

SOCIETY OF ACTUARIES

EXAM MLC Models for Life Contingencies

EXAM MLC SAMPLE SOLUTIONS

The following questions or solutions have been modified since this document was prepared to use with the syllabus effective spring 2012,

Prior to March 1, 2012:

Questions: 151, 181, 289, 300

Solutions: 2, 284, 289, 290, 295, 300

Changed on March 19, 2012: Questions: 20, 158, 199 (all are minor edits)

Changed on April 24, 2012: Solution: 292

Changed on August 20, 2012: Questions and Solutions 38, 54, 89, 180, 217 and 218 were restored from MLC-09-08 and reworded to conform to the current presentation. Question 288 was reworded to resolve an ambiguity. A typo in Question 122 was corrected. Questions and Solutions 301-309 are new questions

Changed on August 23, 2012: Solution 47, initial formula corrected; Solution 72, minus signs added in the first integral

Changed on December 11, 2012: Question 300 deleted

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Question #1**Answer: E**

$${}_2|q_{30:34} = {}_2p_{30:34} - {}_3p_{30:34}$$

$${}_2p_{30} = (0.9)(0.8) = 0.72$$

$${}_2p_{34} = (0.5)(0.4) = 0.20$$

$${}_2p_{30:34} = (0.72)(0.20) = 0.144$$

$${}_2p_{30:34} = 0.72 + 0.20 - 0.144 = 0.776$$

$${}_3p_{30} = (0.72)(0.7) = 0.504$$

$${}_3p_{34} = (0.20)(0.3) = 0.06$$

$${}_3p_{30:34} = (0.504)(0.06) = 0.03024$$

$$\begin{aligned} {}_3p_{30:34} &= 0.504 + 0.06 - 0.03024 \\ &= 0.53376 \end{aligned}$$

$$\begin{aligned} {}_2|q_{30:34} &= 0.776 - 0.53376 \\ &= 0.24224 \end{aligned}$$

Alternatively,

$$\begin{aligned} {}_2|q_{30:34} &= {}_2|q_{30} + {}_2|q_{34} - {}_2|q_{30:34} \\ &= {}_2p_{30}q_{32} + {}_2p_{34}q_{36} - {}_2p_{30:34}(1 - p_{32:36}) \\ &= (0.9)(0.8)(0.3) + (0.5)(0.4)(0.7) - (0.9)(0.8)(0.5)(0.4) [1 - (0.7)(0.3)] \\ &= 0.216 + 0.140 - 0.144(0.79) \\ &= 0.24224 \end{aligned}$$

Alternatively,

$$\begin{aligned} {}_2|q_{30:34} &= {}_3q_{30} \times {}_3q_{34} - {}_2q_{30} \times {}_2q_{34} \\ &= (1 - {}_3p_{30})(1 - {}_3p_{34}) - (1 - {}_2p_{30})(1 - {}_2p_{34}) \\ &= (1 - 0.504)(1 - 0.06) - (1 - 0.72)(1 - 0.20) \\ &= 0.24224 \end{aligned}$$

(see first solution for ${}_2p_{30}$, ${}_2p_{34}$, ${}_3p_{30}$, ${}_3p_{34}$)

Question #2**Answer: E**

$$\begin{aligned}
1000\bar{A}_x &= 1000 \left[\bar{A}_{x:\overline{10}|}^1 + {}_{10|}\bar{A}_x \right] \\
&= 1000 \left[\int_0^{10} e^{-0.04t} e^{-0.06t} (0.06) dt + e^{-0.4} e^{-0.6} \int_0^{\infty} e^{-0.05t} e^{-0.07t} (0.07) dt \right] \\
&= 1000 \left[0.06 \int_0^{10} e^{-0.1t} dt + e^{-1} (0.07) \int_0^{\infty} e^{-0.12t} dt \right] \\
&= 1000 \left[0.06 \left[\frac{-e^{-0.1t}}{0.10} \right]_0^{10} + e^{-1} (0.07) \left[\frac{-e^{-0.12t}}{0.12} \right]_0^{\infty} \right] \\
&= 1000 \left[\frac{0.06}{0.10} [1 - e^{-1}] + \frac{0.07}{0.12} e^{-1} \right] \\
&= 1000(0.37927 + 0.21460) = 593.87
\end{aligned}$$

Because this is a timed exam, many candidates will know common results for constant force and constant interest without integration.

For example $\bar{A}_{x:\overline{10}|}^1 = \frac{\mu}{\mu + \delta} (1 - {}_{10}E_x)$

$${}_{10}E_x = e^{-10(\mu + \delta)}$$

$$\bar{A}_x = \frac{\mu}{\mu + \delta}$$

With those relationships, the solution becomes

$$\begin{aligned}
1000\bar{A}_x &= 1000 \left[\bar{A}_{x:\overline{10}|}^1 + {}_{10}E_x A_{x+10} \right] \\
&= 1000 \left[\left(\frac{0.06}{0.06 + 0.04} \right) (1 - e^{-(0.06+0.04)10}) + e^{-(0.06+0.04)10} \left(\frac{0.07}{0.07 + 0.05} \right) \right] \\
&= 1000 \left[(0.60)(1 - e^{-1}) + 0.5833 e^{-1} \right] \\
&= 593.86
\end{aligned}$$

Question #3**Answer: D**

$$E[Z] = \int_0^{\infty} b_t v^t {}_t p_x \mu_{x+t} dt = \int_0^{\infty} e^{0.06t} e^{-0.08t} e^{-0.05t} \frac{1}{20} dt$$

$$= \frac{1}{20} \left(\frac{100}{7} \right) \left[-e^{-0.07t} \right]_0^{\infty} = \frac{5}{7}$$

$$E[Z^2] = \int_0^{\infty} (b_t v^t)^2 {}_t p_x \mu_{x+t} dt = \int_0^{\infty} e^{0.12t} e^{-0.16t} e^{-0.05t} \frac{1}{20} dt = \frac{1}{20} \int_0^{\infty} e^{-0.09t} dt$$

$$= \frac{1}{20} \left(\frac{100}{9} \right) \left[e^{-0.09t} \right]_0^{\infty} = \frac{5}{9}$$

$$\text{Var}[Z] = \frac{5}{9} - \left(\frac{5}{7} \right)^2 = 0.04535$$

Question #4**Answer: C**Let ns = nonsmoker and s = smoker

$k =$	$q_{x+k}^{(ns)}$	$p_{x+k}^{(ns)}$	$q_{x+k}^{(s)}$	$p_{x+k}^{(s)}$
0	.05	0.95	0.10	0.90
1	.10	0.90	0.20	0.80
2	.15	0.85	0.30	0.70

$$A_{x:\overline{2}|}^{1(ns)} = v q_x^{(ns)} + v^2 p_x^{(ns)} q_{x+1}^{(ns)}$$

$$\frac{1}{1.02} (0.05) + \frac{1}{1.02^2} (0.95 \times 0.10) = 0.1403$$

$$A_{x:\overline{2}|}^{1(s)} = v q_x^{(s)} + v^2 p_x^{(s)} q_{x+1}^{(s)}$$

$$\frac{1}{1.02} (0.10) + \frac{1}{(1.02)^2} (0.90 \times 0.20) = 0.2710$$

$$A_{x:\overline{2}|}^1 = \text{weighted average} = (0.75)(0.1403) + (0.25)(0.2710)$$

$$= 0.1730$$

Question #5**Answer: B**

$$\mu_x^{(\tau)} = \mu_x^{(1)} + \mu_x^{(2)} + \mu_x^{(3)} = 0.0001045$$

$${}_t p_x^{(\tau)} = e^{-0.0001045t}$$

$$\begin{aligned} \text{APV Benefits} &= \int_0^{\infty} e^{-\delta t} 1,000,000 {}_t p_x^{(\tau)} \mu_x^{(1)} dt \\ &+ \int_0^{\infty} e^{-\delta t} 500,000 {}_t p_x^{(\tau)} \mu_x^{(2)} dt \\ &+ \int_0^{\infty} e^{-\delta t} 200,000 {}_t p_x^{(\tau)} \mu_x^{(3)} dt \\ &= \frac{1,000,000}{2,000,000} \int_0^{\infty} e^{-0.0601045t} dt + \frac{500,000}{250,000} \int_0^{\infty} e^{-0.0601045t} dt + \frac{250,000}{10,000} \int_0^{\infty} e^{-0.0601045t} dt \\ &= 27.5(16.6377) = 457.54 \end{aligned}$$

Question #6**Answer: B**

$$EPV \text{ Benefits} = 1000A_{40:\overline{20}|}^1 + \sum_{k=20}^{\infty} {}_k E_{40} 1000vq_{40+k}$$

$$EPV \text{ Premiums} = \pi \ddot{a}_{40:\overline{20}|} + \sum_{k=20}^{\infty} {}_k E_{40} 1000vq_{40+k}$$

Benefit premiums \Rightarrow Equivalence principle \Rightarrow

$$1000A_{40:\overline{20}|}^1 + \sum_{k=20}^{\infty} {}_k E_{40} 1000vq_{40+k} = \pi \ddot{a}_{40:\overline{20}|} + \sum_{k=20}^{\infty} {}_k E_{40} 1000vq_{40+k}$$

$$\begin{aligned} \pi &= 1000A_{40:\overline{20}|}^1 / \ddot{a}_{40:\overline{20}|} \\ &= \frac{161.32 - (0.27414)(369.13)}{14.8166 - (0.27414)(11.1454)} \\ &= 5.11 \end{aligned}$$

While this solution above recognized that $\pi = 1000P_{40:\overline{20}|}^1$ and was structured to take advantage of that, it wasn't necessary, nor would it save much time. Instead, you could do:

$$EPV \text{ Benefits} = 1000A_{40} = 161.32$$

$$\begin{aligned} EPV \text{ Premiums} &= \pi \ddot{a}_{40:\overline{20}|} + {}_{20}E_{40} \sum_{k=0}^{\infty} {}_kE_{60} 1000vq_{60+k} \\ &= \pi \ddot{a}_{40:\overline{20}|} + {}_{20}E_{40} 1000A_{60} \\ &= \pi [14.8166 - (0.27414)(11.1454)] + (0.27414)(369.13) \\ &= 11.7612\pi + 101.19 \\ 11.7612\pi + 101.19 &= 161.32 \\ \pi &= \frac{161.32 - 101.19}{11.7612} = 5.11 \end{aligned}$$

Question #7

Answer: C

$$A_{70} = \frac{\delta}{i} \bar{A}_{70} = \frac{\ln(1.06)}{0.06} (0.53) = 0.5147$$

$$\ddot{a}_{70} = \frac{1 - A_{70}}{d} = \frac{1 - 0.5147}{0.06/1.06} = 8.5736$$

$$\ddot{a}_{69} = 1 + vp_{69}\ddot{a}_{70} = 1 + \left(\frac{0.97}{1.06}\right)(8.5736) = 8.8457$$

$$\begin{aligned} \ddot{a}_{69}^{(2)} &= \alpha(2)\ddot{a}_{69} - \beta(2) = (1.00021)(8.8457) - 0.25739 \\ &= 8.5902 \end{aligned}$$

Note that the approximation $\ddot{a}_x^{(m)} \cong \ddot{a}_x - \frac{(m-1)}{2m}$ works well (is closest to the exact answer, only off by less than 0.01). Since $m = 2$, this estimate becomes $8.8457 - \frac{1}{4} = 8.5957$

Question #8 - Removed

Question #9 - Removed

Question #10**Answer: E**

$$d = 0.05 \rightarrow v = 0.95$$

At issue

$$A_{40} = \sum_{k=0}^{49} v^{k+1} {}_k|q_{40} = 0.02(v^1 + \dots + v^{50}) = 0.02v(1 - v^{50})/d = 0.35076$$

$$\text{and } \ddot{a}_{40} = (1 - A_{40})/d = (1 - 0.35076)/0.05 = 12.9848$$

$$\text{so } P_{40} = \frac{1000A_{40}}{\ddot{a}_{40}} = \frac{350.76}{12.9848} = 27.013$$

$$E({}_{10}L | K_{40} \geq 10) = 1000A_{50}^{\text{Revised}} - P_{40}\ddot{a}_{50}^{\text{Revised}} = 549.18 - (27.013)(9.0164) = 305.62$$

where

$$A_{50}^{\text{Revised}} = \sum_{k=0}^{24} v^{k+1} {}_k|q_{50}^{\text{Revised}} = 0.04(v^1 + \dots + v^{25}) = 0.04v(1 - v^{25})/d = 0.54918$$

$$\text{and } \ddot{a}_{50}^{\text{Revised}} = (1 - A_{50}^{\text{Revised}})/d = (1 - 0.54918)/0.05 = 9.0164$$

Question #11**Answer: E**

Let NS denote non-smokers and S denote smokers.

The shortest solution is based on the conditional variance formula

$$\text{Var}(X) = E(\text{Var}(X|Y)) + \text{Var}(E(X|Y))$$

Let $Y = 1$ if smoker; $Y = 0$ if non-smoker

$$\begin{aligned} E(\bar{a}_{\overline{T}|Y=1}) &= \bar{a}_x^S = \frac{1 - \bar{A}_x^S}{\delta} \\ &= \frac{1 - 0.444}{0.1} = 5.56 \end{aligned}$$

$$\text{Similarly } E(\bar{a}_{\overline{T}|Y=0}) = \frac{1 - 0.286}{0.1} = 7.14$$

$$\begin{aligned} E(E(\bar{a}_{\overline{T}|Y})) &= E(E(\bar{a}_{\overline{T}}|0)) \times \text{Prob}(Y=0) + E(E(\bar{a}_{\overline{T}}|1)) \times \text{Prob}(Y=1) \\ &= (7.14)(0.70) + (5.56)(0.30) \\ &= 6.67 \end{aligned}$$

$$E\left[\left(E(\bar{a}_{T1}|Y)\right)^2\right] = (7.14^2)(0.70) + (5.56^2)(0.30) \\ = 44.96$$

$$\text{Var}\left(E(\bar{a}_{T1}|Y)\right) = 44.96 - 6.67^2 = 0.47$$

$$E\left(\text{Var}(\bar{a}_{T1}|Y)\right) = (8.503)(0.70) + (8.818)(0.30) \\ = 8.60$$

$$\text{Var}(\bar{a}_{T1}) = 8.60 + 0.47 = 9.07$$

Alternatively, here is a solution based on

$\text{Var}(Y) = E(Y^2) - [E(Y)]^2$, a formula for the variance of any random variable.

This can be

transformed into $E(Y^2) = \text{Var}(Y) + [E(Y)]^2$ which we will use in its conditional form

$$E\left((\bar{a}_{T1})^2 | \text{NS}\right) = \text{Var}(\bar{a}_{T1} | \text{NS}) + \left[E(\bar{a}_{T1} | \text{NS})\right]^2$$

$$\text{Var}[\bar{a}_{T1}] = E\left[(\bar{a}_{T1})^2\right] - \left(E[\bar{a}_{T1}]\right)^2$$

$$E[\bar{a}_{T1}] = E[\bar{a}_{T1} | \text{S}] \times \text{Prob}[\text{S}] + E[\bar{a}_{T1} | \text{NS}] \times \text{Prob}[\text{NS}] \\ = 0.30\bar{a}_x^{\text{S}} + 0.70\bar{a}_x^{\text{NS}} \\ = \frac{0.30(1 - \bar{A}_x^{\text{S}})}{0.1} + \frac{0.70(1 - \bar{A}_x^{\text{NS}})}{0.1} \\ = \frac{0.30(1 - 0.444) + 0.70(1 - 0.286)}{0.1} = (0.30)(5.56) + (0.70)(7.14) \\ = 1.67 + 5.00 = 6.67$$

$$E\left[(\bar{a}_{T1})^2\right] = E[\bar{a}_{T1}^2 | \text{S}] \times \text{Prob}[\text{S}] + E[\bar{a}_{T1}^2 | \text{NS}] \times \text{Prob}[\text{NS}] \\ = 0.30\left(\text{Var}(\bar{a}_{T1} | \text{S}) + \left(E[\bar{a}_{T1} | \text{S}]\right)^2\right) \\ + 0.70\left(\text{Var}(\bar{a}_{T1} | \text{NS}) + \left(E[\bar{a}_{T1} | \text{NS}]\right)^2\right) \\ = 0.30\left[8.818 + (5.56)^2\right] + 0.70\left[8.503 + (7.14)^2\right] \\ = 11.919 + 41.638 = 53.557$$

$$\text{Var}[\bar{a}_{T1}] = 53.557 - (6.67)^2 = 9.1$$

Alternatively, here is a solution based on $\bar{a}_{\overline{T}|} = \frac{1-v^T}{\delta}$

$$\begin{aligned}\text{Var}(\bar{a}_{\overline{T}|}) &= \text{Var}\left(\frac{1}{\delta} - \frac{v^T}{\delta}\right) \\ &= \text{Var}\left(\frac{-v^T}{\delta}\right) \text{ since } \text{Var}(X + \text{constant}) = \text{Var}(X) \\ &= \frac{\text{Var}(v^T)}{\delta^2} \text{ since } \text{Var}(\text{constant} \times X) = \text{constant}^2 \times \text{Var}(X) \\ &= \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{\delta^2} \text{ which is Bowers formula 5.2.9}\end{aligned}$$

This could be transformed into ${}^2A_x = \delta^2 \text{Var}(\bar{a}_{\overline{T}|}) + \bar{A}_x^2$, which we will use to get ${}^2A_x^{\text{NS}}$ and ${}^2A_x^{\text{S}}$.

$$\begin{aligned}{}^2A_x &= E[v^{2T}] \\ &= E[v^{2T} | \text{NS}] \times \text{Prob}(\text{NS}) + E[v^{2T} | \text{S}] \times \text{Prob}(\text{S}) \\ &= \left[\delta^2 \text{Var}(\bar{a}_{\overline{T}|} | \text{NS}) + (\bar{A}_x^{\text{NS}})^2 \right] \times \text{Prob}(\text{NS}) \\ &\quad + \left[\delta^2 \text{Var}(\bar{a}_{\overline{T}|} | \text{S}) + (\bar{A}_x^{\text{S}})^2 \right] \times \text{Prob}(\text{S}) \\ &= \left[(0.01)(8.503) + 0.286^2 \right] \times 0.70 \\ &\quad + \left[(0.01)(8.818) + 0.444^2 \right] \times 0.30 \\ &= (0.16683)(0.70) + (0.28532)(0.30) \\ &= 0.20238\end{aligned}$$

$$\begin{aligned}\bar{A}_x &= E[v^T] \\ &= E[v^T | \text{NS}] \times \text{Prob}(\text{NS}) + E[v^T | \text{S}] \times \text{Prob}(\text{S}) \\ &= (0.286)(0.70) + (0.444)(0.30) \\ &= 0.3334\end{aligned}$$

$$\begin{aligned}\text{Var}(\bar{a}_{\overline{T}|}) &= \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{\delta^2} \\ &= \frac{0.20238 - 0.3334^2}{0.01} = 9.12\end{aligned}$$

Question #12 - Removed

Question #13

Answer: D

Let NS denote non-smokers, S denote smokers.

$$\begin{aligned}\text{Prob}(T < t) &= \text{Prob}(T < t | \text{NS}) \times \text{Prob}(\text{NS}) + \text{Prob}(T < t | \text{S}) \times \text{Prob}(\text{S}) \\ &= (1 - e^{-0.1t}) \times 0.7 + (1 - e^{-0.2t}) \times 0.3 \\ &= 1 - 0.7e^{-0.1t} - 0.3e^{-0.2t}\end{aligned}$$

$$S_0(t) = 0.3e^{-0.2t} + 0.7e^{-0.1t}$$

Want \hat{t} such that $0.75 = 1 - S_0(\hat{t})$ or $0.25 = S_0(\hat{t})$

$$0.25 = 0.3e^{-2\hat{t}} + 0.7e^{-0.1\hat{t}} = 0.3(e^{-0.1\hat{t}})^2 + 0.7e^{-0.1\hat{t}}$$

Substitute: let $x = e^{-0.1\hat{t}}$

$$0.3x^2 + 0.7x - 0.25 = 0$$

This is quadratic, so $x = \frac{-0.7 \pm \sqrt{0.49 + (0.3)(0.25)4}}{2(0.3)}$

$$x = 0.3147$$

$$e^{-0.1\hat{t}} = 0.3147 \quad \text{so } \hat{t} = 11.56$$

Question #14**Answer: A**

At a constant force of mortality, the benefit premium equals the force of mortality and so $\mu = 0.03$.

$${}^2\bar{A}_x = 0.20 = \frac{\mu}{2\delta + \mu} = \frac{0.03}{2\delta + 0.03}$$

$$\Rightarrow \delta = 0.06$$

$$\text{Var}({}_0L) = \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{(\delta \bar{a})^2} = \frac{0.20 - \left(\frac{1}{3}\right)^2}{\left(\frac{0.06}{0.09}\right)^2} = 0.20$$

$$\text{where } A = \frac{\mu}{\mu + \delta} = \frac{0.03}{0.09} = \frac{1}{3} \quad \bar{a} = \frac{1}{\mu + \delta} = \frac{1}{0.09}$$

Question #15 - Removed**Question #16****Answer: A**

$$1000P_{40} = \frac{A_{40}}{\ddot{a}_{40}} = \frac{161.32}{14.8166} = 10.89$$

$$1000 {}_{20}V_{40} = 1000 \left(1 - \frac{\ddot{a}_{60}}{\ddot{a}_{40}} \right) = 1000 \left(1 - \frac{11.1454}{14.8166} \right) = 247.78$$

$${}_{21}V = \frac{({}_{20}V + 5000P_{40})(1+i) - 5000q_{60}}{P_{60}}$$

$$= \frac{(247.78 + (5)(10.89)) \times 1.06 - 5000(0.01376)}{1 - 0.01376} = 255$$

[Note: For this insurance, ${}_{20}V = 1000 {}_{20}V_{40}$ because retrospectively, this is identical to whole life]

Though it would have taken much longer, you can do this as a prospective reserve. The prospective solution is included for educational purposes, not to suggest it would be suitable under exam time constraints.

$$1000P_{40} = 10.89 \text{ as above}$$

$$1000A_{40} + 4000 {}_{20}E_{40} A_{60:\overline{5}|}^1 = 1000P_{40} + 5000P_{40} \times {}_{20}E_{40} \ddot{a}_{60:\overline{5}|} + \pi {}_{20}E_{40} \times {}_5E_{60} \ddot{a}_{65}$$

where $A_{60:\overline{5}|}^1 = A_{60} - {}_5E_{60} A_{65} = 0.06674$

$$\ddot{a}_{40:\overline{20}|} = \ddot{a}_{40} - {}_{20}E_{40} \ddot{a}_{60} = 11.7612$$

$$\ddot{a}_{60:\overline{5}|} = \ddot{a}_{60} - {}_5E_{60} \ddot{a}_{65} = 4.3407$$

$$1000(0.16132) + (4000)(0.27414)(0.06674) =$$

$$= (10.89)(11.7612) + (5)(10.89)(0.27414)(4.3407) + \pi(0.27414)(0.68756)(9.8969)$$

$$\pi = \frac{161.32 + 73.18 - 128.08 - 64.79}{1.86544}$$

$$= 22.32$$

Having struggled to solve for π , you could calculate ${}_{20}V$ prospectively then (as above)

calculate ${}_{21}V$ recursively.

$$\begin{aligned} {}_{20}V &= 4000A_{60:\overline{5}|}^1 + 1000A_{60} - 5000P_{40} \ddot{a}_{60:\overline{5}|} - \pi {}_5E_{60} \ddot{a}_{65} \\ &= (4000)(0.06674) + 369.13 - (5000)(0.01089)(4.3407) - (22.32)(0.68756)(9.8969) \\ &= 247.86 \text{ (minor rounding difference from } 1000{}_{20}V_{40}\text{)} \end{aligned}$$

Or we can continue to ${}_{21}V$ prospectively

$${}_{21}V = 5000A_{61:\overline{4}|}^1 + 1000 {}_4E_{61} A_{65} - 5000P_{40} \ddot{a}_{61:\overline{4}|} - \pi {}_4E_{61} \ddot{a}_{65}$$

$$\text{where } {}_4E_{61} = \frac{l_{65}}{l_{61}} v^4 = \left(\frac{7,533,964}{8,075,403} \right) (0.79209) = 0.73898$$

$$\begin{aligned} A_{61:\overline{4}|}^1 &= A_{61} - {}_4E_{61} A_{65} = 0.38279 - 0.73898 \times 0.43980 \\ &= 0.05779 \end{aligned}$$

$$\begin{aligned} \ddot{a}_{61:\overline{4}|} &= \ddot{a}_{61} - {}_4E_{61} \ddot{a}_{65} = 10.9041 - 0.73898 \times 9.8969 \\ &= 3.5905 \end{aligned}$$

$$\begin{aligned} {}_{21}V &= (5000)(0.05779) + (1000)(0.73898)(0.43980) \\ &\quad - (5)(10.89)(3.5905) - 22.32(0.73898)(9.8969) \\ &= 255 \end{aligned}$$

Finally. A moral victory. Under exam conditions since prospective benefit reserves must equal retrospective benefit reserves, calculate whichever is simpler.

Question #17**Answer: C**

$$\text{Var}(Z) = {}^2A_{41} - (A_{41})^2$$

$$\begin{aligned} A_{41} - A_{40} &= 0.00822 = A_{41} - (vq_{40} + vp_{40}A_{41}) \\ &= A_{41} - (0.0028/1.05 + (0.9972/1.05)A_{41}) \\ &\Rightarrow A_{41} = 0.21650 \end{aligned}$$

$$\begin{aligned} {}^2A_{41} - {}^2A_{40} &= 0.00433 = {}^2A_{41} - (v^2q_{40} + v^2p_{40}{}^2A_{41}) \\ &= {}^2A_{41} - (0.0028/1.05^2 + (0.9972/1.05^2)A_{41}) \\ {}^2A_{41} &= 0.07193 \end{aligned}$$

$$\begin{aligned} \text{Var}(Z) &= 0.07193 - 0.21650^2 \\ &= 0.02544 \end{aligned}$$

Question #18 - Removed**Question #19 - Removed**

Question #20**Answer: D**

$$\mu_x^{(\tau)} = \mu_{x+t}^{(1)} + \mu_{x+t}^{(2)}$$

$$= 0.2\mu_{x+t}^{(\tau)} + \mu_{x+t}^{(2)}$$

$$\Rightarrow \mu_{x+t}^{(2)} = 0.8\mu_{x+t}^{(\tau)}$$

$$q_x^{(1)} = 1 - p_x^{(1)} = 1 - e^{-\int_0^1 0.2k t^2 dt} = 1 - e^{-0.2\frac{k}{3}} = 0.04$$

$$\frac{k}{3} \Rightarrow \ln(1 - 0.04) / (-0.2) = 0.2041$$

$$k = 0.6123$$

$${}_2q_x^{(2)} = \int_0^2 {}_t p_x^{(\tau)} \mu_x^{(2)} dt = 0.8 \int_0^2 {}_t p_x^{(\tau)} \mu_x^{(\tau)}(t) dt$$

$$= 0.8 {}_2q_x^{(\tau)} = 0.8(1 - {}_2p_x^{(\tau)})$$

$${}_2p_x^{(\tau)} = e^{-\int_0^2 \mu_x(t) dt}$$

$$= e^{-\int_0^2 k t^2 dt}$$

$$= e^{\frac{-8k}{3}}$$

$$= e^{\frac{-(8)(0.6123)}{3}}$$

$$= 0.19538$$

$${}_2q_x^{(2)} = 0.8(1 - 0.19538) = 0.644$$

Question #21**Answer: A**

k	$\min(k, 3)$	$f(k)$	$f(k) \times (\min(k, 3))$	$f(k) \times [\min(k, 3)]^2$
0	0	0.1	0	0
1	1	$(0.9)(0.2) = 0.18$	0.18	0.18
2	2	$(0.72)(0.3) = 0.216$	0.432	0.864
3+	3	$1 - 0.1 - 0.18 - 0.216 =$ 0.504	<u>1.512</u>	<u>4.536</u>
			2.124	5.580

$$E[\min(K, 3)] = 2.124$$

$$E\left\{[\min(K, 3)]^2\right\} = 5.580$$

$$\text{Var}[\min(K, 3)] = 5.580 - 2.124^2 = 1.07$$

Note that $E[\min(K, 3)]$ is the temporary curtate life expectancy, $e_{x:\overline{3}|}$ if the life is age x .

Question #22**Answer: B**

$$S_0(60) = \frac{e^{-(0.1)(60)} + e^{-(0.08)(60)}}{2}$$

$$= 0.005354$$

$$S_0(60) = \frac{e^{-(0.1)(61)} + e^{-(0.08)(61)}}{2}$$

$$= 0.00492$$

$$q_{60} = 1 - \frac{0.00492}{0.005354} = 0.081$$

Question #23**Answer: D**

Let q_{64} for Michel equal the standard q_{64} plus c . We need to solve for c .

Recursion formula for a standard insurance:

$${}_{20}V_{45} = ({}_{19}V_{45} + P_{45})(1.03) - q_{64}(1 - {}_{20}V_{45})$$

Recursion formula for Michel's insurance

$${}_{20}V_{45} = ({}_{19}V_{45} + P_{45} + 0.01)(1.03) - (q_{64} + c)(1 - {}_{20}V_{45})$$

The values of ${}_{19}V_{45}$ and ${}_{20}V_{45}$ are the same in the two equations because we are told

Michel's benefit reserves are the same as for a standard insurance.

Subtract the second equation from the first to get:

$$0 = -(1.03)(0.01) + c(1 - {}_{20}V_{45})$$

$$c = \frac{(1.03)(0.01)}{(1 - {}_{20}V_{45})}$$

$$= \frac{0.0103}{1 - 0.427}$$

$$= 0.018$$

Question #24**Answer: B** K is the curtate future lifetime for one insured. L is the loss random variable for one insurance. L_{AGG} is the aggregate loss random variables for the individual insurances. σ_{AGG} is the standard deviation of L_{AGG} . M is the number of policies.

$$L = v^{K+1} - \pi \ddot{a}_{\overline{K+1}|} = \left(1 + \frac{\pi}{d}\right) v^{K+1} - \pi/d$$

$$\begin{aligned} E[L] &= (A_x - \pi \ddot{a}_x) = A_x - \pi \frac{(1 - A_x)}{d} \\ &= 0.24905 - 0.025 \left(\frac{0.75095}{0.056604} \right) = -0.082618 \end{aligned}$$

$$\text{Var}[L] = \left(1 + \frac{\pi}{d}\right)^2 \left({}^2A_x - A_x^2\right) = \left(1 + \frac{0.025}{0.056604}\right)^2 \left(0.09476 - (0.24905)^2\right) = 0.068034$$

$$E[L_{AGG}] = M E[L] = -0.082618M$$

$$\text{Var}[L_{AGG}] = M \text{Var}[L] = M(0.068034) \Rightarrow \sigma_{AGG} = 0.260833\sqrt{M}$$

$$\Pr[L_{AGG} > 0] = \left[\frac{L_{AGG} - E[L_{AGG}]}{\sigma_{AGG}} > \frac{-E[L_{AGG}]}{\sigma_{AGG}} \right]$$

$$\approx \Pr\left(N(0,1) > \frac{0.082618M}{\sqrt{M}(0.260833)} \right)$$

$$\Rightarrow 1.645 = \frac{0.082618\sqrt{M}}{0.260833}$$

$$\Rightarrow M = 26.97$$

\Rightarrow minimum number needed = 27

Question #25**Answer: D**

Annuity benefit: $Z_1 = 12,000 \frac{1-v^{K+1}}{d}$ for $K = 0, 1, 2, \dots$

Death benefit: $Z_2 = Bv^{K+1}$ for $K = 0, 1, 2, \dots$

New benefit: $Z = Z_1 + Z_2 = 12,000 \frac{1-v^{K+1}}{d} + Bv^{K+1}$
 $= \frac{12,000}{d} + \left(B - \frac{12,000}{d} \right) v^{K+1}$

$$\text{Var}(Z) = \left(B - \frac{12,000}{d} \right)^2 \text{Var}(v^{K+1})$$

$$\text{Var}(Z) = 0 \text{ if } B = \frac{12,000}{0.08} = 150,000.$$

In the first formula for $\text{Var}(Z)$, we used the formula, valid for any constants a and b and random variable X ,

$$\text{Var}(a + bX) = b^2 \text{Var}(X)$$

Question #26**Answer: A**

$$\mu_{x+t:y+t} = \mu_{x+t} + \mu_{y+t} = 0.08 + 0.04 = 0.12$$

$$\bar{A}_x = \mu_{x+t} / (\mu_{x+t} + \delta) = 0.5714$$

$$\bar{A}_y = \mu_{y+t} / (\mu_{y+t} + \delta) = 0.4$$

$$\bar{A}_{xy} = \mu_{x+t:y+t} / (\mu_{x+t:y+t} + \delta) = 0.6667$$

$$\bar{a}_{xy} = 1 / (\mu_{x+t:y+t} + \delta) = 5.556$$

$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.5714 + 0.4 - 0.6667 = 0.3047$$

$$\text{Premium} = 0.304762 / 5.556 = 0.0549$$

Question #27**Answer: B**

$$P_{40} = A_{40} / \ddot{a}_{40} = 0.16132 / 14.8166 = 0.0108878$$

$$P_{42} = A_{42} / \ddot{a}_{42} = 0.17636 / 14.5510 = 0.0121201$$

$$a_{45} = \ddot{a}_{45} - 1 = 13.1121$$

$$\begin{aligned} E[{}_3L | K_{42} \geq 3] &= 1000A_{45} - 1000P_{40} - 1000P_{42} a_{45} \\ &= 201.20 - 10.89 - (12.12)(13.1121) \\ &= 31.39 \end{aligned}$$

Many similar formulas would work equally well. One possibility would be $1000 {}_3V_{42} + (1000P_{42} - 1000P_{40})$, because prospectively after duration 3, this differs from the normal benefit reserve in that in the next year you collect $1000P_{40}$ instead of $1000P_{42}$.

