

1958 CET AGE LAST BIRTHDAY  
EXTENDED TERM INSURANCE TABLES

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THE term of extended insurance may be calculated readily from commutation column type functions.<sup>1</sup> The age last birthday basis 1958 CET curtate functions at  $2\frac{1}{2}\%$  and at  $3\%$  are appended.

NOTATION AND FORMULA

$x$  = age at transfer to extended insurance.

CV = cash value per \$1,000 of extended insurance.

$t$  = whole years excluding fractions of a year in the term of extended insurance for which the present value is equal to the cash value.

$n$  = additional number of days of extended insurance following  $t$  whole years. In consequence,  $n/365$  = additional fraction of a year of extended insurance.

The additional number of days is determined by the following formula:

$$n = \left[ \frac{D_x}{1,000} \times CV + (10,000,000 - M_x) - (10,000,000 - M_{x+t}) \right] \div \frac{C_{x+t}}{365}$$

where  $x + t$  is such that  $(10,000,000 - M_{x+t})$  is the largest value which can be subtracted from the sum which precedes it. The procedure for calculating the term of extended insurance is thus of the form  $[A \times B + C - D] \div E$ . With proper setting of decimals in the usual desk calculating machine, this is a continuous process and the calculation requires about 45 seconds.

PURE ENDOWMENT

When the extended insurance extends to the maturity date of an endowment policy, the policy usually provides for a pure endowment benefit at maturity. The amount of the pure endowment may be found from the equation  $[D_x/1,000 \times CV + (10,000,000 - M_x) - (10,000,000 - M_{x+t})] \div D_{x+t}/1,000$  = amount of pure endowment payable at age  $x + t$  per \$1,000 of extended insurance.

<sup>1</sup> *The Actuary's Handbook* (Sarason and Warren), Vols. I & II, pages 13 and 242; Vols. IV & V, pages vi & vii; *RAIA XV*, page 219 (Porter); *TSA XI*, page 346 (Sarason and Havens); *Actuarial Table News*, Vol. I, page 9 (Sarason); *Proceedings*, Conference of Actuaries in Public Practice, Vol. IX, page 406 (Amerman).

TABLE 1

COMMISSIONERS 1958 EXTENDED TERM (CET)—2½%  
AGE LAST BIRTHDAY BASIS EXTENDED INSURANCE FACTORS (MALE)

Age Last Birthday $x$	$D_x/1,000$	$10,000,000-M_x$	$C_x/365$
0.....	9,960.8500	7,704,012.0	137.92716
1.....	9,667.5590	7,754,355.4	61.76102
2.....	9,409.2221	7,776,898.2	56.33673
3.....	9,159.1660	7,797,461.1	53.36918
4.....	8,916.2920	7,816,940.8	50.64606
5.....	8,680.3356	7,835,426.6	48.14451
6.....	8,451.0474	7,852,999.4	45.85702
7.....	8,228.1864	7,869,737.2	43.87509
8.....	8,011.4846	7,885,751.6	42.18632
9.....	7,800.6845	7,901,149.6	40.86631
10.....	7,595.5077	7,916,065.8	39.99485
11.....	7,395.6533	7,930,663.9	39.43494
12.....	7,200.8777	7,945,057.7	39.26412
13.....	7,010.9152	7,959,389.1	39.44658
14.....	6,825.5192	7,973,787.1	39.68002
15.....	6,644.5600	7,988,270.3	39.95980
16.....	6,467.9122	8,002,855.6	40.27976
17.....	6,295.4561	8,017,557.7	40.46899
18.....	6,127.1372	8,032,328.9	40.36967
19.....	5,962.9599	8,047,063.9	40.08561
20.....	5,802.8906	8,061,695.1	39.70656
21.....	5,646.8638	8,076,188.0	39.16619
22.....	5,494.8398	8,090,483.7	38.55249
23.....	5,346.7476	8,104,555.3	37.87272
24.....	5,202.5156	8,118,378.9	37.12928
25.....	5,062.0728	8,131,931.0	36.46446
26.....	4,925.2981	8,145,240.6	35.87328
27.....	4,792.0751	8,158,334.3	35.35236
28.....	4,662.2916	8,171,237.9	34.95468
29.....	4,535.8187	8,183,996.4	34.61413
30.....	4,412.5548	8,196,630.5	34.32037
31.....	4,292.4046	8,209,157.5	34.07506
32.....	4,175.2744	8,221,594.9	33.86981
33.....	4,061.0760	8,233,957.4	33.75843
34.....	3,949.7035	8,246,279.2	33.83436
35.....	3,841.0197	8,258,628.7	34.34041
36.....	3,734.8021	8,271,163.0	35.28731
37.....	3,630.8295	8,284,042.8	36.63392
38.....	3,528.9013	8,297,414.2	38.38645
39.....	3,428.8195	8,311,425.3	40.41433
40.....	3,330.4385	8,326,176.5	42.63603
41.....	3,233.6461	8,341,738.7	44.98364
42.....	3,138.3577	8,358,157.7	47.43109
43.....	3,044.5000	8,375,470.0	49.99954
44.....	2,951.9941	8,393,719.9	52.70037
45.....	2,860.7586	8,412,955.5	55.58285
46.....	2,770.6963	8,433,243.2	58.68137
47.....	2,681.7000	8,454,661.9	62.02696
48.....	2,593.6525	8,477,301.8	65.56907
49.....	2,506.4599	8,501,234.5	69.32396

