

A "SMOOTHED" EQUITY UNIT ANNUITY

JAMES L. CLARE

THE equity unit annuity (popularly termed the "variable annuity") has been commended for affording pensioners the prospect of a greater total income in retirement by enabling more fruitful deployment of assets than would otherwise be the case.

CRITICISM AND CHALLENGE

At the same time, it has been criticized for being prone to undue fluctuations in the dollar amounts of pension payable from year to year. Dr. Cecil J. Nesbitt sharpened the attention of the author of this paper on this matter by remarking, "I should be interested to know about any study of devices for dampening fluctuations in the values of the annuity units."¹

Such dampening, or smoothing, would be desirable in itself, but it would also have as a valuable by-product the following consequence: the greater the degree of smoothing there is, the less will be the need for allocating assets to fixed-interest investments, and hence the total income in retirement may be raised yet further. Mr. Robert J. Myers outlined this possibility as follows:

The various case histories indicate that the unit annuity based on investment in common stocks turns out better in practically all instances (and certainly on the average for all cases) than the fixed annuity arising from a fixed dollar fund. In actual practice, however, the new plan will at best allow a fifty-fifty approach. At first glance it would seem that if the unit annuity basis produces such superior results, it should be followed completely. Perhaps, after some period when experience warrants, this possibility will be offered. . . .²

The smoothing technique outlined in this paper was developed, in large measure, as a direct response to the stimulation afforded by the above remarks of Dr. Nesbitt and Mr. Myers.

PRICES VERSUS "REAL WORTH"

They led the author to realize that there is a considerable difference between having equity unit annuity payments (*a*) follow the ups and downs of common stock *prices* (as is now customary) and (*b*) follow the ups and downs of the "real worth" of common stocks (as is attempted by the smoothing technique set forth here).

Common stock prices tend to over-react—that is to say, their prices

¹ See *TSA*, IV, 779.

² See *TSA*, IV, 772.

tend to advance more quickly in bull markets than do their underlying values and earning capacities, while in bear markets their prices tend to slump proportionately further than do their values and earnings.

Hence, prices alone are not perfect yardsticks for measuring the "real worth" of equity unit annuity assets. However, this does not concern the active lives, who are making periodic contributions to secure "accumulation units" before retirement. They rely on exploiting dollar-averaging over lengthy time spans, with prompt reinvestment of all dividends, to further their single objective of amassing the largest possible total asset value at their retirement date.

It is the retired lives, with their twofold objective of both greater investment profitability and greater unit annuity income stability, who are adversely affected by the imperfections of determining unit annuity values preponderately in relation to common stock prices. Accordingly, this paper applies to retired lives only.

For retired lives, then, not only are the common stock prices relevant, but also the actual dividends on those common stocks are deserving of more attention. The fact that these actual dividends help to determine "real worth" may be given some recognition by linking the amount of each equity unit annuity payment not only to the prices of the common stocks held as assets but also to the dividends being declared on these stocks. This is a natural step to take, since any life annuity payment may be analyzed into two portions, only one of which is return of capital (and hence may properly be linked with stock prices), whereas the remainder is composed of yield on assets (and hence may properly be linked with the dollar totals of dividends payable).

Displacing the single linkage (with prices alone) by a double linkage (with both prices and dividends) leads to a higher degree of smoothing in equity unit annuity payments than hitherto, because dividends actually declared tend to fluctuate less than prices, less than economic conditions, and less than corporate earnings. Past history usually attests to this. For example, see page 52 of *Corporate Earning Power and Market Valuation 1935-1955*, a research study supported by the Life Insurance Association of America, where the authors, Mr. S. Cottle and Professor T. Whitman, state:

... across the twenty-one-year span the composite dividend yield for the 33 industries was more stable than the earnings yield.

In the future, for various economic, political, and social reasons, this may well be no less true.

DISADVANTAGES OF A FIXED INTEREST ASSUMPTION
FOR VALUING LIABILITIES

Additionally, the current practice of linking equity unit annuity payments primarily to prices (instead of to prices and dividends) may be subject to criticism for entailing a fixed interest assumption for valuing liabilities with the following disadvantages:

- (1) since a low interest assumption reduces the payments made in the early years of retirement and increases those made in subsequent years—and vice versa—the selection of any given rate of interest by the actuary exerts a considerable but arbitrary control over the pattern of equity unit annuity payments;
- (2) competitive reasons and the desire to show a high initial payment favor a high interest assumption, but a later ("reserve strengthening") reduction in such a high initial interest assumption to some lower rate could abruptly reduce the equity unit annuity payment levels at the time of the change in the interest assumption;
- (3) excessive caution could lead to too low an interest assumption, and result in a needlessly large excess interest margin which might unduly favor the older annuitants if they were to share proportionately in the total excess interest earnings when the assumed interest rate was eventually adjusted more closely in line with experience;
- (4) some positive margin would presumably be sought between earnings and the assumed interest rate, and customary equity unit annuity valuation techniques would spread the resultant excess interest earnings over the remaining lifetimes of the pensioners, thereby obscuring investment excellence whenever prices remained stable but the dividends payable on the assets of the fund were increased;
- (5) the introduction of any positive margin at all by the assumption of a fixed interest rate reduces the initial competitiveness of an equity unit annuity, since this results in a lower initial payment.

SUGGESTED SMOOTHING TECHNIQUE

It is therefore suggested that equity unit annuities be valued without assuming any fixed future interest rate whatsoever.

Assets are not valued at fixed values, but at fluctuating market values. Similarly, for determining the actuarial liabilities, it is suggested that a fluctuating rate of interest be used, equal to the actual current market yield rate on the assets held. This rate would be calculated by dividing the total of the cash dividends declared by the market value of the corresponding assets.

If this be done, then the following results would be obtained: if the

dollar total of dividends rose as rapidly as prices, then the current market yield rate on assets would remain constant, the valuation basis would not change, and annuity payments would rise in direct proportion to the increases in prices. There would be no dampening, or smoothing, and none would seem to be called for.

