plan B, Men; and Plan B, Women. This characteristic was found in a more extreme form in wartime salary statis-

CHART 2



tics, especially where cost-of-living increases were granted as a flat amount to all persons irrespective of age or salary. In general, increases in compensation due to the generally inflationary economic conditions under which we live may be expected to produce this effect. The commencing salary is more immediately sensitive than is the salary of an established, long-service employee. The real move in the level of compensation of the latter may not be realized in full until the current incumbent is replaced by his successor. This poses a problem to actuaries in dealing with the current statistics of a plan.

In Charts 1 and 2, the graduated salary scale curves are exhibited. The curves on the upper left-hand side of the charts will be all recognized as typical salary scale curves; the curves for the salary scales for men are steeper than the curves for the salary scales for the women. The curve A1 also shows traces of the reverse ogive curves which were typical twenty years or more ago.

The curves derived by Method 2 are startling, since they are practically straight lines and practically all four the same straight line, namely, the line passing through the two points (20, 125) and (65, 1000). By this line the salary at 65 is 8 times the salary at 20, as compared with the approximate maximums of $4\frac{1}{2}$ times for men and $2\frac{1}{2}$ times for women which appear to be currently in use.

CONTRIBUTION RATES

In order to exhibit the effect of the salary scales developed above, contribution rates have been calculated, as a level percentage of salary, for a pension formula of, firstly, one percent of the average salary of the five years preceding retirement multiplied by the years of membership of the plan (referred to as Final Five basis) and, secondly, one percent of the total salary received during employment (referred to as the Career basis).

The other calculation bases employed are:

Service Tables	Men: *Pension Handbook*, T5*
	Women: *Pension Handbook*, T9*
Interest	3%
Pensioners Mortality Table	Men: Ga–1951 Men at $x-1$
	Women: Ga–1951 Men at $x-6$
Normal Retirement Age	65

There is no disability benefit included in the calculations.

* See Appendix 4.

For ready reference, the salary scales will be indicated as follows:

A1M	Plan A, Method 1, Men	(Current Average Scale)
A1F	Plan A, Method 1, Women	(Current Average Scale)
A2M	Plan A, Method 2, Men	(Increase Ratio Scale)
A2F	Plan A, Method 2, Women	(Increase Ratio Scale)
B1M	Plan B, Method 1, Men	(Current Average Scale)
B1F	Plan B, Method 1, Women	(Current Average Scale)
B2M	Plan B, Method 2, Men	(Increase Ratio Scale)
B2F	Plan B, Method 2, Women	(Increase Ratio Scale)

The contribution rates will be found in Table 1.

As could be expected, the effect of the switch from Method 1 scales to Method 2 scales is more marked in the contribution rates for women than for men. The reason is that the Method 2 scales for men and women are very close together, and there is more "lift" to get there from the Method 1 women's scales than there is from the corresponding men's scales. The effect of the switch is more marked for the Final Five contribution rates than for the Career contribution rates. This is again a logical difference.

In the sequence of contribution rates on the same plan, it may be noted that the contribution rates for men start at a higher figure than for women and end with a slightly lower contribution rate. The contribution rates for women at age 60 are raised above the corresponding rate for men by the 5-year setback in the mortality table for pensioners. At the beginning of the table the higher withdrawal rates for women reduce the calculated contribution rates to figures below those for men at the corresponding ages.

The real questions are raised by the ratios in columns (3) and (6) of the table. Taking age 30 as an entry age, it appears that the entry age normal rate for men would be increased by 50% to 75% for the Final Five pension formula and 15% to 25% for the Career formula. For women an entry age of 20 is probably appropriate, but at this age the contribution rates are trifling and even the trebling of them that seems required for the Final Five formula should not cause trouble. For the Career formula the contribution rate needs to be doubled. In both cases it is probable that the contribution rate actually being paid for the whole plan may be adequate on both salary scales, since it is usually higher than the small rates derived in the present calculations.

If age 45 be taken as an average attained age, the increases are indicated for the Final Five formula to be about 20% for men and somewhat higher for women. The Career formula develops a lesser increase at 5% for men and 5% to 8% for womem.

TABLE 1

CONTRIBUTION RATES
INTEREST 3%

AGE	(1)	(2)	(3)	(4)	(5)	(6)
	Final Five					
	CONTRIBUTION RATES		RATIO (2)/(1)	CONTRIBUTION RATES		RATIO (5)/(4)
	A 1 M	A 2 M		A 1 F	A 2 F	
20......	2.22%	5.18%	2.33	0.35%	1.33%	3.80
30......	3.66	6.39	1.75	1.72	3.86	2.24
40......	5.73	7.71	1.35	5.06	7.68	1.52
50......	7.96	8.93	1.12	8.95	10.53	1.18
60......	10.09	10.13	1.00	11.94	11.80	.99
	B 1 M	B 2 M		B 1 F	B 2 F	
20......	2.48%	4.91%	1.98	0.40%	1.01%	2.52
30......	4.01	6.16	1.54	1.88	3.20	1.70
40......	6.03	7.70	1.28	5.30	6.89	1.30
50......	8.11	9.03	1.11	9.10	10.05	1.10
60......	10.07	9.99	.99	11.93	11.84	.99
	Career					
	A 1 M	A 2 M		A 1 F	A 2 F	
20......	1.85%	2.79%	1.51	0.33%	0.72%	2.18
30......	3.32	4.10	1.24	1.68	2.44	1.45
40......	5.44	5.84	1.08	5.00	5.69	1.14
50......	7.79	7.87	1.01	8.92	9.07	1.02
60......	10.09	10.13	1.00	11.94	11.80	.99
	B 1 M	B 2 M		B 1 F	B 2 F	
20......	1.93%	2.70%	1.40	0.35%	0.61%	1.74
30......	3.42	3.99	1.17	1.74	2.24	1.29
40......	5.51	5.82	1.06	5.07	5.50	1.08
50......	7.85	7.87	1.00	8.93	9.03	1.01
60......	10.07	9.99	.99	11.93	11.84	.99

FAMILY OF SALARY SCALES

The term "Family of Salary Scales"[3] has been applied to the series of salary scales derived from one original scale by the addition or subtraction of a fixed amount from each value. If the original scale s be represented by the series

$$s_x,\ s_{x+1},\ s_{x+2},\ \ldots\qquad s_{x+t},\ \ldots,$$

then each member of the derived family of salary scales is represented (k being a constant) by

$$(s_x+k),\ (s_{x+1}+k),\ (s_{x+2}+k),\ \ldots\qquad (s_{x+t}+k),\ \ldots,$$

which can be adjusted to the series s',

$$s'_x,\ s'_{x+1},\ s'_{x+2},\ \ldots\qquad s'_{x+t},\ \ldots,$$

by multiplying by $s_x/(s_x+k)$, which is constant for any given initial age x or retirement age $x = r$.

The addition of a constant to s reduces the slope of the curve of s, and a subtraction of a constant (*i.e.*, if k is negative) increases the slope. The effect of drawing these curves through the retirement point (r, s_r) is to obtain a fan type of a diagram (Chart 3), with all the curves of the same general shape but with increasing slopes as the result of deductions from the original scale and decreasing slopes as the result of additions to the scale. The general shape of the curve will remain unaltered. That is to say, a curve convex to the x axis will remain so and one concave similarly. The curve used for illustrative purposes in Chart 3 is a curve which starts slightly convex to the x axis and ends fully concave. The members of the 'family" of curves retain these characteristics, accentuated or attenuated as the case may be.

Table 2 shows, at quinquennial age points, four of the family of salary scales which can be derived from the original curve. This curve is that of A1M salary scale which possessed the most marked curvilinear shape of the curves we have been examining. The values of the A1M salary scale are recorded under the heading s. The values under the heading $s + 3{,}000$ are obtained by adding 3,000 to each of the values of the s column (from Appendix 2) and by multiplying by the ratio $1{,}000/s_{65}$. The ratio s_{65}/s_x (called the Increase Ratio) is given as a measure of the relative steepness of all the curves.

A family of salary scales as shown in Table 2 gives an actuary a very powerful tool for dealing with the problems of a specific pension plan.

[3] Heywood and Marples, *TFA* XIX, 1.

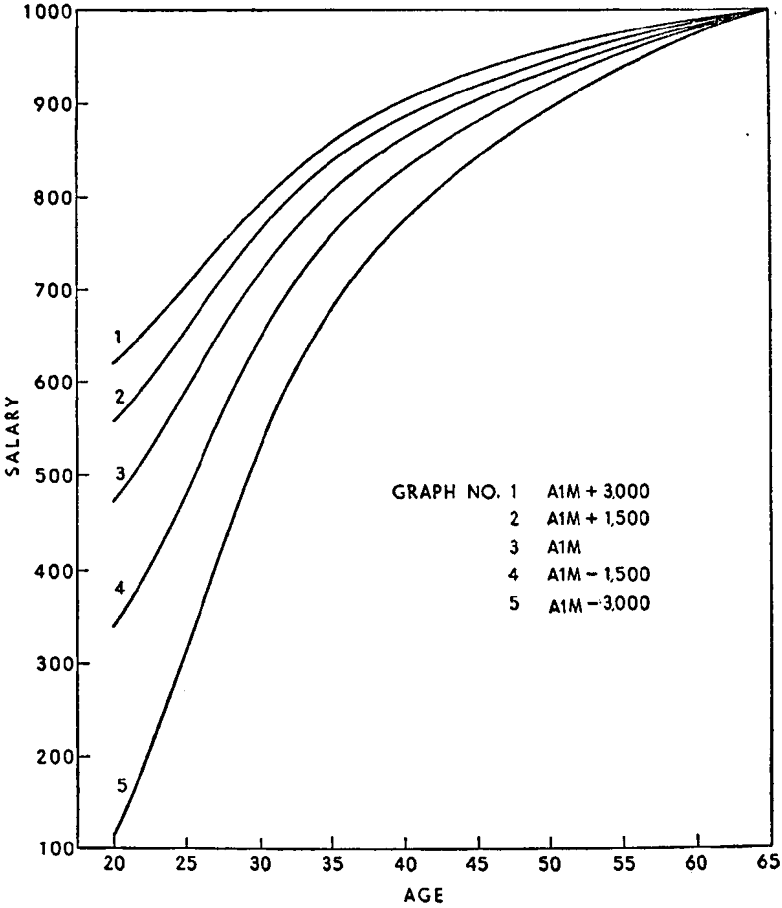
Not only is he able to select the appropriate salary scale from an infinite range of scales once the general shape of the curve is identified, but also he can obtain his valuation multipliers with great speed by a single device.

