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**COMMUTATION FUNCTIONS AND NET SINGLE
PREMIUMS FOR WIDOWS' MONTHLY
BENEFITS—ACTUARIAL NOTE**

A. M. NIESSEN

THERE seems to be a growing interest in extending industrial pension plans to include widows' monthly benefits of one kind or another. In view of this, it was considered worth while to present in the *Transactions* certain commutation functions and net single premiums for such benefits.

In the main, the data presented in the three tables which follow are based on the experience of the Railroad Retirement Board (subsequently referred to as RRB) in the area of survivor benefits. A description of the factors and of the mortality tables with which they are associated is given below.

ABBREVIATIONS AND NOTATIONS

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|----------------------|--|
| TS-8 | Technical supplement to the eighth actuarial valuation of the railroad retirement system (RRB, January 1962, mimeographed). |
| 1959-Ra | Table S-1 of TS-8; mortality table for nondisability retirements. |
| 1956-Rd | Table S-2 of TS-8; postretirement mortality of disability annuitants. Published also in <i>TSA X</i> , pp. 212-16. |
| 1956-Rm | Table S-6 of TS-8; mortality and remarriage table for widows. Published also in <i>TSA XII</i> , pp. 8-12. |
| 1950-Rw | Table S-3 of TS-8; mortality table for women receiving benefits under the Railroad Retirement Act. Currently used for widows with a 1-year rateback in age. |
| G_x^w | Equals $f_x^w \cdot a_y^{(12)}$ where f_x^w is the proportion of men dying between age x and $x + 5$ who leave widows, as shown in Table 1; y is the average age of such widows, which is also shown in Table 1; and $a_y^{(12)}$ is from the 1950-Rw table. This function refers to immediate widows' benefits without a remarriage termination clause. |
| G_x^{wm} | Same as G_x^w except that the annuity value is $a_{[w]}^{m(12)}$ from the 1956-Rm table which is select only with respect to remarriage but not mortality. This function refers to |

immediate widows' benefits with a remarriage termination clause (signified by superscript m).

${}_{60(y)}G_x^{wm}$ and

${}_{62(y)}G_x^{wm}$

Equal $f_x^w \cdot {}_{60-y} | a_{[y]}^{m(12)}$ and $f_x^w \cdot {}_{62-y} | a_{[y]}^{m(12)}$ respectively, the deferment dropping out if y is greater than 60 or 62 as the case may be. The annuity values are from the 1956-Rm table.

TECHNICAL NOTES

1. The single premium for a widow's benefit of \$10 a month has been computed as

$$\frac{120 \sum_{t=0}^{\infty} G_{x+5t}^w \cdot (M_{x+5t} - M_{x+5t+5})}{D_x}$$

where the G_x functions are as defined above, and the D_x with the M_x functions are from the mortality tables identified in the column headings of Tables 2 and 3.

TABLE 1

PERCENT OF DECEASED EMPLOYEES LEAVING WIDOWS, AVERAGE AGES OF SUCH WIDOWS, AND COMMUTATION FUNCTIONS FOR WIDOWS' MONTHLY BENEFITS PAYABLE UNDER SPECIFIED CONDITIONS*

Age of Employee x	Percent Leaving Widows† f_x^w	Average Age of Widow† y	G_x^w	G_x^{wm}	${}_{60(y)}G_x^{wm}$	${}_{62(y)}G_x^{wm}$
30	74.2	31.3	17.8226	9.6586	1.6277	1.4017
35	82.8	36.2	18.7700	12.7187	2.9429	2.5343
40	83.7	40.3	17.9286	13.9554	4.1252	3.5526
45	80.7	45.1	16.0000	13.9117	5.4327	4.6785
50	81.7	49.5	14.9101	13.7242	6.9357	5.9728
55	82.8	54.2	13.6213	12.9605	8.7428	7.5290
60	81.3	58.6	11.9320	11.5640	10.4676	9.0144
65	77.1	62.7	10.0023	9.8085	9.8085	9.8085
70	70.7	66.8	7.9574	7.8651	7.8651	7.8651
75	62.2	70.6	6.0236	5.9856	5.9856	5.9856
80	51.6	74.2	4.2649	4.2596	4.2596	4.2596
85	38.5	77.3	2.7471	2.7471	2.7471	2.7471
90 and over	28.7	78.5	1.9301	1.9301	1.9301	1.9301

* See text for definitions.

† Refers to age interval x to $x + 5$. Based on family composition data for railroad men who died in 1954-56 and who gave rise to survivor benefits payable under the Railroad Retirement Act. The f_x^w factors do not incorporate mortality of employees.