Question #28**Answer: E**

$$\begin{aligned} E[\min(T, 40)] &= 40 - 0.005(40)^2 = 32 \\ 32 &= \int_0^{40} tf(t)dt + \int_{40}^w 40f(t)dt \\ &= \int_0^w tf(t)dt - \int_{40}^w tf(t)dt + 40(.6) \\ &= 86 - \int_{40}^w tf(t)dt \end{aligned}$$

$$\int_{40}^w tf(t)dt = 54$$

$$\dot{e}_{40} = \frac{\int_{40}^w (t-40)f(t)dt}{s(40)} = \frac{54 - 40(.6)}{.6} = 50$$

Question #29**Answer: B**

$$d = 0.05 \Rightarrow v = 0.95$$

Step 1 Determine p_x from Kevin's work:

$$608 + 350vp_x = 1000vq_x + 1000v^2p_x(p_{x+1} + q_{x+1})$$

$$608 + 350(0.95)p_x = 1000(0.95)(1 - p_x) + 1000(0.9025)p_x \quad (1)$$

$$608 + 332.5p_x = 950(1 - p_x) + 902.5p_x$$

$$p_x = 342/380 = 0.9$$

Step 2 Calculate $1000P_{x:\overline{2}|}$, as Kira did:

$$608 + 350(0.95)(0.9) = 1000P_{x:\overline{2}|} [1 + (0.95)(0.9)]$$

$$1000P_{x:\overline{2}|} = \frac{[299.25 + 608]}{1.855} = 489.08$$

The first line of Kira's solution is that the expected present value of Kevin's benefit premiums is equal to the expected present value of Kira's, since each must equal the expected present value of benefits. The expected present value of benefits would also have been easy to calculate as

$$(1000)(0.95)(0.1) + (1000)(0.95^2)(0.9) = 907.25$$

Question #30**Answer: E**Because no premiums are paid after year 10 for (x), ${}_{11}V_x = A_{x+11}$ One of the recursive reserve formulas is ${}_{h+1}V = \frac{({}_hV + \pi_h)(1+i) - b_{h+1}q_{x+h}}{p_{x+h}}$

$${}_{10}V = \frac{(32,535 + 2,078) \times (1.05) - 100,000 \times 0.011}{0.989} = 35,635.642$$

$${}_{11}V = \frac{(35,635.642 + 0) \times (1.05) - 100,000 \times 0.012}{0.988} = 36,657.31 = A_{x+11}$$

Question #31**Answer: B**

The survival function is $S_0(t) = \left(1 - \frac{t}{\omega}\right)$:

Then,

$$e_x^\circ = \frac{\omega - x}{2} \text{ and } {}_t p_x = \left(1 - \frac{t}{\omega - x}\right)$$

$$e_{45}^\circ = \frac{105 - 45}{2} = 30$$

$$e_{65}^\circ = \frac{105 - 65}{2} = 20$$

$$\begin{aligned} e_{45:65}^\circ &= \int_0^{40} {}_t p_{45:65} dt = \int_0^{40} \frac{60-t}{60} \times \frac{40-t}{40} dt \\ &= \frac{1}{60 \times 40} \left(60 \times 40 \times t - \frac{60+40}{2} t^2 + \frac{1}{3} t^3 \right) \Big|_0^{40} \\ &= 15.56 \end{aligned}$$

$$\begin{aligned} e_{45:65}^\circ &= e_{45}^\circ + e_{65}^\circ - e_{45:65}^\circ \\ &= 30 + 20 - 15.56 = 34 \end{aligned}$$

In the integral for $e_{45:65}^\circ$, the upper limit is 40 since 65 (and thus the joint status also) can survive a maximum of 40 years.

Question #32**Answer: E**

$$\begin{aligned} \mu_4 &= -S_0'(4) / S_0(4) \\ &= \frac{-(-e^4 / 100)}{1 - e^4 / 100} \\ &= \frac{e^4 / 100}{1 - e^4 / 100} \\ &= \frac{e^4}{100 - e^4} = 1.202553 \end{aligned}$$

Question # 33**Answer: A**

$$q_x^{(i)} = q_x^{(\tau)} \left[\frac{\ln p_x^{(i)}}{\ln p_x^{(\tau)}} \right] = q_x^{(\tau)} \left[\frac{\ln e^{-\mu^{(i)}}}{\ln e^{-\mu^{(\tau)}}} \right]$$

$$= q_x^{(\tau)} \times \frac{\mu^{(i)}}{\mu^{(\tau)}}$$

$$\mu_x^{(\tau)} = \mu_x^{(1)} + \mu_x^{(2)} + \mu_x^{(3)} = 1.5$$

$$q_x^{(\tau)} = 1 - e^{-\mu^{(\tau)}} = 1 - e^{-1.5}$$

$$= 0.7769$$

$$q_x^{(2)} = \frac{(0.7769)\mu^{(2)}}{\mu^{(\tau)}} = \frac{(0.5)(0.7769)}{1.5}$$

$$= 0.2590$$

Question # 34**Answer: D**

$${}_{2|2}A_{[60]} = v^3 \times {}_2P_{[60]} \times q_{[60]+2} +$$

↓	↓	↓
pay at end	live	then die
of year 3	2 years	in year 3

$$+ v^4 \times {}_3P_{[60]} \times q_{60+3}$$

pay at end	live	then die
of year 4	3 years	in year 4

$$= \frac{1}{(1.03)^3} (1-0.09)(1-0.11)(0.13) + \frac{1}{(1.03)^4} (1-0.09)(1-0.11)(1-0.13)(0.15)$$

$$= 0.19$$

Question # 35**Answer: B**

$$\bar{a}_x = \bar{a}_{x:\overline{5}|} + {}_5E_x \bar{a}_{x+5}$$

$$\bar{a}_{x:\overline{5}|} = \frac{1 - e^{-0.07(5)}}{0.07} = 4.219, \text{ where } 0.07 = \mu + \delta \text{ for } t < 5$$

$${}_5E_x = e^{-0.07(5)} = 0.705$$

$$\bar{a}_{x+5} = \frac{1}{0.08} = 12.5, \text{ where } 0.08 = \mu + \delta \text{ for } t \geq 5$$

$$\therefore \bar{a}_x = 4.219 + (0.705)(12.5) = 13.03$$

Question #36**Answer: D**

$$p_x^{(\tau)} = p_x^{(1)} p_x^{(2)} = 0.8(0.7) = 0.56$$

$$q_x^{(1)} = \left[\frac{\ln(p_x^{(1)})}{\ln(p_x^{(\tau)})} \right] q_x^{(\tau)} \text{ since UDD in double decrement table}$$

$$= \left[\frac{\ln(0.8)}{\ln(0.56)} \right] 0.44$$

$$= 0.1693$$

$${}_{0.3}q_{x+0.1}^{(1)} = \frac{0.3q_x^{(1)}}{1 - 0.1q_x^{(\tau)}} = 0.053$$

To elaborate on the last step:

$${}_{0.3}q_{x+0.1}^{(1)} = \frac{\left(\begin{array}{l} \text{Number dying from cause} \\ \text{1 between } x + 0.1 \text{ and } x + 0.4 \end{array} \right)}{\text{Number alive at } x + 0.1}$$

Since UDD in double decrement,

$$= \frac{l_x^{(\tau)}(0.3)q_x^{(1)}}{l_x^{(\tau)}(1 - 0.1q_x^{(\tau)})}$$

Question #37**Answer: E**

The benefit premium is $\frac{1}{\bar{a}_x} - \delta = \frac{1}{12} - 0.04 = 0.04333$

$${}_oL = v^T - (0.04333 + 0.0066)\bar{a}_{\overline{T}|} + 0.02 + 0.003\bar{a}_{\overline{T}|}$$

$$= v^T - 0.04693 \left(\frac{1 - v^T}{\delta} \right) + 0.02$$

$$= v^T \left(1 + \frac{0.04693}{\delta} \right) - \frac{0.04693}{\delta} + 0.02$$

$$\text{Var}({}_oL) = \text{Var}(v^T) \left(1 + \frac{0.04693}{\delta} \right)^2 = 0.1(4.7230) = 0.4723$$

Question #38**Answer: D**

$$T = \begin{pmatrix} 0.7 & 0.1 & 0.2 \\ 0.3 & 0.6 & 0.1 \\ 0 & 0 & 1 \end{pmatrix} \quad T^2 = \begin{pmatrix} 0.52 & 0.13 & 0.35 \\ 0.39 & 0.39 & 0.22 \\ 0 & 0 & 1 \end{pmatrix}$$

Actuarial present value (APV) prem = $800(1 + (0.7 + 0.1) + (0.52 + 0.13)) = 1,960$

APV claim = $500(1 + 0.7 + 0.52) + 3000(0 + 0.1 + 0.13) = 1800$

Difference = 160

Question # 39 - Removed

Question # 40**Answer: D**

Use Mod to designate values unique to this insured.

$$\ddot{a}_{60} = (1 - A_{60}) / d = (1 - 0.36933) / [(0.06) / (1.06)] = 11.1418$$

$$1000P_{60} = 1000A_{60} / \ddot{a}_{60} = 1000(0.36933 / 11.1418) = 33.15$$

$$A_{60}^{Mod} = v(q_{60}^{Mod} + p_{60}^{Mod} A_{61}) = \frac{1}{1.06} [0.1376 + (0.8624)(0.383)] = 0.44141$$

$$\ddot{a}^{Mod} = (1 - A_{60}^{Mod}) / d = (1 - 0.44141) / [0.06 / 1.06] = 9.8684$$

$$\begin{aligned} E[{}_0L^{Mod}] &= 1000(A_{60}^{Mod} - P_{60}\ddot{a}_{60}^{Mod}) \\ &= 1000[0.44141 - 0.03315(9.8684)] \\ &= 114.27 \end{aligned}$$

Question # 41**Answer: D**

The prospective reserve at age 60 per 1 of insurance is A_{60} , since there will be no future premiums. Equating that to the retrospective reserve per 1 of coverage, we have:

$$A_{60} = P_{40} \frac{\ddot{s}_{40:\overline{10}|}}{{}_{10}E_{50}} + P_{50}^{Mod} \ddot{s}_{50:\overline{10}|} - {}_{20}k_{40}$$

$$A_{60} = \frac{A_{40}}{\ddot{a}_{40}} \times \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40} {}_{10}E_{50}} + P_{50}^{Mod} \frac{\ddot{a}_{50:\overline{10}|}}{{}_{10}E_{50}} - \frac{A_{40}^1}{{}_{20}E_{40}}$$

$$0.36913 = \frac{0.16132}{14.8166} \times \frac{7.70}{(0.53667)(0.51081)} + P_{50}^{Mod} \frac{7.57}{0.51081} - \frac{0.06}{0.27414}$$

$$0.36913 = 0.30582 + 14.8196 P_{50}^{Mod} - 0.21887$$

$$1000 P_{50}^{Mod} = 19.04$$

Alternatively, you could equate the retrospective and prospective reserves at age 50. Your equation would be:

$$A_{50} - P_{50}^{Mod} \ddot{a}_{50:\overline{10}|} = \frac{A_{40}}{\ddot{a}_{40}} \times \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40}} - \frac{A_{40}^1}{{}_{10}E_{40}}$$

where $A_{40}^1 = A_{40} - {}_{10}E_{40} A_{50}$

$$= 0.16132 - (0.53667)(0.24905)$$

$$= 0.02766$$

$$0.24905 - (P_{50}^{Mod})(7.57) = \frac{0.16132}{14.8166} \times \frac{7.70}{0.53667} - \frac{0.02766}{0.53667}$$

$$1000P_{50}^{Mod} = \frac{(1000)(0.14437)}{7.57} = 19.07$$

Alternatively, you could set the expected present value of benefits at age 40 to the expected present value of benefit premiums. The change at age 50 did not change the benefits, only the pattern of paying for them.

$$A_{40} = P_{40} \ddot{a}_{40:\overline{10}|} + P_{50}^{Mod} {}_{10}E_{40} \ddot{a}_{50:\overline{10}|}$$

$$0.16132 = \left(\frac{0.16132}{14.8166} \right) (7.70) + (P_{50}^{Mod})(0.53667)(7.57)$$

$$1000P_{50}^{Mod} = \frac{(1000)(0.07748)}{4.0626} = 19.07$$

Question # 42

Answer: A

$$d_x^{(2)} = q_x^{(2)} \times l_x^{(\tau)} = 400$$

$$d_x^{(1)} = 0.45(400) = 180$$

$$q_x^{(2)} = \frac{d_x^{(2)}}{l_x^{(\tau)} - d_x^{(1)}} = \frac{400}{1000 - 180} = 0.488$$

$$p_x^{(2)} = 1 - 0.488 = 0.512$$

Note: The UDD assumption was not critical except to have all deaths during the year so that 1000 - 180 lives are subject to decrement 2.

Question #43

Answer: D

Use “age” subscripts for years completed in program. E.g., p_0 applies to a person newly hired (“age” 0).

Let decrement 1 = fail, 2 = resign, 3 = other.

$$\begin{aligned} \text{Then } q_0^{(1)} &= 1/4, q_1^{(1)} = 1/5, q_2^{(1)} = 1/3 \\ q_0^{(2)} &= 1/5, q_1^{(2)} = 1/3, q_2^{(2)} = 1/8 \\ q_0^{(3)} &= 1/10, q_1^{(3)} = 1/9, q_2^{(3)} = 1/4 \end{aligned}$$

$$\text{This gives } p_0^{(\tau)} = (1 - 1/4)(1 - 1/5)(1 - 1/10) = 0.54$$

$$p_1^{(\tau)} = (1 - 1/5)(1 - 1/3)(1 - 1/9) = 0.474$$

$$p_2^{(\tau)} = (1 - 1/3)(1 - 1/8)(1 - 1/4) = 0.438$$

$$\text{So } l_0^{(\tau)} = 200, l_1^{(\tau)} = 200(0.54) = 108, \text{ and } l_2^{(\tau)} = 108(0.474) = 51.2$$

$$q_2^{(1)} = \left[\log p_2^{(1)} / \log p_2^{(\tau)} \right] q_2^{(\tau)}$$

$$q_2^{(1)} = \left[\log(2/3) / \log(0.438) \right] [1 - 0.438]$$

$$= (0.405 / 0.826)(0.562)$$

$$= 0.276$$

$$d_2^{(1)} = l_2^{(\tau)} q_2^{(1)}$$

$$= (51.2)(0.276) = 14$$

Question #44 - Removed

Question #45**Answer: E**

For the given life table function: $e_x^\circ = \frac{\omega - x}{2}$

$${}_k|q_x = \frac{1}{\omega - x}$$

$$A_x = \sum_{k=b}^{\omega-x-1} v^{k+1} {}_k|q_x = \frac{1}{\omega - x} \sum_{k=b}^{\omega-x-1} v^{k+1}$$

$$A_x = \frac{a_{\overline{\omega-x}|}}{\omega - x}$$

$$\ddot{a}_x = \frac{1 - A_x}{d}$$

$e_{50}^\circ = 25 \Rightarrow \omega = 100$ for typical annuitants

$e_y^\circ = 15 \Rightarrow y = \text{Assumed age} = 70$

$$A_{70} = \frac{a_{\overline{30}|}}{30} = 0.45883$$

$$\ddot{a}_{70} = 9.5607$$

$$500000 = b \ddot{a}_{70} \Rightarrow b = 52,297$$

Question #46**Answer: B**

$$\begin{aligned} {}_{10}E_{30:40} &= {}_{10}p_{30} {}_{10}p_{40} v^{10} = ({}_{10}p_{30} v^{10}) ({}_{10}p_{40} v^{10}) (1+i)^{10} \\ &= ({}_{10}E_{30}) ({}_{10}E_{40}) (1+i)^{10} \\ &= (0.54733)(0.53667)(1.79085) \\ &= 0.52604 \end{aligned}$$

The above is only one of many possible ways to evaluate ${}_{10}p_{30} {}_{10}p_{40} v^{10}$, all of which should give 0.52604

$$\begin{aligned} a_{\overline{30:40:10}|} &= a_{\overline{30:40}|} - {}_{10}E_{30:40} a_{\overline{30+10:40+10}|} \\ &= (\ddot{a}_{\overline{30:40}|} - 1) - (0.52604)(\ddot{a}_{\overline{40:50}|} - 1) \\ &= (13.2068) - (0.52604)(11.4784) \\ &= 7.1687 \end{aligned}$$

Question #47**Answer: A**

Equivalence Principle, where π is annual benefit premium, gives

$$1000A_{35} + (IA)_{35} \times \pi = \ddot{a}_{35}\pi$$

$$\begin{aligned} \pi &= \frac{1000A_{35}}{(\ddot{a}_{35} - (IA)_{35})} = \frac{1000 \times 0.42898}{(11.99143 - 6.16761)} \\ &= \frac{428.98}{5.82382} \\ &= 73.66 \end{aligned}$$

We obtained \ddot{a}_{35} from

$$\ddot{a}_{35} = \frac{1 - A_{35}}{d} = \frac{1 - 0.42898}{0.047619} = 11.99143$$

Question #48 - Removed**Question #49****Answer: C**

$$\mu_{xy} = \mu_x + \mu_y = 0.14$$

$$\bar{A}_x = \bar{A}_y = \frac{\mu}{\mu + \delta} = \frac{0.07}{0.07 + 0.05} = 0.5833$$

$$\bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy} + \delta} = \frac{0.14}{0.14 + 0.05} = \frac{0.14}{0.19} = 0.7368 \text{ and } \bar{a}_{xy} = \frac{1}{\mu_{xy} + \delta} = \frac{1}{0.14 + 0.05} = 5.2632$$

$$P = \frac{\bar{A}_{xy}}{\bar{a}_{xy}} = \frac{\bar{A}_x + \bar{A}_y - \bar{A}_{xy}}{\bar{a}_{xy}} = \frac{2(0.5833) - 0.7368}{5.2632} = 0.0817$$

Question #50**Answer: E**

$$({}_{20}V + P_{20})(1+i) - q_{40}(1 - {}_{21}V) = {}_{21}V$$

$$(0.49 + 0.01)(1+i) - 0.022(1 - 0.545) = 0.545$$

$$(1+i) = \frac{(0.545)(1 - 0.022) + 0.022}{0.50}$$

$$= 1.11$$

$$({}_{21}V + P_{20})(1+i) - q_{41}(1 - {}_{22}V) = {}_{22}V$$

$$(0.545 + 0.01)(1.11) - q_{41}(1 - 0.605) = 0.605$$

$$q_{41} = \frac{0.61605 - 0.605}{0.395}$$

$$= 0.028$$

Question #51**Answer: E**

$$1000 P_{60} = 1000 A_{60} / \ddot{a}_{60}$$

$$= 1000 v(q_{60} + p_{60}A_{61}) / (1 + p_{60} v \ddot{a}_{61})$$

$$= 1000(q_{60} + p_{60} A_{61}) / (1.06 + p_{60} \ddot{a}_{61})$$

$$= (15 + (0.985)(382.79)) / (1.06 + (0.985)(10.9041)) = 33.22$$

Question #52 - Removed

Question #53**Answer: E**

$$g = -\ln(0.96) = 0.04082$$

$$\mu_{x+t:y+t}^{02} = 0.04082 - 0.01 = 0.03082$$

$$h = -\ln(0.97) = 0.03046$$

$$\mu_{x+t:y+t}^{01} = 0.03046 - 0.01 = 0.02046$$

$${}_5P_{xy} = {}_5P_{xy}^{00} = \exp\left(-\int_0^5 \mu_{x+t:y+t}^{01} + \mu_{x+t:y+t}^{02} + \mu_{x+t:y+t}^{03} dt\right) = e^{-5(0.06128)} = 0.736$$

Question #54**Answer: B**

Transform these scenarios into a four-state Markov chain, as shown below.

State	from year $t-3$ to year $t-2$	from year $t-2$ to year $t-1$	Probability that year t will decrease from year $t-1$
0	Decrease (D)	Decrease (I)	0.8
1	Increase	Decrease	0.6
2	Decrease	Increase	0.75
3	Increase	Increase	0.9

The transition probability matrix is

$$\begin{bmatrix} 0.80 & 0.00 & 0.20 & 0.00 \\ 0.60 & 0.00 & 0.40 & 0.00 \\ 0.00 & 0.75 & 0.00 & 0.25 \\ 0.00 & 0.90 & 0.00 & 0.10 \end{bmatrix}$$

Note for example that the value of 0.2 in the first row is a transition from DD to DI, that is, the probability of an increase in year three after two successive years of decrease.

The requested probability is that of starting in state 0 and then being in either state 0 or 1 after two years. That requires the first row of the square of the transition probability matrix. It can be obtained by multiplying the first row of the matrix by each column. The result is [0.64 0.15 0.16 0.05]. The two required probabilities are 0.64 and 0.15 for a total of 0.79. The last two values in the vector do not need to be calculated, but doing so provides a check (in that the four values must sum to one).

Alternatively, the required probabilities are for the successive values DDID and DDDD. The first case is transitions from state 0 to 2 and then 2 to 1 for a probability of $0.2(0.75) = 0.15$. The second case is transitions from 0 to 0 and then 0 to 0 for a probability of $0.8(0.8) = 0.64$ for a total of 0.79.

Question #55**Answer: B**

$$l_x = \omega - x = 105 - x$$

$$\Rightarrow {}_t p_{45} = l_{45+t} / l_{45} = (60 - t) / 60$$

Let K be the curtate future lifetime of (45). Then the sum of the payments is 0 if $K \leq 19$ and is $K - 19$ if $K \geq 20$.

$$\begin{aligned} {}_{20|} \ddot{a}_{45} &= \sum_{K=20}^{60} 1 \times \left(\frac{60 - K}{60} \right) \times 1 \\ &= \frac{(40 + 39 + \dots + 1)}{60} = \frac{(40)(41)}{2(60)} = 13.6\bar{6} \end{aligned}$$

Hence,

$$\text{Prob}(K - 19 > 13.6\bar{6}) = \text{Prob}(K > 32.6\bar{6})$$

$$= \text{Prob}(K \geq 33) \text{ since } K \text{ is an integer}$$

$$= \text{Prob}(T \geq 33)$$

$$= {}_{33} p_{45} = \frac{l_{78}}{l_{45}} = \frac{27}{60}$$

$$= 0.450$$

Question #56**Answer: C**

$${}^2\bar{A}_x = \frac{\mu}{\mu + 2\delta} = 0.25 \rightarrow \mu = 0.04$$

$$\bar{A}_x = \frac{\mu}{\mu + \delta} = 0.4$$

$$(\bar{IA})_x = \int_0^{\infty} {}_s\bar{A}_x ds$$

$$\int_0^{\infty} {}_sE_x \bar{A}_x ds$$

$$= \int_0^{\infty} (e^{-0.1s})(0.4) ds$$

$$= (0.4) \left(\frac{-e^{-0.1s}}{0.1} \right) \Big|_0^{\infty} = \frac{0.4}{0.1} = 4$$

Alternatively, using a more fundamental formula but requiring more difficult integration.

$$\begin{aligned} (\bar{IA})_x &= \int_0^{\infty} t {}_tP_x \mu_x(t) e^{-\delta t} dt \\ &= \int_0^{\infty} t e^{-0.04t} (0.04) e^{-0.06t} dt \\ &= 0.04 \int_0^{\infty} t e^{-0.1t} dt \end{aligned}$$

(integration by parts, not shown)

$$\begin{aligned} &= 0.04 \left(\frac{-t}{0.1} - \frac{1}{0.01} \right) e^{-0.1t} \Big|_0^{\infty} \\ &= \frac{0.04}{0.01} = 4 \end{aligned}$$

Question #57**Answer: E**

Subscripts A and B here distinguish between the tools and do not represent ages.

We have to find ${}^o e_{\overline{AB}}$

$${}^o e_A = \int_0^{10} \left(1 - \frac{t}{10}\right) dt = t - \frac{t^2}{20} \Big|_0^{10} = 10 - 5 = 5$$

$${}^o e_B = \int_0^7 \left(1 - \frac{t}{7}\right) dt = t - \frac{t^2}{14} \Big|_0^7 = 49 - \frac{49}{14} = 3.5$$

$${}^o e_{AB} = \int_0^7 \left(1 - \frac{t}{7}\right) \left(1 - \frac{t}{10}\right) dt = \int_0^7 \left(1 - \frac{t}{10} - \frac{t}{7} + \frac{t^2}{70}\right) dt$$

$$= t - \frac{t^2}{20} - \frac{t^2}{14} + \frac{t^3}{210} \Big|_0^7$$

$$= 7 - \frac{49}{20} - \frac{49}{14} + \frac{343}{210} = 2.683$$

$${}^o e_{\overline{AB}} = {}^o e_A + {}^o e_B - {}^o e_{AB}$$

$$= 5 + 3.5 - 2.683 = 5.817$$

Question #58**Answer: A**

$$\mu_{x+t}^{(\tau)} = 0.100 + 0.004 = 0.104$$

$${}_t p_x^{(\tau)} = e^{-0.104t}$$

Expected present value (EPV) = EPV for cause 1 + EPV for cause 2.

$$2000 \int_0^5 e^{-0.04t} e^{-0.104t} (0.100) dt + 500,000 \int_0^5 e^{-0.04t} e^{-0.104t} (0.400) dt$$

$$= (2000(0.10) + 500,000(0.004)) \int_0^5 e^{-0.144t} dt = \frac{2200}{0.144} (1 - e^{-0.144(5)}) = 7841$$

Question #59**Answer: A**

$$R = 1 - p_x = q_x$$

$$S = 1 - p_x \times e^{(-k)} \text{ since } e^{-\int_0^1 (\mu_{x+t} + k) dt} = e^{-\int_0^1 \mu_{x+t} dt - \int_0^1 k dt}$$
$$= e^{-\int_0^1 \mu_{x+t} dt} e^{-\int_0^1 k dt}$$

$$\text{So } S = 0.75R \Rightarrow 1 - p_x \times e^{-k} = 0.75q_x$$

$$e^{-k} = \frac{1 - 0.75q_x}{p_x}$$

$$e^k = \frac{p_x}{1 - 0.75q_x} = \frac{1 - q_x}{1 - 0.75q_x}$$

$$k = \ln \left[\frac{1 - q_x}{1 - 0.75q_x} \right]$$

Question #60**Answer: C**

$$A_{60} = 0.36913 \quad d = 0.05660$$

$${}^2A_{60} = 0.17741$$

$$\text{and } \sqrt{{}^2A_{60} - A_{60}^2} = 0.202862$$

$$\text{Expected Loss on one policy is } E[L(\pi)] = \left(100,000 + \frac{\pi}{d}\right)A_{60} - \frac{\pi}{d}$$

$$\text{Variance on one policy is } \text{Var}[L(\pi)] = \left(100,000 + \frac{\pi}{d}\right)^2 ({}^2A_{60} - A_{60}^2)$$

On the 10000 lives,

$$E[S] = 10,000E[L(\pi)] \quad \text{and} \quad \text{Var}[S] = 10,000 \text{Var}[L(\pi)]$$

The π is such that $0 - E[S] / \sqrt{\text{Var}[S]} = 2.326$ since $\Phi(2.326) = 0.99$

$$\frac{10,000 \left(\frac{\pi}{d} - \left(100,000 + \frac{\pi}{d}\right)A_{60} \right)}{100 \left(100,000 + \frac{\pi}{d}\right) \sqrt{{}^2A_{60} - A_{60}^2}} = 2.326$$

$$\frac{100 \left(\frac{\pi}{d} - \left(100,000 + \frac{\pi}{d}\right) \right) (0.36913)}{\left(100,000 + \frac{\pi}{d}\right) (0.202862)} = 2.326$$

$$\frac{0.63087 \frac{\pi}{d} - 36913}{100,000 + \frac{\pi}{d}} = 0.004719$$

$$0.63087 \frac{\pi}{d} - 36913 = 471.9 = 0.004719 \frac{\pi}{d}$$

$$\frac{\pi}{d} = \frac{36913 + 471.9}{0.63087 - 0.004719}$$

$$= 59706$$

$$\pi = 59706 \times d = 3379$$

Question #61**Answer: C**

$$\begin{aligned}
{}_1V &= ({}_0V + \pi)(1+i) - (1000 + {}_1V - {}_1V) \times q_{75} \\
&= 1.05\pi - 1000q_{75}
\end{aligned}$$

Similarly,

$${}_2V = ({}_1V + \pi) \times 1.05 - 1000q_{76}$$

$${}_3V = ({}_2V + \pi) \times 1.05 - 1000q_{77}$$

$$1000 = {}_3V = (1.05^3 \pi + 1.05^2 \cdot \pi + 1.05 \pi) - 1000 \times q_{75} \times 1.05^2 - 1000 \times 1.05 \times q_{76} - 1000 \times q_{77} \quad *$$

$$\begin{aligned}
\pi &= \frac{1000 + 1000(1.05^2 q_{75} + 1.05 q_{76} + q_{77})}{(1.05)^3 + (1.05)^2 + 1.05} \\
&= \frac{1000 \times (1 + 1.05^2 \times 0.05169 + 1.05 \times 0.05647 + 0.06168)}{3.310125} \\
&= \frac{1000 \times 1.17796}{3.310125} = 355.87
\end{aligned}$$

* This equation is an algebraic manipulation of the three equations in three unknowns $({}_1V, {}_2V, \pi)$. One method – usually effective in problems where benefit = stated amount plus reserve, is to multiply the ${}_1V$ equation by 1.05^2 , the ${}_2V$ equation by 1.05 , and add those two to the ${}_3V$ equation: in the result, you can cancel out the ${}_1V$, and ${}_2V$ terms. Or you can substitute the ${}_1V$ equation into the ${}_2V$ equation, giving ${}_2V$ in terms of π , and then substitute that into the ${}_3V$ equation.

Question #62**Answer: D**

$$\begin{aligned}
\bar{A}_{28:\overline{2}|}^1 &= \int_0^2 e^{-\delta t} \frac{1}{72} dt \\
&= \frac{1}{72\delta} (1 - e^{-2\delta}) = 0.02622 \text{ since } \delta = \ln(1.06) = 0.05827
\end{aligned}$$

$$\ddot{a}_{28:\overline{2}|} = 1 + v \left(\frac{71}{72} \right) = 1.9303$$

$${}_3V = 500,000 \bar{A}_{28:\overline{2}|}^1 - 6643 \ddot{a}_{28:\overline{2}|} = 287$$

Question #63**Answer: D**

Let \bar{A}_x and \bar{a}_x be calculated with μ_{x+t} and $\delta = 0.06$

Let \bar{A}_x^* and \bar{a}_x^* be the corresponding values with μ_{x+t} increased by 0.03 and δ decreased by 0.03

$$\bar{a}_x = \frac{1 - \bar{A}_x}{\delta} = \frac{0.4}{0.06} = 6.667$$

$$\bar{a}_x^* = \bar{a}_x$$

$$\left[\begin{aligned} \text{Proof: } \bar{a}_x^* &= \int_0^{\infty} e^{-\int_0^t (\mu_{x+s} + 0.03) ds} e^{-0.03t} dt \\ &= \int_0^{\infty} e^{-\int_0^t \mu_{x+s} ds} e^{-0.03t} e^{-0.03t} dt \\ &= \int_0^{\infty} e^{-\int_0^t \mu_{x+s} ds} e^{-0.06t} dt \\ &= \bar{a}_x \end{aligned} \right]$$

$$\begin{aligned} \bar{A}_x^* &= 1 - 0.03\bar{a}_x^* = 1 - 0.03\bar{a}_x \\ &= 1 - (0.03)(6.667) \\ &= 0.8 \end{aligned}$$

Question #64**Answer: A**

Year	bulb ages				# replaced
	0	1	2	3	
0	10000	0	0	0	-
1	1000	9000	0	0	1000
2	100+2700	900	6300	0	2800
3	280+270+3150				3700

The diagonals represent bulbs that don't burn out.