The multiplier to obtain the value of a future annual pension based on the final salary at some percentage for each year of service is basically of the form

$$\frac{s_r}{s_x} \cdot \frac{D_r}{D_x} \cdot \bar{a}_r,$$

CHART 3

FAMILY OF SALARY SCALES
SCALE A1M



where r is the retirement age, if the salary scale s is incorporated into the calculations. If it is not, then the expression for the multiplier is of the type

$$\frac{D_r}{D_x} \cdot \bar{a}_r .$$

TABLE 2

FAMILY OF SALARY SCALES

x	$s+3{,}000$		$s+1{,}500$		s		$s-1{,}500$		$s-3{,}000$	
	Scale (1)	Incr. Ratio (2)	Scale (3)	Incr. Ratio (4)	Scale (5)	Incr. Ratio (6)	Scale (7)	Incr. Ratio (8)	Scale (9)	Incr. Ratio (10)
20...	617	1.62	553	1.81	465	2.15	333	3.00	114	8.77
25...	700	1.43	653	1.54	581	1.72	478	2.09	307	3.26
30...	792	1.26	757	1.32	710	1.41	638	1.57	519	1.93
35...	858	1.16	835	1.20	802	1.25	753	1.33	672	1.49
40...	903	1.11	887	1.13	864	1.16	831	1.20	775	1.29
45...	931	1.07	919	1.09	909	1.11	879	1.14	840	1.19
50...	954	1.05	947	1.06	936	1.07	921	1.09	894	1.12
55...	973	1.03	969	1.03	962	1.04	953	1.05	938	1.07
60...	989	1.01	987	1.01	984	1.02	980	1.02	974	1.03
65...	1,000	1.00	1,000	1.00	1,000	1.00	1,000	1.00	1,000	1.00

Now consider what is necessary if the salary scale is to be modified to scale $s - k$. The expression becomes:

$$\frac{s_r - k}{s_x - k} \cdot \frac{D_r}{D_x} \cdot \bar{a}_r = \frac{1}{s_x - k}\left[\frac{s_r \cdot D_r}{D_x} \cdot \bar{a}_r - k\frac{D_r}{D_x} \cdot \bar{a}_r\right]$$

$$= \frac{1}{s_x - k}\left[s_x \cdot \frac{{}^sD_r}{{}^sD_x} \cdot \bar{a}_r - k \cdot \frac{D_r}{D_x} \cdot \bar{a}_r\right]$$

$$= \frac{1}{s_x - k}\,[\, s_x \times (\text{multiplier on salary scale } s)$$

$$- k \times (\text{multiplier without salary scale})\,] .$$

If the pension formula is the Career formula, the multiplier is of the form

$$\frac{\sum_{t=0}^{r-x-1} s_{x+t}}{s_x} \cdot \frac{D_r}{D_x} \cdot \bar{a}_r ,$$

while without salary scale the expression reduces to

$$(r - x) \cdot \frac{D_r}{D_x} \cdot \bar{a}_r .$$

Now

$$\frac{\sum_{t=0}^{r-x-1}(s_{x+t}-k)}{s_x-k}\cdot\frac{D_r}{D_x}\cdot\bar{a}_r$$

$$=\frac{1}{s_x-k}\left[s_x\cdot\frac{\sum_{t=0}^{r-x-1}s_{x+t}\cdot D_r\cdot\bar{a}_r}{{}^sD_x}-k(r-x)\cdot\frac{D_r}{D_x}\cdot\bar{a}_r\right],$$

which is of the same form as was demonstrated for the final salary formula. In fact, the device can be applied to any pension formula in which a salary scale appears, provided both the multiplier incorporating the original salary scale and the corresponding multiplier with unity in the place of the s_x have been computed.

TABLE 3

CONTRIBUTION RATES: INTEREST 3% PER ANNUM

Age	Final Five			Career		
	A 1 M (1)	A 1 M −3,000 (2)	A 2 M (3)	A 1 M (4)	A 1 M −3,000 (5)	A 2 M (6)
20....	2.22%	3.29%	5.18	1.85%	2.37%	2.79
30....	3.66	4.14	6.39	3.32	3.50	4.10
40....	5.73	6.00	7.71	5.44	5.50	5.84
50....	7.96	8.09	8.93	7.79	7.80	7.86
60....	10.09	10.09	10.13	10.09	10.09	10.13

Disability retirements and other formulas present no difficulties. It is of advantage, in fact, to design the original calculations on a service table and salary scale in such a way that the commutation columns and valuation multipliers are produced both with and without the salary scale. The stage is then set for the development of multipliers on the "family" of salary scales.

For illustration, calculations have been made on the A1M −3,000 scale in Table 2. This scale is illustrated as the extreme right-hand curve in Chart 3, and can be compared with A2M scale in Chart 1. The A1M scale is concave to the x axis, while A2M is almost a straight line. In making the calculations on the A1M −3,000 scale, the value of future pension is derived first by the method described. A separate calculation produces the temporary annuity values; and the contribution rate is the ratio of the values. The contribution rates are given in Table 3 with those on the A1M and A2M scales for comparison.

The marked differences between the contribution rates on A1M−3,000 and A2M under the Final Five formula illustrate the basic difference in the shape of the curve. The contribution rates under the Career formula are somewhat nearer each other.

HIGH-SALARIED MEMBERS

Apart from the general application of the method to the development of multipliers on alternative salary scales, the method has an application to a special problem. This relates to placing a value upon the liabilities in respect of a high-salaried member. Often these members do not merge into the general volume of salaries and so their liabilities cannot be properly estimated by the application of the general multipliers in use for other members. The liabilities for their pensions can be obtained by the application of the "family" salary scale method with a minimum of labor. First an estimate is required as to the salary history between the valuation date and the retirement date. The appropriate deduction or increase in the standard salary scale beginning at his present age is determined and from then on the method described above develops the necessary multipliers very quickly. As an illustration, consider the A1M salary scale in Appendix 2, and assume that the president of the company is now earning $50,000 at age 50 and is expected to be receiving $70,000 at retirement at 65. The salary scale allows for an increase to a salary by age 65 of the following amount:

$$50{,}000 \times 1.07 = 53{,}500 .$$

We require the value of k such that

$$50{,}000 \cdot \frac{1{,}000 - k}{936 - k} = 70{,}000 ,$$

whence, $k = 776$
and the appropriate multipliers can now be produced from the two sets available.

The algebraic generalization of this calculation follows:

Let the ratio of the estimated salary at retirement to the present salary at age x be $(ES)_r/(AS)_x$,
and let the flat deduction necessary to produce this increase be taken as ${}_rk_xs_x$.

Then we require the value of k, such that

$$\frac{(ES)_r}{(AS)_x} = \frac{s_r - {}_rk_x s_x}{s_x - {}_rk_x s_x} ,$$

from which

$${}_rk_x = \frac{(ES)_r/(AS)_x - s_r/s_x}{(ES)_r/(AS)_x - 1} .$$

This demonstration refers to a benefit based on final salary, but the formula is the same in the case of a benefit based on total salary during employment if the benefit is expressed as the unit benefit per year of service. Thus, writing

$$\Sigma \quad \text{for} \quad \Sigma_x^{r-1}$$

the required ratio is

$$\frac{\Sigma(AS)_t}{(r-x)(AS)_x},$$

which is equal to

$$\frac{\Sigma(s_t - ks_x)}{(r-x)(s_x - ks_x)}$$

$$= \frac{\Sigma s_t - (r-x)ks_x}{(r-x)(s_x - ks_x)}$$

$$= \frac{\Sigma s_t/(r-x)s_x - k}{1-k},$$

whence

$$_rk_x = \frac{\Sigma(AS)_t/(r-x)(AS)_x - \Sigma s_t/(r-x)s_x}{\Sigma(AS)_t/(r-x)(AS)_x - 1},$$

which is of the same form as in the final salary formula, namely

$$k = \frac{f' - f}{f' - 1},$$

where f' is the ratio desired and f the ratio by the salary scale.

ADJUSTING THE SALARY SCALE

The comparisons of the salary scales derived from the data developed by Plan A and Plan B on the Current Average method and on the Increase Ratio method indicate that a salary scale produced by the Current Average method cannot be wholly satisfactory in an unmodified condition. The question now arises as to what kind of adjustment can be made to produce, from Current Average salaries, a more satisfactory actuarial tool. In the great majority of plans presented to the actuary for his judgment on the actuarial valuation bases, we have to recognize that, in respect of the salaries of participants, the normal data available are the current average salaries. In few cases only will the employer be able or willing to supply pertinent past salary data in a form suitable to derive a salary scale by the Increase Ratio method. However, when the actuary has surmounted this initial hurdle, he will be able to advise on methods of accumulating the salary statistics in a form suitable for treatment in the future. There are many plans which have been in operation for more than, say, five years, and data may thus be available for a thorough investigation of the salary scale problems. The actuary with such data on hand bears a responsibility to the employer to disclose the situation in the plan and to advise his client as to the effect on his funding program. It should not be overlooked that the valuation bases are, in common parlance, a "package deal," and it may be that taken all together the current bases are adequate. With statistics from a plan available on a standardized basis for a number of years the actuary is adequately supplied with information on which to base his projections for the future, but the actuary who is making his valuation or cost estimate for the first time is on a different footing.

I believe we may assume that the most difficult situation for the actuary is the one in which current salaries only are available. With the techniques described earlier in this paper what can he do to make a more sophisticated estimate of future salary progression? The Family of Salary Scales method offers one approach to the subject. As an example, let us take the B1M and B2M salary scales. Postulate that we have derived the B1M scale from the available data and that it should be adjusted in some manner. It may be noted that the subtraction of a fixed amount from the salary scale values tends to reduce the curvations of the curve but can never reverse the curvations or even flatten it completely. With our foreknowledge that our assumed "true" curve is practically a straight line we are doomed to failure if we judge it on correspondence of contribution rates derived at each age from the two scales, *i.e.*, B1M$-k$ and B2M. Contribution rates based on B1M$-k$ will be less than those on B2M because of the concavity of the curve, except when the deduction k is extremely large, and even then the effect will be larger contribution rates than B2M rates at the young ages and lesser rates only for later ages.

The suggestion is made that a modus operandi can be developed through the adjustments to the Increase Ratio either on the Final Five formula or on the Career formula. In order to illustrate the operation I have calculated in Table 4 the deduction necessary to produce two times, three times and four times the increase ratio from age 20 to age 65 by formula.

The deductions for the Career formula are slightly higher than for the Final Five formula and have been used in the ensuing calculations.

