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ANALYSIS OF THE RAPIDLY EXPANDING COMPANY

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THIS paper deals with the long-term effects of the "surplus strain" felt by insurers on account of the rapid growth of the business. This phrase is generally used in reference to the difficulty experienced in setting up reserves when there is a marked temporary increase in the rate of expansion in the volume of new business written. Thus actuaries tend to regard it as a matter of transitory significance that can be handled, for instance, by drawing on contingency funds. Undoubtedly, fluctuations in the rate of new business expansion can be provided for in this way. Such fluctuations are not our concern here, however. Our object is rather to determine whether the long-term trends in the rate of company expansion are such as to affect policyholder costs and, if so, by how much. To gauge the importance of the problem, we shall, before making a finer analysis, first estimate roughly the average cost for United States and Canadian life insurance companies as a whole.

SURPLUS STRAIN: AN INDUSTRY COST ESTIMATE

The assets of a company, or of the entire industry, may be regarded as composed of two parts—an *asset share account*, the total equity of all policyholders ("asset share" subsequently is given a precise meaning but for the present purpose may be assumed to have the value of the conventional asset share, with profit and contingency margin deducted), and a *corporate share account*, this last complementing the asset share account so that together they form the total assets of the company (or industry). The asset share account is built up from funds contributed by present policyholders. The corporate share account, on the other hand, is in large part passed on from one generation of policyholders to the next.

The growth of the corporate share account may take a somewhat erratic course, being influenced primarily by (*a*) occasional reduction from losses on account of experience radically different from that expected; (*b*) contributions from policyholders; and (*c*) increase on account of interest earned.[1]

Under (*b*) fall the contingency charges included in premium rates and

[1] The introduction of new capital and surplus is not mentioned, as funds are generally not thus raised by well-established stock companies, and it is a source of minor importance for the industry as a whole.

intended to balance the outgo under (*a*). If these are the only payments made by policyholders to the corporate share account, then evidently this account, under the influence of (*c*), will maintain a long-term average rate of expansion equal to the average rate of interest earned. Thus, if the company's (or industry's) long-term rate of expansion is such that the corporate share account is not required to grow at an average rate higher than the average net rate of interest earned, there will be no sustained "surplus strain" to be borne by the policyholders. However, should the company's (or industry's) rate of expansion, as measured by the long-term trend in the rate of growth of the corporate share account, be greater than the average net rate of interest earned, then the policyholders must contribute the necessary funds to maintain the extra growth. We may express the required annual contribution as the product of the size of the corporate share account and the amount by which the average rate of expansion of this account exceeds the average net rate of interest earned. Hence, in forming a cost estimate, we need to appraise (1) the average level (in relation to obligations) at which the corporate share account is maintained and (2) the excess, over the long term, of the average rate of expansion of the corporate share account over the average net rate of interest earned.

To form an estimate of its size, we may divide the industry's corporate share account into two parts—(1) the excess of policyholder reserves over asset share account and (2) "surplus," here taken to include capital and earmarked contingency funds such as security valuation reserves. At the earliest durations the reserve is greater than the asset share, the difference being especially large under net level premium valuation, where it will frequently be more than the annual gross premium. In those companies and at those durations where provision is made for the payment of terminal dividends the asset share will normally exceed the reserve. Otherwise no sizable difference need be expected at the later durations. On balance, with the larger proportion of business in force at earlier durations, total policyholder reserves will exceed the asset share account for the industry as a whole. One-fifth of the gross annual premium might perhaps be taken as a rough, not too generous, estimate of the average difference. The average level of "surplus" over an extended period might be conservatively assessed at 7 per cent of assets.

The average long-term rate of expansion of the corporate share account may be regarded as the resultant of two determining factors: (i) the average rate of expansion of the assets and (ii) any long-term changes in the underlying financial structure of companies that result in a changing ratio of corporate share account to assets.

For United States and Canadian companies Table 1 compares the average net interest rate earned with the average rate of growth of policy reserves, assets, and premium income during successive decades since 1920. The rate of growth of assets has, in each of the four decades, exceeded the rate of interest earned, the difference averaging 3¼ per cent over the entire forty-year period for both United States and Canadian

TABLE 1*

LIFE INSURANCE INDUSTRY GROWTH VERSUS INTEREST EARNED
(Per Cent)

Period	Average Net Interest Rate Earned	Average Growth Rate			Excess of Growth Rate over Interest Rate		
		Policy Reserves	Assets	Premium Income	Policy Reserves	Assets	Premium Income
	United States Companies						
1950–60.......	3.61	6.01	6.45	7.81	2.40	2.84	4.20
1940–50.......	3.15	7.27	7.59	7.74	4.12	4.44	4.59
1930–40.......	3.94	5.31	5.02	1.01	1.37	1.08	−2.93
1920–30.......	5.09	9.86	9.94	9.80	4.77	4.85	4.71
1920–60.......	3.95	7.10	7.23	6.53	3.15	3.28	2.58
	Canadian Companies (Federally Registered)						
1950–60.......	4.28	6.35	6.44	7.15	2.07	2.16	2.87
1940–50.......	3.63	6.01	6.51	6.06	2.38	2.88	2.43
1930–40.......	4.40	4.97	4.97	−0.75	0.57	0.57	−5.15
1920–30.......	6.07	13.35	13.66	13.96	7.28	7.59	7.89
1920–60.......	4.60	7.62	7.84	6.48	3.02	3.24	1.88