E.g., of the initial 10,000, $(10,000)(1-0.1) = 9000$ reach year 1.

$(9000)(1-0.3) = 6300$ of those reach year 2.

Replacement bulbs are new, so they start at age 0.

At the end of year 1, that's $(10,000)(0.1) = 1000$

At the end of 2, it's $(9000)(0.3) + (1000)(0.1) = 2700 + 100$

At the end of 3, it's $(2800)(0.1) + (900)(0.3) + (6300)(0.5) = 3700$

$$\begin{aligned} \text{Expected present value} &= \frac{1000}{1.05} + \frac{2800}{1.05^2} + \frac{3700}{1.05^3} \\ &= 6688 \end{aligned}$$

Question #65**Answer: E**

$$\begin{aligned} e^{\circ}_{25:\overline{25}|} &= \int_0^{15} {}_tP_{25} dt + {}_{15}P_{25} \int_0^{10} {}_tP_{40} dt \\ &= \int_0^{15} e^{-.04t} dt + \left(e^{-\int_0^{15} .04 ds} \right) \int_0^{10} e^{-.05t} dt \\ &= \frac{1}{.04} (1 - e^{-.60}) + e^{-.60} \left[\frac{1}{.05} (1 - e^{-.50}) \right] \\ &= 11.2797 + 4.3187 \\ &= 15.60 \end{aligned}$$

Question #66**Answer: C**

$$\begin{aligned} {}_5P_{[60]+1} &= \\ &= (1 - q_{[60]+1})(1 - q_{[60]+2})(1 - q_{63})(1 - q_{64})(1 - q_{65}) \\ &= (0.89)(0.87)(0.85)(0.84)(0.83) \\ &= 0.4589 \end{aligned}$$

Question # 67**Answer: E**

$$12.50 = \bar{a}_x = \frac{1}{\mu + \delta} \Rightarrow \mu + \delta = 0.08 \Rightarrow \mu = \delta = 0.04$$

$$\bar{A}_x = \frac{\mu}{\mu + \delta} = 0.5$$

$${}^2\bar{A}_x = \frac{\mu}{\mu + 2\delta} = \frac{1}{3}$$

$$\begin{aligned} \text{Var}(\bar{a}_{T|}) &= \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2} \\ &= \frac{\frac{1}{3} - \frac{1}{4}}{0.0016} = 52.083 \end{aligned}$$

$$\text{S.D.} = \sqrt{52.083} = 7.217$$

Question # 68**Answer: D**

$$v = 0.90 \Rightarrow d = 0.10$$

$$A_x = 1 - d\ddot{a}_x = 1 - (0.10)(5) = 0.5$$

$$\begin{aligned} \text{Benefit premium } \pi &= \frac{5000A_x - 5000vq_x}{\ddot{a}_x} \\ &= \frac{(5000)(0.5) - 5000(0.90)(0.05)}{5} = 455 \end{aligned}$$

$$10\text{th benefit reserve for fully discrete whole life} = 1 - \frac{\ddot{a}_{x+10}}{\ddot{a}_x}$$

$$0.2 = 1 - \frac{\ddot{a}_{x+10}}{5} \Rightarrow \ddot{a}_{x+10} = 4$$

$$A_{x+10} = 1 - d\ddot{a}_{x+10} = 1 - (0.10)(4) = 0.6$$

$${}_{10}V = 5000A_{x+10} - \pi\ddot{a}_{x+10} = (5000)(0.6) - (455)(4) = 1180$$

Question #69**Answer: D**

v is the lowest premium to ensure a zero % chance of loss in year 1 (The present value of the payment upon death is v , so you must collect at least v to avoid a loss should death occur).

Thus $v = 0.95$.

$$\begin{aligned} E(Z) &= vq_x + v^2p_xq_{x+1} = 0.95 \times 0.25 + (0.95)^2 \times 0.75 \times 0.2 \\ &= 0.3729 \end{aligned}$$

$$\begin{aligned} E(Z^2) &= v^2q_x + v^4p_xq_{x+1} = (0.95)^2 \times 0.25 + (0.95)^4 \times 0.75 \times 0.2 \\ &= 0.3478 \end{aligned}$$

$$\text{Var}(Z) = E(Z^2) - (E(Z))^2 = 0.3478 - (0.3729)^2 = 0.21$$

Question #70**Answer: D**

Expected present value (EPV) of future benefits =

$$\begin{aligned}
&= (0.005 \times 2000 + 0.04 \times 1000) / 1.06 + (1 - 0.005 - 0.04)(0.008 \times 2000 + 0.06 \times 1000) / 1.06^2 \\
&= 47.17 + 64.60 \\
&= 111.77
\end{aligned}$$

$$\begin{aligned}
\text{EPV of future premiums} &= \left[1 + (1 - 0.005 - 0.04) / 1.06 \right] 50 \\
&= (1.9009)(50) \\
&= 95.05
\end{aligned}$$

$$E[{}_1L | K_{55} \geq 1] = 111.77 - 95.05 = 16.72$$

Question #71 - Removed**Question #72****Answer: A**Let Z be the present value random variable for one life.Let S be the present value random variable for the 100 lives.

$$\begin{aligned}
E(Z) &= 10 \int_5^{\infty} e^{-\delta t} e^{-\mu t} \mu dt \\
&= 10 \frac{\mu}{\delta + \mu} e^{-(\delta + \mu)5} \\
&= 2.426
\end{aligned}$$

$$\begin{aligned}
E(Z^2) &= 10^2 \left(\frac{\mu}{2\delta + \mu} \right) e^{-(2\delta + \mu)5} \\
&= 10^2 \left(\frac{0.04}{0.16} \right) (e^{-0.8}) = 11.233
\end{aligned}$$

$$\begin{aligned}
\text{Var}(Z) &= E(Z^2) - (E(Z))^2 \\
&= 11.233 - 2.426^2 \\
&= 5.348
\end{aligned}$$

$$E(S) = 100 E(Z) = 242.6$$

$$\text{Var}(S) = 100 \text{Var}(Z) = 534.8$$

$$\frac{F - 242.6}{\sqrt{534.8}} = 1.645 \rightarrow F = 281$$

Question #73**Answer: D**

Prob{only 1 survives} = 1 - Prob{both survive} - Prob{neither survives}

$$\begin{aligned}
 &= 1 - {}_3p_{50} \times {}_3p_{[50]} - (1 - {}_3p_{50})(1 - {}_3p_{[50]}) \\
 &= 1 - \underbrace{(0.9713)(0.9698)(0.9682)}_{=0.912012} \underbrace{(0.9849)(0.9819)(0.9682)}_{0.936320} - (1 - 0.912012)(1 - 0.93632) \\
 &= 0.140461
 \end{aligned}$$

Question # 74 - Removed**Question #75 - Removed****Question # 76****Answer: C**

This solution applies the equivalence principle to each life. Applying the equivalence principle to the 100 life group just multiplies both sides of the first equation by 100, producing the same result for P .

$$\begin{aligned}
 EPV(\text{Prens}) = P &= EPV(\text{Benefits}) = 10q_{70}v + 10p_{70}q_{71}v^2 + Pp_{70}p_{71}v^2 \\
 P &= \frac{(10)(0.03318)}{1.08} + \frac{(10)(1 - 0.03318)(0.03626)}{1.08^2} + \frac{P(1 - 0.03318)(1 - 0.03626)}{1.08^2} \\
 &= 0.3072 + 0.3006 + 0.7988P \\
 P &= \frac{0.6078}{0.2012} = 3.02
 \end{aligned}$$

(EPV above means Expected Present Value).

Question #77**Answer: E**

Level benefit premiums can be split into two pieces: one piece to provide term insurance for n years; one to fund the reserve for those who survive.

Then,

$$P_x = P_{x:\overline{n}|}^1 + P_{x:\overline{n}|}^{\text{1}} \cdot nV$$

And plug in to get

$$0.090 = P_{x:\overline{n}|}^1 + (0.00864)(0.563)$$

$$P_{x:\overline{n}|}^1 = 0.0851$$

Another approach is to think in terms of retrospective reserves. Here is one such solution:

$$\begin{aligned} nV &= \left(P_x - P_{x:\overline{n}|}^1 \right) \ddot{s}_{x:\overline{n}|} \\ &= \left(P_x - P_{x:\overline{n}|}^1 \right) \frac{\ddot{a}_{x:\overline{n}|}}{nE_x} \\ &= \left(P_x - P_{x:\overline{n}|}^1 \right) \frac{\ddot{a}_{x:\overline{n}|}}{P_{x:\overline{n}|}^1 \ddot{a}_{x:\overline{n}|}} \\ &= \frac{\left(P_x - P_{x:\overline{n}|}^1 \right)}{\left(P_{x:\overline{n}|}^1 \right)} \end{aligned}$$

$$0.563 = \left(0.090 - P_{x:\overline{n}|}^1 \right) / 0.00864$$

$$\begin{aligned} P_{x:\overline{n}|}^1 &= 0.090 - (0.00864)(0.563) \\ &= 0.0851 \end{aligned}$$

Question #78**Answer: A**

$$\delta = \ln(1.05) = 0.04879$$

$$\begin{aligned}\bar{A}_x &= \int_0^{\omega-x} {}_t p_x \mu_{x+t} e^{-\delta t} dt \\ &= \int_0^{\omega-x} \frac{1}{\omega-x} e^{-\delta t} dt \text{ for the given mortality function} \\ &= \frac{1}{\omega-x} \bar{a}_{\omega-x}|\end{aligned}$$

From here, many formulas for the reserve could be used. One approach is:

Since

$$\bar{A}_{50} = \frac{\bar{a}_{50}}{50} = \frac{18.71}{50} = 0.3742 \text{ so } \bar{a}_{50} = \left(\frac{1 - \bar{A}_{50}}{\delta} \right) = 12.83$$

$$\bar{A}_{40} = \frac{\bar{a}_{60}}{60} = \frac{19.40}{60} = 0.3233 \text{ so } \bar{a}_{40} = \left(\frac{1 - \bar{A}_{40}}{\delta} \right) = 13.87$$

$$\bar{P}(\bar{A}_{40}) = \frac{0.3233}{13.87} = 0.02331$$

$$\text{reserve} = [\bar{A}_{50} - \bar{P}(\bar{A}_{40})\bar{a}_{50}] = [0.3742 - (0.02331)(12.83)] = 0.0751.$$

Question #79**Answer: D**

$$\begin{aligned}\bar{A}_x &= E[v^{T_x}] = E[v^{T_x} | NS] \times \text{Prob}(NS) + E[v^{T_x} | S] \times \text{Prob}(S) \\ &= \left(\frac{0.03}{0.03 + 0.08} \right) \times 0.70 + \left(\frac{0.6}{0.06 + 0.08} \right) \times 0.30 \\ &= 0.3195\end{aligned}$$

$$\text{Similarly, } {}^2\bar{A}_x = \left(\frac{0.03}{0.03 + 0.16} \right) \times 0.70 + \left(\frac{0.06}{0.06 + 0.16} \right) \times 0.30 = 0.1923.$$

$$\text{Var} \left(\bar{a}_{T(x)} \right) = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2} = \frac{0.1923 - 0.3195^2}{0.08^2} = 14.1.$$

Question #80**Answer: B**

$$\begin{aligned}
{}_2|q_{\overline{80:84}} &= {}_2|q_{80} + {}_2|q_{84} - {}_2|q_{80:84} \\
&= 0.5 \times 0.4 \times (1 - 0.6) + 0.2 \times 0.15 \times (1 - 0.1) \\
&= 0.10136
\end{aligned}$$

Using new p_{82} value of 0.3

$$\begin{aligned}
&0.5 \times 0.4 \times (1 - 0.3) + 0.2 \times 0.15 \times (1 - 0.1) \\
&= 0.16118
\end{aligned}$$

$$\text{Change} = 0.16118 - 0.10136 = 0.06$$

Alternatively,

$$\begin{aligned}
{}_2P_{80} &= 0.5 \times 0.4 = 0.20 \\
{}_3P_{80} &= {}_2P_{80} \times 0.6 = 0.12 \\
{}_2P_{84} &= 0.20 \times 0.15 = 0.03 \\
{}_3P_{84} &= {}_2P_{84} \times 0.10 = 0.003 \\
{}_2P_{\overline{80:84}} &= {}_2P_{80} + {}_2P_{84} - {}_2P_{80} {}_2P_{84} \text{ since independent} \\
&= 0.20 + 0.03 - (0.20)(0.03) = 0.224 \\
{}_3P_{\overline{80:84}} &= {}_3P_{80} + {}_3P_{84} - {}_3P_{80} {}_3P_{84} \\
&= 0.12 + 0.003 - (0.12)(0.003) = 0.12264 \\
{}_2|q_{\overline{80:84}} &= {}_2P_{\overline{80:84}} - {}_3P_{\overline{80:84}} \\
&= 0.224 - 0.12264 = 0.10136
\end{aligned}$$

Revised

$$\begin{aligned}
{}_3P_{80} &= 0.20 \times 0.30 = 0.06 \\
{}_3P_{\overline{80:84}} &= 0.06 + 0.003 - (0.06)(0.003) \\
&= 0.06282 \\
{}_2|q_{\overline{80:84}} &= 0.224 - 0.06282 = 0.16118 \\
\text{change} &= 0.16118 - 0.10136 = 0.06
\end{aligned}$$

Question #81 - Removed

Question #82**Answer: A**

$$\begin{aligned}
{}_5p_{50}^{(\tau)} &= {}_5p_{50}^{(1)} {}_5p_{50}^{(2)} \\
&= \left(\frac{100-55}{100-50}\right) e^{-(0.05)(5)} \\
&= (0.9)(0.7788) = 0.7009
\end{aligned}$$

Similarly

$$\begin{aligned}
{}_{10}p_{50}^{(\tau)} &= \left(\frac{100-60}{100-50}\right) e^{-(0.05)(10)} \\
&= (0.8)(0.6065) = 0.4852
\end{aligned}$$

$$\begin{aligned}
{}_{5|5}q_{50}^{(\tau)} &= {}_5p_{50}^{(\tau)} - {}_{10}p_{50}^{(\tau)} = 0.7009 - 0.4852 \\
&= 0.2157
\end{aligned}$$

Question #83**Answer: C**Only decrement 1 operates before $t = 0.7$

$${}_{0.7}q_{40}^{(1)} = (0.7)q_{40}^{(1)} = (0.7)(0.10) = 0.07 \text{ since UDD}$$

Probability of reaching $t = 0.7$ is $1 - 0.07 = 0.93$ Decrement 2 operates only at $t = 0.7$, eliminating 0.125 of those who reached 0.7

$$q_{40}^{(2)} = (0.93)(0.125) = 0.11625$$

Question #84**Answer: C**

$$\pi(1+{}_2p_{80}v^2) = 1000A_{80} + \frac{\pi v q_{80}}{2} + \frac{\pi v^3 {}_2p_{80}q_{82}}{2}$$

$$\pi\left(1 + \frac{0.83910}{1.06^2}\right) = 665.75 + \pi\left(\frac{0.08030}{2(1.06)} + \frac{0.83910 \times 0.09561}{2(1.06)^3}\right)$$

$$\pi(1.74680) = 665.75 + \pi(0.07156)$$

$$\pi(1.67524) = 665.75$$

$$\pi = 397.41$$

$$\text{Where } {}_2p_{80} = \frac{3,284,542}{3,914,365} = 0.83910$$

$$\text{Or } {}_2p_{80} = (1 - 0.08030)(1 - 0.08764) = 0.83910$$

Question #85**Answer: E**

At issue, expected present value (EPV) of benefits

$$\begin{aligned} &= \int_0^{\infty} b_t v^t {}_t p_{65} \mu_{65+t} dt \\ &= \int_0^{\infty} 1000(e^{0.04t})(e^{-0.04t}) {}_t p_{65} \mu_{65}(t) dt \\ &= 1000 \int_0^{\infty} {}_t p_{65} \mu_{65}(t) dt = 1000 {}_{\infty}q_{65} = 1000 \end{aligned}$$

$$\text{EPV of premiums} = \pi \bar{a}_{65} = \pi \left(\frac{1}{0.04 + 0.02} \right) = 16.667\pi$$

$$\text{Benefit premium } \pi = 1000 / 16.667 = 60$$

$$\begin{aligned} {}_2\bar{V} &= \int_0^{\infty} b_{2+u} v^u {}_u p_{67} \mu_{65}(2+u) du - \pi \bar{a}_{67} \\ &= \int_0^{\infty} 1000 e^{0.04(2+u)} e^{-0.04u} {}_u p_{67} \mu_{65}(2+u) du - (60)(16.667) \\ &= 1000 e^{0.08} \int_0^{\infty} {}_u p_{67} \mu_{65}(2+u) du - 1000 \\ &= 1083.29 {}_{\infty}q_{67} - 1000 = 1083.29 - 1000 = 83.29 \end{aligned}$$

Question #86**Answer: B**

$$(1) \quad a_{x:\overline{20}|} = \ddot{a}_{x:\overline{20}|} - 1 + {}_{20}E_x$$

$$(2) \quad \ddot{a}_{x:\overline{20}|} = \frac{1 - A_{x:\overline{20}|}}{d}$$

$$(3) \quad A_{x:\overline{20}|} = A_{x:\overline{20}|}^1 + A_{x:\overline{20}|}^{\frac{1}{2}}$$

$$(4) \quad A_x = A_{x:\overline{20}|}^1 + {}_{20}E_x A_{x+20}$$

$$0.28 = A_{x:\overline{20}|}^1 + (0.25)(0.40)$$

$$A_{x:\overline{20}|}^1 = 0.18$$

Now plug into (3): $A_{x:\overline{20}|} = 0.18 + 0.25 = 0.43$

Now plug into (2): $\ddot{a}_{x:\overline{20}|} = \frac{1 - 0.43}{(0.05 / 1.05)} = 11.97$

Now plug into (1): $a_{x:\overline{20}|} = 11.97 - 1 + 0.25 = 11.22$

Question #87 - Removed**Question #88****Answer: B**

$$e_x = p_x + p_x e_{x+1} \Rightarrow p_x = \frac{e_x}{1 + e_{x+1}} = \frac{8.83}{9.29} = 0.95048$$

$$\ddot{a}_x = 1 + v p_x + v^2 {}_2p_x + \dots$$

$$\ddot{a}_{x:\overline{2}|} = 1 + v + v^2 {}_2p_x + \dots$$

$$\ddot{a}_{x:\overline{2}|} - \ddot{a}_x = v q_x = 5.6459 - 5.60 = 0.0459$$

$$v(1 - 0.95048) = 0.0459$$

$$v = 0.9269$$

$$i = \frac{1}{v} - 1 = 0.0789$$

Question #89**Answer: E**

One approach is to enumerate the possible paths ending in F and add the probabilities:

$$FFFF - 0.2^3 = 0.008$$

$$FFGF - 0.2(0.8)(0.5) = 0.080$$

$$FGFF - 0.8(0.5)(0.2) = 0.080$$

$$FGHF - 0.8(0.5)(0.75) = 0.300$$

The total is 0.468.

An alternative is to use matrix multiplication. The desired probability is the value in the upper left corner of the transition probability matrix raised to the third power. Only the first row needs to be evaluated.

The first row of the matrix squared is [0.44 0.16 0.40 0.00], obtained by multiplying the first row by each column, in turn. The first row of the matrix cubed is obtained by multiplying the first row of the squared matrix by each column. The result is [0.468 0.352 0.080 0.100]. Note that only the first of the four calculations is necessary, though doing the other three and observing that the sum is 1 provides a check.

Either way, the required probability is 0.468 and the actuarial present value is $500v^3(0.468) = 500(0.9)^3(0.468) = 171$.

Question #90 – Removed

Question #91**Answer: E**

$$\mu_{60}^M = \frac{1}{75-60} = \frac{1}{15}$$

$$\mu_{60}^F = \frac{1}{\omega-60} = \frac{1}{15} \times \frac{3}{5} = \frac{1}{25} \Rightarrow \omega = 85$$

$${}_tP_{65}^M = 1 - \frac{t}{10}$$

$${}_tP_{60}^F = 1 - \frac{t}{25}$$

Let x denote the male and y denote the female.

$$\overset{\circ}{e}_x = 5 \quad (\text{mean for uniform distribution over } (0,10))$$

$$\overset{\circ}{e}_y = 12.5 \quad (\text{mean for uniform distribution over } (0,25))$$

$$\begin{aligned} \overset{\circ}{e}_{xy} &= \int_0^{10} \left(1 - \frac{t}{10}\right) \left(1 - \frac{t}{25}\right) dt \\ &= \int_0^{10} \left(1 - \frac{7}{50}t + \frac{t^2}{250}\right) dt \\ &= \left(t - \frac{7}{100}t^2 + \frac{t^3}{750}\right) \Big|_0^{10} = 10 - \frac{7}{100} \times 100 + \frac{1000}{750} \\ &= 10 - 7 + \frac{4}{3} = \frac{13}{3} \end{aligned}$$

$$\overset{\circ}{e}_{xy} = \overset{\circ}{e}_x + \overset{\circ}{e}_y - \overset{\circ}{e}_{xy} = 5 + \frac{25}{2} - \frac{13}{3} = \frac{30 + 75 - 26}{6} = 13.17$$

Question #92**Answer: B**

$$\bar{A}_x = \frac{\mu}{\mu + \delta} = \frac{1}{3}$$

$${}^2\bar{A}_x = \frac{\mu}{\mu + 2\delta} = \frac{1}{5}$$

$$\bar{P}(\bar{A}_x) = \mu = 0.04$$

$$\begin{aligned} \text{Var}(L) &= \left(1 + \frac{\bar{P}(\bar{A}_x)}{\delta}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right) \\ &= \left(1 + \frac{0.04}{0.08}\right)^2 \left(\frac{1}{5} - \left(\frac{1}{3}\right)^2\right) \\ &= \left(\frac{3}{2}\right)^2 \left(\frac{4}{45}\right) \\ &= \frac{1}{5} \end{aligned}$$

Question #93**Answer: A**Let π be the benefit premiumLet ${}_kV$ denote the benefit reserve at the end of year k .

$$\text{For any } n, ({}_nV + \pi)(1+i) = (q_{25+n} \times {}_{n+1}V + p_{25+n} \times {}_{n+1}V) = {}_{n+1}V$$

$$\text{Thus } {}_1V = ({}_0V + \pi)(1+i)$$

$${}_2V = ({}_1V + \pi)(1+i) = (\pi(1+i) + \pi)(1+i) = \pi \ddot{s}_{\overline{2}|}$$

$${}_3V = ({}_2V + \pi)(1+i) = (\pi \ddot{s}_{\overline{2}|} + \pi)(1+i) = \pi \ddot{s}_{\overline{3}|}$$

By induction (proof omitted)

$${}_nV = \pi \ddot{s}_{\overline{n}|}$$

For $n = 35$, ${}_{35}V = \ddot{a}_{60}$ (expected present value of future benefits; there are no future premiums)

$$\ddot{a}_{60} = \pi \ddot{s}_{\overline{35}|}$$

$$\pi = \frac{\ddot{a}_{60}}{\ddot{s}_{\overline{35}|}} \quad \text{For } n = 20, \quad {}_{20}V = \pi \ddot{s}_{\overline{20}|} = \left(\frac{\ddot{a}_{60}}{\ddot{s}_{\overline{35}|}}\right) \ddot{s}_{\overline{20}|}$$

Alternatively, as above

$$({}_nV + \pi)(1+i) = {}_{n+1}V$$

Write those equations, for $n=0$ to $n=34$

$$0: ({}_0V + \pi)(1+i) = {}_1V$$

$$1: ({}_1V + \pi)(1+i) = {}_2V$$

$$2: ({}_2V + \pi)(1+i) = {}_3V$$

⋮

$$34: ({}_{34}V + \pi)(1+i) = {}_{35}V$$

Multiply equation k by $(1+i)^{34-k}$ and sum the results:

$$\begin{aligned} &({}_0V + \pi)(1+i)^{35} + ({}_1V + \pi)(1+i)^{34} + ({}_2V + \pi)(1+i)^{33} + \cdots + ({}_{34}V + \pi)(1+i) = \\ &{}_1V(1+i)^{34} + {}_2V(1+i)^{33} + {}_3V(1+i)^{32} + \cdots + {}_{34}V(1+i) + {}_{35}V \end{aligned}$$

For $k=1, 2, \dots, 34$, the ${}_kV(1+i)^{35-k}$ terms in both sides cancel, leaving

$${}_0V(1+i)^{35} + \pi \left[(1+i)^{35} + (1+i)^{34} + \cdots + (1+i) \right] = {}_{35}V$$

Since ${}_0V = 0$

$$\pi \ddot{s}_{\overline{35}|} = {}_{35}V$$

$$= \ddot{a}_{60}$$

(see above for remainder of solution)

Question #94**Answer: B**

$$\mu_{\overline{x+t:y+t}} = \frac{{}_t q_y {}_t p_x \mu_{x+t} + {}_t q_x {}_t p_y \mu_{y+t}}{{}_t q_x \times {}_t p_y + {}_t p_x \times {}_t q_y + {}_t p_x \times {}_t p_y}$$

For $(x) = (y) = (50)$

$$\mu_{\overline{50:50}}(10.5) = \frac{({}_{10.5} q_{50})({}_{10} p_{50})q_{60} \cdot 2}{({}_{10.5} q_{50})({}_{10.5} p_{50}) \cdot 2 + ({}_{10.5} p_{50})^2} = \frac{(0.09152)(0.91478)(0.01376)(2)}{(0.09152)(0.90848)(2) + (0.90848)^2} = 0.0023$$

where

$${}_{10.5} p_{50} = \frac{\frac{1}{2}(l_{60} + l_{61})}{l_{50}} = \frac{\frac{1}{2}(8,188,074 + 8,075,403)}{8,950,901} = 0.90848$$

$${}_{10.5} q_{50} = 1 - {}_{10.5} p_{50} = 0.09152$$

$${}_{10} p_{50} = \frac{8,188,074}{8,950,901} = 0.91478$$

$${}_{10.5} p_{50} \mu(50 + 10.5) = ({}_{10} p_{50})q_{60} \quad \text{since UDD}$$

$$\text{Alternatively, } ({}_{10+t}) p_{50} = {}_{10} p_{50} {}_t p_{60}$$

$$({}_{10+t}) p_{50:50} = ({}_{10} p_{50})^2 ({}_t p_{60})^2$$

$$\begin{aligned} ({}_{10+t}) p_{\overline{50:50}} &= 2 {}_{10} p_{50} {}_t p_{60} - ({}_{10} p_{50})^2 ({}_t p_{60})^2 \\ &= 2 {}_{10} p_{50} (1 - tq_{60}) - ({}_{10} p_{50})^2 (1 - tq_{60})^2 \quad \text{since UDD} \end{aligned}$$

$$\text{Derivative} = -2 {}_{10} p_{50} q_{60} + 2 ({}_{10} p_{50})^2 (1 - tq_{60}) q_{60}$$

Derivative at $10 + t = 10.5$ is

$$-2(0.91478)(0.01376) + (0.91478)^2 (1 - (0.5)(0.01376))(0.01376) = -0.0023$$

$$\begin{aligned} {}_{10.5} p_{\overline{50:50}} &= 2 {}_{10.5} p_{50} - ({}_{10.5} p_{50})^2 \\ &= 2(0.90848) - (0.90848)^2 \\ &= 0.99162 \end{aligned}$$

$$\mu \text{ (for any sort of lifetime)} = \frac{-\frac{dp}{dt}}{p} = \frac{-(-0.0023)}{0.99162} = 0.0023$$

Question #95**Answer: D**

$$\mu_{x+t}^{(\tau)} = \mu_{x+t}^{(1)} + \mu_{x+t}^{(2)} = 0.01 + 2.29 = 2.30$$

$$P = P \int_0^2 v^t {}_t p_x^{(\tau)} \mu_{x+t}^{(2)} dt + 50,000 \int_0^2 v^t {}_t p_x^{(\tau)} \mu_{x+t}^{(1)} dt + 50,000 \int_2^\infty v^t {}_t p_x^{(\tau)} \mu_{x+t}^{(\tau)} dt$$

$$P = P \int_0^2 e^{-0.1t} e^{-2.3t} \times 2.29 dt + 50,000 \int_0^2 e^{-0.1t} e^{-2.3t} \times 0.01 dt + 50,000 \int_2^\infty e^{-0.1t} e^{-2.3t} \times 2.3 dt$$

$$P \left[1 - 2.29 \times \frac{1 - e^{-2(2.4)}}{2.4} \right] = 50000 \left[0.01 \times \frac{1 - e^{-2(2.4)}}{2.4} + 2.3 \times \frac{e^{-2(2.4)}}{2.4} \right]$$

$$P = 11,194$$

Question #96**Answer: B**

$$e_x = p_x + {}_2p_x + {}_3p_x + \dots = 11.05$$

$$\text{Annuity} = v^3 {}_3p_x 1000 + v^4 {}_4p_x \times 1000 \times (1.04) + \dots$$

$$= \sum_{k=3}^{\infty} 1000 (1.04)^{k-3} v^k {}_k p_x$$

$$= 1000 v^3 \sum_{k=3}^{\infty} {}_k p_x$$

$$= 1000 v^3 (e_x - 0.99 - 0.98) = 1000 \left(\frac{1}{1.04} \right)^3 \times 9.08 = 8072$$

Let π = benefit premium.

$$\pi(1 + 0.99v + 0.98v^2) = 8072$$

$$2.8580\pi = 8072$$

$$\pi = 2824$$

Question #97**Answer: B**

$$\pi \ddot{a}_{30:\overline{10}|} = 1000A_{30} + P(IA)_{30:\overline{10}|} + (10\pi)({}_{10|}A_{30})$$

$$\begin{aligned} \pi &= \frac{1000A_{30}}{\ddot{a}_{30:\overline{10}|} - (IA)_{30:\overline{10}|} - 10{}_{10|}A_{30}} \\ &= \frac{1000(0.102)}{7.747 - 0.078 - 10(0.088)} \\ &= \frac{102}{6.789} \\ &= 15.024 \end{aligned}$$

Question #98**Answer: E**

For the general survival function $S_0(t) = 1 - \frac{t}{\omega}$, $0 \leq t \leq \omega$,

$$\begin{aligned} \dot{e}_{30} &= \int_0^{\omega-30} \left(1 - \frac{t}{\omega-30}\right) dt \\ &= \left[t - \frac{t^2}{2(\omega-30)} \right]_0^{\omega-30} \\ &= \frac{\omega-30}{2} \end{aligned}$$

Prior to medical breakthrough $\omega = 100 \Rightarrow \dot{e}_{30} = \frac{100-30}{2} = 35$

After medical breakthrough $\dot{e}'_{30} = \dot{e}_{30} + 4 = 39$

$$\text{so } \dot{e}'_{30} = 39 = \frac{\omega' - 30}{2} \Rightarrow \omega' = 108$$

Question #99**Answer: A**

$$\begin{aligned} L &= 100,000v^{2.5} - 4000\ddot{a}_{\overline{3}|} \quad @5\% \\ &= 77,079 \end{aligned}$$

Question #100**Answer: D**

$$\mu^{(accid)} = 0.001$$

$$\mu^{(total)} = 0.01$$

$$\mu^{(other)} = 0.01 - 0.001 = 0.009$$

$$\begin{aligned} \text{Expected present value} &= \int_0^{\infty} 500,000 e^{-0.05t} e^{-0.01t} (0.009) dt \\ &\quad + 10 \int_0^{\infty} 50,000 e^{0.04t} e^{-0.05t} e^{-0.01t} (0.001) dt \\ &= 500,000 \left[\frac{0.009}{0.06} + \frac{0.001}{0.02} \right] = 100,000 \end{aligned}$$

Question #101 Removed**Question #102****Answer: D**

$$\begin{aligned} 1000 {}_{20}V &= 1000 A_{x+20} = \frac{1000({}_{19}V + {}_{20}P_x)(1.06) - q_{x+19}(1000)}{P_{x+19}} \\ &= \frac{(342.03 + 13.72)(1.06) - 0.01254(1000)}{0.98746} = 369.18 \\ \ddot{a}_{x+20} &= \frac{1 - 0.36918}{(0.06/1.06)} = 11.1445 \\ \text{so } 1000 P_{x+20} &= 1000 \frac{A_{x+20}}{\ddot{a}_{x+20}} = \frac{369.18}{11.1445} = 33.1 \end{aligned}$$

Question #103**Answer: B**

$$\begin{aligned}
{}_k P_x^{(\tau)} &= e^{-\int_0^k \mu_{x+t}^{(\tau)} dt} = e^{-\int_0^k 2\mu_{x+t}^{(1)} dt} \\
&= \left(e^{-\int_0^k \mu_{x+t}^{(1)} dt} \right)^2 \\
&= ({}_k P_x)^2 \text{ where } {}_k P_x \text{ is from Illustrative Life Table, since } \mu^{(1)} \text{ follows I.L.T.} \\
{}_{10} P_{60} &= \frac{6,616,155}{8,188,074} = 0.80802 \\
{}_{11} P_{60} &= \frac{6,396,609}{8,188,074} = 0.78121 \\
{}_{10|} q_{60}^{(\tau)} &= {}_{10} P_{60}^{(\tau)} - {}_{11} P_{60}^{(\tau)} \\
&= ({}_{10} P_{60})^2 - ({}_{11} P_{60})^2 \text{ from I.L.T.} \\
&= 0.80802^2 - 0.78121^2 = 0.0426
\end{aligned}$$

Question #104**Answer: C**

$P_s = \frac{1}{\ddot{a}_s} - d$, where s can stand for any of the statuses under consideration.

$$\ddot{a}_s = \frac{1}{P_s + d}$$

$$\ddot{a}_x = \ddot{a}_y = \frac{1}{0.1 + 0.06} = 6.25$$

$$\ddot{a}_{xy} = \frac{1}{0.06 + 0.06} = 8.333$$

$$\ddot{a}_{xy} + \ddot{a}_{xy} = \ddot{a}_x + \ddot{a}_y$$

$$\ddot{a}_{xy} = 6.25 + 6.25 - 8.333 = 4.167$$

$$P_{xy} = \frac{1}{4.167} - 0.06 = 0.18$$

Question #105**Answer: A**

$$d_0^{(\tau)} = 1000 \int_0^1 e^{-(\mu+0.04)t} (\mu + 0.04) dt$$

$$= 1000(1 - e^{-(\mu+0.04)}) = 48$$

$$e^{-(\mu+0.04)} = 0.952$$

$$\mu + 0.04 = -\ln(0.952)$$

$$= 0.049$$

$$\mu = 0.009$$

$$d_3^{(1)} = 1000 \int_3^4 e^{-0.049t} (0.009) dt$$

$$= 1000 \frac{0.009}{0.049} (e^{-(0.049)(3)} - e^{-(0.049)(4)}) = 7.6$$

Question #106**Answer: B**

This is a graph of $l_x \mu_x$.