The effect on contribution rates is illustrated by the figures in Table 5. Based on the B1M salary scale, the deductions actually used in the calculations are:

Increase Ratio Multiple 2: $.663 \times 4025 = 2669$
Increase Ratio Multiple 3: $.797 \times 4025 = 3208$
Increase Ratio Multiple 4: $.855 \times 4025 = 3441$.

TABLE 4

CALCULATION OF DEDUCTION TO ADJUST INCREASE RATIO BETWEEN AGE 20 AND AGE 65: SALARY SCALE B 1 M

Multiple	Increase Ratio (1)	f' (2)	$f'-f$ (3)	$f'-1$ (4)	$k=(3)/(4)$ (5)	ks_{20} $=(5)\times 376$ (6)
	Final Five					
2........	2.663	5.326	2.663	4.326	.614	231
3........	2.663	7.989	5.326	6.989	.762	287
4........	2.663	10.652	7.989	9.652	.828	311
	Career					
2........	2.030	4.060	2.030	3.060	.663	249
3........	2.030	6.090	4.060	5.090	.797	300
4........	2.030	8.120	6.090	7.120	.855	321

TABLE 5

CONTRIBUTION RATES ON ADJUSTED SALARY SCALES—INTEREST 3%

Age	B 1 M (1)	B 1 M —2.669 (2)	B 1 M —3.208 (3)	B 1 M —3.441 (4)	B 2 M (5)
	Final Five				
20.....	2.48	3.21	3.51	3.67	4.91
30.....	4.01	4.44	4.59	4.66	6.16
40.....	6.03	6.31	6.39	6.43	7.70
50.....	8.11	8.23	8.26	8.28	9.03
60.....	10.07	10.06	10.06	10.06	9.99
	Career				
20.....	1.93	2.25	2.38	2.45	2.70
30.....	3.42	3.57	3.62	3.64	3.99
40.....	5.51	5.57	5.59	5.60	5.82
50.....	7.85	7.88	7.89	7.89	7.87
60.....	10.07	10.06	10.06	10.06	9.99

The deductions were obtained by applying the Career ratio in Column (5) of Table 4 to s_{20} (4025) as shown above.

The recalculation of the contribution rates has not produced as satisfactory figures as could have been wished. The Career contribution rates are closer to the B2M rates than the Final Five rates. Again this stems from the basic shape of the curve of the salary scale which affects the Final Five more than the Career rates.

Where the actuary can derive a salary scale by the Increase Ratio method, he is most likely to be well on the conservative side if inflation is current during the period over which the data are collected. If the inflation is marked in this exposure period, he may end with an estimate of future salary increments which would be too high in later periods of lesser inflation. If deflation tendencies were in action in the exposure period, he may not develop a salary scale which will represent a firm basis for cost estimates. The most likely situation is an exposure period, such as the period of the current date, involving a perceptible measure of inflation and in such case, if his only data are the current average salaries, he may wish to adjust his salary scale before use.

The Family of Salary Scales method will enable him to make a temporary adjustment to his Current Average salary scale, but if there is a suspicion that the corresponding Increase Ratio scale might exhibit a different shape, his corrections cannot be more than a gesture in the right direction. Pending establishment of the Increase Ratio scale, however, the method may be helpful and in any case will have its value as a method of developing a pilot survey.

INFLATION

Bacon's Increase Ratio Method may seem more satisfying statistically but the end product includes such inflation-time increment as may happen to have occurred in the period over which the salaries were recorded. Equally so the Current Average Method reflects the current level of salaries but excludes the inflation-time increment. It may even exhibit a slight "compression" effect in certain stages of correction for inflation. This compression effect has been visible in salary statistics after both World War I and World War II. It will be found that where an initial flat increment was granted as a "cost of living bonus" the effect is to reduce the slope of the salary curve, since there is a greater proportion of the higher salaries at the higher ages and the effect of a flat increase is less on higher salaries than on lower salaries. The flat cost of living increase would have the same effect in the Increase Ratio Method if both five-year points were within the period affected by such temporary increases.

Over the past 47 years the Consumers Price Index of the Bureau of Labor Statistics has shown an average increase in the cost of living equal to 2.4 percent *per annum.* In the past seven or eight years the five-year average increase was kept down to 1.7 percent per annum, but the increase which occurred in 1959–60 moved the five-year average up to 2.2 percent. The scale of the annual increase is important. If a salary of $4,000 at age 20 were increased at the rate of 2 percent per annum, the salary received at age 65 would be nearly $10,000. If a modest salary increase ratio of 4.0 were applied for the effect of promotion, the college graduates now being hired at $4,000 could expect in forty-five years' time to average $40,000 a year if salaries kept pace with promotion and inflation. This calculation may be less startling if a little reflection is given to the levels of salaries in 1920 and 1960.

In the course of the calculations on the B2M salary scale, contribution rates have been produced at 2%, 3% and 4%. On the B1M scale, the contribution rates at 3% and 4% have been calculated. With these figures it is possible to obtain estimates of the increase in contributions required to meet pensions on salaries increased by an inflationary rise of 1% per annum or 2% per annum over and above the normal rise on the salary scale.

The ratios required are to allow for the 1% or 2% inflationary rise up to the age at retirement. In consequence, some preliminary adjustment is required. The annuity value was established at 3% per annum for the increase at 1% per annum and 4% for the increase at 2% per annum; with this adjustment the ratios taken were as follows:

1% per annum

B1M	${}^{s_1}_{.03}c_x / ({}^{s_1}_{.04}c_x \times {}^{.03}_{.04}(ar)_{65})$	3%/4%
B2M	${}^{s_2}_{.02}c_x \times {}^{.03}_{.02}(ar)_{65} / {}^{s_2}_{.03}c_x$	2%/3%
	${}^{s_2}_{.03}c_x / {}^{s_2}_{.04}c_x \times {}^{.03}_{.04}(ar)_{65}$	3%/4%

2% per annum

B2M	${}^{s_2}_{.02}c_x \times {}^{.04}_{.02}(ar)_{65} / {}^{s_2}_{.04}c_x$	2%/4%

Table 6 exhibits the ratios obtained.

The cost increases indicated are impressively consistent.

For an Inflationary Increase of:			1% per annum	2% per annum
The Normal Contribution Rate increases:				
At Entry Age	35	by	+20%	+40%
At Attained Age	45	by	+12½%	+25%

The attained age rate probably indicates the level of increase on the gross liabilities of a plan, and the accrued liability should show about the same rate of increase if the attained age normal contribution rate is used, or a lesser rate if the entry age normal contribution rate is employed.

Let us consider the situation if we do not recommend the higher contribution rates. If benefits are based on the final years' salary, the contributions may be inadequate to the extent indicated. If benefits are

TABLE 6

COST RATIOS OF INFLATIONARY ALLOWANCES
MEN

Infl. Allow. per Annum	Age	Final Five			Career		
		Contribution Rates		Ratio (1)/(2)	Contribution Rates		Ratio (4)/(5)
		(1)	(2)	(3)	(4)	(5)	(6)
		B 1 M 3%	B 1 M 4% adj.		B 1 M 3%	B 1 M 4% adj.	
1%......	20	2.48	1.81	1.37	1.93	1.41	1.37
	30	4.01	3.19	1.26	3.42	2.72	1.26
	40	6.03	5.20	1.16	5.51	4.76	1.16
	50	8.11	7.48	1.08	7.85	7.24	1.08
	60	10.07	9.82	1.03	10.07	9.82	1.03
		B 2 M 2% adj.	B 2 M 3%		B 2 M 2% adj.	B 2 M 3%	
1%......	20	6.42	4.91	1.31	3.53	2.70	1.31
	30	7.53	6.16	1.22	4.87	3.99	1.22
	40	8.80	7.70	1.14	6.66	5.82	1.14
	50	9.74	9.03	1.08	8.49	7.87	1.08
	60	10.25	9.99	1.03	10.25	9.99	1.03
		B 2 M 3%	B 2 M 4% adj.		B 2 M 3%	B 2 M 4% adj.	
1%......	20	4.91	3.72	1.32	2.70	2.04	1.32
	30	6.16	5.01	1.23	3.99	3.24	1.23
	40	7.70	6.71	1.15	5.82	5.07	1.15
	50	9.03	8.36	1.08	7.87	7.29	1.08
	60	9.99	9.75	1.03	9.99	9.75	1.03
		B 2 M 2% adj.	B 2 M 4%		B 2 M 2% adj.	B 2 M 4%	
2%......	20	5.93	3.44	1.72	3.27	1.89	1.73
	30	6.96	4.63	1.50	4.50	3.00	1.50
	40	8.14	6.21	1.31	6.16	4.69	1.31
	50	9.01	7.73	1.17	7.85	6.74	1.16
	60	9.48	9.02	1.05	9.48	9.02	1.05

TABLE 6—*Continued*

COST RATIOS OF INFLATIONARY ALLOWANCES

WOMEN

Infl. Allow. per Annum	Age	Final Five: Contribution Rates (1)	Final Five: Contribution Rates (2)	Final Five: Ratio (1)/(2) (3)	Career: Contribution Rates (4)	Career: Contribution Rates (5)	Career: Ratio (4)/(5) (6)
		B 1 F 3%	B 1 F 4% adj.		B 1 F 3%	B 1 F 4% adj.	
1%....	20	0.40	0.27	1.48	0.35	0.24	1.46
	30	1.88	1.45	1.30	1.74	1.34	1.30
	40	5.30	4.51	1.18	5.07	4.31	1.18
	50	9.10	8.38	1.09	8.93	8.22	1.09
	60	11.93	11.63	1.03	11.93	11.63	1.03
		B 2 F 2% adj.	B 2 F 3%		B 2 F 2% adj.	B 2 F 3%	
1%....	20	1.47	1.01	1.46	0.89	0.61	1.46
	30	4.16	3.20	1.30	2.89	2.24	1.29
	40	8.00	6.89	1.16	6.39	5.50	1.16
	50	10.87	10.05	1.08	9.77	9.03	1.08
	60	12.13	11.84	1.02	12.13	11.84	1.02
		B 2 F 3%	B 2 F 4% adj.		B 2 F 3%	B 2 F 4% adj.	
1%....	20	1.01	0.70	1.44	0.61	0.43	1.42
	30	3.20	2.47	1.30	2.24	1.73	1.29
	40	6.89	5.91	1.17	5.50	4.72	1.17
	50	10.05	9.25	1.09	9.03	8.31	1.09
	60	11.84	11.56	1.02	11.84	11.56	1.02
		B 2 F 2% adj.	B 2 F 4%		B 2 F 2% adj.	B 2 F 4%	
2%....	20	1.34	0.64	2.09	0.81	0.39	2.08
	30	3.75	2.26	1.66	2.64	1.58	1.67
	40	7.30	5.40	1.35	5.83	4.31	1.35
	50	9.93	8.45	1.18	8.93	7.59	1.18
	60	11.08	10.56	1.05	11.08	10.56	1.05

based on the average of the last, say, ten years, the contributions will be less inadequate but the benefits themselves will start to be insufficient. If the benefits are based on each year's salary, the contributions will still be inadequate and the benefits will also be inadequate. This is the effect of regular inflation. If contributions are adequate, they may be an immediate financial strain; if they are not adequate, the strain materializes when a request for increased pension "to meet the cost of living" is made.