TABLE 1—Continued

Age Last Birthday <i>x</i>	$D_x/1,000$	$10,000,000-M_x$	$C_x/365$
50.....	2,420.0235	8,526,537.7	73.27039
51.....	2,334.2549	8,553,281.4	77.31462
52.....	2,249.1020	8,581,501.3	81.46365
53.....	2,164.5116	8,611,235.5	85.68565
54.....	2,080.4434	8,642,510.8	89.97168
55.....	1,996.8612	8,675,350.4	94.35645
56.....	1,913.7172	8,709,790.5	98.86140
57.....	1,830.9567	8,745,874.9	103.45945
58.....	1,748.5365	8,783,637.6	108.07182
59.....	1,666.4431	8,823,083.9	112.65233
60.....	1,584.6800	8,864,202.0	117.15288
61.....	1,503.2685	8,906,962.8	121.48778
62.....	1,422.2604	8,951,305.8	125.62827
63.....	1,341.7168	8,997,160.1	129.52497
64.....	1,261.7154	9,044,436.7	133.14751
65.....	1,182.3430	9,093,035.6	136.45829
66.....	1,103.6981	9,142,842.9	139.40682
67.....	1,025.8951	9,193,726.3	141.91877
68.....	949.0729	9,245,526.7	143.74359
69.....	873.4584	9,297,993.1	144.57920
70.....	799.3831	9,350,764.5	144.16140
71.....	727.2671	9,403,383.4	142.33015
72.....	657.5783	9,455,333.9	139.07596
73.....	590.7771	9,506,096.7	134.64029
74.....	527.2242	9,555,240.4	129.38335
75.....	467.1402	9,602,465.3	123.57078
76.....	410.6432	9,647,568.6	117.37835
77.....	357.7844	9,690,411.7	110.84994
78.....	308.5977	9,730,872.0	103.86905
79.....	263.1587	9,768,784.2	96.26553
80.....	221.6033	9,803,921.1	88.01683
81.....	184.0722	9,836,047.2	79.21513
82.....	150.6691	9,864,960.7	70.05256
83.....	121.4251	9,890,529.9	60.82573
84.....	96.2621	9,912,731.3	51.83637
85.....	74.9940	9,931,651.6	43.33517
86.....	57.3475	9,947,468.9	35.50939
87.....	42.9879	9,960,429.9	28.49962
88.....	31.5370	9,970,832.2	22.38881
89.....	22.5959	9,979,004.1	17.19859
90.....	15.7673	9,985,281.6	12.89814
91.....	10.6749	9,989,989.4	9.41887
92.....	6.9766	9,993,427.3	6.67091
93.....	4.3716	9,995,862.2	4.55405
94.....	2.6028	9,997,524.4	2.97965
95.....	1.4517	9,998,612.0	1.85948
96.....	.7376	9,999,290.7	1.09962
97.....	.3182	9,999,692.1	.59329
98.....	.0939	9,999,908.6	.22202
99.....	.0106	9,999,989.7	.02829