However, suppose now that prices rose more rapidly than the dollar total of dividends declared. Some smoothing now seems to be in order—and some would be invoked if the technique of this paper be applied. Because prices had risen more rapidly than dividends, the current market yield rate on assets would have dropped, and the interest rate assumed in annuity calculations would therefore be lowered equally. Because of this reduction in the valuation interest rate, liabilities would be valued more conservatively. This would cause annuity payments to increase proportionately less than the prices of the common stocks had increased. In this way, a measure of dampening, or smoothing, would be achieved.

Similar smoothing would be achieved whenever prices fell more rapidly than dividends, with the annuity payments falling proportionately less than the prices.

ADVANTAGES OF SMOOTHING

Some of the advantages of valuing equity unit annuity liabilities at a fluctuating rate of interest equal to the current market yield rate on assets include:

1. It is simple, and in keeping with the general concept of the equity unit annuity.
2. It obviates the need for ever having abrupt "reserve strengthening" on account of a sharp adjustment in a fixed interest assumption.
3. It enhances the competitiveness of the payments by maximizing initial payments to those entering retirement.
4. It dampens, or smoothes, some of the overreactions of stock market prices so that the payments will more nearly be a true reflection of the underlying "real worth" of the assets, and so that cuts in payments to pensioners at times of falling common stock prices will be less.
5. As a result of the fourth advantage noted immediately above, it justifies allocating a more generous proportion of assets to equity investments since the over-all fluctuations would still then be tolerable, thanks to the smoothing; hence it affords an actuarial expectation not only of more stability in the *individual* payments, but also of a greater *total* retirement income than could be expected from an investment program divided 50-50 as between common stocks and fixed-interest securities.

This smoothing technique, therefore, goes some way towards achieving the goals staked out by Dr. Nesbitt and Mr. Myers.

LIMITATIONS OF SMOOTHED EQUITY UNIT ANNUITIES

It is true, of course, that the fluctuating rate of interest employed for valuing liabilities makes the technique prohibitively laborious for equity annuity funds which lack a high speed computer. This can be overcome by renting time on a service bureau computer.

There may be some question, also, as to the practical difficulties of obtaining the cash dividend rate, or market yield rate. Since the denominator employed in the calculation of the cash dividend rate is the market value of the total portfolio for retired lives, and since this is constantly changing with acquisitions and dispositions of securities, it may require some carefully planned accounting to measure it to within tolerable limits—always remembering that an equity unit annuity is a continuing mechanism with built-in self-corrections and that minor discrepancies in one year will be counteracted automatically in future years.

The natural time interval for the calculation of the cash dividend rate would appear to be one year. Adoption of a year, rather than a month, may be expected of itself to impart a certain further degree of smoothing.

Finally, there may be some question as to whether the cash dividend rate without adjustment is the best choice for this purpose. An equity unit annuity is a special device for achieving greater investment freedom via automatic and self-correcting calculation of the annuity payments. Use of a fluctuating and self-determining cash dividend rate would appear to be in keeping with the character of the equity unit annuity. All the same, this is not to deny that there is always room for further improvement, and any additional adjustment which would achieve an even greater degree of smoothing would be welcomed.

CONCLUSION

For an individual who cannot enjoy the advantages of a group pension plan, hitherto his best actuarial expectation of achieving both greater total retirement income and tolerable stability in the level of his payments seems to have been widely regarded as being a 50-50 division of his contributions between a conventional fixed-dollar annuity and an equity unit annuity.

If he allocates fewer of his contributions to a fixed-dollar annuity, and more to a smoothed equity unit annuity, he may expect to receive an even greater total retirement income, with fluctuations in his variable annuity payments that will be less acute.

APPENDIX

Hypothetical Illustration Comparing a Fixed-Interest Equity Unit Annuity with a Smoothed Equity Unit Annuity

For any one retired individual, the second payment of annual pension (whether from a fixed-interest conventional annuity or from a fixed-interest equity unit annuity) has a smaller proportion attributable to investment earnings, and a larger proportion derived from liquidation of capital, than the first payment. Similarly, the third payment of annual pension has a smaller proportion attributable to investment earnings, and a larger proportion derived from liquidation of capital, than the second payment. These trends continue until the final payment (at the highest age in the mortality table) is composed almost entirely of liquidated capital. To avoid the extraneous influence of these trends, the construction of the following hypothetical examples does not depend on the assumed history of one individual retired life. Rather, it depends upon the stationary group concept, which will cause more constant proportions of the pension payments to be derived on the one hand from investment earnings, and on the other hand from liquidation of capital.

For simplicity, a single annuity value, at age 70, was considered to be sufficiently representative of a possible stationary group. The effect of survivorship was given special treatment to allow for this.

The annuity values were derived from the Canadian Life Table 1951—Males, at integral interest intervals of 1%, with linear interpolation for values at intermediate rates of interest.

The fixed-interest equity unit annuity was valued at 4%.

The smoothed equity unit annuities were valued at the interest rates shown.

The Industrial Common Stock Price Index and the Industrial Common Stock Dividend Index were derived from the Indices of Standard and Poor.

For simplicity, the price index was considered to apply at the year-end. The dividend index for the same year was divided by the price index, to obtain the current market yield rate for calculating the annuity value used to determine the following year's annuity payment.

For simplicity, in carrying forward the asset shares, it was assumed that annuity payments were made once a year, immediately after the calculation of the new annuity value; that the fund then received in advance the full dividend with respect to the coming year; and that this was promptly invested and carried forward, thereby receiving the full impact of capital appreciation (or depreciation) during that year.

The illustration covers years in which equity unit annuity payments have actually been made.

The results of the calculations are as shown in the accompanying table, starting in both cases with assets of \$85.27 on January 1st, 1952.

The reserves on January 1st, 1961, immediately after the payments had been made, were \$164.50 for the fixed-interest annuity and \$158.16 for the smoothed annuity.