The counsel of perfection is to face the estimate of the cost and pay what is required. In most cases only sporadic and temporary efforts are made to meet the situation and the lack of serious effort is rationalized on the combined basis of lack of finances and a doubt as to Treasury approval for directly incorporating such effects in cost estimates. In most cases, the employer realizes that he is meeting the problems of inflation as they arise in his business activities and that its effect on his pension plan is only one of its manifestations. The total cost of employment is the total cost of salary and fringe benefits, and relating the potential rise in pension cost due to inflation to this total figure, the increase may be reduced to its relative size. Philosophically, this reduction to percentages may be comforting, but the hard fact of increased dollar outlay (on top of all the other outlays) remains.

CONCLUSION

It seems clear that the salary scale is not the instrument of precision that it was conceived to be by its originators. The reason why this is so is the same today as it was then, namely the creeping evil of inflation. This was less clearly documented and exhibited at the beginning of this century than it is today, and that may account for the lack of reference to it in the early statements on the subject of salary scales. The degree of public interest and concern at the present time on the subject of inflation must not, however, lead us to expect any greater willingness, on the part of the employer, to pay contributions which include an inflation component in advance of an estimate of its effect. The result is to place the actuary squarely on the horns of a dilemma. If he includes in his costs an allowance for inflation, he deters the employer from making any provision at all, and if he ignores inflation, with such cautionary verbiage as appeals to him, the employer will inevitably complain that he was not told the truth about the situation.

Furthermore, as any consultant will testify, competition is frequently encountered presenting figures innocent of any consideration of a salary scale, let alone an adjustment to meet the cost of inflation. In consequence, the practical result is a conspiracy of silence unless the client specifically requests a review of the effects of inflation upon his pension funding program.

The actuary's freedom of action, in his professional judgment, is not increased by the Internal Revenue Service, which has indicated that it requires to be consulted on the inclusion of specific margins designated to meet inflationary pension costs. This is the difficult situation in which the salary scale technique is attempting to perform some services. I believe that the technique has consistently enabled actuaries to approach closer to the expected events than they would otherwise have done, although it has been disparaged for failing to meet demands upon it beyond its capacity as a mathematical tool. May I quote in conclusion the opinion of Mr. H. W. Manly who said:

The more experience I have of these funds, the more I am convinced that my remark about making an "intelligent graduation" [of the salary scale data] was correct. By "intelligent" I mean involving the exercise of experience, wisdom and skill.

If we can manage to include all this in our advice on pension matters, we shall not fail to live up to the highest standards of the professional actuary.

APPENDIX 1: PLAN A: DATA

Central Age of Group x	Annual Rate of Increase of Survivors $x-1$ to x: Increase 1956	Increase 1957	Increase 1958	Increase 1959	Increase 1960	Average Rate of Increase	Average Salary 1955	Average Salary* 1955	Average Salary 1960	Average Salary* 1960
Men										
20......	1.122	1.125	1.166	1.122	1.115	1.130	2,634	1,000	3,492	1,000
25......	1.107	1.108	1.111	1.091	1.090	1.102	3,279	1,245	4,215	1,207
30......	1.096	1.102	1.102	1.079	1.082	1.092	4,025	1,528	5,185	1,484
35......	1.082	1.086	1.088	1.091	1.067	1.083	4,419	1,678	5,643	1,616
40......	1.078	1.072	1.078	1.063	1.065	1.071	5,062	1,922	5,958	1,706
45......	1.070	1.066	1.072	1.053	1.057	1.064	5,023	1,907	6,091	1,744
50......	1.060	1.061	1.060	1.047	1.046	1.057	5,273	2,002	6,400	1,833
55......	1.057	1.055	1.055	1.039	1.043	1.050	5,444	2,067	6,683	1,914
60......	1.047	1.053	1.052	1.035	1.038	1.045	5,787	2,197	6,745	1,932
65......	1.052	1.049	1.048	1.031	1.035	1.043	5,384	2,044	6,869	1,967
All Ages	1.067	1.067	1.069	1.053	1.054	1.062	4,880	1,853	6,017	1,723
Women										
20......	1.155	1.168	1.171	1.135	1.145	1.155	2,716	1,000	3,363	1,000
25......	1.191	1.093	1.092	1.062	1.072	1.102	2,972	1,094	4,772	1,122
30......	1.077	1.083	1.079	1.059	1.067	1.073	3,264	1,202	3,989	1,186
35......	1.075	1.074	1.075	1.053	1.055	1.066	3,281	1,208	4,119	1,225
40......	1.074	1.078	1.073	1.049	1.052	1.065	3,265	1,202	4,176	1,242
45......	1.066	1.070	1.065	1.047	1.052	1.060	3,324	1,224	4,107	1,221
50......	1.061	1.061	1.059	1.039	1.046	1.053	3,419	1,259	4,233	1,259
55......	1.056	1.055	1.056	1.037	1.074	1.056	3,751	1,381	4,369	1,299
60......	1.050	1.051	1.051	1.051	1.042	1.049	3,468	1,277	4,445	1,322
65......	1.053	1.065	1.055	1.036	1.041	1.050	3,381	1,245	4,412	1,312
All Ages	1.082	1.086	1.084	1.061	1.067	1.076	3,158	1,163	3,991	1,187

* Radix 1,000 at age 20.

APPENDIX 1: PLAN B: DATA

Age x	Ratio of Average Salary of Survivors at age $x+5$ to Average Salary at age x	Average Salaries 1954	Average Salaries* 1954	Average Salaries 1959	Average Salaries* 1959
Men					
20	2.011	2,873	1,000	3,950	1,000
25	1.666	3,743	1,303	4,830	1,223
30	1.408	4,784	1,665	5,813	1,472
35	1.357	5,331	1,856	6,677	1,690
40	1.325	5,708	1,987	7,183	1,818
45	1.431	6,263	2,180	7,482	1,894
50	1.176	6,951	2,419	8,965	2,270
55	1.248	7,034	2,448	8,173	2,069
60	1.221	6,953	2,402	8,779	2,223
65	....	6,874	2,393	8,489	2,149
All Ages	1.374	5,349	1,862	6,642	1,681
Women					
20	1.554	2,510	1,000	3,024	1,000
25	1.385	2,911	1,160	3,530	1,167
30	1.323	3,165	1,261	3,846	1,272
35	1.315	3,261	1,299	4,000	1,323
40	1.284	3,340	1,331	4,103	1,357
45	1.259	3,577	1,425	4,145	1,371
50	1.209	3,731	1,486	4,462	1,476
55	1.204	3,777	1,505	4,512	1,492
60	1.209	3,840	1,530	4,546	1,503
65	....	3,795	1,512	4,643	1,535
All Ages	1.396	3,043	1,212	3,762	1,244

* Radix 1,000 at age 20.

APPENDIX 2: PLAN A: SALARY SCALES

AGE x	CURRENT SALARY METHOD				INCREASE RATIO METHOD			
	s_x	s_{65}/s_x	$\Sigma s_x/(65-x)s_x$	s_x/s_{65}	s_x	s_{65}/s_x	$\Sigma s_x/(65-x)s_x$	s_x/s_{65}
	Scale A 1 M				Scale A 2 M			
Men								
20...	3,522	2.151	1.776	.465	3,522	9.231	4.626	.108
25...	4,403	1.721	1.488	.581	5,445	5.971	3.269	.168
30...	5,375	1.409	1.266	.710	7,847	4.143	2.476	.241
35...	6,075	1.247	1.151	.802	10,758	3.022	1.968	.331
40...	6,547	1.157	1.089	.864	14,120	2.303	1.629	.434
45...	6,843	1.107	1.059	.903	17,790	1.828	1.397	.547
50...	7,093	1.068	1.036	.936	21,559	1.508	1.239	.663
55...	7,291	1.039	1.020	.962	25,272	1.286	1.129	.777
60...	7,456	1.016	1.007	.984	28,887	1.125	1.050	.889
65...	7,576	1.000	....	1.000	32,512	1.000	....	1.000
	Scale A 1 F				Scale A 2 F			
Women								
20...	3,000	1.287	1.211	.777	3,000	8.042	3.964	.124
25...	3,261	1.184	1.134	.844	4,655	5.183	2.776	.193
30...	3,462	1.116	1.083	.896	6,383	3.780	2.194	.265
35...	3,612	1.096	1.049	.935	8,274	2.916	1.831	.343
40...	3,714	1.040	1.028	.961	10,311	2.340	1.587	.427
45...	3,780	1.022	1.014	.979	12,536	1.925	1.409	.520
50...	3,816	1.012	1.008	.988	15,013	1.607	1.269	.622
55...	3,843	1.005	1.003	.995	17,761	1.358	1.156	.736
60...	3,857	1.001	1.001	.998	20,792	1.160	1.063	.862
65...	3,862	1.000	....	1.000	24,127	1.000	....	1.000

APPENDIX 2: PLAN B: SALARY SCALES

Age x	Current Salary Method				Increase Ratio Method			
	s_x	s_{65}/s_x	$\Sigma s_x/(65-x)s_x$	s_x/s_{65}	s_x	s_{65}/s_x	$\Sigma s_x/(65-x)s_x$	s_x/s_{65}
	Scale B 1 M				Scale B 2 M			
Men								
20...	4,025	2.663	2.030	.376	4,025	10.560	5.315	.095
25...	5,530	1.938	1.511	.516	7,708	5.514	3.033	.181
30...	6,629	1.617	1.355	.618	11,437	3.723	2.221	.269
35...	7,506	1.428	1.241	.700	15,220	2.792	1.806	.358
40...	8,266	1.297	1.163	.771	19,127	2.222	1.549	.450
45...	8,926	1.201	1.107	.833	23,188	1.833	1.373	.546
50...	9,494	1.129	1.073	.886	27,468	1.547	1.244	.646
55...	9,977	1.074	1.035	.931	32,066	1.325	1.142	.754
60...	10,379	1.033	1.013	.968	37,058	1.147	1.058	.872
65...	10,719	1.000	....	1.000	42,502	1.000	....	1.000
	Scale B 1 F				Scale B 2 F			
Women								
20...	3,000	1.572	1.370	.636	3,000	5.492	3.126	.182
25...	3,412	1.382	1.240	.723	4,356	3.782	2.320	.264
30...	3,723	1.267	1.162	.789	5,777	2.852	1.878	.351
35...	3,973	1.187	1.110	.842	7,217	2.283	1.607	.438
40...	4,176	1.130	1.073	.885	8,692	1.896	1.422	.528
45...	4,332	1.089	1.048	.918	10,204	1.615	1.286	.619
50...	4,457	1.058	1.031	.945	11,728	1.405	1.184	.712
55...	4,563	1.034	1.017	.967	13,250	1.243	1.170	.804
60...	4,652	1.014	1.007	.986	14,815	1.112	1.044	.899
65...	4,717	1.000	....	1.000	16,476	1.000	....	1.000