**TABLE 2**  
**COMMISSIONERS 1958 EXTENDED TERM (CET)—2½%**  
**AGE LAST BIRTHDAY BASIS EXTENDED INSURANCE FACTORS**  
**FOR PERMISSIBLE FEMALE EXTENSION AT AGES 0 TO 14**

Age Last Birthday* $x$	$D_x/1,000$	$10,000,000 - M_x$	$C_x/365$
0.....	10,771.4434	7,656,736.2	135.11325
1.....	10,459.4089	7,706,052.5	64.02459
2.....	10,180.9324	7,729,421.5	57.96438
3.....	9,911.4600	7,750,578.5	54.83996
4.....	9,649.7005	7,770,595.1	51.97428
5.....	9,395.3713	7,789,565.7	49.34552
6.....	9,148.2048	7,807,576.8	46.95058
7.....	8,907.9409	7,824,713.7	44.88285
8.....	8,674.2918	7,841,096.0	43.24005
9.....	8,446.9411	7,856,878.6	41.99412
10.....	8,225.5903	7,872,206.5	41.00585
11.....	8,009.9990	7,887,173.6	40.24921
12.....	7,799.9422	7,901,864.6	39.71698
13.....	7,595.2030	7,916,361.3	39.38305
14.....	7,395.5794	7,930,736.1	39.23734
15.....	7,200.8777	7,945,057.7	39.26412

\* For ages 15 and over, use factors opposite age  $x - 3$  in CET Table and, when solving for age at expiry, add 3 to (male) age at expiry.

TABLE 3

 COMMISSIONERS 1958 EXTENDED TERM (CET)—3%  
 AGE LAST BIRTHDAY BASIS EXTENDED INSURANCE FACTORS (MALE)

Age Last Birthday $x$	$D_x/1,000$	$10,000,000-M_x$	$C_x/365$
0.....	9,960.8500	8,214,132.1	137.25761
1.....	9,620.6291	8,264,231.1	61.16285
2.....	9,318.0922	8,286,555.6	55.52027
3.....	9,026.4266	8,306,820.5	52.34041
4.....	8,744.4167	8,325,924.7	49.42867
5.....	8,471.6835	8,343,966.2	46.75916
6.....	8,207.8683	8,361,033.3	44.32128
7.....	7,952.6269	8,377,210.5	42.19987
8.....	7,705.5940	8,392,613.5	40.37861
9.....	7,466.4211	8,407,351.7	38.92528
10.....	7,234.7448	8,421,559.4	37.91029
11.....	7,010.1868	8,435,396.7	37.19812
12.....	6,792.4293	8,448,974.0	36.85719
13.....	6,581.1387	8,462,426.9	36.84872
14.....	6,376.0052	8,475,876.6	36.88684
15.....	6,176.8326	8,489,340.3	36.96661
16.....	5,983.4321	8,502,833.2	37.08171
17.....	5,795.6225	8,516,368.0	37.07506
18.....	5,613.2856	8,529,900.4	36.80454
19.....	5,436.3582	8,543,334.0	36.36815
20.....	5,264.7433	8,556,608.4	35.84938
21.....	5,098.3162	8,569,693.4	35.18985
22.....	4,936.9773	8,582,537.7	34.47031
23.....	4,780.6002	8,595,119.4	33.69813
24.....	4,629.0596	8,607,419.2	32.87627
25.....	4,482.2327	8,619,419.0	32.13087
26.....	4,339.9545	8,631,146.8	31.45650
27.....	4,202.0664	8,642,628.4	30.84924
28.....	4,068.4162	8,653,888.4	30.35413
29.....	3,938.8394	8,664,967.7	29.91250
30.....	3,813.1978	8,675,885.7	29.51467
31.....	3,691.3610	8,686,658.6	29.16145
32.....	3,573.2017	8,697,302.5	28.84509
33.....	3,458.5994	8,707,831.0	28.61067
34.....	3,347.4206	8,718,273.9	28.53583
35.....	3,239.5073	8,728,689.4	28.82202
36.....	3,134.6327	8,739,209.5	29.47299
37.....	3,032.5751	8,749,967.1	30.44919
38.....	2,933.1337	8,761,081.1	31.75096
39.....	2,836.1135	8,772,670.2	33.26604
40.....	2,741.3662	8,784,812.3	34.92440
41.....	2,648.7731	8,797,559.7	36.66853
42.....	2,558.2404	8,810,943.7	38.47589
43.....	2,469.6848	8,824,987.4	40.36251
44.....	2,383.0199	8,839,719.7	42.33626
45.....	2,298.1589	8,855,172.5	44.43512
46.....	2,215.0034	8,871,391.3	46.68446
47.....	2,133.4489	8,888,431.1	49.10653
48.....	2,053.3857	8,906,355.0	51.65881
49.....	1,974.7229	8,925,210.4	54.35198