YEAR OF PAYMENT	PRICE INDEX USED IN DETERMINING PAYMENT	MARKET YIELD RATE USED IN DETERMINING PAYMENT	DIVIDEND INDEX FOR YEAR Preceding YEAR OF PAYMENT	4% FIXED-INTEREST EQUITY UNIT ANNUITY			SMOOTHED EQUITY UNIT ANNUITY		
				Pay-ment	In-crease	De-crease	Pay-ment	In-crease	De-crease
1952.....	186.0	6.55%	12.18	\$10.00			\$11.41		
1953.....	214.1	5.65%	12.10	11.32	\$ 1.32		12.12	\$.71	
1954.....	230.1	5.37%	12.35	11.89	.57		12.39	.27	
1955.....	222.3	5.93%	13.19	11.22		\$.67	11.92		\$.47
1956.....	320.1	4.41%	14.11	15.87	4.65		15.34	3.42	
1957.....	407.9	3.67%	14.95	19.55	3.68		18.09	2.75	
1958.....	421.3	3.67%	15.46	19.36		.19	17.96		.13
1959.....	382.6	3.93%	15.04	16.82		2.54	15.88		2.08
1960.....	515.9	3.09%	15.96	21.82	5.00		19.65	3.77	
1961.....	541.7	3.10%	16.80	21.86	.04		19.83	.18	
.....	\$15.26	\$3.40	\$11.10	\$2.68

It may be seen that:

1. The smoothed equity unit annuity offered a more attractive initial payment—\$11.41 versus \$10.00 of the fixed-interest equity unit annuity.
2. Each of the three decreases in the smoothed annuity was less than the corresponding decrease in the fixed-interest annuity—giving a total of \$2.68 in decreases for the smoothed annuity as against a total of \$3.40 for the fixed-interest annuity.
3. In the final years, with the stock market prices advancing faster than the cost-of-living, the smoothed annuity held down the total increase for the two changes between 1959-60 and 1960-61—giving a total

increase for these two changes of \$3.95 for the smoothed annuity payments as contrasted with a total of \$5.04 for the corresponding increases in the fixed-interest annuity payments.

4. In 1961, the fixed-interest annuity was valued at 4%, although the current market yield rate was only 3.10%, and this raises the question of "reserve strengthening"—a concern never invoked in the case of the smoothed annuity.

DISCUSSION OF PRECEDING PAPER

ROBERT M. DUNCAN:

It is always gratifying to have new tools developed for assistance with our problems, and Mr. Clare is to be commended for doing that in his paper. What he has added is a mechanical method for the estimation of prospective dividend earnings. In its simplest terms, he assumes for valuation purposes that whatever the current rate happens to be at a valuation date will be the rate during the full durations of the annuities being valued, regardless of whether such rate is "high" or "low" at the moment. Because of the well-known lags of dividend payment changes in relation to price fluctuations and trend changes, a smoothing effect is certainly to be expected.

However, I do not think a new dimension has been added. A prospective dividend earnings rate decision is made consciously or otherwise at every unit-annuity revaluation date under customary plans. The criteria for the rate selected can be several, but one example will suffice. If the rate selected is to represent management's best investment judgment as to the expected long-term average rate for the durations of the annuities being valued, the decision could easily be made more than once to continue the same rate at the times of two or more successive valuations. Since variations in the dividend rate actually experienced from that assumed will be reflected in any event in future annuity unit values, the important point is that the double linkage of prices and dividends is already present in customary plans, whether or not the dividend rate assumption is changed annually.

Although not related directly to the smoothing technique as such, there will be some who will question Mr. Clare's statement that "the greater the degree of smoothing there is, the less will be the need for allocating assets to fixed-interest investments." The balancing of equity and fixed-interest investments involves considerations of prudent investment safeguards which are independent of smoothing techniques.

I had trouble following Mr. Clare's concept of "real worth," which apparently may be something different than market value, because somehow or other dividends are involved. The rates of current dividends payable are common knowledge in the marketplace, where prices are determined, and it would therefore appear to me that the "real worth" of common stocks, at the date of valuation, is what you can get for them, no more or no less. For this reason as well, equity considerations may also be raised if

very powerful smoothing methods are introduced into the valuation process.

The "advantages" or "disadvantages" stated and implied later in the remarks after the Appendix seem for the most part to depend so much on timing and particular levels of dividend rates which happen to be juxtaposed at the moment that what appears favorable in a particular example could very easily turn out to be unfavorable in a different but equally plausible set of circumstances. This is particularly true about such things as the level of initial payments and "reserve strengthening."

Mr. Clare has contributed an alternative technique for consideration of the actuary faced with the problem of choosing an appropriate basis for obtaining annuity unit values, and for this he is to be congratulated.

GEORGE C. CAMPBELL:

The position of the Metropolitan is so well known that I hardly need say that our formula for a "smoothed" annuity is the traditional guaranteed annuity.

I want to make just one point. The thought runs through this paper, from the introductory remarks to the conclusion, that it is almost axiomatic for the annuitant to receive a greater total return from an equity annuity than from a fixed-dollar annuity. We feel this is a debatable proposition, even during the accumulation period, and much more so in respect to retirement annuities during the shorter payout period, which is the subject of this paper.

The expectation of life is around fifteen years at age 65 on some recent annuity tables for male lives. If accumulated pension funds were applied at age 65 to set up equity retirement annuities at a peak in the stock market, such as 1929 just before the crash, the total dollar pension payments to individual annuitants dying fairly early certainly would be less than they would have received from fixed-dollar annuities.

There have been some eight occasions since 1929 where stock prices have dropped one-sixth or more and remained below their original level for extended periods. Hence one need not become so pessimistic as to expect another 1929 before questioning equity annuities.

However, the main purpose of these remarks is not to debate whether equity annuities will be higher or lower than fixed dollar annuities, or under what circumstances, but rather to put forward the point that the issue is at least debatable and not a proper assumption to be used as a firm foundation from which to draw unalterable conclusions.

WILLIAM M. ANDERSON:

Mr. Clare is to be congratulated on having brought formally to the attention of the Society the concept of a flexible valuation assumption in the operation of variable annuities. His examination of the particular use of the dividend yield at market for this purpose is most helpful.