* Source: *Life Insurance Fact Book, 1961* and reports of the Superintendent of Insurance (Ottawa) and of the Canadian Life Insurance Officers Association.

companies. With rising interest rates and a trend to lower premium forms of insurance, the excess of the growth rate over the interest rate has, in recent years, declined. It may be noted that the assets of stock companies are growing at a much higher rate than those of mutual ones. Thus in the decade 1950–60 the twenty-one United States mutual companies with assets (1960) in excess of a half-billion dollars experienced an average annual rate of growth of their combined assets of 5.5 per cent as against 8.6 per cent for the fourteen corresponding stock companies.

It has been suggested above that the corporate share account for the industry may be approximated roughly as 7 per cent of assets plus 20 per cent of annual premium receipts. If we assume that the excess during the long term of the rate of expansion of this account over the rate of interest earned is $2\frac{1}{2}$ per cent and apply these rates to the business of 1960,[2] we obtain required contributions to meet long-term surplus strain of about $300 million in the case of United States companies, and about $20 million in the case of Canadian companies. For United States and Canadian companies these amounts represent 18 and 26 per cent, respectively, of the total commissions to agents.[3]

CALCULATION OF GROSS PREMIUMS AND POLICYHOLDER EQUITIES

The above estimate applies to the industry as a whole. The cost for individual companies will vary widely depending on their financial structure and rate of growth. The actuary not only should be interested in assessing the long-term over-all cost for his company but should be able to allocate this cost equitably among different lines of the business and among different risk categories and forms of contract within individual lines. Taking insurance on individual lives as an example, premium and asset share formulas that satisfy this requirement are first developed. It is then shown that the value of these formulas stems not only from their taking account of the strain arising from a continued high rate of expansion but also from their adaptability (*a*) to distribute contingency and profit charges in accordance with appropriate financial criteria and (*b*) to take account of interest rate varying by policy duration. The necessary adjustments are discussed briefly in the body of the paper, the technique for incorporating the contingency charge being given in more detail in Appendix II. With the aid of the approximations suggested in Appendix III, the calculations may be readily performed on an ordinary desk machine.

The application of the theory of the paper to both nonparticipating and participating assurance is illustrated. While these illustrations generally serve to show the high cost of a continuing rapid rate of company expansion, it is seen that, in the case of participating assurance, the cost varies widely according to the methods used to value liabilities and distribute surplus. The paper concludes with a discussion of the policy that a mutual

[2] For United States companies premium income totaled $17,365 million and assets $119,576 million (*Life Insurance Fact Book, 1961*). For Canadian companies premium income totaled $996 million and assets $8,610 million (*Report of Superintendent of Insurance* [Ottawa]).

[3] Commissions in 1960 totaled $1,633 million for United States companies and $77 million for Canadian companies (Institute of Life Insurance and *Report of Superintendent of Insurance* [Ottawa]).

company should follow in order to minimize the cost of sustained rapid expansion.

Basic Gross Premium Theory

Conventional premium theory is developed with respect to a block of business of a certain type issued at one time. It equates the value at issue of anticipated receipts and disbursements on this business. The transactions associated with each generation of policies are thus regarded in isolation from one another. However, the funds held by insurers do not all derive from present policyholders. The theory of this paper recognizes the cost of maintaining the size of the balance, the corporate share account, when the company is "rapidly expanding," that is, when the long-term trend in the rate of growth of the corporate share account is greater than the long-term average of the rate of interest earned.

The cost of maintaining the size of the corporate share account is a function of the rate of company growth. A premium theory that is to take account of this cost must, then, be developed not from a consideration of the transactions arising from business issued at only one time but rather with reference to a model office. Since the intention of such a model is to reflect the over-all long-term effect of rapid company expansion, it should be based on a rate of expansion of the business equal to the anticipated average long-term rate of growth of the corporate share account and on a level of assets that approximates the average level over an extended period. In the case of a multiple-line company, or one operating in different territories each having its own premium rates or dividend scale, the rate of expansion used should be that appropriate to the line or territory in question.

For one unit of an m year endowment with n level annual premiums and unit death benefit:

$_tMV$ = Mean reserve, policy year t
D_t = Policyholder dividend, policy year t
$_tA$ = Assets allocable to policy year t
F_m = Pure endowment (≤ 1) payable at end of m years
F'_t = Asset share at end of policy year t
CV_t = Cash value at end of policy year t
π = Gross premium
i = Rate of interest earnings during the year
q_t = Probability of death during policy year t
$_{1/2}q_t$ = Probability of death during the first half of policy year t
w_t = Probability of voluntary withdrawal during policy year t
r = Annual rate of expansion of the business

C_t = Per premium expense factor, commissions included, policy year t (when $t > n$, C_t is to be taken as 1)

E_t = Per unit volume expense factor, including distributed per policy expenses, at beginning of policy year t

Q = Claim expense

W = Other withdrawal expense

${}_tp$ = Probability of new business remaining in force at least t years

$a_{t+1} = (1 + r)^{-t} \cdot {}_tp$.