μ_x would be increasing in the interval (80,100).

The graphs of $l_x p_x$, l_x and l_x^2 would be decreasing everywhere.

Question #107**Answer: B**

$$\text{Variance} = v^{30} {}_{15}p_x {}_{15}q_x \qquad \text{Expected value} = v^{15} {}_{15}p_x$$

$$v^{30} {}_{15}p_x {}_{15}q_x = 0.065 \quad v^{15} {}_{15}p_x$$

$$v^{15} {}_{15}q_x = 0.065 \Rightarrow {}_{15}q_x = 0.3157$$

Since μ is constant

$${}_{15}q_x = (1 - (p_x)^{15})$$

$$(p_x)^{15} = 0.6843$$

$$p_x = 0.975$$

$$q_x = 0.025$$

Question #108**Answer: E**

$$(1) \quad {}_{11}V^A = \left({}_{10}V^A + 0 \right) \frac{(1+i)}{P_{x+10}} - \frac{q_{x+10}}{P_{x+10}} \times 1000$$

$$(2) \quad {}_{11}V^B = \left({}_{10}V^B + \pi^B \right) \frac{(1+i)}{P_{x+10}} - \frac{q_{x+10}}{P_{x+10}} \times 1000$$

$$\begin{aligned} (1)-(2) \quad {}_{11}V^A - {}_{11}V^B &= \left({}_{10}V^A - {}_{10}V^B - \pi^B \right) \frac{(1+i)}{P_{x+10}} \\ &= (101.35 - 8.36) \frac{(1.06)}{1 - 0.004} \\ &= 98.97 \end{aligned}$$

Question #109**Answer: A**

$$\begin{aligned} EPV(x\text{'s benefits}) &= \sum_{k=0}^2 v^{k+1} b_{k+1} {}_k p_x q_{x+k} \\ &= 1000 \left[300v(0.02) + 350v^2(0.98)(0.04) + 400v^3(0.98)(0.96)(0.06) \right] \\ &= 36,829 \end{aligned}$$

Question #110**Answer: E** π denotes benefit premium ${}_{19}V = EPV \text{ future benefits} - EPV \text{ future premiums}$

$$0.6 = \frac{1}{1.08} - \pi \Rightarrow \pi = 0.326$$

$$\begin{aligned} {}_{11}V &= \frac{({}_{10}V + \pi)(1.08) - (q_{65})(10)}{P_{65}} \\ &= \frac{(5.0 + 0.326)(1.08) - (0.10)(10)}{1 - 0.10} \\ &= 5.28 \end{aligned}$$

Question #111**Answer: A**

$$\begin{aligned} \text{Expected present value Benefits} &= \frac{(0.8)(0.1)(10,000)}{1.06^2} + \frac{(0.8)(0.9)(0.097)(9,000)}{1.06^3} \\ &= 1,239.75 \end{aligned}$$

$$\begin{aligned} 1,239.75 &= P \left(1 + \frac{(0.8)}{1.06} + \frac{(0.8)(0.9)}{1.06^2} \right) \\ &= P(2.3955) \\ P &= 517.53 \Rightarrow 518 \end{aligned}$$

Question #112**Answer: A**

$$\begin{aligned} 1180 &= 70\bar{a}_{30} + 50\bar{a}_{40} - 20\bar{a}_{30:40} \\ 1180 &= (70)(12) + (50)(10) - 20\bar{a}_{30:40} \\ \bar{a}_{30:40} &= 8 \\ \bar{a}_{30:40} &= \bar{a}_{30} + \bar{a}_{40} - \bar{a}_{30:40} = 12 + 10 - 8 = 14 \\ 100\bar{a}_{30:40} &= 1400 \end{aligned}$$

Question #113**Answer: B**

$$\begin{aligned} \bar{a} &= \int_0^{\infty} \bar{a}_{\overline{t}|} f(t) dt = \int_0^{\infty} \frac{1 - e^{-0.05t}}{0.05} \frac{1}{\Gamma(2)} te^{-t} dt \\ &= \frac{1}{0.05} \int_0^{\infty} (te^{-t} - te^{-1.05t}) dt \\ &= \frac{1}{0.05} \left[-(t+1)e^{-t} + \left(\frac{t}{1.05} + \frac{1}{1.05^2} \right) e^{-1.05t} \right] \Bigg|_0^{\infty} \\ &= \frac{1}{0.05} \left[1 - \left(\frac{1}{1.05} \right)^2 \right] = 1.85941 \end{aligned}$$

$$20,000 \times 1.85941 = 37,188$$

Question #114**Answer: C**

<u>Event</u>	<u>Prob</u>	<u>Present Value</u>
$x = 0$	(0.05)	15
$x = 1$	$(0.95)(0.10) = 0.095$	$15 + 20/1.06 = 33.87$
$x \geq 2$	$(0.95)(0.90) = 0.855$	$15 + 20/1.06 + 25/1.06^2 = 56.12$

$$E[X] = (0.05)(15) + (0.095)(33.87) + (0.855)(56.12) = 51.95$$

$$E[X^2] = (0.05)(15)^2 + (0.095)(33.87)^2 + (0.855)(56.12)^2 = 2813.01$$

$$\text{Var}[X] = E(X^2) - E(X)^2 = 2813.01 - (51.95)^2 = 114.2$$

Question #115**Answer: B**

Let K be the curtate future lifetime of $(x + k)$

$${}_k L = 1000v^{K+1} - 1000P_{x:\overline{3}|} \times \ddot{a}_{\overline{K+1}|}$$

When (as given in the problem), (x) dies in the second year from issue, the curtate future lifetime of $(x+1)$ is 0, so

$${}_1 L = 1000v - 1000P_{x:\overline{3}|} \ddot{a}_{\overline{1}|}$$

$$= \frac{1000}{1.1} - 279.21$$

$$= 629.88 \approx 630$$

The premium came from

$$P_{x:\overline{3}|} = \frac{A_{x:\overline{3}|}}{\ddot{a}_{x:\overline{3}|}}$$

$$A_{x:\overline{3}|} = 1 - d \ddot{a}_{x:\overline{3}|}$$

$$P_{x:\overline{3}|} = 279.21 = \frac{1 - d \ddot{a}_{x:\overline{3}|}}{\ddot{a}_{x:\overline{3}|}} = \frac{1}{\ddot{a}_{x:\overline{3}|}} - d$$

Question #116**Answer: D**

Let M = the force of mortality of an individual drawn at random; and T = future lifetime of the individual.

$$\begin{aligned}
 \Pr[T \leq 1] &= E\{\Pr[T \leq 1 | M]\} \\
 &= \int_0^{\infty} \Pr[T \leq 1 | M = \mu] f_M(\mu) d\mu \\
 &= \int_0^2 \int_0^1 \mu e^{-\mu t} dt \frac{1}{2} d\mu \\
 &= \int_0^2 (1 - e^{-\mu}) \frac{1}{2} d\mu = \frac{1}{2} (2 + e^{-2} - 1) = \frac{1}{2} (1 + e^{-2}) \\
 &= 0.56767
 \end{aligned}$$

Question #117**Answer: E**

For this model:

$$\mu_{40+t}^{(1)} = \frac{1/60}{1-t/60} = \frac{1}{60-t}; \mu_{40+20}^{(1)} = 1/40 = 0.025$$

$$\mu_{40+t}^{(2)} = \frac{1/40}{1-t/40} = \frac{1}{40-t}; \mu_{40+20}^{(2)} = 1/20 = 0.05$$

$$\mu_{40+20}^{(\tau)} = 0.025 + 0.05 = 0.075$$

Question #118**Answer: D**Let π = benefit premium

Expected present value of benefits =

$$\begin{aligned}
&= (0.03)(200,000)v + (0.97)(0.06)(150,000)v^2 + (0.97)(0.94)(0.09)(100,000)v^3 \\
&= 5660.38 + 7769.67 + 6890.08 \\
&= 20,320.13
\end{aligned}$$

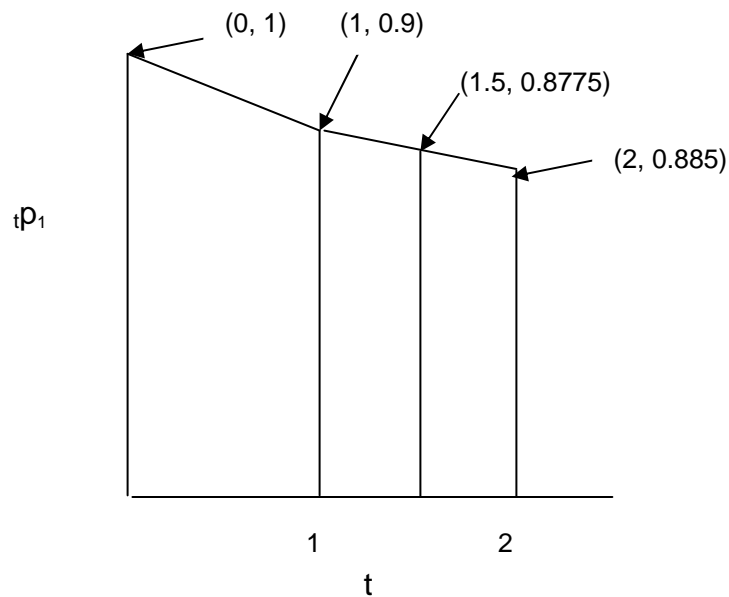
Expected present value of benefit premiums

$$\begin{aligned}
&= \ddot{a}_{x:\overline{3}|} \pi \\
&= [1 + 0.97v + (0.97)(0.94)v^2] \pi \\
&= 2.7266 \pi \\
\pi &= \frac{20,320.13}{2.7266} = 7452.55 \\
{}_1V &= \frac{(7452.55)(1.06) - (200,000)(0.03)}{1 - 0.03} \\
&= 1958.46
\end{aligned}$$

$$\begin{aligned}
\text{Initial reserve, year 2} &= {}_1V + \pi \\
&= 1958.56 + 7452.55 \\
&= 9411.01
\end{aligned}$$

Question #119**Answer: A**Let π denote the premium.

$$\begin{aligned}
L &= b_T v^T - \pi \bar{a}_{\overline{T}|} = (1+i)^T \times v^T - \pi \bar{a}_{\overline{T}|} \\
&= 1 - \pi \bar{a}_{\overline{T}|} \\
E[L] &= 1 - \pi \bar{a}_x = 0 \quad \Rightarrow \pi = \frac{1}{\bar{a}_x} \\
\Rightarrow L &= 1 - \pi \bar{a}_{\overline{T}|} = 1 - \frac{\bar{a}_{\overline{T}|}}{\bar{a}_x} = \frac{\delta \bar{a}_x - (1 - v^T)}{\delta \bar{a}_x} \\
&= \frac{v^T - (1 - \delta \bar{a}_x)}{\delta \bar{a}_x} = \frac{v^T - \bar{A}_x}{1 - \bar{A}_x}
\end{aligned}$$

Question #120**Answer: D**

$${}_1p_1 = (1 - 0.1) = 0.9$$

$${}_2p_1 = (0.9)(1 - 0.05) = 0.855$$

$$\text{since uniform, } {}_{1.5}p_1 = (0.9 + 0.855) / 2 \\ = 0.8775$$

$$\begin{aligned} \overset{\circ}{e}_{1:\overline{1.5}|} &= \text{Area between } t = 0 \text{ and } t = 1.5 \\ &= \left(\frac{1+0.9}{2}\right)(1) + \left(\frac{0.9+0.8775}{2}\right)(0.5) \\ &= 0.95 + 0.444 \\ &= 1.394 \end{aligned}$$

Alternatively,

$$\begin{aligned}
 \ddot{e}_{1:\overline{1.5}|} &= \int_0^{1.5} {}_t p_1 dt \\
 &= \int_0^1 {}_t p_1 dt + {}_1 p_1 \int_0^{0.5} {}_x p_2 dx \\
 &= \int_0^1 (1 - 0.1t) dt + 0.9 \int_0^{0.5} (1 - 0.05x) dx \\
 &= \left[t - \frac{0.1t^2}{2} \right]_0^1 + 0.9 \left[x - \frac{0.05x^2}{2} \right]_0^{0.5} \\
 &= 0.95 + 0.444 = 1.394
 \end{aligned}$$

Question #121

Answer: A

$$10,000A_{63}(1.12) = 5233$$

$$A_{63} = 0.4672$$

$$A_{x+1} = \frac{A_x(1+i) - q_x}{p_x}$$

$$\begin{aligned}
 A_{64} &= \frac{(0.4672)(1.05) - 0.01788}{1 - 0.01788} \\
 &= 0.4813
 \end{aligned}$$

$$\begin{aligned}
 A_{65} &= \frac{(0.4813)(1.05) - 0.01952}{1 - 0.01952} \\
 &= 0.4955
 \end{aligned}$$

$$\begin{aligned}
 \text{Single gross premium at 65} &= (1.12)(10,000)(0.4955) \\
 &= 5550
 \end{aligned}$$

$$(1+i)^2 = \frac{5550}{5233} \quad i = \sqrt{\frac{5550}{5233}} - 1 = 0.02984$$

Question #122A

Answer: C

Because your original survival function for (x) was correct, you must have

$$\mu_{x+t} = 0.06 = \mu_{x+t:y+t}^{02} + \mu_{x+t:y+t}^{03} = \mu_{x+t:y+t}^{02} + 0.02$$

$$\mu_{x+t:y+t}^{02} = 0.04$$

Similarly, for (y)

$$\mu_{y+t} = 0.06 = \mu_{x+t:y+t}^{01} + \mu_{x+t:y+t}^{03} = \mu_{x+t:y+t}^{01} + 0.02$$

$$\mu_{x+t:y+t}^{01} = 0.04$$

The first-to-die insurance pays as soon as State 0 is left, regardless of which state is next. The force of transition from State 0 is

$$\mu_{x+t:y+t}^{01} + \mu_{x+t:y+t}^{02} + \mu_{x+t:y+t}^{03} = 0.04 + 0.04 + 0.02 = 0.10.$$

With a constant force of transition, the expected present value is

$$\int_0^{\infty} e^{-\delta t} {}_tP_{xy}^{00} (\mu_{x+t:y+t}^{01} + \mu_{x+t:y+t}^{02} + \mu_{x+t:y+t}^{03}) dt = \int_0^{\infty} e^{-0.05t} e^{-0.10t} (0.10) dt = \frac{0.10}{0.15}$$

Question #122B

Answer: E

Because (x) is to have a constant force of 0.06 regardless of (y)'s status (and vice-versa) it must be that $\mu_{x+t:y+t}^{13} = \mu_{x+t:y+t}^{23} = 0.06$.

There are three mutually exclusive ways in which both will be dead by the end of year 3:

1: Transition from State 0 directly to State 3 within 3 years. The probability of this is

$$\int_0^3 {}_tP_{xy}^{00} \mu_{x+t:y+t}^{03} dt = \int_0^3 e^{-0.10t} 0.02 dt = -\frac{0.02}{0.10} e^{-0.10t} \Big|_0^3 = 0.2(1 - e^{-0.3}) = 0.0518$$

2: Transition from State 0 to State 1 and then to State 3 all within 3 years. The probability of this is

$$\begin{aligned} \int_0^3 {}_tP_{xy}^{00} \mu_{x+t:y+t}^{01} {}_{3-t}P_{x+t:y+t}^{13} dt &= \int_0^3 e^{-0.10t} 0.04(1 - e^{-0.06(3-t)}) dt \\ &= \int_0^3 0.04 [e^{-0.10t} - e^{-0.18} e^{-0.04t}] dt = -\frac{0.04}{0.10} e^{-0.10t} + \frac{0.04 e^{-0.18}}{0.04} e^{-0.04t} \Big|_0^3 \\ &= 0.4(1 - e^{-0.3}) - e^{-0.18}(1 - e^{-0.12}) = 0.00922 \end{aligned}$$

3: Transition from State 0 to State 2 and then to State 3 all within 3 years. By symmetry, this probability is 0.00922.

The answer is then $0.0518 + 2(0.00922) = 0.0702$.

Question #122C**Answer: D**

Because the original survival function continues to hold for the individual lives, with a constant force of mortality of 0.06 and a constant force of interest of 0.05, the expected present values of the individual insurances are

$$\bar{A}_x = \bar{A}_y = \frac{0.06}{0.06 + 0.05} = 0.54545,$$

Then,

$$\bar{A}_{xy} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.54545 + 0.54545 - 0.66667 = 0.42423$$

Alternatively, the answer can be obtained by using the three mutually exclusive outcomes used in the solution to Question 122B.

$$1: \int_0^{\infty} e^{-0.05t} {}_tP_{xy}^{00} \mu_{x+t;y+t}^{03} dt = \int_0^{\infty} e^{-0.05t} e^{-0.10t} 0.02 dt = \frac{0.02}{0.15} = 0.13333$$

$$2 \text{ and } 3: \int_0^{\infty} e^{-0.05t} {}_tP_{xy}^{00} \mu_{x+t;y+t}^{01} \int_0^{\infty} e^{-0.05r} {}_rP_{x+t;y+t}^{11} \mu_{x+t+r;y+t+r}^{13} dr dt \\ = \int_0^{\infty} e^{-0.05t} e^{-0.10t} 0.04 \int_0^{\infty} e^{-0.05r} e^{-0.06r} 0.06 dr dt = \frac{0.04}{0.15} \frac{0.06}{0.11} = 0.14545$$

The solution is $0.13333 + 2(0.14545) = 0.42423$.

The fact that the double integral factors into two components is due to the memoryless property of the exponential transition distributions.

Question #123**Answer: B**

$$\begin{aligned}
{}_5|q_{\overline{35:45}} &= {}_5|q_{35} + {}_5|q_{45} - {}_5|q_{35:45} \\
&= {}_5p_{35}q_{40} + {}_5p_{45}q_{50} - {}_5p_{35:45}q_{40:50} \\
&= {}_5p_{35}q_{40} + {}_5p_{45}q_{50} - {}_5p_{35} \times {}_5p_{45} (1 - p_{40:50}) \\
&= {}_5p_{35}q_{40} + {}_5p_{45}q_{50} - {}_5p_{35} \times {}_5p_{45} (1 - p_{40}p_{50}) \\
&= (0.9)(0.03) + (0.8)(0.05) - (0.9)(0.8)[1 - (0.97)(0.95)] \\
&= 0.01048
\end{aligned}$$

Alternatively,

$${}_6p_{35} = {}_5p_{35} \times p_{40} = (0.90)(1 - 0.03) = 0.873$$

$${}_6p_{45} = {}_5p_{45} \times p_{50} = (0.80)(1 - 0.05) = 0.76$$

$$\begin{aligned}
{}_5|q_{\overline{35:45}} &= {}_5p_{\overline{35:45}} - {}_6p_{\overline{35:45}} \\
&= ({}_5p_{35} + {}_5p_{45} - {}_5p_{35:45}) - ({}_6p_{35} + {}_6p_{45} - {}_6p_{35:45}) \\
&= ({}_5p_{35} + {}_5p_{45} + {}_5p_{35} \times {}_5p_{45}) - ({}_6p_{35} + {}_6p_{45} - {}_6p_{35} \times {}_6p_{45}) \\
&= (0.90 + 0.80 - 0.90 \times 0.80) - (0.873 + 0.76 - 0.873 \times 0.76) \\
&= 0.98 - 0.96952 \\
&= 0.01048
\end{aligned}$$

Question #124 – Removed**Question #125 - Removed**

Question #126**Answer: E**Let Y = present value random variable for payments on one life $S = \sum Y$ = present value random variable for all payments

$$E[Y] = 10\ddot{a}_{40} = 148.166$$

$$\begin{aligned} \text{Var}[Y] &= 10^2 \frac{({}^2A_{40} - A_{40}^2)}{d^2} \\ &= 100(0.04863 - 0.16132^2)(1.06/0.06)^2 \\ &= 705.55 \end{aligned}$$

$$E[S] = 100E[Y] = 14,816.6$$

$$\text{Var}[S] = 100 \text{Var}[Y] = 70,555$$

$$\text{Standard deviation } [S] = \sqrt{70,555} = 265.62$$

By normal approximation, need

$$\begin{aligned} E[S] + 1.645 \text{ Standard deviations} &= 14,816.6 + (1.645)(265.62) \\ &= 15,254 \end{aligned}$$

Question #127**Answer: B**

$$\begin{aligned} \text{Initial Benefit Prem} &= \frac{5A_{30} - 4(A_{30:\overline{20}|}^1)}{5\ddot{a}_{30:\overline{35}|} - 4\ddot{a}_{30:\overline{20}|}} \\ &= \frac{5(0.10248) - 4(0.02933)}{5(14.835) - 4(11.959)} \\ &= \frac{0.5124 - 0.11732}{74.175 - 47.836} = \frac{0.39508}{26.339} = 0.015 \end{aligned}$$

Where

$$A_{30:\overline{20}|}^1 = (A_{30:\overline{20}|} - A_{30:\overline{20}|} \cdot \frac{1}{1.06}) = 0.32307 - 0.29374 = 0.02933$$

and

$$\ddot{a}_{30:\overline{20}|} = \frac{1 - A_{30:\overline{20}|}}{d} = \frac{1 - 0.32307}{\left(\frac{0.06}{1.06}\right)} = 11.959$$

Comment: the numerator could equally well have been calculated as

$$A_{30} + 4 \cdot {}_{20}E_{30} \cdot A_{50} = 0.10248 + (4)(0.29374)(0.24905) = 0.39510$$

Question #128**Answer: B**

$$\begin{aligned} {}_{0.75}p_x &= 1 - (0.75)(0.05) \\ &= 0.9625 \end{aligned}$$

$$\begin{aligned} {}_{0.75}p_y &= 1 - (0.75)(.10) \\ &= 0.925 \end{aligned}$$

$$\begin{aligned} {}_{0.75}q_{xy} &= 1 - {}_{0.75}p_{xy} \\ &= 1 - ({}_{0.75}p_x)({}_{0.75}p_y) \text{ since independent} \\ &= 1 - (0.9625)(0.925) \\ &= 0.1097 \end{aligned}$$

Question #129**Answer: D**

Let G be the expense-loaded premium.

Expected present value (EPV) of benefits = $100,000A_{35}$

EPV of premiums = $G\ddot{a}_{35}$

EPV of expenses = $[0.1G + 25 + (2.50)(100)]\ddot{a}_{35}$

Equivalence principle:

$$G\ddot{a}_{35} = 100,000A_{35} + (0.1G + 25 + 250)\ddot{a}_{35}$$

$$G = 100,000 \frac{A_{35}}{\ddot{a}_{35}} + 0.1G + 275$$

$$0.9G = 100,000P_{35} + 275$$

$$G = \frac{(100)(8.36) + 275}{0.9}$$

$$= 1234$$

Question #130**Answer: A**

The person receives K per year guaranteed for 10 years $\Rightarrow K\ddot{a}_{\overline{10}|} = 8.4353K$

The person receives K per years alive starting 10 years from now $\Rightarrow {}_{10|}\ddot{a}_{40}K$

*Hence we have $10000 = (8.4353 + {}_{10}E_{40}\ddot{a}_{50})K$

Derive ${}_{10}E_{40}$:

$$A_{40} = A_{40:\overline{10}|}^1 + ({}_{10}E_{40})A_{50}$$
$${}_{10}E_{40} = \frac{A_{40} - A_{40:\overline{10}|}^1}{A_{50}} = \frac{0.30 - 0.09}{0.35} = 0.60$$

Derive $\ddot{a}_{50} = \frac{1 - A_{50}}{d} = \frac{1 - 0.35}{\frac{.04}{1.04}} = 16.90$

Plug in values:

$$10,000 = (8.4353 + (0.60)(16.90))K$$
$$= 18.5753K$$
$$K = 538.35$$

Question #131**Answer: D**

$$\text{STANDARD: } \ddot{e}_{25:\overline{11}|} = \int_0^{11} \left(1 - \frac{t}{75}\right) dt = t - \frac{t^2}{2 \times 75} \Big|_0^{11} = 10.1933$$

$$\text{MODIFIED: } p_{25} = e^{-\int_0^1 0.1 ds} = e^{-0.1} = 0.90484$$

$$\begin{aligned} \ddot{e}_{25:\overline{11}|} &= \int_0^1 {}_t p_{25} dt + p_{25} \int_0^{10} \left(1 - \frac{t}{74}\right) dt \\ &= \int_0^1 e^{-0.1t} dt + e^{-0.1} \int_0^{10} \left(1 - \frac{t}{74}\right) dt \\ &= \frac{1 - e^{-0.1}}{0.1} + e^{-0.1} \left(t - \frac{t^2}{2 \times 74} \right) \Big|_0^{10} \\ &= 0.95163 + 0.90484(9.32432) = 9.3886 \end{aligned}$$

$$\text{Difference} = 0.8047$$

Question #132**Answer: B**

Comparing B & D: Prospectively at time 2, they have the same future benefits. At issue, B has the lower benefit premium. Thus, by formula 7.2.2, B has the higher reserve.

Comparing A to B: use formula 7.3.5. At issue, B has the higher benefit premium. Until time 2, they have had the same benefits, so B has the higher reserve.

Comparing B to C: Visualize a graph C* that matches graph B on one side of t=2 and matches graph C on the other side. By using the logic of the two preceding paragraphs, C's reserve is lower than C*'s which is lower than B's.

Comparing B to E: Reserves on E are constant at 0.