In order to understand the underlying argument, it is useful to visualize a fund which provides unit perpetuities rather than unit annuities. In this situation valuation of the future unit obligations at the dividend yield at market has the effect of producing a current payout equal to the dividends received by the fund. Now it is quite obvious that in the long run the payout under unit perpetuities must be equal in value to the dividends received by the fund, since these dividends are the only source of income of a fund held continuously at market valuation (the net profits both realized and unrealized being reflected in the valuation of the obligations rather than in the payout). While such a fund might, over short periods, modulate its payout (either for the purpose of changing the amplitude of the dividend series or to alter the phasing of the series through time or both), the long-term constraining equation of dividends and payout is a compelling one, since departures from it involve a philosophy of capital accumulation or dissipation. At the same time it should be observed that a payout equal to earnings must, at the expense of smoothness, achieve the same result, since, in the ultimate, earnings and dividends must be equivalent.

The foreshortening of the payout period to a life annuity (or annuity certain) is a process of orderly liquidation of the capital values of the units such that the combined payout follows some contemplated form. For example, one may visualize a life annuity where the payout consisted of the dividends on a block of stocks together with an appropriate one-year term insurance premium on the current market value of the block (to provide for release of the stocks to the fund at death). Such a payout plan would combine the stability of the dividend series with the strong (and smooth) upward movement in the probability of dying which should be sufficient to offset all but the most extreme downward market movements. The difficulty with such an arrangement would be that smoothness had been secured at the expense of a marked upward movement in the mortality factor and that the combined long-term upward movement in market values, mortality, and dividends would produce a payout which was too low in the early years.

The effect of Mr. Clare's technique appears to be the equivalent of substituting for the term insurance premium on the market value of the

block of stock (as reinvested from time to time) a level whole life insurance premium calculated at an interest assumption equal to the dividend yield at market, with appropriate adjustment each year to correct for the market movement which has occurred. Thus the upward movement in mortality rates is removed, but, in consequence, the market fluctuations of the underlying stocks are accentuated by the application of whole life premiums rather than term insurance premiums to them.

In further exploration of the possibility of achieving greater smoothness without marked consequent upward bias, a word should be said about the use of earnings (and the earnings yield at market) rather than dividends. It is evident that, apart from long-term movement in interest rates, the earnings yield at market should have no long-term trend. Accordingly, a process of distributing full earnings (rather than dividends alone) should produce long-term stability. However, it suffers from the notoriously greater short-term fluctuation of earnings as compared with dividends. A multiple of dividends (such as one and a half times) could be used as a conservative but smooth representation of earnings, since the long-term dividends have averaged less than two-thirds of earnings. With this technique a life annuity might consist of one and a half times the dividends on a block of stock (the value of the block being reduced each year by one-half the dividends received), together with the appropriate one-year term insurance premium on the current market value of the narrowing block. It is suggested that the long-term upward bias in market prices and dividends, caused by retained earnings, would be removed by this process but replaced by a somewhat larger (but smoother) long-term upward bias in the individual mortality rates. The result might well be a considerably smoother annuity with slightly lower initial rates than those achieved by combining equated mortality and dividend yields at market.

Another area which requires exploration is that of the time phasing of variable annuity fluctuations as distinct from their amplitude and longer term trends. Here we are concerned with the current direction of movement of the annuity payments as compared with a cost-of-living index (base weighted) or preferably an implicit price index (currently weighted) obtained from the seasonally adjusted totals of consumer expenditure in current and constant dollars. I have formed the impression that the order of phase reversals tends to be (1) implicit consumer prices; (2) stock-market prices; (3) earnings; and (4) dividends. Thus, emerging current earnings and dividends reflect the results achieved under product-price structure at some time in the past, which in turn influenced stock-market prices, which tended to discount prospective earnings and dividends. If this is the case, it suggests that earnings and dividend yields at market,

computed with an antedated denominator, may produce results in closer phase with the implicit price index.

To those persons who may be interested in further exploration of equity annuity smoothing possibilities, attention is directed to a recently published book entitled *Valuation of Securities Holdings of Life Insurance Companies* by Dr. Harold G. Fraine (a volume in the "Irwin Series on Risk and Insurance" financed by a L.I.A.A. investment research grant). This book draws together common-stock performance data from both outside and inside the life insurance business. The data given provide ample evidence of the shorter-term smoothness of the dividend series, and the opinion is advanced that the long-term smoothness of the dividend yield at market is such that equity as between generations of policyholders is better served if unrealized market appreciation is not released to surplus as it arises. Thus a valuation concept related to dividend yield at market is given corroboration from an entirely different point of view.

Mr. Clare has referred to what he calls the "real worth" of common stocks and to the fact that actual dividends help to determine this "real worth." While it is true that dividends do in fact represent that portion of earnings which a corporation decides that it can turn over to its shareholders, this decision can, in many instances, involve a substantial departure from earnings, and it is prospective earnings, measured according to the best information available, which is the best measure of the "real worth" of any stock from an investment, as distinct from a speculative, point of view. Accordingly, the smoothness of the dividend series (which is a composite result of many causes) and the dampening effect on valuation of the dividend yield at market should not be relied on too heavily to justify an extremely high proportion of post retirement savings placed in equity form.

Finally, it may be said that the typical retirement span of years is too short to permit heavy reliance on variable annuities based upon volatile low payout stocks. Such funds probably should concentrate more on the high payout "defensive" stocks, which not only exhibit smoother dividend series but by the mere fact of the high payout demonstrate the confidence of the corporation in its ability to maintain shorter-term stability in earnings. It is stocks such as these where the prospective estimates of dividends based on present and past performance may be used with confidence as an approximate measure of "real worth."

DONALD S. GRUBBS, JR.:

Mr. Clare has demonstrated considerable ingenuity in his approach to an important current problem. Those of us who have valued equity

annuity retirement plans with valuation dates in June or later of this year have been impressed with the need for smoothing equity annuities. In addition to legal problems, serious problems of equity would be raised by application of a smoothing technique to individual variable annuity contracts issued by an insurer, but smoothing is both desirable and practical for pension plans.