To calculate a gross premium for a given plan and issue age, we postulate a model office composed of business issued on this plan and issue age and with a rate of expansion of the business and a level of assets in accordance with anticipated long-term trends as described above. The premium rate emerges from the relationship between the assets held on account of this business at consecutive annual valuations and the transactions of the intervening year.

It may be assumed, for simplicity, that (1) a valuation year's new business is issued at the middle of the year; (2) deaths in the first half of the policy year occur immediately after the preceding anniversary, while those in the last half of the policy year occur immediately before the following anniversary; and (3) all voluntary withdrawals occur at the end of the policy year. In what follows "year" means "valuation year" unless specifically designated as "policy year."

Consider the transactions of a valuation year y in which 1 unit of business is issued. In year $y - (t - 1)$ the volume issued is $(1 + r)^{-(t-1)}$, and, of this, the volume still in force on the anniversary in year y is $(1 + r)^{-(t-1)} \cdot {}_{t-1}p$ or a_t. By the end of year y this has been reduced to $(1 - {}_{1/2}q_t)a_t$. Hence the total business in force at the end of year y, arising from issues in years $y - (m - 1)$ to y, amounts to

$$\sum_1^m (1 - {}_{1/2}q_t)\, a_t\,.$$

The assets held at the beginning and end of year y are thus, respectively,

$$\frac{1}{1+r}\sum_1^m {}_tA\,(1 - {}_{1/2}q_t)\, a_t \quad \text{and} \quad \sum_1^m {}_tA\,(1 - {}_{1/2}q_t)\, a_t\,.$$

The value of these amounts at the middle of year y is

$$\frac{(1+i)^{1/2}}{1+r}\sum_1^m {}_tA\,(1 - {}_{1/2}q_t)\, a_t \quad \text{and} \quad v^{1/2}\sum_1^m {}_tA\,(1 - {}_{1/2}q_t)\, a_t\,.$$

During the year the assets evidently receive, in addition to interest, a contribution which at the middle of the year is worth the difference of these last amounts, namely,

$$v^{1/2}\left(1-\frac{1+i}{1+r}\right)\sum_{1}^{m}{}_{t}A\,(1-{}_{1/2}q_t)\,a_t$$

or

$$v^{1/2}\frac{e}{1+e}\sum_{1}^{m}{}_{t}A\,(1-{}_{1/2}q_t)a_t\,,$$

where e, *the rate of excess expansion* over the interest rate, is defined by

$$1+e=\frac{1+r}{1+i}\,.$$

Since all transactions are assumed to occur at the middle of the valuation year, on the policy anniversary, this contribution to assets equals the excess of receipts over disbursements for the year.

The transactions in year y may be classified into three groups. The first group occurs at the beginning of each policy year and arises from issues of years $y-(m-1)$ to y. The second group occurs at the end of each policy year and arises from issues of years $y-m$ to $y-1$. The third group are maturities and arise from issues of year $y-m$. Expressions for the volume of transactions in the first group from issues of year $y-(t-1)$ and in the second group from issues of year $y-t$ are first determined. The total transactions for the two groups are obtained by summing these amounts for values of t from 1 to m.

In interpreting what follows, it should be borne in mind that the second half of a (valuation) year is the first half of a policy year and vice versa.

In the first group, from the $(1+r)^{-(t-1)}$ units of volume issued in year $y-(t-1)$, we have

Gross premium receivable less expense (per premium and per unit volume)

$$\{(1-C_t)\pi-E_t\}(1+r)^{-(t-1)}\cdot{}_{t-1}p=\{(1-C_t)\pi-E_t\}a_t\,.$$

Death benefit and claim expense payable in the second half of year y

$$(1+Q)(1+r)^{-(t-1)}\cdot{}_{t-1}p\cdot{}_{1/2}q_t=(1+Q)a_t\cdot{}_{1/2}q_t\,.$$

In the second group, from the $(1+r)^{-t}$ units of volume issued in year $y-t$, we have

Death benefit and claim expense payable in the first half of year y

$$(1+Q)(1+r)^{-t}\cdot{}_{t-1}p(q_t-{}_{1/2}q_t)=(1+Q)(1+r)^{-1}a_t(q_t-{}_{1/2}q_t)\,.$$

Dividend payable, including payment on death during the first half only of year y (equivalent to a pro rata dividend on death)

$$D_t(1+r)^{-t} \cdot {}_{t-1}p(1 - {}_{1/2}q_t) = D_t(1+r)^{-1}a_t(1 - {}_{1/2}q_t) \, .$$