TABLE 3—Continued

Age Last Birthday $x$	$D_x/1,000$	$10,000,000-M_x$	$C_x/365$
50.....	1,897.3682	8,945,048.9	57.16723
51.....	1,821.2390	8,965,915.0	60.02981
52.....	1,746.2824	8,987,825.8	62.94422
53.....	1,672.4451	9,010,800.5	65.88502
54.....	1,599.6851	9,034,848.5	68.84479
55.....	1,527.9640	9,059,976.9	71.84946
56.....	1,457.2351	9,086,201.9	74.91440
57.....	1,387.4476	9,113,545.7	78.01809
58.....	1,318.5599	9,142,022.3	81.10064
59.....	1,250.5535	9,171,624.0	84.12762
60.....	1,183.4231	9,202,330.6	87.06389
61.....	1,117.1761	9,234,108.9	89.84716
62.....	1,051.8428	9,266,903.1	92.45827
63.....	987.4593	9,300,650.4	94.86336
64.....	924.0732	9,335,275.5	97.04310
65.....	861.7378	9,370,696.2	98.97333
66.....	800.5133	9,406,821.5	100.62108
67.....	740.4707	9,443,548.2	101.93690
68.....	681.6966	9,480,755.2	102.74642
69.....	624.3390	9,518,257.6	102.84204
70.....	568.6170	9,555,795.0	102.04705
71.....	514.8081	9,593,042.1	100.26170
72.....	463.2182	9,629,637.7	97.49377
73.....	414.1412	9,665,222.9	93.92614
74.....	367.7958	9,699,505.9	89.82070
75.....	324.2987	9,732,290.5	85.36905
76.....	283.6934	9,763,450.2	80.69736
77.....	245.9760	9,792,904.7	75.83915
78.....	211.1303	9,820,586.0	70.71813
79.....	179.1688	9,846,398.1	65.22319
80.....	150.1438	9,870,204.6	59.34493
81.....	124.1098	9,891,865.5	53.15115
82.....	101.0948	9,911,265.7	46.77515
83.....	81.0773	9,928,338.6	40.41710
84.....	63.9636	9,943,090.8	34.27670
85.....	49.5896	9,955,601.8	28.51620
86.....	37.7368	9,966,010.2	23.25311
87.....	28.1503	9,974,497.6	18.57221
88.....	20.5516	9,981,276.5	14.51918
89.....	14.6535	9,986,576.0	11.09917
90.....	10.1755	9,990,627.2	8.28345
91.....	6.8556	9,993,650.6	6.01963
92.....	4.4588	9,995,847.8	4.24270
93.....	2.7803	9,997,396.4	2.88232
94.....	1.6473	9,998,448.4	1.87671
95.....	.9143	9,999,133.4	1.16549
96.....	.4623	9,999,558.8	.68587
97.....	.1985	9,999,709.2	.36826
98.....	.0583	9,999,943.6	.13714
99.....	.0065	9,999,993.7	.01739

**TABLE 4**  
**COMMISSIONERS 1958 EXTENDED TERM (CET)—3%**  
**AGE LAST BIRTHDAY BASIS EXTENDED INSURANCE FACTORS**  
**FOR PERMISSIBLE FEMALE EXTENSION AT AGES 0 TO 14**