Mr. Clare sees the purpose of equity annuities as providing the largest possible income. I prefer to think of an object of providing an amount of income which is initially adequate and which will vary to meet changes in the cost of living and perhaps in the standard of living also. The purpose of smoothing then is to reduce or eliminate decreases in the ratio of retirement income to the cost of living.

Because over a long period the market values of equities have fluctuated much more than the cost of living, most plans provide only one-fourth to one-half of the income as an equity annuity and provide the balance as a fixed annuity. Another way of accomplishing this same objective is to provide the entire income as a variable annuity but invest the assets of the fund three-fourths to one-half in fixed-dollar investments.

In deciding the proportion of income to be derived from equity annuities, one must decide whether to consider the social security retirement benefit as part of the fixed-dollar income. Whether the social security income itself should be geared to the cost of living is a subject worth our future consideration.

The author has assumed that a money-purchase plan is in effect. Therefore, units of retirement income are determined only at retirement, and actuarial assumptions do not affect the cost to the employer.

If benefits are determined under a unit benefit formula, it will be necessary to allocate units of assumed income for active members from each year's payment to the trust. In either the case of a unit benefit formula or a final pay formula the actuarial assumptions used in computing the value of benefits credited in the current year will, of course, directly affect the cost to the employer. In this case the valuation interest assumption takes on an entirely different role from that discussed by Mr. Clare. Increasing the interest assumption directly lowers the employer cost, and it also lowers the annual increase in the unit value for the employee, since each year the unit value increases by $(1 + i')/(1 + i)$, where i' is the actual rate of investment income (including realized and unrealized appreciation in assets) and i is the assumed rate.

The assumed interest rate need not be based solely on expected investment income to be received, but it may also contemplate part of the appreciation in equity values. Thus, we currently assume 5 per cent for

some plans. Since the long-range prospect of investment income plus asset appreciation is well in excess of any assumptions being used, it is not necessary to strengthen reserves based on the rate of investment income received.

The author points out that market values often fail to reflect the real worth of common stocks. If a stock is to be retained, its value is related primarily to future dividends and to the future price for which it may be sold, but neither of these may be closely reflected by the current year's dividend.

The proposed smoothing method is illustrated by an example which shows that, although the method has smoothed the fluctuations somewhat, it still leaves something to be desired. A 12 per cent decrease in retirement payments in 1959 does not reflect any corresponding decrease in the cost of living.

Since the unit value is the ratio of assets to liabilities measured in units, a smoothing process can adjust either the liabilities or the assets. The freshness of Mr. Clare's approach in adjusting the value of liabilities for the purpose of smoothing retirement payments is appealing. However, the adjustment of asset values can accomplish the objective better while avoiding the laborious valuation work at odd interest rates, which is especially important for smaller plans.

Adjustment of asset values can be made either by one of the methods of asset valuation used by fixed benefit plans to gradually recognize asset appreciation or by setting aside part of the assets in a reserve for asset fluctuation and excluding these assets from the numerator when determining the unit value. This latter process has been suggested by Mr. G. Ashley Cooper (*TSA*, XIII).

I have prepared Table 1 to illustrate the method and its results. I assume that the unit value should increase at least twice as fast as the Consumer Price Index, which would be appropriate where only half of the retirement benefit is provided under a variable plan. For simplicity the liabilities are assumed to be constant in value as measured in benefit units, and the assets are assumed to appreciate and to earn investment income in accordance with Standard and Poor's five hundred stock average. The total increase from both appreciation and investment income is reduced by a valuation assumed interest rate of $3\frac{1}{2}$ per cent to provide the percentage by which the unit value would increase if there were no smoothing process. In years in which this total increase over $3\frac{1}{2}$ per cent exceeds twice the increase in the Consumer Price Index, half of the excess is withdrawn into a reserve for asset fluctuation, thus making the increase in the unit value less than it would be under an unsmoothed plan. In years in

which the total increase over $3\frac{1}{2}$ per cent is less than twice the increase in the Consumer Price Index, a sufficient amount is withdrawn from the reserve for asset fluctuation to make the unit value increase twice as fast as the Consumer Price Index. Some maximum limit should be placed on the reserve for asset fluctuation per unit, such as 40 per cent of the unit value. The various arbitrary ratios used in this formula can be adjusted in accordance with the judgment of the actuary.

This process has produced unit values far smoother than the technique used by Mr. Clare. One weakness of my method is that the amount of smoothing is limited to the amount in the reserve for asset fluctuation

TABLE 1

Year	Consumer Price Index*	Twice Increase in C.P.I.	Composite Stock Price Index†	Increase in Stock Index	Investment Income Composite Yield†	Total Increase over $3\frac{1}{2}$ %	Unit Value	Reserve for Asset Fluctuation per Unit
1952.	113.1	23.41	\$10.00	\$0.00
1953.	114.1	1.8%	26.04	11.23%	5.47%	12.75%	10.73	0.55
1954.	114.9	1.4	24.83	-4.65	5.83	-2.24	10.88	0.15
1955.	114.3	-1.0	34.97	40.84	4.45	40.38	12.91	2.57
1956.	114.7	0.6	45.37	29.74	4.15	29.36	14.84	5.18
1957.	118.0	5.8	46.44	2.36	4.24	3.00	15.70	4.92
1958.	121.6	6.2	40.33	-13.16	4.64	-11.61	16.67	1.56
1959.	123.7	3.4	53.49	32.63	3.33	31.36	19.57	4.38
1960.	125.5	3.0	59.06	10.41	3.18	9.75	20.82	5.47
1961.	127.5	3.2	56.80	-3.83	3.41	-3.79	21.48	3.81
1962.	128.2	1.0	71.74	26.30	2.85	24.78	24.26	7.30

* December of previous year.