Cash value and withdrawal expense payable

$$(CV_t + W)(1+r)^{-t} \cdot {}_{t-1}p \cdot w_t = (CV_t + W)(1+r)^{-1}a_t w_t \, .$$

In the third group, from the $(1+r)^{-m}$ units of volume issued in year $y - m$, we have

Pure endowment benefit payable

$$F_m(1+r)^{-m} \cdot {}_m p = F_m a_{m+1} \, .$$

Combining the death benefit and claim expense payable in each half of year y gives

$$\begin{aligned}(1+Q)a_t \cdot {}_{1/2}q_t + (1+Q)(1+r)^{-1}a_t(q_t - {}_{1/2}q_t) \\ &= (1+Q)(1+r)^{-1}a_t\{(1+r)_{1/2}q_t + q_t - {}_{1/2}q_t\} \\ &= (1+Q)(1+r)^{-1}a_t(q_t + r \cdot {}_{1/2}q_t) \, .\end{aligned}$$

Equating the required contribution to assets with the excess of receipts over disbursements for the year gives

$$\begin{aligned} v^{1/2} \cdot \frac{e}{1+e} \sum_1^m {}_tA\,(1 - {}_{1/2}q_t)\,a_t = \sum_1^m \Big\{ &(1 - C_t)\pi - E_t \\ &- \frac{1+Q}{1+r}(q_t + r \cdot {}_{1/2}q_t) - D_t \frac{1 - {}_{1/2}q_t}{1+r} \quad (1) \\ &- (CV_t + W)\frac{w_t}{1+r} \Big\} \, a_t - F_m a_{m+1} \, . \end{aligned}$$

It should be noted that this equation expresses a generalization of conventional theory. The right-hand side represents, under conventional theory, the value at issue of anticipated receipts less anticipated disbursements, assuming as interest rate the rate of expansion r. In the special case when the rate of expansion equals the rate of interest, no contribution to assets is required, and the relationships of conventional gross premium theory can be seen to hold. Dividing both sides of the equation by

$$\sum_1^m (1 - C_t)\,a$$

and rearranging the terms, we may write

$$\pi = {}^{r}\pi' + \frac{v^{1/2} \cdot \frac{e}{1+e} \sum_{1}^{m} {}_{t}A\,(1 - {}_{1/2}q_t)\,a_t}{\sum_{1}^{m} (1 - C_t)\,a_t}, \qquad (2)$$

where ${}^{r}\pi'$ is a conventional Jenkins[4] type gross premium computed so as to reflect the incidence of expense, mortality, and withdrawal rates, but at a rate of interest equal to the rate of expansion r. If no gain or loss from surrender need be assumed, then w_t may be taken to be zero and ${}^{r}\pi'$ a conventional Cammack[5] type gross premium computed so as to reflect the incidence of expense and mortality, but again at a rate of interest equal to the rate of expansion r.

Asset Shares

In what follows, the "characteristics" of business of a certain policy year are to be understood to include the gross premium charged at the beginning of the policy year, any benefits payable during or at the end of the policy year, the mortality interest and expense rates applicable to the policy year, the average rate of business expansion and the level of assets allocable to business of the given duration, and, where appropriate and desired, the policyholder dividend, the cash value and withdrawal rate, and the profit and contingency charges applicable to the policy year.

To define the retrospective asset share at, say, duration t, a model office is postulated consisting of a block of business maturing t years from issue, and with the characteristics in policy year $t'(t' \leq t)$ as experienced in policy year t' under the original contract. The retrospective asset share is the maturity value emerging under this office. On the other hand, to define the prospective asset share at duration t, a model office is postulated consisting of a block of business commencing with a single premium and with characteristics in policy year t'' as anticipated for policy year $t + t''$ under the original contract. The prospective asset share is the single premium required under this office. (In accordance with the definition of "characteristics," this single premium will exclude any simultaneously payable annual premium. It is not to be regarded as subject to per premium expenses.) When the characteristics both experienced in the past and antici-

[4] *RAIA* XXI, 8.

[5] *TASA* XX, 379.

pated for the future are those assumed in determining the gross premium, the retrospective and prospective shares are, of course, equal.

Since the asset share has been defined as an endowment value appropriate to a model office, it may be regarded as determined by equation (1) in terms of the characteristics of this office (including the gross premium). Thus the relationship between asset shares at any two consecutive durations $s-1$ and s may be found by setting the term of the endowment period equal to s in equation (1), deducting from each side the corresponding functions for an endowment period of $s-1$, and dividing through by a_s. Hence

$$v^{1/2}\cdot\frac{e}{1+e}(1-{}_{1/2}q_s)\,{}_sA=(1-C_s)\pi-E_s-\frac{1+Q}{1+r}(q_s+r\cdot{}_{1/2}q_s) \tag{3}$$
$$-D_s\frac{1-{}_{1/2}q_s}{1+r}-(CV_s+W)\frac{w_s}{1+r}-\frac{F'_s(1-q_s-w_s)}{1+r}+F'_{s-1}\,.$$