Question #133**Answer: C**

Since only decrements (1) and (2) occur during the year, probability of reaching the end of the year is

$$p'_{60}^{(1)} \times p'_{60}^{(2)} = (1 - 0.01)(1 - 0.05) = 0.9405$$

Probability of remaining through the year is

$$p'_{60}^{(1)} \times p'_{60}^{(2)} \times p'_{60}^{(3)} = (1 - 0.01)(1 - 0.05)(1 - 0.10) = 0.84645$$

Probability of exiting at the end of the year is

$$q_{60}^{(3)} = 0.9405 - 0.84645 = 0.09405$$

Question #134 - Removed**Question #135****Answer: D**

$$\begin{aligned} \text{EPV of regular death benefit} &= \int_0^{\infty} (100000)(e^{-\delta t})(0.008)(e^{-\mu t}) dt \\ &= \int_0^{\infty} (100000)(e^{-0.06t})(0.008)(e^{-0.008t}) dt \\ &= 100000[0.008 / (0.06 + 0.008)] = 11,764.71 \end{aligned}$$

$$\begin{aligned} \text{EPV of accidental death benefit} &= \int_0^{30} (100000)(e^{-\delta t})(0.001)(e^{-\mu t}) dt \\ &= \int_0^{30} (100000)(e^{-0.06t})(0.001)(e^{-0.008t}) dt \\ &= 100[1 - e^{-2.04}] / 0.068 = 1,279.37 \end{aligned}$$

$$\text{Total EPV} = 11765 + 1279 = 13044$$

Question #136**Answer: B**

$$l_{[60]+.6} = (.6)(79,954) + (.4)(80,625)$$

$$= 80,222.4$$

$$l_{[60]+1.5} = (.5)(79,954) + (.5)(78,839)$$

$$= 79,396.5$$

$${}_{0.9}q_{[60]+.6} = \frac{80222.4 - 79,396.5}{80,222.4}$$

$$= 0.0103$$

Question #137 - Removed**Question #138****Answer: A**

$$q_{40}^{(\tau)} = q_{40}^{(1)} + q_{40}^{(2)} = 0.34$$

$$= 1 - p_{40}'^{(1)} p_{40}'^{(2)}$$

$$0.34 = 1 - 0.75 p_{40}'^{(2)}$$

$$p_{40}'^{(2)} = 0.88$$

$$q_{40}'^{(2)} = 0.12 = y$$

$$q_{41}'^{(2)} = 2y = 0.24$$

$$q_{41}^{(\tau)} = 1 - (0.8)(1 - 0.24) = 0.392$$

$$l_{42}^{(\tau)} = 2000(1 - 0.34)(1 - 0.392) = 803$$

Question #139**Answer: C**

$$\Pr[L(\pi') > 0] < 0.5$$

$$\Pr\left[10,000v^{K+1} - \pi' \ddot{a}_{\overline{K+1}|} > 0\right] < 0.5$$

From Illustrative Life Table, ${}_{47}p_{30} = 0.50816$ and ${}_{48}p_{30} = .47681$

Since L is a decreasing function of K , to have

$$\Pr[L(\pi') > 0] < 0.5 \text{ means we must have } L(\pi') \leq 0 \text{ for } K \geq 47.$$

Highest value of $L(\pi')$ for $K \geq 47$ is at $K = 47$.

$$\begin{aligned} L(\pi')[\text{at } K = 47] &= 10,000 v^{47+1} - \pi' \ddot{a}_{\overline{47+1}|} \\ &= 609.98 - 16.589\pi' \end{aligned}$$

$$L(\pi') \leq 0 \Rightarrow (609.98 - 16.589\pi') \leq 0$$

$$\Rightarrow \pi' > \frac{609.98}{16.589} = 36.77$$

Question #140**Answer: B**

$$\Pr(K = 0) = 1 - p_x = 0.1$$

$$\Pr(K = 1) = {}_1p_x - {}_2p_x = 0.9 - 0.81 = 0.09$$

$$\Pr(K > 1) = {}_2p_x = 0.81$$

$$E(Y) = .1 \times 1 + .09 \times 1.87 + .81 \times 2.72 = 2.4715$$

$$E(Y^2) = .1 \times 1^2 + .09 \times 1.87^2 + .81 \times 2.72^2 = 6.407$$

$$\text{VAR}(Y) = 6.407 - 2.4715^2 = 0.299$$

Question #141**Answer: E**

$$E[Z] = b\bar{A}_x$$

since constant force $\bar{A}_x = \mu/(\mu + \delta)$

$$E(Z) = \frac{b\mu}{\mu + \delta} = \frac{b(0.02)}{(0.06)} = b/3$$

$$\begin{aligned} \text{Var}[Z] &= \text{Var}[bv^T] = b^2 \text{Var}[v^T] = b^2 \left({}^2\bar{A}_x - \bar{A}_x^2 \right) \\ &= b^2 \left(\frac{\mu}{\mu + 2\delta} - \left(\frac{\mu}{\mu + \delta} \right)^2 \right) \\ &= b^2 \left[\frac{2}{10} - \frac{1}{9} \right] = b^2 \left(\frac{4}{45} \right) \end{aligned}$$

$$\text{Var}(Z) = E(Z)$$

$$b^2 \left[\frac{4}{45} \right] = \frac{b}{3}$$

$$b \left[\frac{4}{45} \right] = \frac{1}{3} \Rightarrow b = 3.75$$

Question #142**Answer: B**

$$\text{In general } \text{Var}(L) = \left(1 + \frac{p}{\delta}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$\text{Here } \bar{P}(\bar{A}_x) = \frac{1}{\bar{a}_x} - \delta = \frac{1}{5} - .08 = .12$$

$$\text{So } \text{Var}(L) = \left(1 + \frac{.12}{.08}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right) = .5625$$

$$\text{and } \text{Var}(L^*) = \left(1 + \frac{\frac{5}{4}(.12)}{.08}\right)^2 \left({}^2\bar{A}_x - \bar{A}_x^2\right)$$

$$\text{So } \text{Var}(L^*) = \frac{\left(1 + \frac{15}{8}\right)^2}{\left(1 + \frac{12}{8}\right)^2} (.5625) = .744$$

$$E[L^*] = \bar{A}_x - .15\bar{a}_x = 1 - \bar{a}_x(\delta + .15) = 1 - 5(.23) = -.15$$

$$E[L^*] + \sqrt{\text{Var}(L^*)} = .7125$$

Question #143 - Removed

Question #144

Answer: B

Let $l_0^{(\tau)}$ = number of students entering year 1
superscript (f) denote academic failure
superscript (w) denote withdrawal
subscript is "age" at start of year; equals year - 1

$$p_0^{(\tau)} = 1 - 0.40 - 0.20 = 0.40$$

$$l_2^{(\tau)} = 10l_2^{(f)} q_2^{(f)} \Rightarrow q_2^{(f)} = 0.1$$

$$q_2^{(w)} = q_2^{(\tau)} - q_2^{(f)} = (1.0 - 0.6) - 0.1 = 0.3$$

$$l_1^{(\tau)} q_1^{(f)} = 0.4 \left[l_1^{(\tau)} (1 - q_1^{(f)} - q_1^{(w)}) \right]$$

$$q_1^{(f)} = 0.4(1 - q_1^{(f)} - 0.3)$$

$$q_1^{(f)} = \frac{0.28}{1.4} = 0.2$$

$$p_1^{(\tau)} = 1 - q_1^{(f)} - q_1^{(w)} = 1 - 0.2 - 0.3 = 0.5$$

$$\begin{aligned} {}_3q_0^{(w)} &= q_0^{(w)} + p_0^{(\tau)} q_1^{(w)} + p_0^{(\tau)} p_1^{(\tau)} q_2^{(w)} \\ &= 0.2 + (0.4)(0.3) + (0.4)(0.5)(0.3) \\ &= 0.38 \end{aligned}$$

Question #145**Answer: D**

$$e_{25} = p_{25}(1 + e_{26})$$

$$e_{26}^N = e_{26}^M \text{ due to having the same } \mu$$

$$p_{25}^N = \exp\left[-\int_0^1 \mu_{25+t}^M + 0.1(1-t)dt\right] = p_{25}^M e^{-0.05}$$

$$e_{25}^N = p_{25}^N(1 + e_{26}^M) = e^{-0.05} p_{25}^M(1 + e_{26}^M) = 0.951 e_{25}^M = 9.51$$

Question #146**Answer: D**

$$\begin{aligned} E[Y_{AGG}] &= 100E[Y] = 100(10,000)\bar{a}_x \\ &= 100(10,000)\left(\frac{1 - \bar{A}_x}{\delta}\right) = 10,000,000 \end{aligned}$$

$$\begin{aligned} \sigma_Y &= \sqrt{\text{Var}[Y]} = \sqrt{(10,000)^2 \frac{1}{\delta^2} (\bar{A}_x - \bar{A}_x^2)} \\ &= \frac{(10,000)}{\delta} \sqrt{(0.25) - (0.16)} = 50,000 \end{aligned}$$

$$\sigma_{AGG} = \sqrt{100}\sigma_Y = 10(50,000) = 500,000$$

$$0.90 = \Pr\left[\frac{F - E[Y_{AGG}]}{\sigma_{AGG}} > 0\right]$$

$$\Rightarrow 1.282 = \frac{F - E[Y_{AGG}]}{\sigma_{AGG}}$$

$$F = 1.282\sigma_{AGG} + E[Y_{AGG}]$$

$$F = 1.282(500,000) + 10,000,000 = 10,641,000$$

Question #147**Answer: A**

$$A_{30:\overline{3}|}^1 = 1000vq_{30} + 500v^2 {}_1|q_{30} + 250v^3 {}_2|q_{30}$$

$$= 1000\left(\frac{1}{1.06}\right)\left(\frac{1.53}{1000}\right) + 500\left(\frac{1}{1.06}\right)^2 (0.99847)\left(\frac{1.61}{1000}\right) + 250\left(\frac{1}{1.06}\right)^3 (0.99847)(0.99839)\left(\frac{1.70}{1000}\right)$$

$$= 1.4434 + 0.71535 + 0.35572 = 2.51447$$

$$\ddot{a}_{30:\overline{1}|}^{(2)} = \frac{1}{2} + \frac{1}{2}\left(\frac{1}{1.06}\right)^{\frac{1}{2}} (1 - \frac{1}{2}q_{30}) = \frac{1}{2} + \frac{1}{2}(0.97129)\left(1 - \frac{0.00153}{2}\right)$$

$$= \frac{1}{2} + \frac{1}{2}(0.97129)(0.999235)$$

$$= 0.985273$$

$$\text{Annualized premium} = \frac{2.51447}{0.985273}$$

$$= 2.552$$

$$\text{Each semiannual premium} = \frac{2.552}{2}$$

$$= 1.28$$

Question: #148**Answer: E**

$$(DA)_{80:\overline{20}|}^1 = 20vq_{80} + vp_{80}((DA)_{81:\overline{19}|}^1)$$

$$q_{80} = .2 \quad 13 = \frac{20(.2)}{1.06} + \frac{.8}{1.06}(DA)_{81:\overline{19}|}^1$$

$$\therefore (DA)_{81:\overline{19}|}^1 = \frac{13(1.06) - 4}{.8} = 12.225$$

$$q_{80} = .1 \quad DA_{80:\overline{20}|}^1 = 20v(.1) + v(.9)(12.225)$$

$$= \frac{2 + .9(12.225)}{1.06} = 12.267$$

Question #149 - Removed

Question #150**Answer: A**

$${}_t p_x = \exp\left[-\int_0^t \frac{ds}{100-x-s}\right] = \exp\left[\ln(100-x-s)\Big|_0^t\right] = \frac{100-x-t}{100-x}$$

$$\overset{\circ}{e}_{50:\overline{60}} = \overset{\circ}{e}_{50} + \overset{\circ}{e}_{60} - \overset{\circ}{e}_{50:60}$$

$$\overset{\circ}{e}_{50} = \int_0^{50} \frac{50-t}{50} dt = \frac{1}{50} \left[50t - \frac{t^2}{2} \right]_0^{50} = 25$$

$$\overset{\circ}{e}_{60} = \int_0^{40} \frac{40-t}{40} dt = \frac{1}{40} \left[40t - \frac{t^2}{2} \right]_0^{40} = 20$$

$$\begin{aligned} \overset{\circ}{e}_{50:60} &= \int_0^{40} \left(\frac{50-t}{50} \right) \left(\frac{40-t}{40} \right) dt = \int_0^{40} \frac{1}{2000} (2000 - 90t + t^2) dt \\ &= \frac{1}{2000} \left(2000t - 45t^2 + \frac{t^3}{3} \Big|_0^{40} \right) = 14.67 \end{aligned}$$

$$\overset{\circ}{e}_{50:\overline{60}} = 25 + 20 - 14.67 = 30.33$$

Question #151**Answer: C**Ways to go $0 \rightarrow 2$ in 2 years

$$0-0-2; p = (0.7)(0.1) = 0.07$$

$$0-1-2; p = (0.2)(0.25) = 0.05$$

$$0-2-2; p = (0.1)(1) = 0.1$$

Total = 0.22

Binomial $m = 100$ $q = 0.22$ Var = $(100)(0.22)(0.78) = 17$

Question #152**Answer: A**

For death occurring in year 2

$$EPV = \frac{0.3 \times 1000}{1.05} = 285.71$$

For death occurring in year 3, two cases:

(1) State 2 → State 1 → State 4:	(0.2 × 0.1) = 0.02
(2) State 2 → State 2 → State 4:	(0.5 × 0.3) = <u>0.15</u>
Total	0.17

$$EPV = \frac{0.17 \times 1000}{1.05^2} = 154.20$$

$$\text{Total EPV} = 285.71 + 154.20 = 439.91$$

Question #153 - Removed**Question #154****Answer: C**Let π denote the single benefit premium.

$$\begin{aligned} \pi &= {}_{30}\ddot{a}_{35} + \pi A_{35:\overline{30}|}^1 \\ \pi &= \frac{{}_{30}\ddot{a}_{35}}{1 - A_{35:\overline{30}|}^1} = \frac{(A_{35:\overline{30}|} - A_{35:\overline{30}|}^1)\ddot{a}_{65}}{1 - A_{35:\overline{30}|}^1} = \\ &= \frac{(.21 - .07)9.9}{(1 - .07)} = \\ &= \frac{1.386}{.93} \\ &= 1.49 \end{aligned}$$

Question #155**Answer: E**

$$\begin{aligned}
{}_{0.4}P_0 = .5 &= e^{-\int_0^{0.4} (F+e^{2x}) dx} \\
&= e^{-.4F - \left[\frac{e^{2x}}{2}\right]_0^{0.4}} \\
&= e^{-.4F - \left(\frac{e^{0.8}-1}{2}\right)} \\
.5 &= e^{-.4F - .6128}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \ln(.5) &= -.4F - .6128 \\
\Rightarrow -.6931 &= -.4F - .6128 \\
\Rightarrow F &= 0.20
\end{aligned}$$

Question #156**Answer: C**

$$\begin{aligned}
({}_9V + P)(1.03) &= q_{x+9}b + (1 - q_{x+9}) {}_{10}V \\
&= q_{x+9}(b - {}_{10}V) + {}_{10}V \\
(343)(1.03) &= 0.02904(872) + {}_{10}V \\
\Rightarrow {}_{10}V &= 327.97 \\
b &= (b - {}_{10}V) + {}_{10}V = 872 + 327.97 = 1199.97 \\
P &= b \left(\frac{1}{\ddot{a}_x} - d \right) = 1200 \left(\frac{1}{14.65976} - \frac{0.03}{1.03} \right) \\
&= 46.92
\end{aligned}$$

$${}_9V = \text{benefit reserve at the start of year ten} - P = 343 - 46.92 = 296.08$$

Question #157**Answer: B**

$$d = 0.06 \Rightarrow V = 0.94$$

Step 1 Determine p_x

$$\begin{aligned}
668 + 258vp_x &= 1000vq_x + 1000v^2p_x(p_{x+1} + q_{x+1}) \\
668 + 258(0.94)p_x &= 1000(0.94)(1 - p_x) + 1000(0.8836)p_x(1) \\
668 + 242.52p_x &= 940(1 - p_x) + 883.6p_x \\
p_x &= 272/298.92 = 0.91
\end{aligned}$$

Step 2 Determine $1000P_{x:\overline{2}|}$

$$668 + 258(0.94)(0.91) = 1000P_{x:\overline{2}|} [1 + (0.94)(0.91)]$$

$$1000P_{x:\overline{2}|} = \frac{[220.69 + 668]}{1.8554} = 479$$

Question #158

Answer: D

$$\begin{aligned} 100,000(IA)_{40:\overline{10}|}^1 &= 100,000v p_{40} \left[(IA)_{41:\overline{10}|}^1 - 10v^{10} {}_9p_{41} q_{50} \right] + A_{40:\overline{10}|}^1 (100,000) \quad [\text{see comment}] \\ &= 100,000 \frac{0.99722}{1.06} \left[0.16736 - \frac{10 \left(\frac{8,950,901}{9,287,264} \right)}{1.06^{10}} \times (0.00592) \right] \\ &\quad + (0.02766 \times 100,000) \\ &= 15,513 \end{aligned}$$

$$\begin{aligned} \text{Where } A_{40:\overline{10}|}^1 &= A_{40} - {}_{10}E_{40} A_{50} \\ &= 0.16132 - (0.53667)(0.24905) \\ &= 0.02766 \end{aligned}$$

Comment: the first line comes from comparing the benefits of the two insurances. At each of age 40, 41, 42, ..., 49 $(IA)_{40:\overline{10}|}^1$ provides a death benefit 1 greater than $(IA)_{41:\overline{10}|}^1$. Hence the $A_{40:\overline{10}|}^1$ term. But $(IA)_{41:\overline{10}|}^1$ provides a death benefit at 50 of 10, while $(IA)_{40:\overline{10}|}^1$ provides 0. Hence a term involving ${}_9q_{41} = {}_9p_{41} q_{50}$. The various v 's and p 's just get all expected present values at age 40.

Question #159**Answer: A**

$$1000_1V = \pi(1+i) - q_x(1000 - 1000_1V)$$

$$40 = 80(1.1) - q_x(1000 - 40)$$

$$q_x = \frac{88 - 40}{960} = 0.05$$

$$\begin{aligned} {}_1AS &= \frac{(G - \text{expenses})(1+i) - 1000q_x}{p_x} \\ &= \frac{(100 - (0.4)(100))(1.1) - (1000)(0.05)}{1 - 0.05} \\ &= \frac{60(1.1) - 50}{0.95} = 16.8 \end{aligned}$$

Question #160**Answer: C**

At any age, $p_x^{(1)} = e^{-0.02} = 0.9802$

$q_x^{(1)} = 1 - 0.9802 = 0.0198$, which is also $q_x^{(1)}$, since decrement 2 occurs only at the end of the year.

Expected present value (EPV) at the start of each year for that year's death benefits

$$= 10,000 * 0.0198 \quad v = 188.1$$

$$p_x^{(\tau)} = 0.9802 * 0.96 = 0.9410$$

$$E_x = p_x^{(\tau)}v = 0.941 \quad v = 0.941 * 0.95 = 0.8940$$

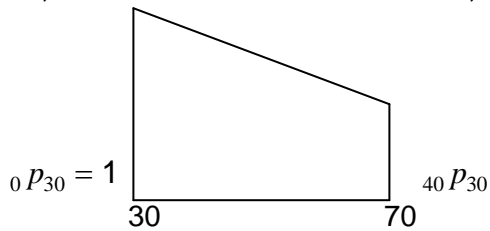
$$\text{EPV of death benefit for 3 years } 188.1 + E_{40} * 188.1 + E_{40} * E_{41} * 188.1 = 506.60$$

Question #161**Answer: B**

$$\begin{aligned}
e_{\overline{30:40}|}^{\circ} &= \int_0^{40} {}_tP_{30} dt \\
&= \int_0^{40} \frac{\omega - 30 - t}{\omega - 30} dt \\
&= t - \frac{t^2}{2(\omega - 30)} \Big|_0^{40} \\
&= 40 - \frac{800}{\omega - 30} \\
&= 27.692
\end{aligned}$$

$$\omega = 95$$

Or, with a linear survival function, it may be simpler to draw a picture:



$$e_{\overline{30:40}|}^{\circ} = \text{area} = 27.692 = 40 \frac{(1 + {}_{40}P_{30})}{2}$$

$${}_{40}P_{30} = 0.3846$$

$$\frac{\omega - 70}{\omega - 30} = 0.3846$$

$$\omega = 95$$

$${}_tP_{30} = \frac{65 - t}{65}$$

$$\text{Var} = E(T)^2 - (E(T))^2$$

Using a common formula for the second moment:

$$\begin{aligned} \text{Var}(T) &= \int_0^{\infty} 2t {}_t p_x dt - \overset{\circ}{e}_x^2 \\ &= 2 \int_0^{65} t \left(1 - \frac{t}{65}\right) dt - \left(\int_0^{65} \left(1 - \frac{t}{65}\right) dt \right)^2 \\ &= 2 * (2112.5 - 1408.333) - (65 - 65/2)^2 \\ &= 1408.333 - 1056.25 = 352.08 \end{aligned}$$

Another way, easy to calculate for a linear survival function is

$$\begin{aligned} \text{Var}(T) &= \int_0^{\infty} t^2 {}_t p_x \mu_x(t) dt - \left(\int_0^{\infty} t {}_t p_x \mu_x(t) dt \right)^2 \\ &= \int_0^{65} t^2 \times \frac{1}{65} dt - \left(\int_0^{65} t \times \frac{1}{65} dt \right)^2 \\ &= \frac{t^3}{3 \times 65} \Big|_0^{65} - \left(\frac{t^2}{2 \times 65} \Big|_0^{65} \right)^2 \\ &= 1408.33 - (32.5)^2 = 352.08 \end{aligned}$$

With a linear survival function and a maximum future lifetime of 65 years, you probably didn't need to integrate to get $E(T_{30}) = \overset{\circ}{e}_{30} = 32.5$

Likewise, if you realize (after getting $\omega = 95$) that T_{30} is uniformly distributed on (0, 65), its variance is just the variance of a continuous uniform random variable:

$$\text{Var} = \frac{(65-0)^2}{12} = 352.08$$

Question #162

Answer: E

$${}_1V = \frac{218.15(1.06) - 10,000(0.02)}{1 - 0.02} = 31.88$$

$${}_2V = \frac{(31.88 + 218.15)(1.06) - (9,000)(0.021)}{1 - 0.021} = 77.66$$

Question #163**Answer: D**

$$e_x = e_y = \sum_{k=1}^{\infty} {}_k p_x = 0.95 + 0.95^2 + \dots$$

$$= \frac{0.95}{1-0.95} = 19$$

$$e_{xy} = p_{xy} + {}_2 p_{xy} + \dots$$

$$= 1.02(0.95)(0.95) + 1.02(0.95)^2(0.95)^2 + \dots$$

$$= 1.02[0.95^2 + 0.95^4 + \dots] = \frac{1.02(0.95)^2}{1-0.95^2} = 9.44152$$

$$e_{\overline{xy}} = e_x + e_y - e_{xy} = 28.56$$

Question #164 - Removed**Question #165 - Removed****Question #166****Answer: E**

$$\bar{a}_x = \int_0^{\infty} e^{-0.08t} dt = 12.5$$

$$\bar{A}_x = \int_0^{\infty} e^{-0.08t} (0.03) dt = \frac{3}{8} = 0.375$$

$${}^2\bar{A}_x = \int_0^{\infty} e^{-0.13t} (0.03) dt = \frac{3}{13} = 0.23077$$

$$\sigma(\bar{a}_{\overline{T}|}) = \sqrt{\text{Var}[\bar{a}_{\overline{T}|}]} = \sqrt{\frac{1}{\delta^2} [{}^2\bar{A}_x - (\bar{A}_x)^2]} = \sqrt{400 [0.23077 - (0.375)^2]} = 6.0048$$

$$\Pr[\bar{a}_{\overline{T}|} > \bar{a}_x - \sigma(\bar{a}_{\overline{T}|})] = \Pr[\bar{a}_{\overline{T}|} > 12.5 - 6.0048]$$

$$= \Pr\left[\frac{1-v^T}{0.05} > 6.4952\right] = \Pr[0.67524 > e^{-0.05T}]$$

$$= \Pr\left[T > \frac{-\ln 0.67524}{0.05}\right] = \Pr[T > 7.85374]$$

$$= e^{-0.03 \times 7.85374} = 0.79$$

Question #167**Answer: A**

$${}_5p_{50}^{(\tau)} = e^{-(0.05)(5)} = e^{-0.25} = 0.7788$$

$$\begin{aligned} {}_5q_{55}^{(1)} &= \int_0^5 \mu_{55+t}^{(1)} \times e^{-(0.03+0.02)t} dt = -(0.02/0.05)e^{-0.05t} \Big|_0^5 \\ &= 0.4(1 - e^{-0.25}) \\ &= 0.0885 \end{aligned}$$

$$\begin{aligned} \text{Probability of retiring before 60} &= {}_5p_{50}^{(\tau)} \times {}_5q_{55}^{(1)} \\ &= 0.7788 \times 0.0885 \\ &= 0.0689 \end{aligned}$$

Question #168**Answer: C**

Complete the table:

$$l_{81} = l_{[80]} - d_{[80]} = 910$$

$$l_{82} = l_{[81]} - d_{[81]} = 830 \quad (\text{not really needed})$$

$$\overset{\circ}{e}_x = e_x + \frac{1}{2} \quad \left(\frac{1}{2} \text{ since UDD} \right)$$

$$\overset{\circ}{e}_{[x]} = e_{[x]} + \frac{1}{2}$$

$$\overset{\circ}{e}_{[x]} = \left[\frac{l_{81} + l_{82} + l_{83} + \dots}{l_{[80]}} \right] + \frac{1}{2}$$

$$\left[\overset{\circ}{e}_{[80]} - \frac{1}{2} \right] l_{[80]} = l_{81} + l_{82} + \dots \quad \text{Call this equation (A)}$$

$$\left[\overset{\circ}{e}_{[81]} - \frac{1}{2} \right] l_{[81]} = l_{82} + \dots \quad \text{Formula like (A), one age later. Call this (B)}$$

Subtract equation (B) from equation (A) to get

$$l_{81} = \left[\overset{\circ}{e}_{[80]} - \frac{1}{2} \right] l_{[80]} - \left[\overset{\circ}{e}_{[81]} - \frac{1}{2} \right] l_{[81]}$$

$$910 = [8.5 - 0.5]1000 - \left[\overset{\circ}{e}_{[81]} - 0.5 \right] 920$$

$$\overset{\circ}{e}_{[81]} = \frac{8000 + 460 - 910}{920} = 8.21$$

Alternatively, and more straightforward,

$$p_{[80]} = \frac{910}{1000} = 0.91$$

$$p_{[81]} = \frac{830}{920} = 0.902$$

$$p_{81} = \frac{830}{910} = 0.912$$

$$\dot{e}_{[80]} = \frac{1}{2}q_{[80]} + p_{[80]}\left(1 + \dot{e}_{81}\right)$$

where $q_{[80]}$ contributes $\frac{1}{2}$ since UDD

$$8.5 = \frac{1}{2}(1-0.91) + (0.91)\left(1 + \dot{e}_{81}\right)$$

$$\dot{e}_{81} = 8.291$$

$$\dot{e}_{81} = \frac{1}{2}q_{81} + p_{81}\left(1 + \dot{e}_{82}\right)$$

$$8.291 = \frac{1}{2}(1-0.912) + 0.912\left(1 + \dot{e}_{82}\right)$$

$$\dot{e}_{82} = 8.043$$

$$\dot{e}_{[81]} = \frac{1}{2}q_{[81]} + p_{[81]}\left(1 + \dot{e}_{82}\right)$$

$$= \frac{1}{2}(1-0.902) + (0.902)(1+8.043)$$

$$= 8.206$$

Or, do all the recursions in terms of e , not \dot{e} , starting with $e_{[80]} = 8.5 - 0.5 = 8.0$, then

final step $\dot{e}_{[81]} = e_{[81]} + 0.5$

Question #169**Answer: A**

T	p_{x+t}	${}_t p_x$	v^t	$v^t {}_t p_x$
0	0.7	1	1	1
1	0.7	0.7	0.95238	0.6667
2	–	0.49	0.90703	0.4444
3	–	–	–	–

From above $\ddot{a}_{x:\overline{3}|} = \sum_{t=0}^2 v^t {}_t p_x = 2.1111$

$$1000 {}_2V_{x:\overline{3}|} = 1000 \left(1 - \frac{\ddot{a}_{x+\overline{2}|}}{\ddot{a}_{x:\overline{3}|}} \right) = 1000 \left(1 - \frac{1}{2.1111} \right) = 526$$

Alternatively,

$$P_{x:\overline{3}|} = \frac{1}{\ddot{a}_{x:\overline{3}|}} - d = 0.4261$$

$$\begin{aligned} 1000 {}_2V_{x:\overline{3}|} &= 1000(v - P_{x:\overline{3}|}) \\ &= 1000(0.95238 - 0.4261) \\ &= 526 \end{aligned}$$

You could also calculate $A_{x:\overline{3}|}$ and use it to calculate $P_{x:\overline{3}|}$.

Question #170**Answer: E**Let G be the gross premium.Expected present value (EPV) of benefits = $1000A_{50}$.EPV of expenses, except claim expense = $15 + 1 \times \ddot{a}_{50}$ EPV of claim expense = $50A_{50}$ (50 is paid when the claim is paid)EPV of premiums = $G\ddot{a}_{50}$ Equivalence principle: $G\ddot{a}_{50} = 1000A_{50} + 15 + 1 \times \ddot{a}_{50} + 50A_{50}$

$$G = \frac{1050A_{50} + 15 + \ddot{a}_{50}}{\ddot{a}_{50}}$$

For the given survival function,

$$A_{50} = \frac{1}{l_{50}} \sum_{k=1}^{50} v^k (l_{50+k-1} - l_{50+k}) = \frac{1}{100} \sum_{k=1}^{50} v^k (2) = \frac{a_{\overline{50}|}}{50} = \frac{1 - 1.05^{-50}}{0.05(50)} = 0.36512$$

$$\ddot{a}_{50} = \frac{1 - A_{50}}{d} = 13.33248$$

Solving for G , $G = 30.88$ **Question #171****Answer: A**

$${}_4p_{50} = e^{-(0.05)(4)} = 0.8187$$

$${}_{10}p_{50} = e^{-(0.05)(10)} = 0.6065$$

$${}_8p_{60} = e^{-(0.04)(8)} = 0.7261$$

$${}_{18}p_{50} = ({}_{10}p_{50})({}_8p_{60}) = 0.6065 \times 0.7261 \\ = 0.4404$$

$${}_{4|14}q_{50} = {}_4p_{50} - {}_{18}p_{50} = 0.8187 - 0.4404 = 0.3783$$

Question #172**Answer: D**

$$\begin{aligned}\ddot{a}_{40:\overline{5}|} &= \ddot{a}_{40} - {}_5E_{40} \ddot{a}_{45} \\ &= 14.8166 - (0.73529)(14.1121) \\ &= 4.4401\end{aligned}$$

$$\begin{aligned}\pi \ddot{a}_{40:\overline{5}|} &= 100,000 A_{45} v^5 {}_5P_{40} + \pi (IA)_{40:\overline{5}|}^1 \\ \pi &= 100,000 A_{45} \times {}_5E_{40} / (\ddot{a}_{40:\overline{5}|} - (IA)_{40:\overline{5}|}^1) \\ &= 100,000(0.20120)(0.73529) / (4.4401 - 0.04042) \\ &= 3363\end{aligned}$$

Question #173**Answer: B**

Calculate the probability that both are alive or both are dead.

$$P(\text{both alive}) = {}_kP_{xy} = {}_kP_x \cdot {}_kP_y$$

$$P(\text{both dead}) = {}_kq_{xy} = {}_kq_x \cdot {}_kq_y$$

$$P(\text{exactly one alive}) = 1 - {}_kP_{xy} - {}_kq_{xy}$$

Only have to do two year's worth so have table

Pr(both alive)	Pr(both dead)	Pr(only one alive)
1	0	0
$(0.91)(0.91) = 0.8281$	$(0.09)(0.09) = 0.0081$	0.1638
$(0.82)(0.82) = 0.6724$	$(0.18)(0.18) = 0.0324$	0.2952

$$EPV \text{ Annuity} = 30,000 \left(\frac{1}{1.05^0} + \frac{0.8281}{1.05^1} + \frac{0.6724}{1.05^2} \right) + 20,000 \left(\frac{0}{1.05^0} + \frac{0.1638}{1.05^1} + \frac{0.2952}{1.05^2} \right) = 80,431$$

Alternatively,

$$\ddot{a}_{xy:\overline{3}|} = 1 + \frac{0.8281}{1.05} + \frac{0.6724}{1.05^2} = 2.3986$$

$$\ddot{a}_{x:\overline{3}|} = \ddot{a}_{y:\overline{3}|} = 1 + \frac{0.91}{1.05} + \frac{0.82}{1.05^2} = 2.6104$$

$$EPV = 20,000 \ddot{a}_{x:\overline{3}|} + 20,000 \ddot{a}_{y:\overline{3}|} - 10,000 \ddot{a}_{xy:\overline{3}|}$$

(it pays 20,000 if x alive and 20,000 if y alive, but 10,000 less than that if both are alive)

$$= (20,000)(2.6104) + (20,000)(2.6104) - (10,000)2.3986 = 80,430$$

Question #174**Answer: C**

Let P denote the gross premium.

$$P = \bar{a}_x = \int_0^{\infty} e^{-\delta t} e^{-\mu t} dt = \int_0^{\infty} e^{-0.05t} dt = 20$$

$$E[L] = \bar{a}_x^{IMP} - P$$

$$\begin{aligned} \bar{a}_x^{IMP} &= \int_0^{10} e^{-0.03t} e^{-0.02t} dt + e^{-0.03(10)} e^{-0.02(10)} \int_0^{\infty} e^{-0.03t} e^{-0.01t} dt \\ &= \frac{1 - e^{-0.5}}{0.05} + \frac{e^{-0.5}}{0.04} = 23 \end{aligned}$$

$$E[L] = 23 - 20 = 3$$

$$\frac{E[L]}{P} = \frac{3}{20} = 15\%$$

Question #175**Answer: C**

$$\begin{aligned} A_{30:\overline{2}|}^1 &= 1000vq_{30} + 500v^2 {}_1q_{30} \\ &= 1000\left(\frac{1}{1.06}\right)(0.00153) + 500\left(\frac{1}{1.06}\right)^2 (0.99847)(0.00161) \\ &= 2.15875 \end{aligned}$$

$$\text{Initial fund} = 2.15875 \times 1000 \text{ participants} = 2158.75$$

Let F_n denote the size of Fund 1 at the end of year n .

$$F_1 = 2158.75(1.07) - 1000 = 1309.86$$

$$F_2 = 1309.86(1.065) - 500 = 895.00$$

Expected size of Fund 2 at end of year 2 = 0 (since the amount paid was the single benefit premium). Difference is 895.

Question #176**Answer: C**

$$\text{Var}[Z] = E[Z^2] - E[Z]^2$$

$$\begin{aligned} E[Z] &= \int_0^{\infty} (v^t b_t)_t p_x \mu_{x+t} dt = \int_0^{\infty} e^{-0.08t} e^{0.03t} e^{-0.02t} (0.02) dt \\ &= \int_0^{\infty} (0.02) e^{-0.07t} dt = \frac{0.02}{0.07} = 2/7 \end{aligned}$$

$$\begin{aligned} E[Z^2] &= \int_0^{\infty} (v^t b_t)_t^2 p_x \mu_{x+t} dt = \int_0^{\infty} (e^{-0.05t})^2 e^{-0.02t} (0.02) dt \\ &= \int_0^{\infty} 0.02 e^{-0.12t} \mu_{x+t} dt = 2/12 = 1/6 \end{aligned}$$

$$\text{Var}[Z] = \frac{1}{6} - \left(\frac{2}{7}\right)^2 = \frac{1}{6} - \frac{4}{49} = 0.08503$$

Question #177**Answer: C**

From $A_x = 1 - d\ddot{a}_x$ we have $A_x = 1 - \frac{0.1}{1.1}(8) = 3/11$

$$A_{x+10} = 1 - \frac{0.1}{1.1}(6) = 5/11$$

$$\bar{A}_x = A_x \times i/\delta$$

$$\bar{A}_x = \frac{3}{11} \times \frac{0.1}{\ln(1.1)} = 0.2861$$

$$\bar{A}_{x+10} = \frac{5}{11} \times \frac{0.1}{\ln(1.1)} = 0.4769$$

$$\begin{aligned} {}_{10}V &= \bar{A}_{x+10} - P(\bar{A}_x) \times \ddot{a}_{x+10} \\ &= 0.4769 - \left(\frac{0.2861}{8}\right) 6 \\ &= 0.2623 \end{aligned}$$

There are many other equivalent formulas that could be used.