† December monthly average of previous year, Standard and Poor's five hundred stock average.

unless that reserve is allowed to become negative, and thus it provides no protection at all against a fall in market values in the plan's first year unless the employer makes an initial contribution to that reserve.

HENRY G. NEEBE:

In July, 1954, one of our clients established an approved variable annuity retirement plan which recognized the need for dampening the undue fluctuations in the dollar amounts of pensions payable from year to year. Although this plan is not an equity unit type in the strictest sense, it does rely on the dollar-averaging method of investing in common stocks. It has a combined investment fund balanced internally between fixed income and equity securities from which a single variable annuity is produced. The balance of investments between fixed income and equity securities, on the basis of cost, has varied from 90-10 per cent in 1954 to 55-45 per

cent in 1962. Current contributions are invested on a 50-50 basis. The average fixed income-equity ratio over the life of the plan has been 67-33 per cent.

Our client's plan employs the following smoothing techniques:

1. The pension payments are adjusted in the following year to reflect the average market value of both debt and equity securities held at the end of the year. Average market value is based on the arithmetic average of the month-end market values for the current year of each security.

2. The actual rate of return from interest and dividends in the current year is used to compute and adjust payments in the following year.

Pension payment results for a male employee age 65, starting July 1, 1955, have been as shown in Table 1.

TABLE 1

Year of Payment (July 1)	Index of Investment Fund	Capital Appreciation (or Depreciation)	Rate of Return Used in Determining Payments	Payments	Purchasing Power*
1955-56.....	100.00	2.40%	3.14%	\$10.00	\$10.00
1956-57.....	102.46	2.46	3.54	10.50	10.38
1957-58.....	100.07	(2.33)	3.96	10.46	9.93
1958-59.....	97.54	(2.53)	4.02	10.53	9.77
1959-60.....	105.15	7.80	4.14	11.42	10.56
1960-61.....	105.18	0.03	3.80	11.09	10.05
1961-62.....	108.36	3.02	3.73	11.32	10.18
1962-63.....	114.33	5.51	3.55	11.71	10.40

* Bureau of Labor Statistics Consumer Price Index, where July, 1955 = \$10.00.

G. ASHLEY COOPER:

The criticism and challenge in Mr. Clare's paper have, I am sure, been occupying the minds of most actuaries who deal with variable annuity plans. I suspect that many of these actuaries have come up with solutions of their own and that those solutions are by no means all along the same lines.

I would like to develop my comments in three phases: (a) First, to point out what appear to be major imperfections in Mr. Clare's solution. (b) Second, to give some principles which have a somewhat different approach to the problem. (c) Finally, to describe briefly another solution.

In my view there are some important objections to the proposed "smoothed" equity unit annuity. Here are some of them:

a) The proposal mentions "real worth" of common stocks. The definition of "real worth" has baffled a lot of people for a long time and may possibly be indeterminate. However, I seriously doubt that the method

proposed by Mr. Clare gives an adequate definition. For example, the proposal certainly does not take into account the fact that market value often discounts future increases (or decreases) in dividends.

- b) Along the same lines, some peculiar results will be obtained in practice in circumstances which are not at all unusual. Consider, for example, speculative stocks which may miss a dividend entirely, or stocks which have a very high price to earnings ratio, or even the holding of substantial amounts of cash which might be a very smart move for a trustee, for example, during May, 1962.
- c) I particularly dislike the idea of the unit value being affected by the trustee's investment *methods* or even worse, affecting the trustee's investment methods. In order to increase the unit value, the trustee will be tempted to invest in high-dividend yield stocks, while in fact other stocks may be a more profitable investment in the long run.
- d) Finally, I dislike the principle of assuming that current "yield" will always be earned in the future.

Now I should like to set out some general thoughts which take a different direction:

- a) I see no reason why it is necessary in a normal variable annuity plan for separate assets to be maintained for retired lives.
- b) Nor, in most instances, do I see why it is necessary to have a different unit value for "accumulation" before retirement and for actual benefits after retirement.
- c) Turning to the design of the plan, it is quite simple to arrange for the initial interest assumption *not* to penalize pensioners in the early years. In fact, the choice of the basic interest assumption, and even changing it, need not affect the benefits directly, although, of course, they may affect the unit value in the long run.
- d) Perhaps the most important point is not to lose sight of the original objective which is to give pension benefits varying with purchasing power or, which is really saying the same thing, with the cost of living.

The method I am going to describe is specifically built around these principles. It is a suggested solution to the problem of smoothing equity annuities, and, moreover, it has worked well in actual practice. It is, of course, not perfect and by no means the only solution. The procedure is as follows:

- a) In the first place there is only one unit value; it applies to both active and retired lives.
- b) The unit value is determined using both the gain or loss on assets

(realized and unrealized) and the difference in actual earnings from the rate assumed.

- c) First, an "unadjusted" share value is computed. This is a simple comparison between the actual assets on the determination date and the expected assets. Expected assets are computed on the basis of an assumed rate of interest and anticipating no capital gains or losses. Thus if the assets turn out to be 10 per cent higher than expected, last year's unit value can be increased 10 per cent.
- d) The next step is to take the ratio of Consumer Price Index (as published by the Labor Department) on the current determination date to the index on the prior determination date. This measures, although unfortunately not perfectly, the increase or decrease in the cost of living.
- e) We then split the difference between the increase in cost of living and the assets increase. This mean increase or decrease will be used to compute the next year's unit value, if possible.
- f) The difference between the "unadjusted" unit value and the actual unit value used causes a surplus or deficit of assets which is handled through a reserve fund. Thus, if the final unit value is lower than the "unadjusted" unit value, assets are released into the reserve fund. If the actual unit value is higher than the "unadjusted" value, then assets are withdrawn from the reserve fund. The reserve fund is not allowed to go negative.
- g) An example or two may make this clearer. Let us assume the unadjusted unit value is 10 per cent larger than last year's unit value. If the Consumer Price Index went up 5 per cent, the final unit value will be $7\frac{1}{2}$ per cent greater, and the reserve fund will get the balance of $2\frac{1}{2}$ per cent. However, if the Consumer Price Index increased 15 per cent, then the net unit value would be $12\frac{1}{2}$ per cent higher if there are sufficient assets in the reserve fund to support it. If there are not sufficient assets, the new value will be what can be supported by liquidating the reserve fund.