For more detailed information on the family compositions of deceased male railroad employees, see tables S-11 and S-11a of the technical supplement to the eighth actuarial valuation of the railroad retirement system (Railroad Retirement Board, January 1962, mimeographed).

2. The ${}_{60(y)}G_x^{wm}$ and ${}_{62(y)}G_x^{wm}$ functions shown in Table 1 are based on proportions leaving widows and the average ages of such widows which are shown in that table. These values are somewhat different from those used in the railroad retirement valuation where special refinements were introduced by considering two average ages, one for widows under

TABLE 2
NET SINGLE PREMIUMS FOR AN IMMEDIATE WIDOW'S BENEFIT OF \$10 A MONTH WITH AND WITHOUT A REMARRIAGE TERMINATION CLAUSE*

AGE OF EMPLOYEE	WITH REMARRIAGE CLAUSE			WITHOUT REMARRIAGE CLAUSE		
	1959-Ra	1956-Rd†	1958 CSO	1959-Ra	1956-Rd†	1958 CSO
30.....	\$349.37	\$777.54	\$361.54	\$383.83	\$1,099.32	\$400.60
35.....	398.08	879.22	409.82	429.61	1,110.07	443.64
40.....	447.12	916.61	458.11	474.56	1,065.16	486.48
45.....	493.20	910.58	503.58	515.53	993.38	526.00
50.....	531.05	889.33	543.37	548.78	938.38	561.19
55.....	552.86	843.31	569.72	566.17	872.34	583.34
60.....	553.26	771.89	575.64	562.72	789.12	585.56
65.....	528.68	691.87	556.36	534.77	701.89	562.98
70.....	483.82	565.76	508.34	487.31	570.49	512.18
75.....	422.44	464.27	436.67	424.03	466.28	438.38
80.....	344.16	364.72	351.34	344.42	365.00	351.65
85.....	259.52	270.32	261.88	259.52	270.32	261.88
90.....	207.66	211.58	209.76	207.66	211.58	209.76

* See text for identification of mortality tables.
† Applies to employees retired because of disability.

TABLE 3
NET SINGLE PREMIUMS FOR A DEFERRED WIDOW'S BENEFIT OF \$10 A MONTH SUBJECT TO A REMARRIAGE TERMINATION CLAUSE*

AGE OF EMPLOYEE	DEFERMENT TO AGE 60†			DEFERMENT TO AGE 62†		
	1959-Ra	1956-Rd†	1958 CSO	1959-Ra	1956-Rd†	1958 CSO
30.....	\$255.89	\$299.88	\$262.79	\$238.92	\$266.57	\$245.70
35.....	297.40	382.39	305.82	277.81	340.24	286.12
40.....	344.41	463.39	354.19	322.07	413.65	331.78
45.....	396.78	547.14	408.10	371.82	490.97	383.15
50.....	452.17	629.92	465.72	425.51	569.80	439.09
55.....	504.35	702.89	521.48	478.54	643.88	495.56
60.....	538.86	737.29	561.02	519.78	691.40	541.66

* See text for identification of mortality tables.
† Applies to employees retired because of disability.
‡ For ages of employee 65 and over, the average age of widow is over 62. There is, therefore, no deferment and the values are the same as those shown in Table 2 (with remarriage clause).

60-62 and the other for widows that are older. However, the differences are not very significant.

3. The values for deferred widows' benefits may be useful in private pension work in cases where an adjustment in the benefit is made on account of railroad retirement or social security benefits to aged widows.
4. All monetary functions are at 3 percent interest.
5. The net single premiums associated with the 1958 CSO table (M_x and D_x functions published in *TSA XII*, pp. 338-339) have been inserted only for purposes of comparison with the values based in part on the 1959-Ra table. The age last birthday basis was selected for the 1958 CSO table in order to measure the effect of a somewhat more conservative mortality standard for employees on net single premiums for widows' monthly benefits.
6. The single premiums shown in Tables 2 and 3 are only illustrative. The main purpose of this note was to present the G_x functions which can be associated with any mortality table for employees that is considered appropriate. If more refined calculations are desired, the G_x functions, treating each one as applying to the middle age of the 5-year period, may be interpolated for single ages and then associated with C_x functions rather than using quinquennial functions as was done for purposes of this presentation. If still further refinements are desired, the f_x^w and y functions may be similarly interpolated for single ages of the employee and single age G_x functions would then be constructed to be associated with the appropriate C_x functions from the mortality table.