Question #178**Answer: C**

$$\text{Regular death benefit} = \int_0^{\infty} 100,000 \times e^{-0.06t} \times e^{-0.001t} 0.001 dt$$

$$= 100,000 \left(\frac{0.001}{0.06 + 0.001} \right)$$

$$= 1639.34$$

$$\text{Accidental death} = \int_0^{10} 100,000 e^{-0.06t} e^{-0.001t} (0.0002) dt$$

$$= 20 \int_0^{10} e^{-0.061t} dt$$

$$= 20 \left[\frac{1 - e^{-0.61}}{0.061} \right] = 149.72$$

$$\text{Expected Present Value} = 1639.34 + 149.72 = 1789.06$$

Question #179**Answer: C**

$$p_{61}^{00} = 560/800 = 0.70$$

$$p_{61}^{01} = 160/800 = 0.20$$

$$p_{61}^{10} = 0, \text{ once dead, stays dead}$$

$$p_{61}^{11} = 1, \text{ once dead by cause 1, stays dead by cause 1}$$

$$p_{61}^{00} + p_{61}^{01} + p_{61}^{10} + p_{61}^{11} = 0.70 + 0.20 + 0 + 1 = 1.90$$

Question #180**Answer: C**

The solution requires determining the element in row 2 column 3 of the matrix obtained by multiplying the first three transition probability matrices. Only the second row needs to be calculated. After one period, the row is [0.0 0.7 0.3]. For the second period, multiply this row by the columns of Q_1 . The result is [0.07 0.49 0.44]. For the third period, multiply this row by the columns of Q_2 . The result is [0.1645 0.3465 0.4890] with the final entry providing the answer.

An alternative is to enumerate the transitions that result in extinction. Label the states S (sustainable), E (endangered) and X (extinct). The possible paths are:

$$EX = 0.3$$

$$EEX = 0.7(0.2) = 0.14$$

$$EEEX = 0.7(0.7)(0.1) = 0.049$$

$$\text{Total} = 0.489$$

Note that if the species is not extinct by time 3, it will never become extinct.

Question #181**Answer: B**

$$\text{Pr(dies in year 1)} = p^{02} = 0.1$$

$$\text{Pr(dies in year 2)} = p^{00}p^{02} + p^{01}p^{12} = 0.8(0.1) + 0.1(0.2) = 0.10$$

$$\text{Pr(dies in year 3)} = p^{00}p^{00}p^{02} + p^{00}p^{01}p^{12} + p^{01}p^{11}p^{12} + p^{01}p^{10}p^{02} = 0.095$$

$$EPV(\text{benefits}) = 100,000[0.9(0.1) + 0.9^2(0.10) + 0.9^3(0.095)] = 24,025.5$$

$$\text{Pr(in State 0 at time 0)} = 1$$

$$\text{Pr(in State 0 at time 1)} = p^{00} = 0.8$$

$$\text{Pr(in State 0 at time 2)} = p^{00}p^{00} + p^{01}p^{10} = 0.8(0.8) + 0.1(0.1) = 0.65$$

$$EPV(\$1 \text{ of premium}) = 1 + 0.9(0.8) + 0.9^2(0.65) = 2.2465$$

$$\text{Benefit premium} = 24,025.5/2.2465 = 10,695.$$

Question #182 - Removed**Answer: A****Question #183 - Removed**

Question #184**Answer: B**

$$1000P_{45:\overline{15}|} \ddot{a}_{45:\overline{15}|} + \pi \ddot{a}_{60:\overline{15}|} \times {}_{15}E_{45} = 1000A_{45}$$

$$1000 \frac{A_{45}}{\ddot{a}_{45}} (\ddot{a}_{45} - {}_{15}E_{45} \ddot{a}_{60}) + \pi (\ddot{a}_{60} - {}_{15}E_{60} \ddot{a}_{75}) ({}_{15}E_{45}) = 1000A_{45}$$

$$\frac{201.20}{14.1121} (14.1121 - (0.72988)(0.51081)(11.1454))$$

$$+ \pi (11.1454 - (0.68756)(0.39994)(7.2170)) \times (0.72988)(0.51081) = 201.20$$

where ${}_{15}E_x$ was evaluated as ${}_5E_x \times {}_{10}E_{x+5}$

$$14.2573(9.9568) + (\pi)(3.4154) = 201.20$$

$$\pi = 17.346$$

Question #185**Answer: A**

$${}_1V = ({}_0V + \pi)(1+i) - (1000 + {}_1V - {}_1V)q_x$$

$${}_2V = ({}_1V + \pi)(1+i) - (2000 + {}_2V - {}_2V)q_{x+1} = 2000$$

$$((\pi(1+i) - 1000q_x) + \pi)(1+i) - 2000q_{x+1} = 2000$$

$$((\pi(1.08) - 1000 \times 0.1) + \pi)(1.08) - 2000 \times 0.1 = 2000$$

$$\pi = 1027.42$$

Question #186**Answer: A**

Let Y be the present value of payments to 1 person.
 Let S be the present value of the aggregate payments.

$$E[Y] = 500\ddot{a}_x = 500 \frac{(1 - A_x)}{d} = 5572.68$$

$$\sigma_Y = \sqrt{\text{Var}[Y]} = \sqrt{(500)^2 \frac{1}{d^2} ({}^2A_x - A_x^2)} = 1791.96$$

$$S = Y_1 + Y_2 + \dots + Y_{250}$$

$$E(S) = 250E[Y] = 1,393,170$$

$$\sigma_S = \sqrt{250} \times \sigma_Y = 15.811388 \sigma_Y = 28,333$$

$$0.90 = \Pr(S \leq F) = \Pr\left[\frac{S - 1,393,170}{28,333} \leq \frac{F - 1,393,170}{28,333}\right]$$

$$\approx \Pr\left[N(0,1) \leq \frac{F - 1,393,170}{28,333}\right]$$

$$0.90 = \Pr(N(0,1) \leq 1.28)$$

$$F = 1,393,170 + 1.28(28,333)$$

$$= 1.43 \text{ million}$$

Question #187**Answer: A**

$$q'_{41}{}^{(1)} = 1 - p'_{41}{}^{(1)} = 1 - \left(p_{41}{}^{(\tau)}\right)^{q_{41}{}^{(1)}/q_{41}{}^{(\tau)}}$$

$$l_{41}{}^{(\tau)} = l_{40}{}^{(\tau)} - d_{40}{}^{(1)} - d_{40}{}^{(2)} = 1000 - 60 - 55 = 885$$

$$d_{41}{}^{(1)} = l_{41}{}^{(\tau)} - d_{41}{}^{(2)} - l_{42}{}^{(\tau)} = 885 - 70 - 750 = 65$$

$$p_{41}{}^{(\tau)} = \frac{750}{885} \qquad \frac{q_{41}{}^{(1)}}{q_{41}{}^{(\tau)}} = \frac{65}{135}$$

$$q'_{41}{}^{(1)} = 1 - \left(\frac{750}{885}\right)^{65/135} = 0.0766$$

Question #188**Answer: D**

$$S_0(t) = \left(1 - \frac{t}{\omega}\right)^\alpha$$

$$\mu_t = \frac{d}{dt} \log(S_0(t)) = \frac{\alpha}{\omega - t}$$

$$e_x^\circ = \int_0^{\omega-x} \left(1 - \frac{t}{\omega-x}\right)^\alpha dt = \frac{\omega-x}{\alpha+1}$$

$$e_0^{\text{new}} = \frac{1}{2} \times \frac{\omega}{\alpha^{\text{old}} + 1} = \frac{\omega}{\alpha^{\text{new}} + 1} \Rightarrow \alpha^{\text{new}} = 2\alpha^{\text{old}} + 1$$

$$\mu_0^{(\text{new})} = \frac{2\alpha^{\text{old}} + 1}{\omega} = \frac{9}{4} \times \frac{\alpha^{\text{old}}}{\omega} \Rightarrow \alpha^{\text{old}} = 4$$

Question #189**Answer: C**

Constant force implies exponential lifetime

$$\text{Var}[T] = E[T^2] - (E[T])^2 = \frac{2}{\mu^2} - \left(\frac{1}{\mu}\right)^2 = \frac{1}{\mu^2} = 100$$

$$\mu = 0.1$$

$$\begin{aligned} E[\min(T, 10)] &= \int_0^{10} t(0.1)e^{-.1t} dt + \int_{10}^{\infty} 10(0.1)e^{-.1t} dt \\ &= -te^{-.1t} - 10e^{-.1t} \Big|_0^{10} - 10e^{-.1t} \Big|_{10}^{\infty} \\ &= -10e^{-1} - 10e^{-1} + 10 + 10e^{-1} \\ &= 10(1 - e^{-1}) = 6.3 \end{aligned}$$

Question #190**Answer: A**

% premium amount for 15 years

$$G\ddot{a}_{x:\overline{15}} = 100,000A_x + \overbrace{(0.08G + 0.02G\ddot{a}_{x:\overline{15}})} + \underbrace{((x-5) + 5\ddot{a}_x)}$$

Per policy for life

$$4669.95(11.35) = 51,481.97 + (0.08)(4669.95) + (0.02)(11.35)(4669.95) + ((x-5) + 5\ddot{a}_x)$$

$$\ddot{a}_x = \frac{1 - Ax}{d} = \frac{1 - 0.5148197}{0.02913} = 16.66$$

$$53,003.93 = 51,481.97 + 1433.67 + (x-5) + 83.30$$

$$4.99 = (x-5)$$

$$x = 9.99$$

The % of premium expenses could equally well have been expressed as $0.10G + 0.02G a_{x:\overline{14}}$.

The per policy expenses could also be expressed in terms of an annuity-immediate.

Question #191**Answer: D**

For the density where $T_x \neq T_y$,

$$\begin{aligned} \Pr(T_x < T_y) &= \int_{y=0}^{40} \int_{x=0}^y 0.0005 dx dy + \int_{y=40}^{50} \int_{x=0}^{40} 0.0005 dx dy \\ &= \int_{y=0}^{40} 0.0005 x \Big|_0^y dy + \int_{y=40}^{50} 0.0005 x \Big|_0^{40} dy \\ &= \int_0^{40} 0.0005 y dy + \int_{y=40}^{50} 0.02 dy \\ &= \frac{0.0005 y^2}{2} \Big|_0^{40} + 0.02 y \Big|_{40}^{50} \\ &= 0.40 + 0.20 = 0.60 \end{aligned}$$

For the overall density,

$$\Pr(T_x < T_y) = 0.4 \times 0 + 0.6 \times 0.6 = 0.36$$

where the first 0.4 is the probability that $T_x = T_y$ and the first 0.6 is the probability that $T_x \neq T_y$.

Question #192**Answer: B**

The conditional expected value of the annuity, given μ , is $\frac{1}{0.01 + \mu}$.

The unconditional expected value is

$$\bar{a}_x = 100 \int_{0.01}^{0.02} \frac{1}{0.01 + \mu} d\mu = 100 \ln \left(\frac{0.01 + 0.02}{0.01 + 0.01} \right) = 40.5$$

100 is the constant density of μ on the interval $[0.01, 0.02]$. If the density were not constant, it would have to go inside the integral.

Question #193**Answer: E**

Recall $\overset{\circ}{e}_x = \frac{\omega - x}{2}$

$$\overset{\circ}{e}_{x:x} = \overset{\circ}{e}_x + \overset{\circ}{e}_x - \overset{\circ}{e}_{x:x}$$

$$\overset{\circ}{e}_{x:x} = \int_0^{\omega-x} \left(1 - \frac{t}{\omega-x}\right) \left(1 - \frac{t}{\omega-y}\right) dt$$

Performing the integration we obtain

$$\overset{\circ}{e}_{x:x} = \frac{\omega - x}{3}$$

$$\overset{\circ}{e}_{x:x} = \frac{2(\omega - x)}{3}$$

$$(i) \quad \frac{2(\omega - 2a)}{3} = 3 \times \frac{2(\omega - 3a)}{3} \Rightarrow 2\omega = 7a$$

$$(ii) \quad \frac{2}{3}(\omega - a) = k \times \frac{2(\omega - 3a)}{3}$$

$$3.5a - a = k(3.5a - 3a)$$

$$k = 5$$

The solution assumes that all lifetimes are independent.

Question #194**Answer: B**

Although simultaneous death is impossible, the times of death are dependent as the force of mortality increases after the first death. There are two ways for the benefit to be paid. The first is to have (x) die prior to (y). That is, the transitions are State 0 to State 2 to State 3. The EPV can be written with a double integral

$$\int_0^{\infty} e^{-0.04t} {}_tP_{xy} {}^0\mu_{x+t:y+t} \int_0^{\infty} e^{-0.04r} {}_rP_{x+t:y+t} {}^2\mu_{x+t+r:y+t+r} dr dt$$

$$= \int_0^{\infty} e^{-0.04t} e^{-0.12t} 0.06 \int_0^{\infty} e^{-0.04r} e^{-0.10r} 0.10 dr dt = \frac{0.06}{0.16} \frac{0.10}{0.14} = 0.26786$$

By symmetry, the second case (State 0 to State 1 to State 3) has the same EPV. Thus the total EPV is $10,000(0.26786 + 0.26786) = 5,357$.

Question #195**Answer: E**

Let ${}_k p_0$ = Probability someone answers the first k problems correctly.

$${}_2 p_0 = (0.8)^2 = 0.64$$

$${}_4 p_0 = (0.8)^4 = 0.41$$

$${}_2 p_{0:0} = ({}_2 p_0)^2 = 0.64^2 = 0.41$$

$${}_4 p_{0:0} = (0.41)^2 = 0.168$$

$${}_2 \overline{p}_{0:0} = {}_2 p_0 + {}_2 p_0 - {}_2 p_{0:0} = 0.87$$

$${}_4 \overline{p}_{0:0} = 0.41 + 0.41 - 0.168 = 0.652$$

$$\begin{aligned} \text{Prob(second child loses in round 3 or 4)} &= {}_2 \overline{p}_{0:0} - {}_4 \overline{p}_{0:0} \\ &= 0.87 - 0.652 \\ &= 0.218 \end{aligned}$$

$$\begin{aligned} \text{Prob(second loses in round 3 or 4} \mid \text{second loses after round 2)} &= \frac{{}_2 \overline{p}_{0:0} - {}_4 \overline{p}_{0:0}}{{}_2 \overline{p}_{0:0}} \\ &= \frac{0.218}{0.87} = 0.25 \end{aligned}$$

Question #196**Answer: E**

If (40) dies before 70, he receives one payment of 10, and $Y = 10$. The probability of this is $(70 - 40)/(110 - 40) = 3/7$

If (40) reaches 70 but dies before 100, he receives 2 payments.

$$Y = 10 + 20v^{30} = 16.16637$$

The probability of this is also $3/7$.

If (40) survives to 100, he receives 3 payments.

$$Y = 10 + 20v^{30} + 30v^{60} = 19.01819$$

The probability of this is $1 - 3/7 - 3/7 = 1/7$

$$E(Y) = (3/7) \times 10 + (3/7) \times 16.16637 + (1/7) \times 19.01819 = 13.93104$$

$$E(Y^2) = (3/7) \times 10^2 + (3/7) \times 16.16637^2 + (1/7) \times 19.01819^2 = 206.53515$$

$$\text{Var}(Y) = E(Y^2) - [E(Y)]^2 = 12.46$$

Since everyone receives the first payment of 10, you could have ignored it in the calculation.

Question #197**Answer: C**

$$\begin{aligned}
E(Z) &= \sum_{k=0}^2 (v^{k+1} b_{k+1}) {}_k p_x q_{x+k} \\
&= \left[v(300) \times 0.02 + v^2(350)(0.98)(0.04) + v^3 400(0.98)(0.96)(0.06) \right] \\
&= 36.8
\end{aligned}$$

$$\begin{aligned}
E(Z^2) &= \sum_{k=0}^2 (v^{k+1} b_{k+1})^2 {}_k p_x q_{x+k} \\
&= v^2(300)^2 \times 0.02 + v^4(350)^2(0.98)(0.04) + v^6 400^2(0.98)(0.96)0.06 \\
&= 11,773
\end{aligned}$$

$$\begin{aligned}
\text{Var}[Z] &= E(Z^2) - E(Z)^2 \\
&= 11,773 - 36.8^2 \\
&= 10,419
\end{aligned}$$

Question #198**Answer: A**

$$\begin{array}{rcl}
& \text{Benefits +} & \text{Expenses} & \text{– Premiums} \\
{}_0L_e = & 1000v^3 + & (0.20G + 8) + (0.06G + 2)v + (0.06G + 2)v^2 & - G\ddot{a}_{\overline{3}|}
\end{array}$$

at $G = 41.20$ and $i = 0.05$,

$${}_0L \text{ (for } K = 2) = 770.59$$

Question #199

Answer: D

$$P = 1000P_{40}$$

$$(235 + P)(1 + i) - 0.015(1000 - 255) = 255 \quad [\text{A}]$$

$$(272 + P)(1 + i) - 0.020(1000 - 272) = 272 \quad [\text{B}]$$

Subtract [A] from [B]

$$20(1 + i) - 3.385 = 17$$

$$1 + i = \frac{20.385}{20} = 1.01925$$

Plug into [A] $(235 + P)(1.01925) - 0.015(1000 - 255) = 255$

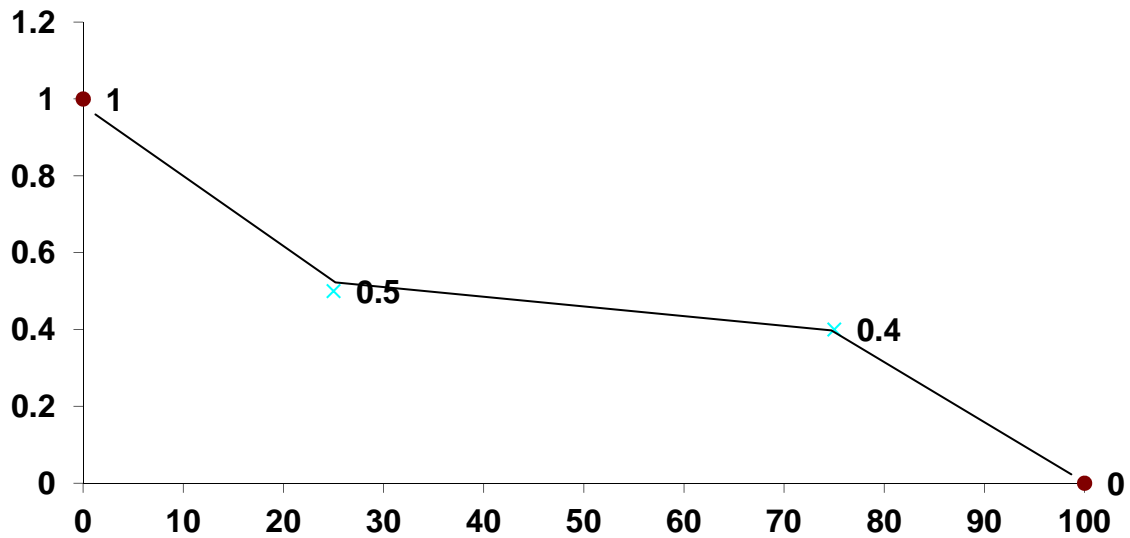
$$235 + P = \frac{255 + 11.175}{1.01925}$$

$$P = 261.15 - 235 = 26.15$$

$$1000 {}_{25}V = \frac{(272 + 26.15)(1.01925) - (0.025)(1000)}{1 - 0.025}$$

$$= 286$$

Question #200
 Answer: A



	Give n		Give n		Give n		Given
x	0	15	25	35	75	90	100
$s(x)$	1	0.70	0.50	0.48	0.4	0.16	0
		Linear Interpolation		Linear Interpolation		Linear Interpolation	

$${}_{55}q_{35} = 1 - \frac{s(90)}{s(35)} = 1 - \frac{0.16}{0.48} = \frac{32}{48} = 0.6667$$

$${}_{20|55}q_{15} = \frac{s(35) - s(90)}{s(15)} = \frac{0.48 - 0.16}{0.70} = \frac{32}{70} = 0.4571$$

$$\frac{{}_{20|55}q_{15}}{{}_{55}q_{35}} = \frac{0.4571}{0.6667} = 0.6856$$

Alternatively,

$$\begin{aligned}\frac{{}_{20|55}q_{15}}{{}_{55}q_{35}} &= \frac{{}_{20}p_{15} \times {}_{55}q_{35}}{{}_{55}q_{35}} = {}_{20}p_{15} = \frac{s(35)}{s(15)} \\ &= \frac{0.48}{0.70} \\ &= 0.6856\end{aligned}$$

Question #201

Answer: A

$$S_0(80) = \frac{1}{2} * (e^{(-0.16 * 50)} + e^{(-0.08 * 50)}) = 0.00932555$$

$$S_0(81) = \frac{1}{2} * (e^{(-0.16 * 51)} + e^{(-0.08 * 51)}) = 0.008596664$$

$$p_{80} = s(81) / s(80) = 0.008596664 / 0.00932555 = 0.9218$$

$$q_{80} = 1 - 0.9218 = 0.078$$

Alternatively (and equivalent to the above)

For non-smokers, $p_x = e^{-0.08} = 0.923116$

$${}_{50}p_x = 0.018316$$

For smokers, $p_x = e^{-0.16} = 0.852144$

$${}_{50}p_x = 0.000335$$

So the probability of dying at 80, weighted by the probability of surviving to 80, is

$$\frac{0.018316 \times (1 - 0.923116) + 0.000335 \times (1 - 0.852144)}{0.018316 + 0.000335} = 0.078$$

Question #202**Answer: B**

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
40	2000	20	60
41	1920	30	50
42	1840	40	

because $2000 - 20 - 60 = 1920$; $1920 - 30 - 50 = 1840$

Let premium = P

$$\text{EPV premiums} = \left(\frac{2000}{2000} + \frac{1920}{2000} v + \frac{1840}{2000} v^2 \right) P = 2.749P$$

$$\text{EPV benefits} = 1000 \left(\frac{20}{2000} v + \frac{30}{2000} v^2 + \frac{40}{2000} v^3 \right) = 40.41$$

$$P = \frac{40.41}{2.749} = 14.7$$

Question #203**Answer: A**

$$\begin{aligned}
\bar{a}_{30} &= \int_0^{10} e^{-0.08t} e^{-0.05t} dt + {}_{10}E_x \int_0^{\infty} e^{-0.08t} e^{-0.08t} dt \\
&= \int_0^{10} e^{-0.13t} dt + e^{-1.3} \int_0^{\infty} e^{-0.16t} dt \\
&= \left. \frac{-e^{-0.13t}}{0.13} \right|_0^{10} + \left(e^{-1.3} \right) \left. \frac{-e^{-0.16t}}{0.16} \right|_0^{\infty} \\
&= \frac{-e^{-1.3}}{0.13} + \frac{1}{0.13} + \frac{e^{-1.3}}{0.16} \\
&= 7.2992 \\
\bar{A}_{30} &= \int_0^{10} e^{-0.08t} e^{-0.05t} (0.05) dt + e^{-1.3} \int_0^{\infty} e^{-0.16t} (0.08) dt \\
&= 0.05 \left(\frac{1}{0.13} - \frac{e^{-1.3}}{0.13} \right) + (0.08) \frac{e^{-1.3}}{0.16} \\
&= 0.41606 \\
&= \bar{P}(\bar{A}_{30}) = \frac{\bar{A}_{30}}{\bar{a}_{30}} = \frac{0.41606}{7.29923} = 0.057
\end{aligned}$$

$$\begin{aligned}
\bar{a}_{40} &= \frac{1}{0.08 + 0.08} = \frac{1}{0.16} \\
\bar{A}_{40} &= 1 - \delta \bar{a}_{40} \\
&= 1 - (0.08/0.16) = 0.5 \\
{}_{10}\bar{V}(\bar{A}_{30}) &= \bar{A}_{40} - \bar{P}(\bar{A}_{30}) \bar{a}_{40} \\
&= 0.5 - \frac{(0.057)}{0.16} = 0.14375
\end{aligned}$$

Question #204**Answer: C**

Let T be the future lifetime of Pat, and $[T]$ denote the greatest integer in T . ($[T]$ is the same as K , the curtate future lifetime).

$$\begin{aligned}
L &= 100,000v^T - 1600\ddot{a}_{\overline{[T]+1}|} & 0 < T \leq 10 \\
&= 100,000v^T - 1600\ddot{a}_{\overline{10}|} & 10 < t \leq 20 \\
&\quad - 1600\ddot{a}_{\overline{10}|} & 20 < t
\end{aligned}$$

$$\begin{aligned}
\text{Minimum is } & -1600\ddot{a}_{\overline{10}|} & \text{when evaluated at } i = 0.05 \\
& = -12,973
\end{aligned}$$

Question #205 - Removed

Question #206

Answer: A

$P\ddot{a}_{x:\overline{3}|} = EPV$ (stunt deaths)

$$P \left[\frac{2500 + 2486/1.08 + 2466/(1.08)^2}{2500} \right] = 500000 \left(\frac{4/1.08 + 5/(1.08)^2 + 6/(1.08)^3}{2500} \right)$$

$$P(2.77) = 2550.68$$

$$\Rightarrow p = 921$$

Question #207

Answer: D

$$\begin{aligned} \dot{e}_{30:\overline{50}|} &= \frac{\int_{30}^{80} s(x) dx}{s(30)} = \frac{\int_{30}^{80} \left(1 - \frac{x^2}{10,000} \right) dx}{1 - \left(\frac{30}{100} \right)^2} \\ &= \frac{\left(x - \frac{x^3}{30,000} \right) \Big|_{30}^{80}}{0.91} \\ &= \frac{33.833}{0.91} \\ &= 37.18 \end{aligned}$$

Question #208

Answer: B

$$\begin{aligned} A_{60} &= v \times (p_{60} \times A_{61} + q_{60}) \\ &= (1/1.06) \times (0.98 \times 0.440 + 0.02) \\ &= 0.42566 \end{aligned}$$

$$\begin{aligned} \ddot{a}_{60} &= (1 - A_{60}) / d \\ &= (1 - 0.42566) / (0.06/1.06) \\ &= 10.147 \end{aligned}$$

$$\begin{aligned} 1000 {}_{10}V &= 1000A_{60} - 1000P_{50} \times \ddot{a}_{60} \\ &= 425.66 - 10.147 \times 25 \\ &= 172 \end{aligned}$$

Question #209**Answer: E**

Let Y_{65} = present value random variable for an annuity due of one on a single life age 65.

$$\text{Thus } E(Y_{65}) = \ddot{a}_{65}$$

Let Y_{75} = present value random variable for an annuity due of one on a single life age 75.

$$\text{Thus } E(Y_{75}) = \ddot{a}_{75}$$

$$\begin{aligned} E(X) &= 50(2)\ddot{a}_{65} + 30(1)\ddot{a}_{75} \\ &= 100(9.8969) + 30(7.217) = 1206.20 \end{aligned}$$

$$\text{Var}(X) = 50 \times 2^2 \text{Var}[Y_{65}] + 30(1)^2 \text{Var}[Y_{75}] = 200(13.2996) + 30(11.5339) = 3005.94$$

$$\text{where } \text{Var}[Y_{65}] = \frac{1}{d^2} ({}^2A_{65} - A_{65}^2) = \frac{1}{(0.05660)^2} [0.23603 - (0.4398)^2] = 13.2996$$

$$\text{and } \text{Var}[Y_{75}] = \frac{1}{d^2} ({}^2A_{75} - A_{75}^2) = \frac{1}{(0.05660)^2} [0.38681 - (0.59149)^2] = 11.5339$$

$$\begin{aligned} 95^{\text{th}} \text{ percentile} &= E(X) + 1.645\sqrt{\text{Var}[X]} \\ &= 1206.20 + 1.645(54.826) \\ &= 1296.39 \end{aligned}$$

Question #210**Answer: C**

$$\bar{a} = \int_0^{\infty} e^{-\delta t} \times e^{-\mu t} dt = \frac{1}{\delta + \mu}$$

$$\begin{aligned} EPV &= 50,000 \times \frac{1}{0.5} \int_{0.5}^1 \frac{1}{\delta + \mu} d\mu = 100,000 \times [\ln(\delta + 1) - \ln(\delta + 0.5)] \\ &= 100,000 \times \ln\left(\frac{0.045 + 1}{0.045 + 0.5}\right) \\ &= 65,099 \end{aligned}$$

Question #211 - Removed**Question #212 - Removed**

Question #213 - Removed

Question #214

Answer: A

Let π be the benefit premium at issue.

$$\begin{aligned}\pi &= 10,000 \frac{A_{45:\overline{20}|}}{\ddot{a}_{45:\overline{20}|}} = 10,000 \frac{[0.20120 - 0.25634(0.43980) + 0.25634]}{14.1121 - 0.25634(9.8969)} \\ &= 297.88\end{aligned}$$

The expected prospective loss at age 60 is

$$\begin{aligned}10,000 {}_{15}V_{45:\overline{20}|} &= 10,000 A_{60:\overline{5}|} - 297.88 \ddot{a}_{60:\overline{5}|} \\ &= 10,000 \times 0.7543 - 297.88 \times 4.3407 \\ &= 6250\end{aligned}$$

$$\text{where } A_{60:\overline{5}|}^1 = 0.36913 - 0.68756(0.4398) = 0.06674$$

$$A_{60:\overline{5}|}^{\frac{1}{2}} = 0.68756$$

$$A_{60:\overline{5}|} = 0.06674 + 0.68756 = 0.7543$$

$$\ddot{a}_{60:\overline{5}|} = 11.1454 - 0.68756 \times 9.8969 = 4.3407$$

After the change, expected prospective loss = $10,000 A_{60:\overline{5}|}^1 + (\text{Reduced Amount}) A_{60:\overline{5}|}^{\frac{1}{2}}$

Since the expected prospective loss is the same

$$6250 = (10,000)(0.06674) + (\text{Reduced Amount})(0.68756)$$

$$\text{Reduced Amount} = 8119$$

Question #215**Answer: D**

$$\bar{A}_x = \bar{A}_{x:\overline{5}|}^1 + {}_5E_x \bar{A}_{x+5:\overline{7}|}^1 + {}_{12}E_x \bar{A}_{x+12}$$

where

$${}_5E_x = e^{-5(0.04+0.02)} = 0.7408$$

$$\bar{A}_{x:\overline{5}|}^1 = \frac{0.04}{0.04+0.02} \times (1-0.7408) = 0.1728$$

$${}_7E_{x+5} = e^{-7(0.05+0.02)} = 0.6126$$

$$\bar{A}_{x+5:\overline{7}|}^1 = \left(\frac{0.05}{0.05+0.02} \right) (1-0.6126) = 0.2767$$

$${}_{12}E_x = {}_5E_x \times {}_7E_{x+5} = 0.7408 \times 0.6126 = 0.4538$$

$$\bar{A}_{x+12} = \frac{0.05}{0.05+0.03} = 0.625$$

$$\begin{aligned} \bar{A}_x &= 0.1728 + (0.7408)(0.2767) + (0.4538)(0.625) \\ &= 0.6614 \end{aligned}$$

Question #216**Answer: A**

EPV of Accidental death benefit and related settlement expense =

$$(2000 \times 1.05) \times \frac{0.004}{0.004 + 0.04 + 0.05} = 89.36$$

$$\text{EPV of other DB and related settlement expense} = (1000 \times 1.05) \times \frac{0.04}{0.094} = 446.81$$

EPV of Initial expense = 50

$$\text{EPV of Maintenance expense} = \frac{3}{0.094} = 31.91$$

$$\text{EPV of future premiums} = \frac{100}{0.094} = 1063.83$$

$$\begin{aligned} \text{EPV of } {}_0L &= 89.36 + 446.81 + 50 + 31.91 - 1063.83 \\ &= -445.75 \end{aligned}$$

Question #217**Answer: C**

One approach is to enumerate all the paths from state 1 to state 3 and evaluate their probabilities. The paths are:

$$113 - (0.8)(0.05) = 0.04$$

$$123 - (0.15)(0.05) = 0.0075$$

$$133 - (0.05)(1) = 0.05$$

The sum is 0.0975.