Since

$$v^{1/2}\left(\frac{e}{1+e}\right)=v^{1/2}e\left(\frac{1+i}{1+r}\right)$$
$$=\frac{(1+i)^{1/2}}{1+r}\,e$$

$$F'_s=\frac{1+r}{1-q_s-w_s}\left\{F'_{s-1}+(1-C_s)\pi-E_s-\frac{1+Q}{1+r}(q_s+r\cdot{}_{1/2}q_s)\right.$$
$$\left.-D_s\frac{1-{}_{1/2}q_s}{1+r}-(CV_s+W)\frac{w_s}{1+r}\right\}-\frac{(1+i)^{1/2}(1-{}_{1/2}q_s)}{1-q_s-w_s}e\cdot{}_sA\,.$$

It would appear from the accumulation factor $(1+r)/(1-q_s-w_s)$ of this last equation that the asset share is accumulated not at the rate of interest i but rather at the rate of expansion r. Offsetting this difference, however, a contribution to assets at the rate of excess expansion e is required on valuation in the middle of the policy year. The net effect is that the asset share accumulates at the rate of interest i, with deductions each year at rate e on the *corporate share*—the excess of allocable assets (represented here by ${}_sA$) over the asset share of the same duration. We may thus express the deduction as

$${}^aF'_{s-1/2}-{}^bF'_{s-1/2}=e({}_sA-{}^aF'_{s-1/2})\,, \tag{4}$$

where ${}^aF'_{s-1/2}$ and ${}^bF'_{s-1/2}$ are the values of the asset share immediately before and after the deduction in policy year S. An explicit proof of equation (4) is given in Appendix I.

The premium and asset share theory developed above applies whether the corporate share be positive or negative. However, to be solvent, a private insurer must be able to meet his obligations from assets held and premiums payable under existing contracts. Thus, while equation (2) expresses a necessary condition—a minimum value for the gross premium if the company is to maintain a given rate of expansion—it is not a condition sufficient to ensure that the gross premium is adequate. The further requirement that the assets held be not less than the prospective asset share must be met.

The Contingency Margin

Contingency margins have been described as "charges levied to meet the cost of unpredictable events of major financial consequence for which provision has not elsewhere been made."[6] Using conventional techniques, however, it is not possible to relate these charges directly to the "major financial consequences" which they are intended to provide for. Such financial consequences, expressed as an average reduction in the ratio of surplus and contingency funds to liabilities occurring at a certain average frequency, may be more readily assessed by the actuary from his business experience than "some kind of estimate, however crude, of the probable impact of [each] given contingency."[7] Furthermore, a method based on the rate of restoration of surplus ensures that the contingency margin is in accordance with actual company policy (and legal requirements) as regards accumulation of surplus, contingency funds, and security valuation reserves.

If, to compensate for occasional major setbacks, the asset level for policy duration s is subject to an average annual increase of $\Delta({}_sA)$ per unit volume between such setbacks, then the contingency charge enters the asset share accumulation as a deduction of this amount at valuation. Thus equation (4) may be rewritten

$$\begin{aligned} {}^aF'_{s-1/2} - {}^bF'_{s-1/2} &= e\,({}_sA - {}^aF'_{s-1/2}) + \Delta({}_sA) \\ &= e\,({}^c_sA - {}^aF'_{s-1/2}), \end{aligned} \qquad (5)$$

where

$${}^c_sA = {}_sA + \frac{\Delta({}_sA)}{e} \qquad \text{and} \qquad e \neq 0 .$$

By reversing the process by which equation (4) was obtained, it may be shown that equation (4) implies equations (1)–(3). Hence an allowance for

[6] James C. H. Anderson, "Gross Premium Calculations and Profit Measurement for Nonparticipating Insurance," *TSA* XI, 363.

[7] *Ibid.*

contingencies can be incorporated in the gross premium and asset share formulas by substituting ${}_t^cA$ for ${}_tA$.

In order to apply the theory, ${}_t^cA$ is expressed as a linear function of ${}_t\mathrm{MV}$. Appendix II demonstrates how this may be done.

The Profit Margin

Like the rest of the assets, stockholders' capital is assumed to increase in proportion to the size of the company. Provision is thus automatically made for the declaration of stock dividends when appropriate.

The charge for dividend disbursements should be levied in proportion to the stockholders' stake in the business. The asset share account is by definition built up by the policyholders exclusively. The corporate share account of a stock company, on the other hand, will have been raised, in the first instance, from the capital and surplus subscribed by the stockholders. Subsequently, the policyholders may contribute much, but, to the extent that the stockholders have an interest in the business, it must be presumed to be proportional to the size of the corporate share. Equation (4) shows that a charge for profit may be levied in this proportion simply by increasing the rate of company expansion assumed.

Interest Rate Varying by Policy Duration

The gross premium and asset share formulas developed above may readily be extended to take account of an interest rate varying by policy duration. Thus, with rate of excess expansion e_t based on the interest rate i_t appropriate to policy year t, equation (2) becomes

$$\pi = {}^r\pi' + \frac{\sum_1^m v_t^{1/2} \cdot \frac{e_t}{1+e_t} \cdot {}_tA\,(1 - {}_{1/2}q_t)\,a_t}{\sum_1^m (1 - C_t)\,a_t},$$

where $v_t = (1 + i_t)^{-1}$.