As I say, the method has been successful in practice, and our experience covers practically all the permutations of increases and decreases. What I believe to be of particular significance is (a) the direct link with increases (or decreases) in the cost of living; (b) due recognition of "yield," whether it be from actual dividends or from capital gains; and (c) considerable flexibility (that is to say, the final unit value need not necessarily be half-way between the values produced by assets performance and the ratio of Consumer Price Index).

COLIN E. JACK:

The author is to be commended for a paper which may be expected to provoke thought about investment policy particularly as related to annuity funds.

It is, of course, not surprising that rates of dividend on common stocks are more stable than the prices of the stocks, since the latter are established daily in the bustle of the market place, while the former are determined quarterly in the calm of the board room with stability an important factor in their determination.

It is open to question whether investment in common stocks will maximize accumulation. It may be that more often than not more money can be accumulated by equity investment than by fixed interest investment, particularly over long periods, but surely the maximum will be obtained by investing in one at some times and in the other at other times and by making switches from one to the other as opportunity offers. It may be difficult to invest large funds in a particular medium and to obtain better results than average in that medium, but surely better results can be obtained if funds may be invested in different media than if restricted to one.

In the investment of funds which are to be used for payment of vested annuities it seems necessary to be conscious of the fact that the term of the investments is relatively short. In these circumstances the attractions of equities are less bright than when investments are made for a long term. There is no advantage in investing for "growth" if the annuitant will be dead before the "growth" is achieved. The fact that the fund may be "open," that is, that other annuitants may enter the fund, does not, I believe, make any difference. If the fund is "closed," investments must be sold in the market to buy the annuities; if the fund is "open," the securities are in effect sold to new annuitants, and, unless they are sold at market prices, there will be inequity between the generations of annuitants.

We thus reach the conclusion that immediate annuity funds should be invested in equities only if the ratio of dividends to prices is relatively high or if increases in dividends and/or prices seem likely in the near future. (This is perhaps good advice for anyone thinking of investing in the stock market.) Similarly, deferred annuity funds if in common stocks might well be switched out as vesting approaches unless these conditions prevail.

The author indicates that the smoothing technique maximizes initial payments. It does in his example, but surely it is a matter of the cycle.

If the annuity were taken out when the dividend yield was less than 4 per cent, the smoothed annuity would have shown the lower initial payment. One might perhaps argue that, if the dividend yield were below 4 per cent, the "fixed interest" annuity would not be valued at so high a rate; but this does not seem correct. At a time when bond yields were, say, 5 per cent and common-stock yields, say, 3 per cent, it would not be quite reasonable to use 4 per cent.

In the Appendix the author appears to have been led into error by assuming an open fund. One may wonder why, although the price index increased nearly threefold (186.0 to 541.7) the annuities only doubled (\$10.00 to \$21.86 and \$11.41 to \$19.83). This, I think, is due to a compound error of about 5 per cent per annum. It appears that the author starts with a fund of \$85.27; deducts, in the first case, the payment of \$10.00, adds interest and revalues, bringing the fund to \$92.28. Dividing this by the probability of survival at age 70 (0.95565) gives \$96.56 for each survivor. On the assumption that the annuity value remains constant, this is divided by 8.527 to give the second payment of \$11.32. If the fund were closed, we would divide by 8.191 (\ddot{a}_{71}) and find a second payment of \$11.79. Theoretically, the open fund and the closed fund must give the same result, or there is inequity between entrants of different years. The error appears to be that, if the annuity value is to remain constant, entrants younger than the survivors must come in and they must bring in more than \$96.56 each if there is to be equity. While the error does not affect the author's conclusions regarding smoothness, it does have a compound effect on the actual results. I have calculated for 1961 payments of \$31.37 and \$28.96 for the two forms which are much more in line with the price increases.

It may be shown for a group of the same age that the ratio of the payment at the end of the year to that at the beginning of the year is

$$\frac{(1+g) \cdot \ddot{a}_{x+1}^{i_1}}{(1+i_1) \cdot \ddot{a}_{x+1}^{i_2}}$$

where i_1 and i_2 are the valuation rates at beginning and end, g the continued effect of interest and change in capital value, and x the age at the beginning of the year. If i_1 and i_2 are equal, the result is independent of age, but otherwise not. Thus, although the fixed interest method is equitable for a fund composed of annuitants of different ages, the smoothing method is theoretically, at least, inequitable.

HAROLD GILBERT:

Mr. Clare has suggested that smoothing of equity annuity payments decreases the need for fixed-dollar investments, while earlier discussions of this paper suggest that the point is, at least, debatable. Smoothing does not change the investment base or the stability of that base. To the extent that fluctuation may be considered short term or spurious rather than a reflection of fundamental instability, the need for fixed-dollar balancing is lessened. However, changing the incidence of payments does not change the stability of the investment base, and this stability, or lack of it, creates the need for fixed-dollar balancing of equity investments.

CECIL J. NESBITT:

Although my ideas concerning this paper are still in a formative stage, I wish to submit a few comments. Since my remark in discussion of Mr. Duncan's paper had some influence on the present paper, perhaps it is worthwhile to venture observations before one has reached a solid understanding.

In the Appendix of Mr. Duncan's paper (see *TSA*, IV, 343) there are listed common-stock net yields for a seventy-year period. One notes abrupt changes from year to year in some cases, for example, 1906-7, 1932-33, 1940-41, and 1941-42. A strict adherence to Mr. Clare's proposed use of current market yield rate for valuation purposes might have produced problems in those instances.

While undoubtedly Mr. Clare's proposal would in some periods exert a dampening effect on changes in unit values, it goes against one's actuarial grain to choose the interest assumption for a long-term transaction on the basis of current yield alone and, moreover, to revise the assumption yearly. It is not clear to me how equitably such a system would work.