APPENDIX 3

Contribution Rates on Plan A Salary Scales

Age	Final Five		Career	
	A 1 M 3%	A 1 M 4%	A 1 M 3%	A 1 M 4%
20	2.22%	1.49%	1.85%	1.24%
30	3.66	2.69	3.32	2.42
40	5.73	4.56	5.44	4.33
50	7.96	6.78	7.79	6.64
60	10.09	9.10	10.09	9.10
	A 2 M 3%	A 2 M 4%	A 2 M 3%	A 2 M 4%
20	5.18%	3.64%	2.79%	1.96%
30	6.39	4.83	4.10	3.09
40	7.71	6.22	5.84	4.72
50	8.93	7.65	7.87	6.74
60	10.13	9.04	10.13	9.04
	A 1 F 3%	A 1 F 4%	A 1 F 3%	A 1 F 4%
20	0.35%	0.22%	0.33%	0.20%
30	1.72	1.20	1.68	1.17
40	5.06	3.02	5.00	3.88
50	8.95	7.49	8.92	7.46
60	11.94	10.63	11.94	10.63
	A 2 F 3%	A 2 F 4%	A 2 F 3%	A 2 F 4%
20	1.33%	0.85%	0.72%	0.46%
30	3.86	2.75	2.44	1.74
40	7.68	6.05	5.69	4.47
50	10.53	8.86	9.07	7.64
60	11.80	10.51	11.80	10.51

APPENDIX 3—*Continued*

CONTRIBUTION RATES ON PLAN B SALARY SCALES

AGE	FINAL FIVE			CAREER		
		B 1 M 3%	B 1 M 4%		B 1 M 3%	B 1 M 4%
20....		2.48%	1.67%		1.93%	1.30%
30....		4.01	2.95		3.42	2.52
40....		6.03	4.81		5.51	4.40
50....		8.11	6.92		7.85	6.70
60....		10.07	9.08		10.07	9.08
	B 2 M 2%	B 2 M 3%	B 2 M 4%	B 2 M 2%	B 2 M 3%	B 2 M 4%
20....	6.97%	4.91%	3.44%	3.84%	2.70%	1.89%
30....	8.18	6.16	4.63	5.29	3.99	3.00
40....	9.56	7.70	6.21	7.23	5.82	4.69
50....	10.58	9.03	7.73	9.22	7.87	6.74
60....	11.13	9.99	9.02	11.13	9.99	9.02
		B 1 F 3%	B 1 F 4%		B 1 F 3%	B 1 F 4%
20....		0.40%	0.25%		0.35%	0.22%
30....		1.88	1.32		1.74	1.22
40....		5.30	4.12		5.07	3.94
50....		9.10	7.65		8.93	7.51
60....		11.93	10.62		11.93	10.62
	B 2 F 2%	B 2 F 3%	B 2 F 4%	B 2 F 2%	B 2 F 3%	B 2 F 4%
20....	1.62%	1.01%	0.64%	0.98%	0.61%	0.39%
30....	4.53	3.20	2.26	3.18	2.24	1.58
40....	8.81	6.89	5.40	7.04	5.50	4.31
50....	11.98	10.05	8.45	10.76	9.03	7.59
60....	13.37	11.84	10.56	13.37	11.84	10.56

APPENDIX 4

SUMMARY OF ACTUARIAL BASES

1. Service Table

AGE	SEPARATIONS PER 1,000,000			
	Men		Women	
	By Death	By Withdrawal	By Death	By Withdrawal
20	616	79,384	616	179,384
25	758	77,242	758	172,242
30	991	72,219	991	158,295
35	1,374	62,764	1,374	136,896
40	2,000	51,504	2,000	112,500
45	3,580	39,753	3,580	84,319
50	6,475	25,627	6,475	50,645
55	10,436	8,394	10,436	17,264
60	15,555	901	15,555	1,614

2. Pensioners Mortality Table

Men: Ga-1951 Table (men) at age $x - 1$
Women: Ga-1951 Table (men) at age $x - 6$

3. Ratios of Annuity Values: ${}^{i}_{j}(ar)_{65}$

j	MEN			WOMEN		
	$i=2\%$	$i=3\%$	$i=4\%$	$i=2\%$	$i=3\%$	$i=4\%$
2%	1.0000	.9205	.8514	1.0000	.9076	.8288
3%	1.0864	1.0000	.9250	1.1019	1.0000	.9133
4%	1.1745	1.0811	1.0000	1.2065	1.0950	1.0000

DISCUSSION OF PRECEDING PAPER

KENNETH ALTMAN:

I would like to commend Mr. Marples for his important contribution to our actuarial literature in an area which has long deserved further attention. Our own experience is given here, to supplement the material presented in his paper.

A study of salary experience of the 225,000 members of the New York State Employees' Retirement System was conducted for the period 1955–1960. Employees were grouped into three broad occupational classifications: clerical and administrative, laborers, and firemen-policemen. Working with 5-year age groups, we took the salaries of members employed both at the beginning and end of each year. The average rate of salary increase was determined at quinquennial ages within each classification. We removed the effect of inflation from our unadjusted salary scales by referring to the actual statewide salary increases granted to New York State employees over and above regular annual increments and promotions.

Up to this point our efforts have been directed at deriving salary scales free of the effects of inflation. The way in which we will finally treat the element of inflation still requires a great deal of additional consideration on our part.

Our newly devised basic salary scales (*i.e.*, without inflation) were compared with our former salary scales. The following characteristics were noted:

1. Clerical employees experienced generally higher salary increase ratios.
2. There was a marked rise in salary increase ratios at all ages among our laboring group. This is due in part to the change in attitude toward laborers and also to the changing composition of the laborer group. Our laborer group now includes many skilled job categories.
3. Fire and police salary experience has a completely different pattern from either our clerical or laborer classifications. Ratios of salary increase begin at a higher level at the early ages and decline very rapidly.

Although most of our employees may retire as early as age 55, few actually do, giving us an average retirement age of 64.

Retirements at ages shortly after 55 are often associated with disability, leaving healthier than average lives in active service. These employees may reasonably be expected to experience salary increases which are no

NEW YORK STATE EMPLOYEES' RETIREMENT SYSTEM
1955–60 BASIC SALARY SCALES

$$\frac{S_{x+1}}{S_x}$$

Age	Clerical and Administrative	Laborers	Fire-Police
20	1.0619	1.0614	1.0678
21	1.0582	1.0581	1.0638
22	1.0549	1.0550	1.0598
23	1.0519	1.0521	1.0554
24	1.0491	1.0494	1.0513
25	1.0465	1.0469	1.0474
26	1.0441	1.0444	1.0437
27	1.0419	1.0421	1.0402
28	1.0399	1.0400	1.0369
29	1.0381	1.0379	1.0338
30	1.0365	1.0359	1.0309
31	1.0351	1.0340	1.0282
32	1.0338	1.0322	1.0257
33	1.0326	1.0305	1.0234
34	1.0315	1.0290	1.0213
35	1.0305	1.0277	1.0194
36	1.0296	1.0265	1.0177
37	1.0288	1.0254	1.0162
38	1.0280	1.0243	1.0149
39	1.0272	1.0233	1.0137
40	1.0264	1.0224	1.0126
41	1.0256	1.0216	1.0116
42	1.0248	1.0209	1.0107
43	1.0240	1.0203	1.0099
44	1.0232	1.0197	1.0092
45	1.0224	1.0191	1.0086
46	1.0216	1.0185	1.0081
47	1.0208	1.0179	1.0077
48	1.0200	1.0173	1.0074
49	1.0192	1.0167	1.0072
50	1.0184	1.0161	1.0070
51	1.0176	1.0155	1.0068
52	1.0168	1.0149	1.0066
53	1.0160	1.0143	1.0065
54	1.0152	1.0137	1.0064
55	1.0144	1.0131	1.0063
56	1.0136	1.0125	1.0062
57	1.0128	1.0119	1.0061
58	1.0120	1.0113	1.0060
59	1.0112	1.0107	1.0059
60	1.0104	1.0101	1.0058
61	1.0096	1.0095	1.0057
62	1.0088	1.0089	1.0056
63	1.0080	1.0083	1.0055
64	1.0072	1.0078	1.0055
65	1.0064	1.0073	1.0055
66	1.0057	1.0069	1.0055
67	1.0051	1.0065	1.0055
68	1.0046	1.0065	1.0055
69	1.0042	1.0057	1.0055

less favorable than employees covered under plans with a normal retirement age of 65.

The accompanying table summarizes our basic salary scales for each occupational classification.

S_x = salary earned at exact age x.

GEOFFREY N. CALVERT:

The general burden of this paper is that, unless the Increase Ratio method of constructing salary scales is used, or something approximate to it, then pension costs will be seriously understated, the employer will not be informed of the facts, pension plans will be heavily underfunded, and the actuary will have failed to do his duty. "The counsel of perfection," states the author, "is to face the estimate of the cost and pay what is required."

In my opinion, the contents of this paper do not support this conclusion at all. The treatment of the general subject is seriously incomplete. Most important of all, the whole approach to the question of inflation is limited to a consideration of the *liabilities* to be met from a pension fund. Nothing at all is said, and no hint of any kind is given, as to the equally important composition of the *assets* in the pension fund, and the effect of inflation on the value of those assets. Given a reasonably adequate representation of equity forms of investment in the portfolio of the pension fund, the inflation which affects the liability side of the pension fund will also in the long run affect the asset side, and the author's conclusion will simply fall to the ground.

Perhaps this paper, and the very serious consequences which would follow from its general adoption by the actuarial profession, might serve instead as a stepping stone in encouraging both a greater attention to the study of economics and investments in the examination course provided by the Society of Actuaries, and also a more attentive approach to the investment of pension funds both by consulting actuaries and by trustees and employers. The consequences of a long-term inflationary trend are not always understood or translated into funding policy in either the assets or the liabilities of these funds.