The Slowly Expanding or Contracting Company

While the company expanding (positively or negatively) at a rate lower than its net earned rate of interest is not the topic of this paper, it is natural to ask how far the analysis here presented is applicable in such a situation. It *can* be applied *mutatis mutandis*, with equation (4) the criterion for determining the amounts transferred from corporate share account to asset share. However, it is not possible to justify any particular distribution of funds so released on grounds of equity. In the case of the rapidly expanding company the policyholder, by his very act of taking out a policy, contributes to the excess rate of expansion. He thus renders himself liable to any charges that such a rate of expansion entails. In the present case, on the other hand, the policyholder does not, by virtue of becoming a policyholder, reduce the rate of expansion and cannot therefore be in any sense said to have earned the funds released. If such funds are not used to build up additional surplus (and sooner or later a limit would have to be set on the amount of surplus accumulated), then the management would appear to have a wide area of discretion to use them in the best interests of the company as a whole without regard to policyholder equity.

ILLUSTRATIONS

Nonparticipating Premiums

The additional premium necessitated by a company's rapid rate of expansion is illustrated for the whole life plan, issue age 35, in Table 2. Using assumptions stated in Appendix IV, the effect on the premium of (A) policy size, (B) surplus level, (C) valuation basis, (D) contingency margin, and (E) variation in interest rate for earlier and later periods since issue are compared for rates of expansion (including profit margin) 4 per cent, 8 per cent, and 12 per cent.

Panel (A) of Table 2 shows that, even when a comparatively low reserve basis is used, the increase in cost resulting from a rate of expansion just a few percentage points higher than the interest rate is substantial. The increase is greatest for the smaller policy sizes, where the higher excess initial expense per unit face amount results in lower asset shares and hence a larger corporate share. The quantity discount thus increases as the rate of expansion increases, the increase being here about 6 per cent of the discount shown by the conventional Jenkins premiums for each 1 per cent increase in rate of expansion. Each unit of the smaller policies with its larger corporate share needs more financial backing and hence may be expected to yield a larger profit. If a rate of expansion of the business of 8 per cent and an annual profit of 1 per cent of the corporate share are assumed, the premiums appropriate to the 9 per cent total rate of expansion may be obtained by interpolation from Table 2. The profit (loaded for per premium expenses) is thus seen to range from $0.39 per $1,000 for $3,000 policies to $0.25 per $1,000 for $30,000 policies.

Panel (E) shows the effect on the premium of a change in interest, analyzed according to the durations at which the change is presumed to occur. While the effect of a change in interest increases steadily with policy duration when the rate of expansion is 4 per cent, with the 12 per cent rate the experience of the later durations is so heavily discounted that after about the first ten years the effect of a change in interest diminishes with increasing duration. Consequent upon this heavy rate of discount, the effect of an interest-rate change applicable to all durations is only a little more than half as great for a 12 per cent as for a 4 per cent rate of expansion.

Equation (2) shows that the effect on the premium of a change in interest is almost exactly proportional to the amount of the change in interest assumed. Thus Panel (E) can be used to determine the effect on the premium of practically any interest assumption desired.

The conventional Jenkins premium differences shown in the first column of Panel (E) are less than those calculated by the method of this paper in the next column for a company expanding at (roughly) the rate of interest earned. This is explained by the fact that the former arise from

TABLE 2

ILLUSTRATIVE WHOLE LIFE NONPARTICIPATING PREMIUMS—AGE 35*

	Standard Jenkins Premium	Increase in Premium for Rate of Expansion		
		4%	8%	12%
Assets: 115% of 1958 CSO 3% CRVM reserves (no contingency margin)				
(A) Premiums by policy amount:				
$3,000	$18.39	$0.16	$1.91	$3.34
6,000	17.23	.15	1.62	2.77
12,000	16.65	.15	1.49	2.49
30,000	16.25	0.15	1.37	2.26
(B) Addition if surplus higher by 1% of reserves	0	0.01	0.06	0.10
(C) Addition for other valuation base, with same surplus:				
1958 CSO 2½% CRVM	0	0.05	0.38	0.59
1958 CSO 3% NLP	0	.01	.52	1.08
1958 CSO 2½% NLP	0	0.06	0.95	1.77
(D) Addition for contingencies (replenishes surplus by ½% of reserve every year)	....	0.48	0.40	0.36
(E) Effect of ½% change in interest earned:				
Policy years 1–7	0.03†	0.06	0.08	0.09
Policy years 8–14	.09†	.12	.12	.11
Policy years 15–21	.11†	.13	.10	.07
Policy years after 21	0.27†	0.28	0.13	0.06
All durations	0.50†	0.59	0.43	0.33

* The specified rates of expansion are assumed to include the profit loading. The Jenkins premiums contain no profit margin. It should be noted that these premiums incorporate no charge for federal income tax.