Alternatively, the required probability is in row 1 column 3 of the square of the transition probability matrix. Multiplying the first row of the matrix by each column produces the first row of the square, [0.6475 0.255 0.0975]. While only the third calculation is required, doing all three provides a check as the numbers must sum to one.

The number of the 50 members who will be in state 3 has a binomial distribution, so the variance is $50(0.0975)(0.9025) = 4.40$.

Question #218**Answer: C**

$$Q_0 = \begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad Q_0 \times Q_1 = \begin{pmatrix} 0.36 & 0.18 & 0.46 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q_0 \times Q_1 \times Q_2 = \begin{pmatrix} 0 & 0.108 & 0.892 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Note that after four transitions, everyone must be in state 2. For premiums, the probability is the sum of the first two entries in row 1 (with probability 1 that a premium is paid at time 0). Then,

$$APV(\text{Premiums}) = 1 + 0.9v + 0.54v^2 + 0.108v^3 = 2.35.$$

For benefits, the probabilities are the second entry in row 1. Then,

$$APV(\text{Benefits}) = 4(0.3v + 0.18v^2 + 0.108v^3) = 2.01$$

$$\text{Difference} = 2.35 - 2.01 = 0.34$$

Note that only the first row of each of the products needs to be calculated.

It is also possible to enumerate the transitions that produce the required events, though for this problem the list is relatively long:

Premium at time 1:

00 (0.6)

01 (0.3)

Total = 0.9

Premium at time 2:

000 (0.6)(0.6) = 0.36

001 (0.6)(0.3) = 0.18

Total = 0.54

Premium at time 3:

0001 (0.6)(0.6)(0.3) = 0.108

Benefit at time 1:

01 (0.3)

Benefit at time 2:

001 (0.6)(0.3) = 0.18

Benefit at time 3:

0001 (0.6)(0.6)(0.3) = 0.108

Question #219**Answer: E**

$${}_{0.25|1.5}q_x = {}_{0.25}p_x - {}_{1.75}p_x$$

Let μ be the force of mortality in year 1, so 3μ is the force of mortality in year 2.

Probability of surviving 2 years is 10%

$$\begin{cases} 0.10 = p_x p_{x+1} = e^{-\mu} e^{-3\mu} = e^{-4\mu} \\ \mu = \frac{\ln(0.1)}{4} = 0.5756 \end{cases}$$

$${}_{0.25}p_x = e^{-\frac{1}{4}(0.5756)} = 0.8660$$

$${}_{1.75}p_x = p_x \times {}_{0.75}p_{x+1} = e^{-\mu} e^{-\frac{3}{4}(3\mu)} = e^{-\frac{13}{4}(0.5756)} = 0.1540$$

$${}_{1.5}q_{x+0.25} = \frac{{}_{0.25|1.5}q_x}{{}_{0.25}p_x} = \frac{{}_{0.25}p_x - {}_{1.75}p_x}{{}_{0.25}p_x} = \frac{0.866 - 0.154}{0.866} = 0.82$$

Question #220**Answer: C**

$$\begin{aligned} \mu_x^{NS} &= \frac{500}{500(110-x)} = \frac{1}{110-x} \\ &= \frac{1}{2} \mu_x^S \Rightarrow \mu_x^S = \frac{2}{110-x} \end{aligned}$$

$$\Rightarrow l_x^S = l_0^S (110-x)^2 \text{ [see note below]}$$

$$\text{Thus } {}_t p_{20}^S = \frac{l_{20+t}^S}{l_{20}^S} = \frac{(90-t)^2}{90^2}$$

$${}_t p_{25}^{NS} = \frac{l_{25+t}^{NS}}{l_{25}^{NS}} = \frac{(85-t)}{85}$$

$$\begin{aligned}
\ddot{e}_{20:25} &= \int_0^{85} {}_t p_{20:25} dt \\
&= \int_0^{85} {}_t p_{20}^S {}_t p_{25}^{NS} dt = \int_0^{85} \frac{(90-t)^2 (85-t)}{(90)^2 \cdot 85} dt \\
&= \frac{1}{688,500} \int_0^{85} (90-t)^2 (90-t-5) dt \\
&= \frac{1}{688,500} \left[\int_0^{85} (90-t)^3 dt - 5 \int_0^{85} (90-t)^2 dt \right] \\
&= \frac{1}{688,500} \left[\frac{-(90-t)^4}{4} + \frac{5(90-t)^3}{3} \right]_0^{85} \\
&= \frac{1}{688,500} [-156.25 + 208.33 + 16,402,500 - 1,215,000] \\
&= 22.1
\end{aligned}$$

[There are other ways to evaluate the integral, leading to the same result].

The $S_0(x)$ form is derived as $S_0(x) = e^{-\int_0^x \left(\frac{2}{110-t}\right) dt} = e^{2\ln(110-t)} \Big|_0^x = \left(\frac{110-x}{110}\right)^2$

The l_x form is equivalent.

Question #221

Answer: B

$$\ddot{a}_{30:\overline{20}|} = \ddot{a}_{30:\overline{10}|} + {}_{10}E_{30} \times \ddot{a}_{40:\overline{10}|}$$

$$15.0364 = 8.7201 + {}_{10}E_{30} \times 8.6602$$

$${}_{10}E_{30} = (15.0364 - 8.7201) / 8.6602 = 0.72935$$

Expected present value (EPV) of benefits =

$$= 1000 \times A_{40:\overline{10}|}^1 + 2000 \times {}_{10}E_{30} \times A_{50:\overline{10}|}^1$$

$$= 16.66 + 2 \times 0.72935 \times 32.61 = 64.23$$

$$\text{EPV of premiums} = \pi \times \ddot{a}_{30:\overline{10}|} + 2\pi \times 0.72935 \times \ddot{a}_{40:\overline{10}|}$$

$$= \pi \times 8.7201 + 2 \times \pi \times 0.72935 \times 8.6602$$

$$= 21.3527\pi$$

$$\pi = 64.23 / 21.3527 = 3.01$$

Question #222**Answer: A**

$${}_{15}V = P_{25} \ddot{s}_{25:\overline{15}|} - \frac{A_{25:\overline{15}|}^1}{{}_{15}E_{25}} \quad (\text{this is the retrospective reserve calculation})$$

$$P_{25:\overline{15}|}^1 = P_{25:\overline{15}|} - P_{25:\overline{15}|}^1 = 0.05332 - 0.05107 = 0.00225$$

$$= \frac{A_{25:\overline{15}|}^1}{\ddot{a}_{25:\overline{15}|}}$$

$$0.05107 = P_{25:\overline{15}|}^1 = \frac{{}_{15}E_{25}}{\ddot{a}_{25:\overline{15}|}} = \frac{1}{\ddot{s}_{25:\overline{15}|}}$$

$$\frac{A_{25:\overline{15}|}^1}{{}_{15}E_{25}} = \frac{A_{25:\overline{15}|}^1 / \ddot{a}_{25:\overline{15}|}}{{}_{15}E_{25} / \ddot{a}_{25:\overline{15}|}} = \frac{0.00225}{0.05107} = 0.04406$$

$$P_{25} \ddot{s}_{25:\overline{15}|} = \frac{0.01128}{0.05107} = 0.22087$$

$$25,000 {}_{15}V = 25,000(0.22087 - 0.04406) = 25,000(0.17681) = 4420$$

There are other ways of getting to the answer, for example writing

A: the retrospective reserve formula for ${}_{15}V$.

B: the retrospective reserve formula for the 15th benefit reserve for a 15-year term insurance issued to (25), which = 0

Subtract B from A to get

$$(P_{25} - P_{25:\overline{15}|}^1) \ddot{s}_{25:\overline{15}|} = {}_{15}V$$

Question #223**Answer: C**

ILT:

$$\text{We have } p_{70} = 6,396,609 / 6,616,155 = 0.96682$$

$${}_2p_{70} = 6,164,663 / 6,616,155 = 0.93176$$

$$e_{70:\overline{2}|} = 0.96682 + 0.93176 = 1.89858$$

$$\text{CF: } 0.93176 = {}_2p_{70} = e^{-2\mu} \Rightarrow \mu = 0.03534$$

$$\text{Hence } e_{70:\overline{2}|} = p_{70} + {}_2p_{71} = e^{-\mu} + e^{-2\mu} = 1.89704$$

DM: Since l_{70} and ${}_2p_{70}$ for the DM model equal the ILT, therefore l_{72} for the DM model

also equals the ILT. For DM we have $l_{70} - l_{71} = l_{71} - l_{72} \Rightarrow l_{71}^{(DM)} = 6,390,409$

Hence $e_{70:\overline{2}|} = 6,390,409 / 6,616,155 + 6,164,663 / 6,616,155 = 1.89763$

So the correct order is CF < DM < ILT

You could also work with p 's instead of l 's. For example, with the ILT,

$$p_{70} = (1 - 0.03318) = 0.96682$$

$${}_2p_{70} = (0.96682)(1 - 0.03626) = 0.93176$$

Note also, since $e_{70:\overline{2}|} = p_{70} + {}_2p_{70}$, and ${}_2p_{70}$ is the same for all three, you could just order p_{70} .

Question #224

Answer: D

$$l_{60}^{(\tau)} = 1000$$

$$l_{61}^{(\tau)} = 1000(0.99)(0.97)(0.90) = 864.27$$

$$d_{60}^{(\tau)} = 1000 - 864.27 = 135.73$$

$$d_{60}^{(3)} = 135.73 \times \frac{-\ln(0.9)}{-\ln[(0.99)(0.97)(0.9)]} = 135.73 \times \frac{0.1054}{0.1459} = 98.05$$

$$l_{62}^{(\tau)} = 864.27(0.987)(0.95)(0.80) = 648.31$$

$$d_{61}^{(\tau)} = 864.27 - 648.31 = 215.96$$

$$d_{61}^{(3)} = 215.96 \times \frac{-\ln(0.80)}{-\ln[(0.987)(0.95)(0.80)]} = 215.96 \times \frac{0.2231}{0.2875} = 167.58$$

So $d_{60}^{(3)} + d_{61}^{(3)} = 98.05 + 167.58 = 265.63$

Question #225**Answer: B**

$${}_tP_{40} = e^{-0.05t}$$

$${}_tP_{50} = (60-t)/60$$

$$\mu_{50+t} = 1/(60-t)$$

$$\begin{aligned} \int_0^{10} {}_tP_{40:50} \mu_{50+t} dt &= \int_0^{10} \frac{e^{-0.05t}}{60} dt = -\frac{1}{60} \frac{e^{-0.05t}}{(0.05)} \Big|_0^{10} \\ &= \frac{20}{60} (1 - e^{-0.5}) = 0.13115 \end{aligned}$$

Question #226**Answer: A**

$$\text{Actual payment (in millions)} = \frac{3}{1.1} + \frac{5}{1.1^2} = 6.860$$

$$q_3 = 1 - \frac{0.30}{0.60} = 0.5$$

$${}_1q_3 = \frac{0.30 - 0.10}{0.60} = 0.333$$

$$\text{Expected payment} = 10 \left(\frac{0.5}{1.1} + \frac{0.333}{1.1^2} \right) = 7.298$$

$$\text{Ratio } \frac{6.860}{7.298} = 94\%$$

Question #227**Answer: E**

At duration 1

K_x	${}_1L$	Prob
1	$v - P_{x:2}^1$	q_{x+1}
>1	$0 - P_{x:2}^1$	$1 - q_{x+1}$

$$\text{So } \text{Var}({}_1L) = v^2 q_{x+1} (1 - q_{x+1}) = 0.1296$$

That really short formula takes advantage of $Var(aX + b) = a^2Var(X)$, if a and b are constants.

Here $a = v$; $b = P_{x:\overline{2}|}^1$; X is binomial with $p(X = 1) = q_{x+1}$.

Alternatively, evaluate $P_{x:\overline{2}|}^1 = 0.1303$

$${}_1L = 0.9 - 0.1303 = 0.7697 \text{ if } K_x = 1$$

$${}_1L = 0 - 0.1303 = -0.1303 \text{ if } K_x > 1$$

$$E({}_1L) = (0.2)(0.7697) + (0.8)(-0.1303) = 0.0497$$

$$E({}_1L^2) = (0.2)(0.7697)^2 + (0.8)(-0.1303)^2 = 0.1320$$

$$Var({}_1L) = 0.1320 - (0.0497)^2 = 0.1295$$

Question #228

Answer: C

$$\bar{P}(\bar{A}_x) = \frac{\bar{A}_x}{\bar{a}_x} = \frac{\bar{A}_x}{\left(\frac{1 - \bar{A}_x}{\delta}\right)} = \frac{\delta \bar{A}_x}{1 - \bar{A}_x} = \frac{(0.1)(\frac{1}{3})}{(1 - \frac{1}{3})} = 0.05$$

$$Var(L) = \left(1 + \frac{\bar{P}(\bar{A}_x)}{\delta}\right)^2 ({}^2\bar{A}_x - \bar{A}_x^2)$$

$$\frac{1}{5} = \left(1 + \frac{0.05}{0.10}\right)^2 ({}^2\bar{A}_x - \bar{A}_x^2)$$

$$({}^2\bar{A}_x - \bar{A}_x^2) = 0.08888$$

$$Var[L'] = \left(1 + \frac{\pi}{\delta}\right)^2 ({}^2\bar{A}_x - \bar{A}_x^2)$$

$$\frac{16}{45} = \left(1 + \frac{\pi}{0.1}\right)^2 (0.08888)$$

$$\left(1 + \frac{\pi}{0.1}\right)^2 = 4$$

$$\pi = 0.1$$

Question #229**Answer: E**

Seek g such that $\Pr\left\{\bar{a}_{\overline{T(40)|}} > g\right\} = 0.25$

$\bar{a}_{\overline{T|}}$ is a strictly increasing function of T .

$$\Pr\{T(40) > 60\} = 0.25 \text{ since } {}_{60}p_{40} = \frac{100-40}{120-40} = 0.25$$

$$\therefore \Pr\left\{\bar{a}_{\overline{T(40)|}} > \bar{a}_{\overline{60|}}\right\} = 0.25$$

$$g = \bar{a}_{\overline{60|}} = 19.00$$

Question 230**Answer: B**

$$A_{\overline{51:9|}} = 1 - d\ddot{a}_{\overline{51:9|}} = 1 - \left(\frac{0.05}{1.05}\right)(7.1) = 0.6619$$

$${}_{11}V = (2000)(0.6619) - (100)(7.1) = 613.80$$

$$({}_{10}V + P)(1.05) = {}_{11}V + q_{50}(2000 - {}_{11}V)$$

$$({}_{10}V + 100)(1.05) = 613.80 + (0.011)(2000 - 613.80)$$

$${}_{10}V = 499.09$$

$$\text{where } q_{50} = (0.001)(10) + (0.001) = 0.011$$

Alternatively, you could have used recursion to calculate $A_{\overline{50:10|}}$ from $A_{\overline{51:9|}}$, then

$$\ddot{a}_{\overline{50:10|}}$$

from $A_{\overline{50:10|}}$, and used the prospective reserve formula for ${}_{10}V$.

Question #231**Answer: C**

$$1000A_{81} = (1000A_{80})(1+i) - q_{80}(1000 - A_{81})$$

$$689.52 = (679.80)(1.06) - q_{80}(1000 - 689.52)$$

$$q_{80} = \frac{720.59 - 689.52}{310.48} = 0.10$$

$$q_{[80]} = 0.5q_{80} = 0.05$$

$$\begin{aligned} 1000A_{[80]} &= 1000vq_{[80]} + vP_{[80]}1000A_{81} \\ &= 1000 \times \frac{0.05}{1.06} + 689.52 \times \frac{0.95}{1.06} = 665.14 \end{aligned}$$

Question #232**Answer: D**

	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
42	776	8	16
43	752	8	16

$l_{42}^{(\tau)}$ and $l_{43}^{(\tau)}$ came from $l_{x+1}^{(\tau)} = l_x^{(\tau)} - d_x^{(1)} - d_x^{(2)}$

$$EPV \text{ Benefits} = \frac{2000(8v + 8v^2) + 1000(16v + 16v^2)}{776} = 76.40$$

$$EPV \text{ Premiums} = 34 \left(\frac{776 + 752v}{776} \right) = (34)(1.92) = 65.28$$

$${}_2V = 76.40 - 65.28 = 11.12$$

Question #233**Answer: B**

$$p_{xx} = 1 - q_{xx} = 0.96$$

$$p_x = \sqrt{0.96} = 0.9798$$

$$p_{x+1:x+1} = 1 - q_{x+1:x+1} = 0.99$$

$$p_{x+1} = \sqrt{0.99} = 0.995$$

$$\begin{aligned}\ddot{a}_x &= 1 + vp_x + v^2 \times {}_2p_x = 1 + \frac{0.9798}{1.05} + \frac{(0.9798)(0.995)}{1.05^2} \\ &= 2.8174\end{aligned}$$

$$\ddot{a}_{xx} = 1 + vp_{xx} + v^2 \times {}_2p_{xx} = 1 + \frac{0.96}{1.05} + \frac{(0.96)(0.99)}{1.05^2} = 2.7763$$

$$\begin{aligned}\text{EPV} &= 2000\ddot{a}_x + 2000\ddot{a}_x + 6000\ddot{a}_{xx} \\ &= (4000)(2.8174) + (6000)(2.7763) \\ &= 27,927\end{aligned}$$

Notes: The solution assumes that the future lifetimes are identically distributed. The precise description of the benefit would be a special 3-year temporary life annuity-due.

Question #234**Answer: B**

$${}_tP_x^{(1)} \mu_x^{(1)}(t) = q_x^{(1)} = 0.20$$

$${}_tP_x^{(2)} = 1 - tq_x^{(2)} = 1 - 0.08t$$

$${}_tP_x^{(3)} = 1 - tq_x^{(3)} = 1 - 0.125t$$

$$q_x^{(1)} = \int_0^1 {}_tP_x^{(2)} {}_tP_x^{(3)} {}_tP_x^{(1)} \mu_{x+t}^{(1)} dt = \int_0^1 {}_tP_x^{(2)} {}_tP_x^{(3)} {}_tP_x^{(1)} \mu_{x+t}^{(1)} dt$$

$$= \int_0^1 (1 - 0.08t)(1 - 0.125t)(0.20) dt$$

$$= 0.2 \int_0^1 (1 - 0.205t + 0.01t^2) dt$$

$$= 0.2 \left[t - \frac{0.205t^2}{2} + \frac{0.01t^3}{3} \right]_0^1$$

$$= (0.2) \left[1 - 0.1025 + \frac{0.01}{3} \right] = 0.1802$$

Question #235**Answer: B**

$${}_1AS = \frac{(G - 0.1G - (1.50)(1))(1.06) - 1000q_{40}^{(d)} - 2.93 \times q_{40}^{(w)}}{1 - q_{40}^{(d)} - q_{40}^{(w)}}$$

$$= \frac{(0.9G - 1.50)(1.06) - (1000)(0.00278) - (2.93)(0.2)}{1 - 0.00278 - 0.2}$$

$$= \frac{0.954G - 1.59 - 2.78 - 0.59}{0.79722}$$

$$= 1.197G - 6.22$$

$${}_2AS = \frac{({}_1AS + G - 0.1G - (1.50)(1))(1.06) - 1000q_{41}^{(d)} - {}_2CV \times q_{41}^{(w)}}{1 - q_{41}^{(d)} - q_{41}^{(w)}}$$

$$= \frac{(1.197G - 6.22 + G - 0.1G - 1.50)(1.06) - (1000)(0.00298) - {}_2CV \times 0}{1 - 0.00298 - 0}$$

$$= \frac{(2.097G - 7.72)(1.06) - 2.98}{0.99702}$$

$$= 2.229G - 11.20$$

$$2.229G - 11.20 = 24$$

$$G = 15.8$$

Question #236**Answer: A**

$$\begin{aligned} {}_5AS &= \frac{({}_4AS + G(1 - c_4) - e_4)(1 + i) - 1000q_{x+4}^{(1)} - {}_5CV \times q_{x+4}^{(2)}}{1 - q_{x+4}^{(1)} - q_{x+4}^{(2)}} \\ &= \frac{(396.63 + 281.77(1 - 0.05) - 7)(1 + i) - 90 - 572.12 \times 0.26}{1 - 0.09 - 0.26} \\ &= \frac{(657.31)(1 + i) - 90 - 148.75}{0.65} \\ &= 694.50 \end{aligned}$$

$$(657.31)(1 + i) = 90 + 148.75 + (0.65)(694.50)$$

$$1 + i = \frac{690.18}{657.31} = 1.05$$

$$i = 0.05$$

Question #237 - Removed**Question #238 - Removed**

Question #239**Answer: B**

Let P denote the annual premium

The EPV of benefits is $25,000\bar{A}_{x:\overline{20}|} = 25,000(0.4058) = 10,145$.

The EPV of premiums is $P\ddot{a}_{x:\overline{20}|} = 12.522P$

The EPV of expenses is

$$(0.25 - 0.05)P + 0.05P\ddot{a}_{x:\overline{20}|} + \left[(2.00 - 0.50) + 0.50\ddot{a}_{x:\overline{20}|} \right] \frac{25,000}{1,000} + (15 - 3) + 3\ddot{a}_{x:\overline{20}|}$$

$$= 0.20P + 0.6261P + 194.025 + 12 + 37.566 = 0.8261P + 243.591$$

Equivalence principle:

$$12.522P = 10,145 + 0.8261P + 243.591$$

$$P = \frac{10,389.591}{12.522 - 0.8261}$$

$$= 888.31$$

Question #240**Answer: D**

Let G denote the premium.

Expected present value (EPV) of benefits = $1000\bar{A}_{40:\overline{20}|}$

EPV of premiums = $G\ddot{a}_{40:\overline{10}|}$

$$\text{EPV of expenses} = (0.04 + 0.25)G + 10 + (0.04 + 0.05)G a_{40:\overline{9}|} + 5a_{40:\overline{19}|}$$

$$= 0.29G + 10 + 0.09G a_{40:\overline{9}|} + 5a_{40:\overline{19}|}$$

$$= 0.2G + 10 + 0.09G\ddot{a}_{40:\overline{10}|} + 5a_{40:\overline{19}|}$$

(The above step is getting an $\ddot{a}_{40:\overline{10}|}$ term since all the answer choices have one. It could equally well have been done later on).

Equivalence principle:

$$G \ddot{a}_{40:\overline{10}|} = 1000 \bar{A}_{40:\overline{20}|} + 0.2G + 10 + 0.09G \ddot{a}_{40:\overline{10}|} + 5a_{40:\overline{19}|}$$

$$G(\ddot{a}_{40:\overline{10}|} - 0.2 - 0.09\ddot{a}_{40:\overline{10}|}) = 1000\bar{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{19}|}$$

$$G = \frac{1000\bar{A}_{40:\overline{20}|} + 10 + 5a_{40:\overline{19}|}}{0.91\ddot{a}_{40:\overline{10}|} - 0.2}$$

Question #241 - Removed

Question #242

Answer: C

$$\begin{aligned} {}_{11}AS &= \frac{({}_{10}AS + G - c_{10}G - e_{10})(1+i) - 10,000q_{x+10}^{(d)} - {}_{11}CV q_{x+10}^{(w)}}{1 - q_{x+10}^{(d)} - q_{x+10}^{(w)}} \\ &= \frac{(1600 + 200 - (0.04)(200) - 70)(1.05) - (10,000)(0.02) - (1700)(0.18)}{1 - 0.02 - 0.18} \\ &= \frac{1302.1}{0.8} \\ &= 1627.63 \end{aligned}$$

Question #243**Answer: E**

The benefit reserve at the end of year 9 is the certain payment of the benefit one year from now, less the premium paid at time 9. Thus, it is $10,000v - 76.87$.

The gross premium reserve adds expenses paid at the beginning of the tenth year and subtracts the gross premium rather than the benefit premium. Thus it is $10,000v + 5 + 0.1G - G$ where G is the gross premium.

Then,

$$10,000v - 76.87 - (10,000v + 5 - 0.9G) = 1.67$$

$$0.9G - 81.87 = 1.67$$

$$0.9G = 83.54$$

$$G = 92.82$$

Question #244**Answer: C**

$${}_4AS = \frac{({}_3AS + G - c_4G - e_4)(1+i) - 1000q_{x+3}^{(d)} - {}_4CVq_{x+3}^{(w)}}{1 - q_{x+3}^{(d)} - q_{x+3}^{(w)}}$$

Plugging in the given values:

$${}_4AS = \frac{(25.22 + 30 - (0.02)(30) - 5)(1.05) - 1000(0.013) - 75(0.05)}{1 - 0.013 - 0.05}$$

$$= \frac{35.351}{0.937}$$

$$= 37.73$$

With higher expenses and withdrawals:

$$\begin{aligned}
 {}_4AS^{\text{revised}} &= \frac{25.22 + 30 - (1.2)((0.02)(30) + 5)(1.05) - 1000(0.013) - 75(1.2)(0.05)}{1 - 0.013 - (1.2)(0.05)} \\
 &= \frac{(48.5)(1.05) - 13 - 4.5}{0.927} \\
 &= \frac{33.425}{0.927} \\
 &= 36.06
 \end{aligned}$$

$$\begin{aligned}
 {}_4AS - {}_4AS^{\text{revised}} &= 37.73 - 36.06 \\
 &= 1.67
 \end{aligned}$$

Question #245

Answer: E

Let G denote the gross premium.

EPV (expected present value) of benefits = $1000 {}_{10|20}A_{30}$.

EPV of premiums = $G \ddot{a}_{30:\overline{5}|}$.

$$\begin{aligned}
 \text{EPV of expenses} &= (0.05 + 0.25)G + 20 \text{ first year} \\
 &\quad + [(0.05 + 0.10)G + 10] a_{30:\overline{4}|} \text{ years 2-5} \\
 &\quad + 10 {}_5\ddot{a}_{35:\overline{4}|} \text{ years 6-10 (there is no premium)} \\
 &= 0.30G + 0.15G a_{30:\overline{4}|} + 20 + 10 a_{30:\overline{4}|} + 10 {}_5\ddot{a}_{30:\overline{5}|} \\
 &= 0.15G + 0.15G \ddot{a}_{30:\overline{5}|} + 20 + 10 a_{30:\overline{9}|}
 \end{aligned}$$

(The step above is motivated by the form of the answer. You could equally well put it that form later).

Equivalence principle:

$$\begin{aligned}
 G \ddot{a}_{30:\overline{5}|} &= 1000 {}_{10|20}A_{30} + 0.15G + 0.15G \ddot{a}_{30:\overline{5}|} + 20 + 10 a_{30:\overline{9}|} \\
 G &= \frac{(1000 {}_{10|20}A_{30} + 20 + 10 a_{30:\overline{9}|})}{(1 - 0.15) \ddot{a}_{30:\overline{5}|} - 0.15} \\
 &= \frac{(1000 {}_{10|20}A_{30} + 20 + 10 a_{30:\overline{9}|})}{0.85 \ddot{a}_{30:\overline{5}|} - 0.15}
 \end{aligned}$$

Question #246**Answer: E**Let G denote the gross premium

EPV (expected present value) of benefits

$$\begin{aligned}
&= (0.1)(3000)v + (0.9)(0.2)(2000)v^2 + (0.9)(0.8)1000v^2 \\
&= \frac{300}{1.04} + \frac{360}{1.04^2} + \frac{720}{1.04^2} = 1286.98
\end{aligned}$$

EPV of premium = G

$$\begin{aligned}
\text{EPV of expenses} &= 0.02G + 0.03G + 15 + (0.9)(2)v \\
&= 0.05G + 16.73
\end{aligned}$$

Equivalence principle: $G = 1286.98 + 0.05G + 16.73$

$$G = \frac{1303.71}{1 - 0.05} = 1372.33$$

Question #247**Answer: C**

EPV (expected present value) of benefits = 3499 (given)

$$\begin{aligned}
\text{EPV of premiums} &= G + (0.9)(G)v \\
&= G + \frac{0.9G}{1.05} = 1.8571G
\end{aligned}$$

EPV of expenses, except settlement expenses,

$$\begin{aligned}
&= [25 + (4.5)(10) + 0.2G] + (0.9)[10 + (1.5)(10) + 0.1G]v + (0.9)(0.85)[10 + (1.5)(10)]v^2 \\
&= 70 + 0.2G + \frac{0.9(25 + 0.1G)}{1.05} + \frac{0.765(25)}{1.05^2} \\
&= 108.78 + 0.2857G
\end{aligned}$$

Settlement expenses are $20 + (1)(10) = 30$, payable at the same time the death benefit is paid.

$$\begin{aligned} \text{So EPV of settlement expenses} &= \left(\frac{30}{10,000} \right) \text{EPV of benefits} \\ &= (0.003)(3499) \\ &= 10.50 \end{aligned}$$

Equivalence principle:

$$\begin{aligned} 1.8571G &= 3499 + 108.78 + 0.2857G + 10.50 \\ G &= \frac{3618.28}{1.8571 - 0.2857} = 2302.59 \end{aligned}$$

Question #248

Answer: D

$$\begin{aligned} \ddot{a}_{50:\overline{20}|} &= \ddot{a}_{50} - {}_{20}E_{50} \ddot{a}_{70} \\ &= 13.2668 - (0.23047)(8.5693) \\ &= 11.2918 \end{aligned}$$

$$\begin{aligned} A_{50:\overline{20}|} &= 1 - d \ddot{a}_{50:\overline{20}|} = 1 - \left(\frac{0.06}{1.06} \right) (11.2918) \\ &= 0.36084 \end{aligned}$$

$$\begin{aligned} \text{Expected present value (EPV) of benefits} &= 10,000 A_{50:\overline{20}|} \\ &= 3608.40 \end{aligned}$$

$$\begin{aligned} \text{EPV of premiums} &= 495 \ddot{a}_{50:\overline{20}|} \\ &= 5589.44 \end{aligned}$$

$$\begin{aligned} \text{EPV of expenses} &= (0.35)(495) + 20 + (15)(10) + [(0.05)(495) + 5 + (1.50)(10)] a_{50:\overline{19}|} \\ &= 343.25 + (44.75)(11.2918 - 1) \\ &= 803.81 \end{aligned}$$

$$\begin{aligned} \text{EPV of amounts available for profit and contingencies} &= \text{EPV premium} - \text{EPV benefits} - \text{APV expenses} \\ &= 5589.44 - 3608.40 - 803.81 \\ &= 1177.23 \end{aligned}$$

Question #249**Answer: B**

$$\begin{aligned}
q_{xy}^1 &= \int_0^1 {}_t p_{xy} \mu_{x+t} dt = \int_0^1 {}_t p_x {}_t p_y \mu_{x+t} dt \\
&= \int_0^1 q_x e^{-0.25t} dt \quad (\text{under UDD, } {}_t p_x \mu_{x+t} = q_x) \\
0.125 &= q_x (-4e^{-0.25t}) \Big|_0^1 = q_x (4)(1 - e^{-0.25}) = 0.8848q_x \\
q_x &= 0.1413
\end{aligned}$$

Question #250**Answer: C**

$$\begin{aligned}
{}_2 p_{[x]+1}^{11} &= p_{[x]+1}^{11} p_{[x]+2}^{11} + p_{[x]+1}^{12} p_{[x]+2}^{21} \\
&= \left(0.7 + \frac{0.1}{2}\right) \left(0.7 + \frac{0.1}{3}\right) + \left(0.3 - \frac{0.1}{2}\right) \left(0.4 - \frac{0.2}{3}\right) \\
&= 0.75(0.7333) + 0.25(0.3333) = 0.6333
\end{aligned}$$

Note that Anne might have changed states many times during each year, but the annual transition probabilities incorporate those possibilities.