Other points worth mentioning include:

a) A confusion of identity between the cost-of-living index and the general subject of inflation. One wonders why the author did not choose to use national wage index figures if he was trying to reconcile the Current Average and Increase Ratio methods by eliminating the effects of inflation. Having introduced a largely irrelevant index which takes no account of the inherent difference between wage trends and price trends, the author refers to a lack of precision in the actuarial tools he is discussing, and never does come up with a reconciliation between these two methods of arriving at an earnings scale.

b) The use of the entry age normal or level percentage of pay method of calculating costs would tend to prefund accruing benefits, hence would tend to increase the leverage exercised by the effects of inflation on the asset side of the pension fund balance sheet.

c) There seems to be a serious confusion between entry age and attained age costs implied by the use of factors at different ages taken from the same column in Table 6. If entry age costs refer to the future funding of benefits accruing in the future for a new entrant, surely it does not follow that the additional funding in the future of benefits already accrued in respect of past service (as in the attained age method) would be influenced to the same degree by changes in the salary scale. Under a career-average plan, these past service benefits would already be a matter of record, and would not be influenced at all by future changes in scale. Under a final-average plan, the opposite would be true and the effect would be much greater as to past than as to future service benefits.

d) The general method of constructing and using a family of salary scales, as given by the author, is a helpful device, and can be usefully employed quite out of the general context of this paper.

CONRAD M. SIEGEL:

Mr. Marples' paper is a welcome addition to actuarial literature and should prove helpful to both student and practitioner. His tables adequately demonstrate the substantial effect on contribution rates of using a salary scale, as well as the substantial differences in contribution rates occasioned by different approaches used in constructing the salary scale from the available data.

Throughout the paper one detects a plea for actuarial maturity in providing for, or at least pointing out, the increase in liabilities created by increases in compensation. I think that as actuaries we should be thankful that most of the Taft-Hartley plans developed in recent years provide dollar benefits, and not wage-related benefits. The shaky benefit-contribution structure of many of these plans will topple earlier if benefits are wage-related.

Our clients, being laymen, attach an aura of precision to our mortality assumptions which is often not justified. Disability assumptions usually are not questioned. At the mention of interest rates, ears perk up. Projections of rates of employment termination often evoke quizzical looks. The actuary's projections of future salary increases, a function directly controlled by the client, begin to take on the appearance of teacup reading in the eyes of some clients.

If we estimate final pensions for employee communication purposes, it seems prudent to assume the present salary remains unchanged; thus, the employer doesn't get the full employee relations value for the money he is spending.

I need not point out that the personal preferences of the actuary will carry considerable weight in developing the benefit structure of a new plan. I personally favor the career average approach in establishing a new plan. After the employer has had five or ten years' experience with the plan, he knows something about his pension plan and his ability to pay for it. At that time, and in the light of the current financial situation, the employer can liberalize the career average formula or change over to a final average formula. We have recently amended several plans which we installed ten to fifteen years ago. In all cases active employees were given full past service credit on the new basis. In some cases retired employees were given increased benefits. In other cases the employers felt that they did not wish to augment the increases provided by Social Security.

Pension plans for state and municipal employees do not usually give the actuary much opportunity to influence the benefit structure. In many plans the benefit is based on the highest five years' salary. In one class of plans the salary rate in effect on the day of retirement is used and employees will often delay retiring, if a general pay raise is announced, until the raise is in effect.

Some plans of this type envisage 50-50 sharing of costs. The employee contribution rate varies with entry age and is determined using a salary scale. Employee contributions are accumulated at interest and converted into pension at retirement. The test of the effectiveness of the salary scale assumption is found by comparing the employee pension with the employer pension, the latter being based on the formula. This type of benefit structure presents certain problems in employee communications. Consider the employee whose pension will be $100 per month provided by employee contributions plus $100 per month provided by his employer, assuming his salary increases according to the scale used in determining the employee contribution rate. On joining the plan it seems prudent to project his pension on the basis of his current salary. This might produce an employee pension of $70 and an employer pension of $50 for a total of $120. Thus, the 50-50 sharing basis appears fictitious to the employee.

For the convenience of students, reference might be made here to salary increase methods which are applicable in situations where calendar time is deemed more important than age or service in influencing salary changes. Messrs. Latimer and Musher, in Volume VI of the *Proceedings* of the Conference of Actuaries employed a constant dollar increase per annum. This approach leads to some ingenious calculation methods developed through finite difference transformations.

I recently valued a firemen's pension fund where 95% of the members were earning exactly the same amount, although their ages ranged from 28 to 60 and past service ranged from two years to thirty-five years. I found it convenient to use a flat percentage salary increase factor, producing a geometric progression which was assumed to level off in forty years.

JOHN B. STEARNS:

A common problem for pension actuaries is the evaluation of the cost of a plan which contains a "floor." For example, consider a basic career-average plan with a supplementary proviso that the company will not pay less than a "floor" which is based on the earnings during the five or ten years immediately preceding retirement. This type of a "final salary supplement" has become popular in recent years because of the rising cost of living. It tends to adjust for the effects of inflation up to the time an employee retires.

To determine the cost of a "final salary supplement," it is generally insufficient to use a single salary scale. The amount of the supplementary benefit, being the excess, if any, of a "final salary benefit" over a career-average benefit, is very sensitive to the way an employee's earnings change. If his future earnings increase at a slow or moderate rate, then the supplement might not produce any additional benefit. Extra benefits arise only if his earnings increase markedly just before retirement. If we were to assume that each employee's future earnings follow the average for the company as a whole—that is, if we project the benefits on the basis of only one scale for all employees—we might seriously understate the cost of a "final salary supplement." Use of an average salary scale might even lead us to think that the supplement would have no extra cost at all. However, as the author reminds us in his introductory section, a salary scale represents the average result of a great number of individual salary movements. We know that there will be *some* cost.

To evaluate this cost we should theoretically determine the cost of the supplementary benefits for each of the various salary progressions which might apply to each employee. Many of the possible salary progressions will produce a zero cost, but a few will produce a positive cost. Then we should assign probabilities to the occurrence of the various salary progressions and find an average cost, weighted according to these probabilities.

This is a theoretical objective, but it is obviously such a tremendous job to calculate that it is completely out of the question as a practical matter. The real problem is to find a reasonably accurate approximation.

Broad grouping may be helpful. The form it takes naturally depends on the circumstances of the particular case, such as the age of the group and the conditions surrounding the employment of executives and supervisors and the lower paid employees, etc. Consequently, I have no general approach to offer. However, it might be helpful to explain briefly what we did to estimate the cost for our own retirement plan when we introduced an alternate final salary plan several years ago. We subdivided our employees into five levels, depending on their annual salaries. We called the levels A, B, C, D, and E. On the basis of our own experience we set up a square table of probabilities according to age and salary level which showed, for example, the chance that a 35 year old man now in scale A would retire in scale E. Then we assumed for each of these situations that the salary would increase geometrically from the current amount to the maximum for the level, at retirement. From there on the calculation procedure was obvious, though tedious.

The author's "Family of Salary Scales" approach might have been quite a useful tool to handle this problem and he suggests this, perhaps, in his section concerning high-salaried members. This would have avoided the difficulty of finding an appropriate rate of geometric increase for each of the possible situations and computing figures based on a scale with such an increase rate, and I do believe the results might have been a little nearer the truth.

I have one other general comment on this topic. We must allow a large tolerance for a wide variation in ratio of actual to expected in determining pension costs, so great refinements are unwarranted. It really doesn't matter so much whether we use the "increase ratio" method or something even more refined which might, for example, give more direct recognition to the length of service of an employee. Any actuary would need more luck than good sense to predict the pension costs that have actually emerged under a pension plan which has been kept up to date in recent years.

RAYMOND B. KRIEGER:

We, in our company, thought that it would be useful to investigate the relative effect on costs, and on elements entering into costs, of the portion of a salary scale at the upper range of ages as compared with that at the lower range. In order to do this we used a "model office" distribution of salaries and lives by sample attained ages and we computed costs of an assumed retirement benefit making use of three alternate salary scale assumptions:

(i) No salary scale (0–0).
(ii) Annual salary rate increases of 3% to age 50 with no increases thereafter (3–0).
(iii) Annual salary rate increases of 3% to age 50 with annual increases of 2% thereafter (3–2).

The benefit assumed for our calculations is one equal to 40% of "final" salary, inclusive of the primary Social Security benefit. "Final" salary is taken to be the average annual salary during the five years prior to retirement date. We assume all employees to be males and to retire at age 65;

TABLE I

CENSUS DATA

Attained Age as of Valuation Date	Number of Lives	Annual Salary per Life		Total Salaries	
		Calculation A	Calculation B	Calculation A	Calculation B
30..........	23	$ 5,000	$ 5,000	$115,000	$ 115,000
40..........	33	6,500	8,000	214,500	264,000
50..........	28	8,500	12,000	238,000	336,000
60..........	16	12,000	20,000	192,000	320,000
All.....	100	....	....	$759,500	$1,035,000

an average age at entry of 25 is postulated. Our mortality, interest, loading and withdrawal rate assumptions are those that we have generally been using for cost estimates and valuations. Calculations have been performed using two different sets of current salary data, one varying more sharply by attained age than the other.

Tables I, II and III have been prepared showing our results in some detail. We are presenting them in the hope that they will also be of some value to others. Some of the following points may be of interest:

(1) For the particular salary scale assumptions mentioned above, it is clear that employees under age 50 on the valuation date will be affected, at least to some extent, by the salary scale under age 50, while employees over age 50 will not be affected by the scale below this point. Measuring from the total single-sum liability figure, we note that the effect on costs of the salary scale above 50 is more than double that of the scale below 50, even though the annual rate of salary increase for ages over and under 50 are 2% and 3% respectively.

