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Arithmetic vs. Geometric Mean Returns

by Douglas C. Doll

You have been pricing your variable annuity product using a singe deterministic scenario with an annual fund growth rate of 9 percent. You want to re-price using stochastic scenarios that are consistent with your single scenario. Since the single scenario has a geometric (i.e., compound) rate of return of 9 percent, you want the stochastic scenarios to have a geometric return of 9 percent, right? Wrong! Assuming that you intend to use the mean of the stochastic scenario results, the right answer is that the stochastic scenarios should have an arithmetic mean return of 9 percent.

Here is a simple example. Consider two scenario returns, (6.80) percent and 25.20 percent. These have geometric annual returns of 8.02 percent and arithmetic annual returns of 9.20 percent. Consider an asset charge of 1 percent at the end of the year, assuming an initial fund value of $1,000 and using the two scenario returns described above. The fund values at the end of the year are $932 and $1,252, and the asset charges are $9.32 and $12.52, or an average of $10.92. Note that the $10.92 is the same asset charge we would get on our single deterministic scenario if we assumed a 9.20 percent fund growth. So, the single scenario is equivalent to the stochastic scenario when we use the arithmetic return. This equivalency also works for multiple years of returns.

If the stochastic returns have lognormal distribution, there is a simple formula to relate the geometric and arithmetic returns. The arithmetic return exceeds the geometric return by one-half of the variance (i.e., one-half of the square of the volatility). For example, for an annual volatility of 16 percent, the difference is $.5^{*}.16^{2}$, or 1.28 percent. This is a fairly sizable difference. Running a single deterministic equity scenario at 9 percent is equivalent to having a geometric scenario of 7.70 percent (assuming 16 percent volatility).

I write this article because I find that these differences are sometimes overlooked. If nothing else, it would be good if actuaries could always take the trouble to document whether the mean returns in their stochastic scenarios are arithmetic or geometric. □

Mortality Arbitage—Life and SPIA

by Douglas C. Doll

At the older issue ages, there is an opportunity for a consumer/agent to arbitrage the difference in mortality assumptions between life products and single premium immediate annuities (SPIAs). The arbitrage can exist in at least two scenarios:

1. A "super-select" individual buys a preferred life policy from Company A and also buys a standard SPIA from Company B.
2. An unhealthy individual buys a standard life policy from Company A (available through table-shaving programs) and also a substandard SPIA from Company B.

Here is how it works. The consumer borrows an amount of money and buys an SPIA. The SPIA payments are used to pay loan interest and the remainder is used to purchase a life policy whose face amount exceeds the amount of the loan amount. At death, there is a guaranteed gain equal to the excess of death benefit over loan amount.

Apparently, structures with the preceding characteristics are being designed and sold. Obviously, at least one of the two insurance companies is mispricing the mortality cost for these insureds by more than the amount of expense and profit loads in the products. Insurers might want to take another look at their pricing of these products and/or at least monitor their sales patterns at high issue ages. □

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