† Average values, there being slight differences by size of policy and direction of interest rate change.

a change in interest earned on the asset share account only, while the latter reflect the effect of a change in interest on the entire assets allocable to the business. Since it is the total assets that would be subjected to any interest rate change, the present technique yields more realistic premium differences.

Participating Premiums

Additional premiums necessitated by a company's rapid rate of expansion are illustrated for the whole life plan, issue age 35, and policy amount $12,000 in Table 3A. A comparison is made of three models, each designed so that, when the rate of expansion equals the rate of interest earned, the same gross premiums apply, and, once dividends become payable, the cash values (increased where appropriate by terminal dividends) equal the corresponding asset shares less the termination expense. Table 3B shows samples of the annual dividends, cash values, and terminal dividends, while Appendix IV details how these values were calculated.

TABLE 3A

ILLUSTRATIVE WHOLE LIFE PARTICIPATING PREMIUMS—AGE 35—POLICY AMOUNT $12,000

Model	Valuation Method	Valuation Interest Rate	Ratio of Surplus to Reserve*	Annual Dividends Payable from Year	Cash Value Equals Reserve from Year	Terminal Dividends Payable from Year	Premiums		
							Rate of Expansion		
							4%	8%	12%
I.......	NLP	2½%	8%	2	20	None	$24.58	$25.49	$26.41
II.......	NLP	2½%	8%	2	14	14	24.58	25.34	26.16
III.......	CRVM	2%	10%	5	Issue	5	24.58	24.70	24.85

* Mandatory security valuation reserve treated as surplus.

TABLE 3B

SAMPLE CASH VALUES AND DIVIDENDS

Policy Year	Annual Dividends			Cash Values			Terminal Dividends	
	Model			Model			Model	
	I	II	III	I	II	III	II	III
2......	$4.20	$3.62	....	$ 11	$ 12	$ 18	....	....
5......	4.55	4.01	$ 3.33	64	66	72	....	$ 7
8......	5.14	4.62	4.40	118	123	128	....	10
13......	7.10	6.62	6.99	213	221	224	....	15
18......	9.25	8.74	9.84	312	315	322	$10	20
20......	9.86	9.30	10.70	353	353	361	15	23

The effect of a high rate of expansion is greatest when the asset share is built up least rapidly and a substantial corporate share maintained against all policy durations. Thus under Model I, where the reserve only is accumulated at the end of twenty years, a rate of expansion 4 per cent higher than the interest rate exacts an extra premium of $0.91. The corresponding extra for Model II, where the total allocable assets are accumulated after twenty years, is $0.76. Model III is clearly best fitted to a high rate of expansion. There the allocable assets are accumulated after but four years, and the corresponding extra is only $0.12.

A POLICY FOR THE RAPIDLY EXPANDING MUTUAL COMPANY

The effect of a continuing high rate of expansion on policyholder costs in three models with unchanging characteristics has been shown in Table 3. The differences in premium there revealed arise from the differences in the amounts requiring to be paid into the corporate share account. These amounts may be substantial, as in Models I and II, or insignificant, as in Model III; but in the rapidly expanding company they can never be negative. Similarly, a rapidly expanding company like Models I or II, desirous to obtain for its policyholders the savings shown in Model III, should seek this end without running down the corporate share account or withdrawing interest earned thereon. Thus it should follow such a course that the corporate share account will continue to expand, but only at the net rate of interest earned. In this way it can eliminate the extra cost associated with a rate of expansion higher than the earned interest rate without drawing on accumulated funds. Savings realized would be permanent, the transition from one mode of operation to another being smooth and gradual.

Extensive investigation would precede implementation of the proposed policy. The total corporate share account of a line of business may be estimated from a model office representative of the current in-force on that line. This amount would then be accumulated forward ten years (say) with benefit of expected interest earnings to obtain a projected value of the corporate share account. On the basis of assumptions as to the volume and type of business to be written over the next ten years, model offices representative of the business in force at the end of the ten-year period would be prepared. Several offices would be examined, with, for business not yet issued, varying assumptions as to mode of valuation and dividend distribution. The total assets appropriate to each office would be deduced from assumptions as to valuation basis and surplus level. An estimate would also be formed in each case of the amount of the asset share account and the difference between this and the total assets compared with the projected corporate share account. From the results obtained a plan of action would be drawn up so that, with the rate of expansion of the business anticipated, the desired rate of growth of the corporate share account over the ten-year period would result. It would be advisable to ascertain also what modifications in this program would be needed should the rate of expansion turn out to be either lower or higher than expected.

In the projected model offices of the above investigation the rate of expansion of the corporate share account would be held down in one or both of two ways:

a) *By increasing the asset shares on future business.* To effect this increase, the annual dividends at the earlier durations would be reduced, or a higher gross premium charged. From the larger fund built up, terminal dividends would be payable as soon as asset shares less termination expense exceeded cash values.
b) *By reducing the assets allocable to the shorter durations on future issues.* To effect this reduction, a company valuing on the net level premium basis may adopt the Commissioners Reserve Valuation Method or a modification thereof. The future business may, however, account for a larger share of the company's assets when it reaches the longer durations, as in the case of Model III as compared with Models I and II.