Age Last Birthday* $x$	$D_x/1,000$	$10,000,000 - M_x$	$C_x/365$
0.....	10,929.8445	8,165,740.8	136.43465
1.....	10,561.7008	8,215,539.4	64.33690
2.....	10,230.5955	8,239,022.4	57.96438
3.....	9,911.4600	8,260,179.4	54.57375
4.....	9,602.8573	8,280,098.8	51.47090
5.....	9,304.3755	8,298,885.7	48.63038
6.....	9,015.6242	8,316,635.8	46.04554
7.....	8,736.2266	8,333,442.4	43.80398
8.....	8,465.7849	8,349,430.9	41.99582
9.....	8,203.8802	8,364,759.3	40.58775
10.....	7,950.1177	8,379,573.9	39.44018
11.....	7,704.1652	8,393,969.5	38.52452
12.....	7,465.7106	8,408,031.0	37.83055
13.....	7,234.4546	8,421,839.1	37.33038
14.....	7,010.1167	8,435,464.7	37.01172
15.....	6,792.4293	8,448,974.0	36.85719

\* For ages 15 and over, use factors opposite age  $x - 3$  in CET Table and, when solving for age at expiry, add 3 to (male) age at expiry.

The factors in the foregoing tables were derived from commutation columns presented by Messrs. Irwin T. Vanderhoof and Robert C. Bailey (TSA XII, page 329).

## DISCUSSION OF PRECEDING PAPER

JOHN C. ANTLIFF:

At the Prudential in Newark we recently printed Monetary Tables on 1958 CSO and CET, using age last birthday and assuming  $2\frac{1}{2}\%$  interest and immediate payment of death benefits. Volume I contains basic functions including various net single premiums and certain nonforfeiture functions. Volume II contains net level premiums and terminal reserves. There is a slight difference between the CET  $D_x$  in our tables and the  $D_x$  in Mr. Sarason's paper. He used the commutation columns from *TSA XII*, pages 334 to 337. We followed the calculation formulas and methods described in the booklet, "Specifications for Monetary Tables based on 1958 CSO and CET Mortality Tables," published by the Society. It may be of value to have our alternative tables of commutation functions in the *Transactions*. Therefore,  $D_x \bar{C}_x$  and  $\bar{M}_x$  on age last birthday basis are shown on the following pages.



COMMISSIONERS 1958 EXTENDED TERM  $2\frac{1}{2}\%$

AGE LAST BIRTHDAY $x$	COMMISSIONERS 1958 EXTENDED TERM $2\frac{1}{2}\%$		
	$D_x$	$C_x$	$\bar{M}_x$
Male			
0.....	9,960,850.0	50,970.118	2,324,569.766
1.....	9,667,559.1	22,823.397	2,273,599.648
2.....	9,409,222.2	20,818.884	2,250,776.251
3.....	9,159,166.0	19,722.246	2,229,957.367
4.....	8,916,291.9	18,715.934	2,210,235.121
5.....	8,680,335.6	17,791.503	2,191,519.187
6.....	8,451,047.4	16,946.175	2,173,727.684
7.....	8,228,186.5	16,213.763	2,156,781.509
8.....	8,011,484.6	15,589.689	2,140,567.746
9.....	7,800,684.5	15,101.887	2,124,978.057
10.....	7,595,507.7	14,779.847	2,109,876.170
11.....	7,395,653.3	14,572.936	2,095,096.323
12.....	7,200,877.8	14,509.810	2,080,523.387
13.....	7,010,915.2	14,577.236	2,066,013.577
14.....	6,825,519.3	14,663.501	2,051,436.341
15.....	6,644,560.0	14,766.893	2,036,772.840
16.....	6,467,912.2	14,885.131	2,022,005.947
17.....	6,295,456.2	14,955.060	2,007,120.816
18.....	6,127,137.2	14,918.360	1,992,165.756
19.....	5,962,960.0	14,813.384	1,977,247.396
20.....	5,802,890.6	14,673.309	1,962,434.012
21.....	5,646,863.9	14,473.620	1,947,760.703
22.....	5,494,839.8	14,246.831	1,933,287.083
23.....	5,346,747.7	13,995.625	1,919,040.252
24.....	5,202,515.6	13,720.894	1,905,044.627
25.....	5,062,072.8	13,475.213	1,891,323.733
26.....	4,925,298.1	13,256.745	1,877,848.520
27.....	4,792,075.1	13,064.244	1,864,591.775
28.....	4,662,291.6	12,917.281	1,851,527.531
29.....	4,535,818.7	12,791.435	1,838,610.250
30.....	4,412,554.9	12,682.879	1,825,818.815
31.....	4,292,404.7	12,592.225	1,813,135.936
32.....	4,175,274.4	12,516.376	1,800,543.711
33.....	4,061,076.0	12,475.216	1,788,027.335
34.....	3,949,703.5	12,503.277	1,775,552.119
35.....	3,841,019.8	12,690.281	1,763,048.842
36.....	3,734,802.1	13,040.203	1,750,358.561
37.....	3,630,829.5	13,537.834	1,737,318.358
38.....	3,528,901.3	14,185.471	1,723,780.524
39.....	3,428,819.5	14,934.864	1,709,595.053
40.....	3,330,438.5	15,755.876	1,694,660.189
41.....	3,233,646.1	16,623.421	1,678,904.313
42.....	3,138,357.7	17,527.861	1,662,280.892
43.....	3,044,500.1	18,477.016	1,644,753.031
44.....	2,951,994.1	19,475.091	1,626,276.015