For instance, I conjecture it is true that the higher the interest assumption used for valuation of the fund, the higher is the number of annuity units that could be purchased by a new annuitant for a given sum of money. The number of annuity units determines the new annuitant's *relative* position or share in the total annuity fund for the duration of his contract. Hence, the choice of interest assumption is exceedingly important not only for the determination of unit values but also for the assignment of the number of annuity units to new entrants. For this latter purpose, a long-term average rate which might be greater or less than the current yield rate and which would not be changed too abruptly would seem most appropriate. However, these are conjectural remarks requiring further study.

Mr. Clare has presented a very interesting suggestion and has indicated the need for some searching thought on the mechanics of variable annuities.

(AUTHOR'S REVIEW OF DISCUSSION)

JAMES L. CLARE:

For a given retirement age, and for a given dollar history of net savings, the only route to larger retirement income is via more profitable investment. As Mr. Jack indicates, the expectation of more profitable investment is greater if the scope for the application of skill judgment is enhanced by (a) reducing investment shackles and (b) maximizing investment freedom.

Regarding (b), maximizing investment freedom, equity unit annuities are as untrammeling as conventional annuities. Equity unit annuity assets may be freely invested in fixed-interest securities, to whatever extent considered most appropriate and most profitable from time to time. Messrs. Grubbs and Neebe concur that equity unit annuities are compatible with any suitable degree of investing in fixed-interest securities.

Regarding (a), reducing investment shackles, equity unit annuities make it more viable for *individuals* to have their retirement annuity savings invested in equities (including common stocks). Equities afford an expectation of greater investment profitability, provided certain ground rules such as dollar-averaging (over sufficiently long periods of time) are observed. These rules are relevant to the "pay-out" as well as to the "pay-in" process. The expectation of greater investment profitability may, or may not, actually eventuate, but the present existence of this expectation has been deduced in theory from more than one historical study (including that preceding the introduction of the original equity unit annuities). Of course, I agree that, *if* contrary theoretical conclusions be obtained, then, as Mr. Campbell contends, there would be no justification for adopting equity unit annuities.

Messrs. Duncan, Campbell, Anderson, Grubbs, Jack, and Gilbert helpfully mention various other aspects of appropriate, profitable—and "prudent"—investing. However, the freedom to dollar-average (over sufficiently long periods of time), both when acquiring and when liquidating equity assets, remains a major concern. An individual can now aim at achieving this, for equity unit annuities make it possible to avoid the abrupt selling of his equity assets at the point of his retirement. It seems to me that this is the primary *raison d'être* for equity unit annuities.

Without invoking the complexities of equity unit annuities, some employers already have pension arrangements for *groups* of their employees

which cope with the above problem. They also cope with other problems mentioned by discussants, such as preserving the adequacy of the purchasing power of the pensions despite changes in living costs. Group pension plans can emphasize objectives such as retirement security and investment profitability while softpedaling individual equity. However, for the people considered in the paper, individual equity is an inescapably leading consideration.

Hence, Mr. Cooper's remarks are either extraneous or else peripheral to the central issues raised in the paper. In particular, Mr. Cooper's "reserve fund" appears to lack relevance to retired individuals, since negative reserves are clearly inadmissible, and positive reserves entail either (i) inequitable forfeitures on death or (ii) terminal lump-sum payments, which leave something to be desired when one remembers that the purpose of an annuity is to provide income while the retired individual is actually alive to enjoy it.

It therefore seems to me that an equity unit annuity pay-out formula should attempt to provide the maximum initial payment, with subsequent payments being tethered to suitable anticipated gradients (as mentioned by Mr. Anderson), and, of course, with individual equity being preserved at all times.

Individual equity is discussed further by Dr. Nesbitt. However, I trust that he will find that his pertinent conjectures have satisfactory answers, for the original equity unit annuity made a huge stride toward individual equity by being self-correcting with regard to market values of assets, and the "smoothed" equity unit annuity is really nothing more than a further step toward individual equity by being self-correcting with regard to investment earnings. This is so whether one "smoothes" by considering dividends payable, as in the paper, or whether one perhaps delves into actual earnings, as Mr. Anderson discusses.

Of course, the actual dollar payments to annuitants and the actual amount of "smoothing" achieved using a variable valuation interest rate will depend upon many influences. Among those influences, as Mr. Jack helpfully observes, is the rate of inflow of money from new entrants at various retirement ages. Because of the importance of all these influences, the simplifications employed in the construction of the hypothetical example in the Appendix are carefully set forth therein. As explained there, this single example concentrates, year after year, on maintaining as far as possible the same relative importance in the "smoothing" process afforded to (a) prices and (b) dividends. A typical and realistic balance is preserved between these two factors, while keeping everything else as simple as possible. The hypothetical example therefore treats both types

of annuity impartially, and the results give a fair depiction of the relative measure of "smoothing" enjoyed by each.

Dr. Nesbitt remarks on the abrupt changes in common-stock net yields which have occurred from year to year. However, it is not the yield percentage which matters so much as the actual dollar volume of the dividends in relation to the actual dollar value of the assets. A superficially disquieting tumble in the net yield rate may perhaps only be the result of the total dollar value of assets rising faster than the total dollar value of dividends. Even though the rate may be down, the dollars may be up. The likelihood that this will in fact be the case, *in the future*, is one of the cornerstones of the paper. Mr. Anderson adds considerably to the discussion of this aspect of the problem.

Mr. Duncan relates why (although regularly linking his equity unit annuity payments directly to the major vagaries of asset prices) he does not regularly link his equity unit annuity payments directly to investment earnings rates but instead interposes human judgment by means of adopting fixed rates of interest for valuing liabilities. However, as he kindly hints, I hope he may find the "smoothing" technique useful for supporting his judgment as to whether a fixed-interest rate is to be left at an existing level or is to be adjusted.

My thanks go to all who have helped in the presentation and discussion of this paper, for their courtesy, their additional treatment of various "smoothing" considerations, and their generous application of time.