Questions #251-260 – Removed**Question #261****Answer: A**

The insurance is payable on the death of (y) provided (x) is already dead.

$$\begin{aligned}
E(Z) &= \bar{A}_{xy}^2 = \int_0^{\infty} e^{-\delta t} {}_t q_x {}_t p_y \mu_{y+t} dt \\
&= \int_0^{\infty} e^{-0.06t} (1 - e^{-0.07t}) e^{-0.09t} 0.09 dt \\
&= 0.09 \int_0^{\infty} e^{-0.15t} - e^{-0.22t} dt \\
&= 0.09 \left(\frac{1}{0.15} - \frac{1}{0.22} \right) = 0.191
\end{aligned}$$

Question #262**Answer: C**

$${}_tP_x = \frac{95-x-t}{95-x}, \quad \mu_{x+t} = \frac{1}{95-x-t}, \quad {}_tP_y = e^{-\mu t}$$

$$\begin{aligned} \Pr(x \text{ dies within } n \text{ years and before } y) &= \int_0^n {}_tP_x {}_tP_y \mu_{x+t} dt \\ &= \int_0^n \frac{95-x-t}{95-x} e^{-\mu t} \frac{1}{95-x-t} dt = \frac{1}{95-x} \int_0^n e^{-\mu t} dt = \frac{1-e^{-\mu n}}{\mu(95-x)} \end{aligned}$$

Question #263**Answer: A**

$$\begin{aligned} {}_{0.25}q_{30.5:40.5}^2 &= \int_0^{0.25} {}_tP_{30.5} \mu_{30.5+t} {}_tq_{40.5} dt \\ &= \int_0^{0.25} \frac{0.4}{1-0.5(0.4)} \frac{0.6t}{1-0.5(0.6)} dt \\ &= \frac{0.4(0.6)}{0.8(0.7)} \frac{t^2}{2} \Big|_0^{0.25} = 0.0134 \end{aligned}$$

Question #264 – Removed**Question #265****Answer: D**

$$\begin{aligned} {}_tP_x &= \exp\left[-\int_0^t 5r dr\right] = e^{-2.5t^2} \\ {}_tP_y &= \exp\left[-\int_0^t r dr\right] = e^{-0.5t^2} \\ q_{x,y}^1 &= \int_0^1 {}_tP_y {}_tP_x \mu_{x+t} dt = \int_0^1 e^{-0.5t^2} e^{-2.5t^2} 5t dt = 5 \int_0^1 e^{-3t^2} t dt \\ &= \frac{5}{6} e^{-3t^2} \Big|_0^1 = \frac{5}{6} (1 - e^{-3}) = 0.7918 \end{aligned}$$

Question #266**Answer: B**

$$G = \int_0^5 \frac{1}{30} \frac{t}{25} dt = \frac{t^2}{30(25)(2)} \Big|_0^5 = \frac{1}{60}$$

$$H = {}_5P_{80:85} - {}_{10}P_{80:85} = \frac{25}{30} \frac{20}{25} - \frac{20}{30} \frac{15}{25} = \frac{200}{750} = \frac{4}{15}$$

$$G + H = \frac{1}{60} + \frac{16}{60} = \frac{17}{60} = 0.2833$$

Question #267**Answer: D**

$$S_0(t) = \exp\left[-\int_0^t (80-x)^{-0.5} dx\right] = \exp\left[2(80-x)^{0.5} \Big|_0^t\right] = \exp\left[2((80-t)^{0.5} - 80^{0.5})\right]$$

$$F = S_0(10.5) = \exp\left[2(69.5^{0.5} - 80^{0.5})\right] = 0.29665$$

$$S_0(10) = 0.31495$$

$$S_0(11) = 0.27935$$

$$G = S_0(10.5)^{\exp} = [0.31495(0.27935)]^{0.5} = 0.29662$$

$$F - G = 0.00003$$

Question #268**Answer: A**

$$\begin{aligned} E(Z) &= 500 \int_0^4 0.2(1-0.25t) dt + 1000 \int_0^4 0.25(0.2t) dt \\ &= 500(0.2) \left(t - 0.125t^2\right) \Big|_0^4 + 1000(0.25)(0.1t^2) \Big|_0^4 \\ &= 100(4-2) + 250(1.6) = 600 \end{aligned}$$

Question #269**Answer: A**

$${}_{10}q_{\overline{30:50}} = {}_{10}q_{30} {}_{10}q_{50} = (1 - e^{-0.5})(1 - e^{-0.5}) = 0.1548$$

Question #270**Answer: C**

$$\ddot{e}_{\overline{30:50}} = \ddot{e}_{30} + \ddot{e}_{50} - \ddot{e}_{30:50}$$

$$\ddot{e}_{30} = \ddot{e}_{50} = \int_0^{\infty} e^{-0.05t} dt = 20$$

$$\ddot{e}_{30:50} = \int_0^{\infty} e^{-0.10t} dt = 10$$

$$\ddot{e}_{\overline{30:50}} = 20 + 20 - 10 = 30$$

Question #271**Answer: B**

$$\bar{A}_{30:50}^1 = \int_0^{\infty} e^{-\delta t} {}_t p_{30} {}_t p_{50} \mu_{30+t} dt = \int_0^{\infty} e^{-0.03t} e^{-0.05t} e^{-0.05t} 0.05 dt = \frac{0.05}{0.13} = 0.3846$$

Question #272**Answer: B**

$T_{30:50}$ has the exponential distribution with parameter $0.05 + 0.05 = 0.10$ and so its mean is 10 and its variance is 100.

Question #273**Answer: D**

$$\text{Cov}[T_{30:50}, T_{\overline{30:50}}] = \left(\ddot{e}_{30} - \ddot{e}_{30:50} \right) \left(\ddot{e}_{50} - \ddot{e}_{30:50} \right) = (20 - 10)(20 - 10) = 100$$

See solution to Question #270 for the individual values.

Question #274**Answer: E**

$${}_3V = ({}_2V + \pi_3)(1 + i_3) - q_{x+2}(b_3 - {}_3V)$$

$$96 = (84 + 18)(1.07) - q_{x+2}(240 - 96)$$

$$q_{x+2} = 13.14 / 144 = 0.09125$$

Question #275**Answer: A**

$${}_4V = \frac{({}_3V + \pi_4)(1 + i_4) - q_{x+3}b_4}{P_{x+3}} = \frac{(96 + 24)(1.06) - 0.101(360)}{0.899} = 101.05$$

Question #276**Answer: D**

Under UDD:

$${}_{0.5}q_{x+3.5} = \frac{0.5q_{x+3}}{1 - 0.5q_{x+3}} = \frac{0.5(0.101)}{1 - 0.5(0.101)} = 0.0532$$

Question #277**Answer: E**

$$\begin{aligned} {}_{3.5}V &= v^{0.5} {}_{0.5}P_{3.5} {}_4V + v^{0.5} {}_{0.5}q_{3.5} b_4 \\ &= 1.06^{-0.5} (0.9468)(101.05) + 1.06^{-0.5} (0.0532)(360) \\ &= 111.53 \end{aligned}$$

Question #278**Answer: D**

$$1 - {}_{10}P_{30:40} = 1 - {}_{10}P_{30} {}_{10}P_{40} = 1 - \frac{60}{70} \frac{50}{60} = \frac{2}{7}$$

Question #279**Answer: A**

$${}_{10}q_{30:40}^2 = \int_0^{10} {}_tP_{30} \mu_{30+t} (1 - {}_tP_{40}) dt = \int_0^{10} \frac{1}{70} \frac{t}{60} dt = \frac{50}{70(60)} = 0.0119$$

Question #280**Answer: A**

$$\begin{aligned} &\int_{10}^{20} {}_tP_{30} \mu_{30+t} {}_tq_{40} dt + \int_{10}^{20} {}_tP_{40} \mu_{40+t} {}_tq_{30} dt \\ &= \int_{10}^{20} \frac{1}{70} \frac{t}{60} dt + \int_{10}^{20} \frac{1}{60} \frac{t}{70} dt = \frac{1}{70} \frac{400 - 100}{2(60)} + \frac{1}{60} \frac{400 - 100}{2(70)} = 0.0714 \end{aligned}$$

Question #281**Answer: C**

$$\begin{aligned}
& 140,000 \int_0^{30} {}_tP_{30} \mu_{30+t} {}_tP_{40} dt + 180,000 \int_0^{30} {}_tP_{40} \mu_{40+t} {}_tP_{30} dt \\
&= 140,000 \int_0^{30} \frac{1}{70} \frac{60-t}{60} dt + 180,000 \int_0^{30} \frac{1}{60} \frac{70-t}{70} dt \\
&= 140,000 \frac{1}{70} \frac{60^2 - 30^2}{2(60)} + 180,000 \frac{1}{60} \frac{70^2 - 40^2}{2(70)} = 115,714
\end{aligned}$$

Question #282**Answer: B**

$$\begin{aligned}
P \int_0^{20} {}_tP_{30} {}_tP_{40} dt &= P \int_0^{20} \frac{70-t}{70} \frac{60-t}{60} dt = \frac{P}{4200} \int_0^{20} 4200 - 130t + t^2 dt \\
&= \frac{P}{4200} [4200(20) - 130(200) + 8000/3] = 14.444P
\end{aligned}$$

Question #283**Answer: A**

Note that this is the same as Question 33, but using multi-state notation rather than multiple-decrement notation.

The only way to be in State 2 one year from now is to stay in State 0 and then make a single transition to State 2 during the year.

$$p_x^{02} = \int_0^1 {}_tP_x^{00} \mu_{x+t}^{02} dt = \int_0^1 e^{-(0.3+0.5+0.7)t} 0.5 dt = 0.5 \frac{e^{-1.5t}}{-1.5} \Big|_0^1 = \frac{1}{3} (1 - e^{-1.5}) = 0.259$$

Question #284**Answer: C**

Woolhouse's formula to three terms is $\ddot{a}_{80}^{(m)} = \ddot{a}_{80} - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\delta + \mu_{80})$. Then,

$$\ddot{a}_{80}^{(2)} - \ddot{a}_{80}^{(4)} = \ddot{a}_{80} - \frac{2-1}{2(2)} - \frac{2^2-1}{12(2)^2}(\delta + \mu_{80}) - \ddot{a}_{80} + \frac{4-1}{2(4)} + \frac{4^2-1}{12(4)^2}(\delta + \mu_{80})$$

$$8.29340 - 8.16715 = -(1/4) + (3/8) - [(3/48) - (15/192)](\delta + \mu_{80})$$

$$0.00125 = 0.015625(\delta + \mu_{80})$$

$$\delta + \mu_{80} = 0.08$$

$$\ddot{a}_{80} = \ddot{a}_{80}^{(2)} + 1/4 + (3/48)(0.08) = 8.5484.$$

The answer is:

$$\ddot{a}_{80}^{(12)} = \ddot{a}_{80} - \frac{12-1}{2(12)} - \frac{12^2-1}{12(12)^2}(\delta + \mu_{80}) = 8.54840 - \frac{11}{24} - \frac{143}{1728}(0.08) = 8.08345.$$

Question #285**Answer: B**

$${}_1|a_{70:\overline{2}|} = v^2 {}_2p_{70} + v^3 {}_3p_{70}$$

$${}_t p_{70} = e^{-At} e^{\left(\frac{-B}{\ln c}\right)c^x(c^t-1)} = e^{-0.0002t} e^{\left(\frac{-0.000003}{\ln(1.1)}\right)1.1^{70}(1.1^t-1)} = e^{-0.0002t} (0.9754483)^{1.1^t-1}$$

$${}_2 p_{70} = e^{-0.0002(2)} (0.9754483)^{1.21-1} = 0.9943956$$

$${}_3 p_{70} = e^{-0.0002(3)} (0.9754483)^{1.331-1} = 0.9912108$$

$${}_1|a_{70:\overline{2}|} = 0.9943956/1.05^2 + 0.9912108/1.05^3 = 1.7582.$$

Question #286**Answer: E**

$$A_{50:\overline{2}|}^1 = vq_{50} + v^2 p_{50} q_{51}$$

$$p_x = e^{\left(\frac{-B}{\ln c}\right)c^x(c-1)} = e^{\left(\frac{-0.000005}{\ln(1.2)}\right)1.2^x(0.2)} = e^{-0.0000054848(1.2)^{50}}$$

$$p_{50} = 0.951311$$

$$p_{51} = 0.941861$$

$$A_{50:\overline{2}|}^1 = 0.048689/1.03 + 0.951311(0.058139)/1.03^2 = 0.09940.$$

Question #287**Answer: E**

The reserve at the end of year 2 is $10,000vq_{52} - \pi_3$ where π_3 is the benefit premium in year 3. From the equivalence principle at time zero:

$$10,000(vq_{50} + v^2 p_{50}q_{51} + v^3 p_{50}p_{51}q_{52}) = 0.5\pi_3 + 0.5\pi_3vp_{50} + \pi_3v^2 p_{50}p_{51}$$

$$10,000\left(\frac{0.05}{1.04} + \frac{0.95(0.06)}{1.04^2} + \frac{0.95(0.94)(0.07)}{1.04^3}\right) = \left(0.5 + 0.5\frac{0.95}{1.04} + \frac{0.95(0.94)}{1.04^2}\right)\pi_3$$

$$1563.4779 = 1.78236\pi_3$$

$$\pi_3 = 877.1953$$

$${}_2V = 10,000(0.7)/1.04 - 877.1953 = -204.12$$

Question #288**Answer: C**

The benefit premium in year one is vq_{50} . By the equivalence principle:

$$10,000(vq_{50} + v^2 p_{50}q_{51} + v^3 p_{50}p_{51}q_{52}) = 10,000vq_{50} + (vp_{50} + v^2 p_{50}p_{51})\pi$$

$$10,000\left(\frac{0.95(0.06)}{1.04^2} + \frac{0.95(0.94)(0.07)}{1.04^3}\right) = \left(\frac{0.95}{1.04} + \frac{0.95(0.94)}{1.04^2}\right)\pi$$

$$1082.708665 = 1.739090\pi$$

$$\pi = 622.5719$$

$${}_2V = 10,000(0.07)/1.04 - 622.5719 = 50.51.$$

Question #289**Answer: E**

<i>t</i> -th year	${}_{t-1}V$	<i>P</i>	<i>E</i>	<i>I</i>	<i>EDB</i>	E_tV	<i>Pr</i>	${}_{t-1}p_x$	Π
0*	0	0	1000	0	0	0	-1000	1.000	-1000.0
1	0	14500	100	864	14000	690.2	573.8	1.000	573.8
2	700	14500	100	906	15000	689.5	316.5	0.986	312.1
3	700	14500	100	906	16000	0	6	0.971	5.8

$$NPV = -1000 + 573.8v + 312.1v^2 + 5.8v^3 = -216.08 \text{ using a 10\% discount rate.}$$

*The 1000 at time 0 is neither accumulated nor discounted. The value is treated as occurring at the end of time 0 and not as occurring at the beginning of year 1.

Question #290**Answer: C**

$$NPV = 140v + 0.95(135)p_{67}v^2 + (0.95)^2(130)_2p_{67}v^3 = 314.09 \text{ at a 10\% discount rate.}$$

Question #291**Answer: B**

Solution

$$\begin{aligned} Pr &= (6,000 + 700 - 10)(1.06)^{1/12} - 0.002(200,000) - 0.0015(50,000 + 15,000) \\ &\quad - 0.9965(6,200) = 46.7. \end{aligned}$$

Question #292**Answer: C**

$$245 = p_{40}274 \text{ and } 300 = {}_2p_{40}395$$

Present value of premium:

$$1000[1 + (245/274)(1/1.12) + (300/395)(1/1.12^2)] = 2403.821.$$

Present value of profit:

$$-400 + 150/1.12 + 245/1.12^2 + 300/1.12^3 = 142.775.$$

$$PV \text{ Profit} / PV \text{ premium} = 5.94\%$$

Question #293**Answer: B**

$$\begin{aligned} &P[1 + 0.97(0.98624) + 0.92(0.98624)(0.98499)] \\ &= 100,000[0.97(0.01376) + 0.92(0.98264)(0.01501) + 0.87(0.98264)(0.98499)(0.01638)] \\ &P = 1431.74. \end{aligned}$$

Question #294**Answer: A**

Let I_F equal 1 if the index drops below its current level in the next year and equals 0 otherwise.

$$E[X_N | I_F = 1] = N(1000)vq_x = 48.54N$$

$$E[X_N | I_F = 0] = 0$$

$$E[X_N] = 0.1(48.54N) + 0 = 4.854N$$

$$\text{Var}[X_N | I_F = 1] = (1000v)^2 Nq_x(1 - q_x) = 44,773.31N$$

$$\text{Var}[X_N | I_F = 0] = 0$$

$$E[\text{Var}(X_N | I_F)] = 0.1(44,773.31N) + 0 = 4,477.33N$$

$$\text{Var}[E(X_N | I_F)] = 0.1(48.54N)^2 + 0 - (4.854N)^2 = 212.05N^2$$

$$\text{Var}[X_N] = 4,477.33N + 212.05N^2$$

$$\frac{\sqrt{\text{Var}(X_{10})}}{10} = 25.69$$

$$\lim_{N \rightarrow \infty} \frac{\sqrt{\text{Var}(X_N)}}{N} = \lim_{N \rightarrow \infty} \frac{\sqrt{4,477.33N + 212.05N^2}}{N} = \sqrt{212.05} = 14.56$$

$$25.69 - 14.56 = 11.13.$$

Question #295**Answer: E**

The actuarial present value of the death benefit is

$$\begin{aligned} & 100,000 \frac{S_{62}(4)d_{62}^{(2)}v^{0.5} + S_{63}(4)d_{63}^{(2)}v^{1.5} + S_{64}(4)d_{64}^{(2)}v^{2.5}}{S_{62}I_{62}} \\ &= 100,000 \frac{3.589(4)(213)v^{0.5} + 3.643(4)(214)v^{1.5} + 3.698(4)(215)v^{2.5}}{3.589(52,860)} \\ &= 4,585. \end{aligned}$$

Question #296**Answer: A**

Policy 1:

$$\begin{aligned}
 AV_{37}^1 &= [AV_{36} + G - E - (100,000 - AV_{37}^1) {}_{1/12}q_{63} / 1.004](1.004) \\
 &= (AV_{36} + G - E)1.004 - 100,000 {}_{1/12}q_{63} + AV_{37}^1 {}_{1/12}q_{63} \\
 &= [(AV_{36} + G - E)1.004 - 100,000 {}_{1/12}q_{63}] / (1 - {}_{1/12}q_{63})
 \end{aligned}$$

Policy 2

$$\begin{aligned}
 AV_{37}^2 &= [AV_{36} + G - E - 100,000 {}_{1/12}q_{63} / 1.004](1.004) \\
 &= (AV_{36} + G - E)1.004 - 100,000 {}_{1/12}q_{63}
 \end{aligned}$$

Because the starting account value, G and E are identical for both policies:

$$AV_{36}^1 / AV_{36}^2 = 1 / (1 - {}_{1/12}q_{63}) = 1 / [1 - (1/12)(0.01788)] = 1.0015.$$

Question #297**Answer: E**

The recursive formula for the account values is:

$$AV_{k+1} = (AV_k + 0.95G - 50)1.06 - (100,000 - AV_{k+1})q_{50+k}.$$

This is identical to the recursive formula for benefit reserves for a 20-year term insurance where the benefit premium is $0.95G - 50$. Because the benefit reserve is zero after 20 years, using this premium will ensure that the account value is zero after 20 years. Therefore,

$$\begin{aligned}
 0.95G - 50 &= 100,000 \frac{A_{50:\overline{20}|}^1}{\ddot{a}_{50:\overline{20}|}} = \frac{A_{50} - {}_{20}E_{50}A_{70}}{\ddot{a}_{50} - {}_{20}E_{50}\ddot{a}_{70}} = 100,000 \frac{0.24905 - 0.23047(0.51495)}{13.2668 - 0.23047(8.5693)} = 1154.55 \\
 G &= (1154.55 + 50) / 0.95 = 1,267.95.
 \end{aligned}$$

Question #298**Answer: A**

$$\begin{aligned}
E(Z^2) &= (5,000/1.030)^2(0.005) + \left[\frac{5,000}{1.030(1.032)} \right]^2 (1-0.005)(0.006) \\
&\quad + \left[\frac{5,000}{1.030(1.032)(1.035)} \right]^2 (1-0.005)(1-0.006)(0.007) \\
&= 392,917
\end{aligned}$$

Question #299**Answer: E**

$$\mu_{44.75} = 0.00004(1.1^{44.75}) = 0.002847$$

$$\mu_{44.5} = 0.00004(1.1^{44.5}) = 0.002780$$

$$1000 - E({}_{4.75}L) = 0.25\{0.04E({}_{4.75}L) + 150 - 150(0.05) - 0.002847[10,000 + 100 - E({}_{4.75}L)]\}$$

$$\begin{aligned}
E({}_{4.75}L) &= \{1000 - 0.25[150 - 150(0.05) - 0.002847(10,000 + 100)]\} / [1 + 0.25(0.04 + 0.002847)] \\
&= 961.27
\end{aligned}$$

$$961.27 - E({}_{4.5}L) = 0.25\{0.04E({}_{4.5}L) + 150 - 150(0.05) - 0.002780[10,000 + 100 - E({}_{4.5}L)]\}$$

$$\begin{aligned}
E({}_{4.5}L) &= \{961.27 - 0.25[150 - 150(0.05) - 0.002780(10,000 + 100)]\} / [1 + 0.25(0.04 + 0.002780)] \\
&= 922.795
\end{aligned}$$

Question #300 - Removed**Question #301****Answer: E**

The death benefit is $\max(100,000, 1.3 \times 85,000 = 110,500) = 110,500$.

The withdrawal benefit is $85,000 - 1,000 = 84,000$.

$$\begin{aligned}
{}_{10}AS &= \frac{({}_9AS + P - E)(1+i) - 110,500q_{79}^{(\text{death})} - 84,000q_{79}^{(\text{withdrawal})}}{1 - q_{79}^{(\text{death})} - q_{79}^{(\text{withdrawal})}} \\
&= \frac{(75,000 + 9,000 - 900)(1.08) - 110,500(0.01) - 84,000(0.03)}{1 - 0.01 - 0.03} \\
&= 89,711.
\end{aligned}$$

Note – companies find asset share calculations less useful for universal life policies than for traditional products. This is because policyholders who choose different premium payment patterns have different asset shares. However, for any pattern of premiums, an asset share can be calculated.

Question #302**Answer: B**

$$\begin{aligned}
{}_{10}V &= \frac{({}_9V + G - E)(1+i) - 1000q_{x+9}^{(\text{death})} - {}_{10}CV(1 - q_{x+9}^{(\text{death})})q_{x+9}^{(\text{withdrawal})}}{1 - q_{x+9}^{(\text{death})} - (1 - q_{x+9}^{(\text{death})})q_{x+9}^{(\text{withdrawal})}} \\
&= \frac{(115 + 16 - 3)(1.06) - 1000(0.01) - 110(1 - 0.01)(0.10)}{1 - 0.01 - (1 - 0.01)(0.10)} = 128.83.
\end{aligned}$$

Expected deaths = $1000(0.01) = 10$; actual deaths = 15.

Expected withdrawals = $1000(1 - 0.01)(0.1) = 99$; actual withdrawals = 100.

Gain from mortality and withdrawals is equal to

(Expected deaths – Actual deaths)(Death benefit – End of Year Reserve)
+ (Expected withdrawals – Actual withdrawals)(Withdrawal benefit – End of Year Reserve)

= $(10 - 15)(1000 - 128.83) + (99 - 100)(110 - 128.83) = -4337$.

Question #303**Answer: C**

Because there are no cash flows at the beginning of the year, the only item earning interest is the reserve from the end of the previous year. The gain is $1000(10,994.49)(0.05 - 0.06) = -109,944.90$

Question #304**Answer: B**

Expenses are incurred for all who do not die. Because gain from mortality has not yet been calculated, the anticipated experience should be used. Thus, expenses are assumed to be incurred for $1000(1 - 0.01) = 990$ survivors. The gain is $990(50 - 60) = -9,900$.

Question #305**Answer: E**

The tenth reserve is

$${}_{10}V = \frac{{}_9V(1+i) - (1 - q_{74}^{(d)})(1000 + 50)}{1 - q_{74}^{(d)}}$$

$${}_{10}V = \frac{10,994.49(1.06) - (1 - 0.01)(1050)}{1 - 0.01}$$

$${}_{10}V = 10,721.88.$$

Note that reserves are prospective calculations using anticipated experience.

There were 2 more deaths than expected. For each extra death, an annuity benefit is not paid, an expense is not paid and a reserve does not have to be maintained. Thus, the saving is $1000 + 60 + 10,721.88 = 11,781.88$. The total gain is $2(11,781.88) = 23,563.76$. Because the gain from expenses has already been calculated, the actual value is used.

As an aside, the total gain from questions 303-305 is -96,281. The total gain can be determined by first calculating the assets at the end of the year. Begin with $1000(10,994.49) = 10,994,490$. They earn 5% interest, to accumulate to 11,544,215. At the end of the year, expenses are 60 for each of 988 who did not die, for 59,280. Annuity benefits of 1000 are paid to the same 988 people, for 988,000. Assets at the end of the year are $11,544,215 - 59,280 - 988,000 = 10,496,935$. Reserves must be held for the 988 continuing policyholders. That is, $988(10,721.88) = 10,593,217$. The difference, $10,496,935 - 10,593,217 = -96,282$.

Question #306**Answer: E**

$${}_tV' = \frac{d}{dt} {}_tV = G + \delta({}_tV) - \mu_t(b_t - {}_tV)$$

At $t = 4.5$,

$${}_{4.5}V' = 25 + 0.05({}_{4.5}V) - 0.02(4.5)(100 - {}_{4.5}V) = 16 + 0.14({}_{4.5}V)$$

Euler's formula in this case is ${}_{5.0}V = {}_{4.5}V + (5.0 - 4.5){}_{4.5}V'$.Because the endowment benefit is 100, ${}_{5.0}V = 100$ and thus,

$$100 = {}_{4.5}V + 0.5({}_{4.5}V') = {}_{4.5}V + 0.5[16 + 0.14({}_{4.5}V)] = 8 + 1.07({}_{4.5}V)$$

$${}_{4.5}V = 85.981.$$

Similarly,

$${}_{4.5}V = {}_4V + (4.5 - 4.0){}_4V' \text{ and}$$

$${}_{4.0}V' = 25 + 0.05({}_{4.0}V) - 0.02(4.0)(100 - {}_{4.0}V) = 17 + 0.13({}_{4.0}V)$$

$$85.981 = {}_{4.0}V + 0.5({}_{4.0}V') = {}_{4.0}V + 0.5[17 + 0.13({}_{4.0}V)] = 8.5 + 1.065({}_{4.0}V)$$

$${}_{4.0}V = 72.752.$$

Note that if smaller step sizes were used (which would be inappropriate for an exam question, where the step size must be specified), the estimate of the time 4 reserve converges to its true value of 71.96.

Question #307**Answer: A**

$${}_tV' = \frac{d}{dt} {}_tV = G + \delta({}_tV) - \mu_t(b_t - {}_tV)$$

At $t = 5.0$,

$${}_{5.0}V' = 25 + 0.05({}_{5.0}V) - 0.02(5.0)(100 - {}_{5.0}V) = 15 + 0.15({}_{5.0}V)$$

Because the endowment benefit is 100, ${}_{5.0}V = 100$ and thus,

$${}_{5.0}V' = 15 + 0.15(100) = 30.$$

Euler's formula in this case is ${}_{4.5}V = {}_{5.0}V + (4.5 - 5.0){}_{5.0}V' = 100 - 0.5(30) = 85$.

Similarly,

$${}_{4.0}V = {}_{4.5}V + (4.0 - 4.5){}_{4.5}V' \text{ and}$$

$${}_{4.5}V' = 25 + 0.05({}_{4.5}V) - 0.02(4.5)(100 - {}_{4.5}V) = 16 + 0.14({}_{4.5}V) = 16 + 0.14(85) = 27.9.$$

$${}_{4.0}V = 85 + (4.0 - 4.5)(27.9) = 71.05.$$

Note that if smaller step sizes were used (which would be inappropriate for an exam question, where the step size must be specified), the estimate of the time 4 reserve converges to its true value of 71.96.

Question #308**Answer: A**

$${}_tP_0 = e^{-\int_0^t \mu_s ds} = e^{-\int_0^t (0.05+0.02s) ds} = e^{-(0.05t+0.01t^2)}$$

$${}_1P_0 = e^{-0.06} = 0.94176$$

$$APV = \int_0^4 80(1-0.25t)e^{-\delta t} {}_tP_0 \mu_t dt$$

$$f(1) = 80(1-0.25)e^{-0.05}(0.94176)(0.05+0.02) = 3.7625.$$

Question #309**Answer: D**

Let P be the benefit premium. There are three ways to approach this problem. The first two are intuitive:

The actuarial present value of the death benefit of 1000 is $1000 {}_{10|}A_{40}$. The return of premium benefit can be thought of as two benefits. First, provide a ten-year temporary annuity-due to everyone, with actuarial present value $P\ddot{a}_{40:\overline{10}|}$. However, those who live ten years must then return the accumulated value of the premiums. This forms a pure endowment with actuarial present value $P\ddot{s}_{\overline{10}|10}E_{40}$. The total actuarial present value of all benefits is $1000 {}_{10|}A_{40} + P\ddot{a}_{40:\overline{10}|} - P\ddot{s}_{\overline{10}|10}E_{40}$. Setting this equal to the actuarial present value of benefit premiums ($P\ddot{a}_{40:\overline{10}|}$) and solving gives

$$1000 {}_{10|}A_{40} + P\ddot{a}_{40:\overline{10}|} - P\ddot{s}_{\overline{10}|10}E_{40} = P\ddot{a}_{40:\overline{10}|}$$

$$1000 {}_{10|}A_{40} = P\ddot{s}_{\overline{10}|}$$

$$P = \frac{1000 {}_{10|}A_{40}}{\ddot{s}_{\overline{10}|10}E_{40}} = \frac{1000 {}_{10}E_{40}A_{50}}{\ddot{s}_{\overline{10}|10}E_{40}} = \frac{1000A_{50}}{\ddot{s}_{\overline{10}|}}$$

The second intuitive approach examines the reserve at time 10. Retrospectively, premiums were returned to those who died, so per survivor, the accumulated premiums are only the ones paid by the survivors, that is $P\ddot{s}_{\overline{10}|}$. There are no other past benefits so this is the reserve (it is easy to show this using a recursive formula) Prospectively, the reserve is the actuarial present value of benefits (there are no future premiums), or $1000A_{50}$. Setting the two reserves equal to each other produces the premium:

$$P = \frac{1000A_{50}}{\ddot{s}_{\overline{10}|}}$$

The final approach is to work from basic principles:

$$\text{APV (benefit premiums)} = P \ddot{a}_{40:\overline{10}|}$$

$$\text{APV benefits} = 1000 {}_{10|}A_{40} + P \sum_{k=0}^9 \sum_{j=0}^{9-k} v^{k+j+1} {}_k p_{40} (1+i)^{j+1} {}_j q_{40+k}$$

In that double sum, k is the time the premium is paid, with probability ${}_k p_{40}$ that it is paid. j is the curtate time, from when the premium was paid, until death. The amount of premium refunded, with interest is $P(1+i)^{j+1}$. The probability, given that it was paid, that it will be refunded at time $j+1$ is ${}_j q_{40+k}$. The interest discount factor, from the date of refund to age 40 is $v^{j+k+1} v^{k+j+1}$. Then,

$$\begin{aligned} P \sum_{k=0}^9 \sum_{j=0}^{9-k} v^{k+j+1} {}_k p_{40} (1+i)^{j+1} {}_j q_{40+k} &= P \sum_{k=0}^9 \sum_{j=0}^{9-k} v^k {}_k p_{40} {}_j q_{40+k} \\ &= P \sum_{k=0}^9 v^k ({}_k p_{40} - {}_{10} p_{40}) = P (\ddot{a}_{40:\overline{10}|} - {}_{10} p_{40} \ddot{a}_{\overline{10}|}) \end{aligned}$$

Setting APV benefit premiums = APV benefits:

$$P \ddot{a}_{40:\overline{10}|} = 1000 {}_{10|}A_{40} + P (\ddot{a}_{40:\overline{10}|} - {}_{10} p_{40} \ddot{a}_{\overline{10}|})$$

$$P {}_{10} p_{40} \ddot{a}_{\overline{10}|} = 1000 {}_{10|}A_{40}$$

$$P = \frac{{}_{10}E_{40} (1000 A_{50})}{{}_{10} p_{40} \ddot{a}_{\overline{10}|}} = \frac{v^{10} {}_{10} p_{40} (1000 A_{50})}{{}_{10} p_{40} \ddot{a}_{\overline{10}|}} = \frac{1000 A_{50}}{\ddot{s}_{\overline{10}|}}$$

The numerical answer is

$$P = \frac{249.05}{13.97164} = 17.83.$$