AGE LAST BIRTHDAY <i>x</i>	COMMISSIONERS 1958 EXTENDED TERM 2½%		
	<i>D<sub>x</sub></i>	<i>C<sub>x</sub></i>	<i>M<sub>x</sub></i>
Male— <i>continued</i>			
45.....	2,860,758.6	20,540.294	1,606,800.924
46.....	2,770,696.3	21,685.331	1,586,260.630
47.....	2,681,699.7	22,921.673	1,564,575.299
48.....	2,593,652.5	24,230.637	1,541,653.626
49.....	2,506,459.9	25,618.233	1,517,422.989
50.....	2,420,023.6	27,076.612	1,491,804.756
51.....	2,334,254.9	28,571.132	1,464,728.144
52.....	2,249,102.0	30,104.381	1,436,157.012
53.....	2,164,511.6	31,664.595	1,406,052.631
54.....	2,080,443.4	33,248.470	1,374,388.036
55.....	1,996,861.2	34,868.836	1,341,139.566
56.....	1,913,717.1	36,533.611	1,306,270.730
57.....	1,830,956.7	38,232.791	1,269,737.119
58.....	1,748,536.5	39,937.263	1,231,504.328
59.....	1,666,443.1	41,629.963	1,191,567.065
60.....	1,584,680.1	43,293.112	1,149,937.102
61.....	1,503,268.5	44,895.048	1,106,643.990
62.....	1,422,260.4	46,425.140	1,061,748.942
63.....	1,341,716.8	47,865.142	1,015,323.802
64.....	1,261,715.3	49,203.825	967,458.660
65.....	1,182,343.0	50,427.305	918,254.835
66.....	1,103,698.1	51,516.917	867,827.530
67.....	1,025,895.1	52,445.192	816,310.613
68.....	949,073.0	53,119.544	763,865.421
69.....	873,458.4	53,428.338	710,745.877
70.....	799,383.1	53,273.941	657,317.539
71.....	727,267.1	52,597.217	604,043.598
72.....	657,578.3	51,394.652	551,446.381
73.....	590,777.1	49,755.477	500,051.729
74.....	527,224.2	47,812.804	450,296.252
75.....	467,140.2	45,664.805	402,483.448
76.....	410,643.2	43,376.434	356,818.643
77.....	357,784.4	40,963.903	313,442.209
78.....	308,597.7	38,384.154	272,478.306
79.....	263,158.7	35,574.323	234,094.152
80.....	221,603.3	32,526.067	198,519.829
81.....	184,072.2	29,273.457	165,993.762
82.....	150,669.1	25,887.483	136,720.305
83.....	121,425.1	22,477.768	110,832.822
84.....	96,262.1	19,155.804	88,355.054
85.....	74,994.0	16,014.242	69,199.250
86.....	57,347.5	13,122.273	53,185.008
87.....	42,987.8	10,531.857	40,062.735
88.....	31,537.0	8,273.646	29,530.878
89.....	22,595.9	6,355.630	21,257.232