TABLE II

NORMAL COST BASED ON ENTRY AT AGE 25 WITH AN ANNUAL SALARY OF $4,000

Assumed Annual Rate of Salary Increase: Prior to Age 50	Assumed Annual Rate of Salary Increase: Age 50 and Later	Total Single-Sum Liability	Normal Cost: Percent	Normal Cost: Dollars	Present Value of Future Salaries from Attained Age	Initial Accrued Liability	Normal Cost plus Interest on Initial Accrued Liability
		Calculation A					
0%	0%	$ 961,270	.51%	$ 3,874	$ 8,138,182	$ 919,765	$ 33,766
3%	0%	1,123,253	3.22	24,460	8,533,446	848,476	52,035
3%	2%	1,483,645	3.84	29,169	10,139,280	1,094,297	64,734
		Calculation B					
0%	0%	$1,757,791	.51%	$ 5,280	$10,512,082	$1,704,179	$ 60,666
3%	0%	1,943,392	3.22	33,336	10,953,381	1,590,693	85,034
3%	2%	2,423,949	3.84	39,755	12,897,239	1,928,695	102,438

TABLE III

NORMAL COST BASED ON ENTRY AT AGE 25 WITH ANNUAL SALARY AT AGE 25 DETERMINED BY BACK PROJECTION FROM PRESENT SALARY

Assumed Annual Rate of Salary Increase: Prior to Age 50	Assumed Annual Rate of Salary Increase: Age 50 and Later	Total Single-Sum Liability	Normal Cost: Percent	Normal Cost: Dollars	Present Value of Future Salaries from Attained Age	Initial Accrued Liability	Normal Cost plus Interest on Initial Accrued Liability
		Calculation A					
0%	0%	$ 961,270	1.61%	$12,230	$ 8,138,182	$ 830,245	$ 39,213
3%	0%	1,123,253	3.49	26,510	8,533,446	825,436	53,337
3%	2%	1,483,645	3.94	29,929	10,139,280	1,084,157	65,164
		Calculation B					
0%	0%	$1,757,791	2.04%	$21,120	$10,512,082	$1,543,345	$ 71,279
3%	0%	1,943,392	4.08	42,240	10,953,381	1,496,494	90,876
3%	2%	2,423,949	4.39	45,449	12,897,239	1,857,760	105,826

The costs illustrate, too, the drastic understatement in liabilities that can result from omitting a salary scale.

(2) It is obvious that the present value of salaries from attained age should be relatively unaffected by the salary scale in the range of ages below 50 and affected to a much greater extent by the scale above this point. The increase in present value is in fact, of the order of 5% in going from (0–0) to (3–0) and of the order of 25% in going from (0–0) to (3–2).

(3) We note, as we might expect, that the accrued liability is affected to a very small extent by introducing a salary scale for the lower range of ages. This follows since most of the accrued liability relates to older, longer service employees who are, to a large extent, unaffected by the lower end of the scale. Since the normal cost is a function of age at entry, we are not surprised by the marked jump in the normal cost when comparing the (3–0) with (0–0) scale results. In order to explain this result in normal cost in terms of some of its elements, the introduction of a salary scale increases the projected benefit quite sharply and, at the same time, the present value of salaries measured from attained age decreases. This latter effect is probably due, primarily, to the implicit assumption of higher salaries prior to the valuation date for the (0–0) scale as compared with the (3–0) scale.

(4) In going from the (3–0) to the (3–2) scale, we note the reverse effect of that noted under (3)—*i.e.*, the accrued liability increases to a much greater extent than the normal cost, because the primary effect here is on the older, longer service employees whose liabilities weight our results heavily. Viewing this in terms of the cost elements, the introduction of a salary scale at the higher ages results in a relatively large increase in total liability, benefit and present value of future salaries measured from entry age. The increase in benefit and increase in present value of salaries tend to offset each other in the determination of a normal cost percent. The end result, therefore, is that the increase in total liability goes primarily into the accrued liability and not into the normal cost portion of the costs.

WALTER RIESE:

Mr. Marples has presented a stimulating paper on a fascinating subject. The special techniques described in the paper and the demonstration of the effect of providing for inflation should prove particularly useful.

I agree with Mr. Marples that the salary scale is no instrument of precision, but one of his conclusions puzzles me. After describing and comparing the two main methods of deriving salary scales, Mr. Marples concludes that the Current Average method cannot be wholly satisfactory in an unmodified condition, and we are left with the implication that the Increase Ratio method is satisfactory—at least as modified by Mr. Marples.

The Current Average Method

With regard to the Current Average method I would go further than Mr. Marples; I would suggest that perhaps what the method needs is not to be modified but to be scrapped. We all agree that the Current Average method ignores the effect of future inflation, but this characteristic is not necessarily a weakness as long as we are aware of it. As Mr. Marples has pointed out, once a decision is reached on how much provision for inflation should be made, this is easily provided for. The essential weakness is that

Attained Age	0 Years, Service		5 Years, Service		10 Years, Service		Total	
	Number	Av. Sal.	Number	Av. Sal.	Number	Av. Sal.	Number	Av. Sal.
Company A								
25.....	1	$2,000	1	$3,000			2	$2,500
30.....			2	3,000	1	$4,500	3	3,500
Company B								
25.....	1	$2,000	1	$3,000			2	$2,500
30.....			1	3,000	2	$4,500	3	4,000

the scale produced by the Current Average method is affected by the distribution of employees in such a way that only by chance will this scale represent accurately the underlying pattern of promotional salary increases.

Let us assume, for example, that two companies have the same policy of increasing an employee's salary by fifty percent between his twenty-fifth and thirtieth birthdays (in other words the five-year increase ratio is known to be 1.5), and let us assume distributions of employees as shown in the accompanying table. Clearly, for Company A the Current Average method produces a five-year increase ratio of 1.4. Furthermore, if we attempt to fill in the blank spots for both attained ages, we are likely to increase the average salary for age 25 and decrease the average salary for age 30, which would produce an increase ratio even further below the stipulated 1.5. On the other hand, for the "top-heavy" distribution in Company B, the Current Average method produces a five-year increase ratio of 1.6, which is too high.

The Increase Ratio Method

The Increase Ratio method superficially commends itself as tending to be factual rather than hypothetical. However, it seems to present some problems, quite apart from the possible difficulty of obtaining the necessary salary records.

Firstly, while the initial unadjusted increase ratio makes allowance for inflation in a rough sort of way, this allowance, as Mr. Marples points out, reflects the specific conditions prevailing between two arbitrary dates. There is no assurance that similar conditions will prevail in the future. Mr. Marples has adjusted the initially obtained scale for increases in the Consumer Price Index, but we cannot be sure that such adjustment will remove the allowance for inflation from the salary scale, because increases in general salary levels during the observation period may bear no particular relation to changes in the Consumer Price Index and may reflect such factors as increased productivity and favorable bargaining position in a specific industry or a particular company.

Secondly, since the data on which the ratios are based exclude terminating employees, and since normally the increase ratios applicable to terminating employees might be expected to be smaller than those applicable to continuing employees, it seems reasonable to suppose that the resulting salary scale may unduly inflate salaries of terminating employees. In some situations the reverse may be true. However, this is a minor point.

Thirdly, there is a danger that the basic nature of the increase ratio produced by this method may not be properly understood. Effectively, this ratio seems to be a weighted average obtained by weighting the increase ratio applicable to each duration-of-service group at the first of the two chosen dates by the total salaries in that group. Whether or not this is the proper weighting may depend on the particular plan.

The Current Increase Ratio Method (?)

Now, I would like to describe a third approach which seems to overcome some of the weaknesses of the other two, but which itself, as Mr. Marples would say, is not wholly satisfactory, perhaps mainly because it is an approach rather than a clear-cut method. This approach is based on the assumption that salary is a function of age and service. The initial step is to schedule average current salaries by length of service for various age-at-entry groups and to work out increase ratios from duration to duration for each age-at-entry group. The table below shows five-year increase ratios which were obtained for males with central age 35 (actually 34 to 36 nearest birthday) in one particular plan.

Years of Service at Central Age 35	S_{40}/S_{35} (Crude)
0	1.113
5	1.043
10	1.159
15	1.113

If a conventional salary scale depending on attained age only is to be used, a series of ratios such as the above poses the problem of finding a weighting formula that will produce the right composite increase factor from any one attained age to the next. The weighting formula adopted for the case illustrated above produced an increase ratio of 1.122. For the same data the Current Average method produced an increase ratio of 1.031; this obviously is well below the increase ratio applicable to any of the groups of employees corresponding to the periods of service in the above table. It would seem reasonable to conclude that any arbitrarily chosen increase ratio lying between 1.043 and 1.159, or preferably between 1.11 and 1.16, would be more appropriate than the one produced by the Current Average method.

ROBERT J. MYERS:

Mr. Marples is to be congratulated on making a valuable contribution to pension fund literature by giving a discussion of salary scale techniques under current conditions and usage. He has rendered a valuable service in making readily available illustrations and comparisons of the two general methods for developing salary scales—the Current Average method, which may be termed the static or snapshot method, and the Increase Ratio method, which may be termed the dynamic or cohort method.

Under completely static economic conditions—such as may have existed in the classical days of yore (or may have been thought to exist then)—these two methods would, on the whole, give the same results. This, however, is by no means the case currently or in recent years because of the dynamic economic conditions. Three basic elements are intertwined in the salary scale problem. The first, which is really the only one recognized by the classic theory of salary scales, is the generally upward progression of salary with increasing age—and simultaneously with increasing length of service. The second is the increasing trend of the general price level, which Mr. Marples has taken into account. The third is the increasing trend of the general wage level relative to the general price level—reflecting increasing productivity—which Mr. Marples has also considered, but not separately from the general upward progression of salary with increasing age and service.

Considering Plan A, the annual increase in salaries during 1955–60 was about 5%–6% after excluding those under age 30, who would be expected to have somewhat larger increases at the beginning of their working careers. This increase represents the combined effect of all three of the elements mentioned previously. Data for the OASDI system (after eliminating the dampening effect of the maximum taxable earnings base) indicate that the increase in the general wage level averaged 4.4% per year during 1955–60 (the annual increases being 5.6%, 3.9%, 1.6%, 7.8%, and 2.9%). About half of this increase might be said to be due to the effect of the rise in the general price level, and the remainder might be said to represent the rise in real earnings, resulting from increased productivity. If the trend shown by the OASDI data were applicable to Plan A, then it would appear that the increase in average salaries was due more or less equally to each of the three elements discussed previously.

Mr. Marples derives salary scales on the basis of eliminating the effect of changes in the general price level, although he subsequently considers the effect of a continuing inflation of the general price level at a rate of 1% or 2% a year. It may thus be said that his salary scales include the effect of the inflation of earnings levels due to increases in productivity but under the hypothesis of a constant price level. Even so, he shows the striking result that final salary at age 65 may be some 8 times that for an entrant at age 20. Either this approach or the modification of it to include inflation of the general price level is satisfactory as long as the underlying assumptions are recognized and made clear.

The question then remains as to whether—in the fact of past price and wage trends and the likelihood of their continuance in the future—a salary scale developed on static principles can reasonably be used. Again, in my opinion this depends upon making a clear statement of the underlying assumptions and of the effect on the results of the cost estimates if there is inflation of wages (whether or not accompanied by inflation of prices).