COMMISSIONERS 1958 EXTENDED TERM  $2\frac{1}{2}\%$

AGE LAST BIRTHDAY $x$	$D_x$	$C_x$	$\bar{M}_x$
Male— <i>continued</i>			
90.....	15,767.3	4,766.427	14,901.602
91.....	10,674.9	3,480.686	10,135.175
92.....	6,976.6	2,465.193	6,654.489
93.....	4,371.6	1,682.920	4,189.296
94.....	2,602.7	1,101.111	2,506.376
95.....	1,451.7	687.160	1,405.265
96.....	737.6	406.356	718.105
97.....	381.2	219.247	311.749
98.....	93.9	82.046	92.502
99.....	10.6	10.456	10.456
Female			
0.....	10,771,443.5	49,930.255	2,372,434.112
1.....	10,459,408.9	23,659.885	2,322,503.857
2.....	10,180,932.4	21,420.374	2,298,843.972
3.....	9,911,460.0	20,265.763	2,277,423.598
4.....	9,649,700.5	19,206.770	2,257,157.835
5.....	9,395,371.4	18,235.326	2,237,951.065
6.....	9,148,204.8	17,350.292	2,219,715.739
7.....	8,907,940.9	16,586.174	2,202,365.447
8.....	8,674,291.8	15,979.091	2,185,779.273
9.....	8,446,941.1	15,518.663	2,169,800.182
10.....	8,225,590.3	15,153.453	2,154,281.519
11.....	8,009,999.0	14,873.844	2,139,128.066
12.....	7,799,942.2	14,677.162	2,124,254.222
13.....	7,595,203.0	14,553.759	2,109,577.060
14.....	7,395,579.3	14,499.914	2,095,023.301

## (AUTHOR'S REVIEW OF DISCUSSION)

HARRY M. SARASON:

The two tables of  $D_x$  have both, indirectly, received the sanction of the Society of Actuaries. As Mr. Antliff has said, the differences are slight, but, entirely aside from the desirability of actuaries solving problems in arithmetic correctly, we should recognize that the Commissioners Mortality Tables are used as precision instruments in at least two sets of circumstances—firstly, in determining the duration of extended insurance and, secondly, in producing reserves which are acceptable to certain very meticulous state insurance department officials. Consequently, I think that we should investigate the procedures necessary to produce correct numerical answers to problems involving the commissioners tables. This involves a precise starting point for the commissioners tables, which may be lacking at present, so I make the following suggestions for the *next* commissioners tables and their use:

1. The tables be defined as the values of  $q_x$ .
2. Values of  $v^x$  and of  $l_x$  implicit in the calculation of commutation columns be carried to as many digits as convenient, but not less than ten.
3. Published values of  $l_x$  be carried to whole numbers from a radix of  $l_0 = 10,000,000$ , as at present, and  $l_x$  be labeled as one of the values derived from the mortality table. Published values of  $D_0$  be the same at all interest rates, *i.e.*,  $l_0$ , for the commissioners tables.
4. All initial calculations of reserves and cash values be based upon carrying the last digit in every intermediate factor to the higher or lower digit, whichever will produce a higher reserve or cash value. These calculations will be on electronic computers.
5. Reserves and cash values then be carried to the higher cent.
6. All intermediate factors used in calculating paid-up values be designed to produce the higher answer. Thus, in the tables in the paper,  $D_x/1,000$ , a multiplier, would be to the higher digit;  $10,000,000 - M_x$  would be given to the higher digit for adding and would be listed separately to the lower digit for subtracting;  $C_x/365$  would be given to the lower digit for dividing. These are for individual calculations, but tables of extended insurance would be based upon using  $q_x$  and  $i$  to exhaust the cash value and, in this process, the result of every accumulating step would be taken to the higher digit, while the residual fraction of a day would be carried to the higher day.

The foregoing needs further consideration. There is no need for such precision in using other tables, but the commissioners tables, legally, are precise tools.