I believe that a static salary scale can reasonably be used in conjunction with modified or adjusted assumptions in regard to the interest rate. It may well be viewed that a portion of the interest rate reflects the effect of inflation of prices, with the remainder being the "true" interest rate. Under present conditions, it might be argued that a total interest rate of $4\frac{1}{2}\%$ represents a $1\frac{1}{2}\%$ allowance for inflation of prices, with the remaining 3% representing the "true" interest rate. Accordingly, for a final-salary plan whose assets are earning $4\frac{1}{2}\%$, a valuation rate of 3% might be used along with a static salary scale. The resulting interest "gains" would not only fully permit taking into account the higher salary rates and the resulting higher initial pensions than "expected" from the static scale, but also would permit the adjustment of pensions initially granted for changes in the cost of living occurring after retirement. Similarly, in a career-average plan, sufficient margins would be available to update the plan to counteract the erosion in the average wage due to the inflation of the price level.

Disregarding the element of the price level, there still remains the problem that higher salaries may be experienced than "expected" according to the static salary scale, because of increasing real earnings. If a static salary scale developed by the Current Average method (or otherwise) is used, this element can be taken into account by proper choice of the valuation interest rate. An unduly low rate might be necessary, but from a methodological standpoint this can easily be done. The same procedure could be followed as Mr. Marples did in allowing for the effect of inflation of prices on salaries before retirement—namely, using an interest rate lower than the expected rate by the amount of the inflation rate and an adjusted contribution-rate value (modified by the ratio of the value of the annuity at the expected rate of interest to that at the lower, adjusted rate). This might be easier and more understandable than Mr. Marples' suggestion of using the Family of Salary Scales method, with its accompanying difficulty of obtaining (and explaining) the necessary deduction factor. Then, the allowance for future wage inflation due to increased productivity (and, if included, due to increased prices) would be readily apparent. The allowance for the increasing wage trend might be about 3% per year (see *Actuarial Study No. 53* of the Social Security Administration).

Long-term investment gains may be used to help meet the additional cost arising because of the more rapid increase in salaries than "expected" according to the static salary scale, since equities might increase in value in somewhat the same way as rises in productivity. Of course, still another solution is to do as Mr. Marples suggests—namely, build the productivity factor into the salary scales.

As Mr. Marples demonstrates, the effect of the increasing trend of the general wage level, which apparently is likely to occur even though the price level remains constant, has a very significant effect on pension costs. This must be recognized both by actuaries and by those concerned with pension plans, even though this factor does involve elements of higher cost—and thus bad news. As it so happens, for cash benefits under the OASDI system, the factor of increasing wages is a margin of safety, rather than the reverse. The controlling feature is that the benefit formula is of a bent nature—providing benefits that are a decreasing proportion of average wage as the latter increases. Even if this were not the case, the cost effect would not be too great because the average-wage basis is the career-average type. Nonetheless, it is recognized that rising earnings levels will undoubtedly necessitate changes in OASDI benefit levels so that any resulting reductions in cost (relative to taxable payroll) may be used up in this manner.

One other aspect of salary scales might be mentioned. Undoubtedly, for purposes of simplification, Mr. Marples determines salary scales on an aggregate basis by attained age. It is widely recognized that, in many instances, salary scales should be on a select and ultimate basis. A particular valuation in which I was interested (the United Nations Joint Staff Pension Plan) indicated rather surprisingly that for the particular groups of employees concerned (separately by sex and by professional and general service staff), the salary scales were actually virtually independent of attained age, and rather seemed to depend solely upon length of service. The first analysis of the data was on the basis of attained age and showed the usual picture as to salary scale, but with certain anomalies resulting. The subsequent analysis by duration of service showed that any variation in the salary scale by attained age was due solely to the correlation between length of service and age. Perhaps this particular group is so unique in its composition that this result might not apply for other plans, but nonetheless it may well be worth while to examine this possibility when making any salary scale study.

SAMUEL ECKLER:

The members of the Society and particularly those engaged in pension work are indebted to Mr. Marples for his scholarly revaluation of salary scale techniques and constructive suggestions for improvements in them. He reviews the differences between the Current Average and the Increase Ratio methods of deriving salary scales and includes very useful detailed illustrations of each of these methods from actual salary data.

It is evident that the author, in spite of his efforts to be charitable to the Current Average method strongly favors the Increase Ratio method. Certainly, the illustrations used by Mr. Marples might make one wonder about the validity of results obtained through the Current Average method, which I think is still the most common one used.

I share the author's amazement at the differences between the salary curves derived from these two methods and shown in Charts 1 and 2, even after the salary scales under the Increase Ratio method have been adjusted for annual consumer price increases. I do not understand why the adjustment was based on the consumer price index and not on some index which measures the general increase in wages or salaries. It is the increase in the general salary index over a period of time rather than the consumer price index which would distort the relationship between salary scales derived from current salary data and those derived from increase ratios.

There may be other reasons as well for the wide differences between the salary scales derived from the current salary and the increase ratio methods, aside from the obvious ones such as the peculiarities of the data or the period studied. For example, suppose during the period from 1955 to 1960 the salaries of employees at the younger ages and early service rose a great deal while those at the older ages rose scarcely at all perhaps because of labor shortages in the young age group or the rapid growth of an industry. Under such circumstances, the Increase Ratio method which produces the salary scale at the advanced ages partially from the increments at the young ages over a period of time must result in a salary scale much steeper in slope than one derived by the current salary method.

The salary variable of future pension costs depends on two factors which we like to call "general salary" increases and "seniority" increases. We prefer the phrase "general salary" rather than inflationary increases since it is quite possible to have general salary increases due to increases in productivity without any rise in the consumer price index. It seems to me that if we can isolate the seniority salary increases from the salary data, we can go a long way towards more effectively estimating future pension costs. I think that some of the ideas in Mr. Marples' paper will help in this area.

RALPH E. EDWARDS:

At the present time, the primary interest of the actuary in salary scales (other than his own) arises from pension problems. We are greatly indebted to Mr. Marples for his most useful contribution to our literature on this topic. Without wishing to detract from the praise he deserves, I would suggest, in addition, that salary scales in the future may come to have wider applicability in the planning of other insurance coverages. The nature of salary scales in various employments may also come to be of considerable value to business men and economists.

In this discussion, I will present salary scale data having characteristics unlike those usually encountered. The employer in this instance was a labor union. There were two classes of employee, clerical and nonclerical. The latter class appears not to have been recruited from the former, but rather from among members of the union. The youngest member of this class was born in 1928.

For the clerical employees, who were mostly females, only current salary rates were available. Salary rates for males and females appeared to be equal. The scale is very flat, becoming level at about age 35, as can be seen from Table A on page 49.

For the nonclerical employees, current salaries showed only a slight tendency to increase with age. Since the youngest age was about 35, this was consistent with the scale for clerical employees. As with clerical employees, salary rates for males and females appeared to be equal. The number of higher paid employees—the executives—was relatively low, and their salaries were low enough to have little effect on the average salary, even by age group. Current salary rates are shown in Table B.

For the nonclerical employees salaries for 1956 and later were available, so that it was possible to trace 133 employees for a full five-year period. The data were divided arbitrarily into lower and higher salary ranges and it was observed that the number going from the lower to the higher group was 4 and the number going from higher to lower was 3, so that there was a net of only one "promotion" during the period.

The nonclerical employees did not receive increases at regular intervals, and these intervals were different among the higher salary group. The data, shown in Table C, seem to indicate that the increase ratio was higher for the higher salary group, but additional information shows that this was probably due to timing rather than any inherent difference.

(AUTHOR'S REVIEW OF DISCUSSION)

WILLIAM F. MARPLES:

I am grateful for the large participation in the discussion on my paper, and I take it as evidence of general interest in the subject. My intention was to outline the theories involved in the determination of salary scales, including the development of the family of salary scales. I purposely ignored wider aspects in order to concentrate on producing the tools, leaving each actuary to his own carving of the profile appropriate to the case under consideration. There was no intention to favor one method of development of a salary scale over another, but I wished to emphasize that the final product should not be just a mechanical production but should include adjustments for H. W. Manly's "intelligent graduation." I do not believe that his use of this phrase is limited to smoothing processes. What we are seeking is an approach to a rate of benefit increase which can be applied to the current salary to give as realistic as possible estimate of the benefit emerging at retirement. With this firmly in mind, I believe that the Increase Ratio method may tend to overestimate the rate of bene-

TABLE A

CLERICAL EMPLOYEES—1961 SALARY RATES

Year of Birth	Number of Employees	Average Annual Salary Rate
1894–1923	57	$5,104
1924–1928	19	5,017
1929–1933	30	4,776
1934–1938	34	4,649
1939–1944	44	4,132

TABLE B

NONCLERICAL EMPLOYEES—1961 SALARY RATES

Year of Birth	Number of Employees	Average Annual Salary Rate
1894–1898	3	$ 9,667
1899–1903	16	10,941
1904–1908	47	10,851
1909–1913	49	11,341
1914–1918	30	9,566
1919–1923	47	10,155
1924–1928	27	9,921
Total	219	$10,511

TABLE C

NONCLERICAL EMPLOYEES—RATIO OF 1961 TO 1956 SALARY

Sex	Salary Class	Number	Ratio
Male	Lower	85	1.251
Female	Lower	27	1.259
Male	Higher	12	1.346
Female	Higher	2	1.333
Both	Changed	7	1.384
Total	....	133	1.273

fit increase because it produces the estimate on the assumption of increases continuing at the rate disclosed by the data over a relatively short period of time. If the same increases were developed by the next five years, I would be impressed, and if they were shown again over a third period, I would start to take serious notice of them. We are seeking truth here and it may only emerge slowly from obscurity, and then only if we are looking in the right direction.

I do not understand the adjustments of the crude figures by a salary or wage index, as proposed by Mr. Eckler and Mr. Calvert, as it seems to me that this would tend to take out the very increases we are interested in measuring. Mr. Eckler's seniority increase on top of a general increase seems to be similar to Mr. Stearns's ideas, and suggests a device in use by actuaries dealing with a large employment situation with multi-stage executive levels. The family of salary scales theory can be used to simplify the mathematics.

I found Mr. Myers' discussion to be of great interest, not only for its general summary of the economic aspects of salary scales, but also for his illustration of an extreme special case. I understand that the United Nations Joint Staff has an exceptionally large number of specialists on contract and apparently does show special statistical variations as a result. I am greatly obliged to him for reminding us that our methods and formulas must not be applied blindly.

I am inclined to agree with the remarks of Mr. Reise on the salary increase method and perhaps a salary exposed to risk (by formula) rather than a census approach might give better results. Unfortunately, the requisite data are almost unobtainable. I am somewhat at a loss to grasp the implications of his third method.

Mr. Calvert reminds us of the economic aspects of pension plans. Perhaps he could be prevailed upon one of these days to develop his remarks as a full paper to the Society.