The effect of application of the first method is only felt gradually, as new business has to be in force some years before a substantially larger asset share than accumulated formerly can be built up. The effect of application of the second method, on the other hand, is immediate and substantial, for the greatest difference in asset level, and hence corporate share, induced occurs in the first policy year. A change in valuation basis from net level premium to a preliminary term method on all or the larger part of new issues could not, then, be accomplished without a sudden marked increase in surplus level. If a drastic change of this nature is to be avoided, consideration must be given to ways by which the change in valuation method may be effected more slowly. One approach would be to value at first just one or two plans on the Commissioners Method, possibly modified, subsequently extending this method to all new issues. This, however, may be felt to be too clumsy, as the valuation basis would not be uniform with respect to the issues either of different years or of any single year. A better approach might be to base the reserve on an interpolation between net level premium values and (modified) Commissioners Method values, using a common mortality table and interest rate. The interpolation factor would be varied only by calendar year of issue, the same factor giving consistent values of reserves, net premiums, the required interest, and the expected mortality. The valuation bases would be chosen, of course, so that the reserves produced would cover the cash values desired.

This interpolation method is flexible; the preconceived program may be changed to meet circumstances as they arise. Furthermore, control of the interpolation factors, being very evidently a technical matter, would rest with the actuary. To the advantages of flexibility and secure actuarial control may be added that of adaptability to computer processing.

CONCLUSION

Traditional actuarial theory regards the cost of any given life insurance benefit as determined by three factors: the mortality experienced, the interest earned, and the expenses incurred, "expense" referring to payments disbursed. This paper has introduced a fourth factor—the contribution made to the corporate share account in the rapidly expanding company.

The extension of conventional gross premium theory to embrace wider considerations of asset level and growth enables the actuary to apply appropriate financial criteria to the apportionment of contingency and profit charges. This advantage, and the ease with which a varying interest rate can be allowed for, may recommend the suggested methods even when the business is not expected to expand rapidly.

As previously mentioned, for the past ten years stock companies have been expanding their assets at a much higher rate than mutual companies. During this period of rising interest rates, the increasing amount of surplus thrown up on older issues has been used by stock companies to finance their high rate of growth. Mutual companies, on the other hand, have returned a large part of these earnings to the contributing policyholders by way of increased dividends. Clearly, if the mutual companies are to compete successfully for their share of the life insurance investment dollar, they must be able to hold down the cost of surplus strain while expanding their assets at a high rate. The proposals contained herein could enable them to meet both these indispensable but hitherto conflicting requirements.

It is to be anticipated that the rate of interest earned will level off and that the proportion of business in force in stock companies at highly profitable premium rates will diminish. It will not then be possible for these companies to finance a high rate of asset expansion from the earnings of previous issues. Under such circumstances stock companies that expect to continue to expand rapidly may be forced to take some cognizance of the cost of surplus strain in setting premium rates. Thus the theory of this paper is likely to become increasingly relevant to their operation.

APPENDIX I

THE ASSET SHARE BUILD

This Appendix gives an explicit proof of equation (4) (see p. 122).

Let F'_{s-1} accumulate in the conventional manner with allowance for interest, mortality, and expenses to ${}^aF'_{s-1/2}$ immediately prior to the following valuation. Then

$$ {}^aF'_{s-1/2} = \{F'_{s-1} + (1 - C_s)\pi - E_s - (1+Q)\,{}_{1/2}q_s\}\frac{(1+i)^{1/2}}{1 - {}_{1/2}q_s} . $$

Let F'_s be discounted back in the conventional manner with allowance for interest, mortality, expense, withdrawals, and dividends paid to ${}^bF'_{s-1/2}$ immediately after the preceding valuation and to ${}^bF'_{s-1}$ immediately after the preceding anniversary (and before any death benefit for policy year s is presumed to be payable). Then

$$ {}^bF'_{s-1} = v\{F'_s(1 - q_s - w_s) + (1+Q)(q_s - {}_{1/2}q_s) + D_s(1 - {}_{1/2}q_s) + (CV_s + W)w_s\} + (1+Q)\,{}_{1/2}q_s , $$

and

$$ \begin{aligned} {}^bF'_{s-1/2} &= \frac{(1+i)^{1/2}}{1 - {}_{1/2}q_s}\{{}^bF'_{s-1} - (1+Q)\,{}_{1/2}q_s\} \\ &= \frac{v^{1/2}}{1 - {}_{1/2}q_s}\{F'_s(1 - q_s - w_s) + (1+Q)(q_s - {}_{1/2}q_s) \\ &\qquad + D_s(1 - {}_{1/2}q_s) + (CV_s + W)w_s\} . \end{aligned} $$

Multiplying both sides of equation (3) by $(1+i)^{1/2}(1+e)(1 - {}_{1/2}q_s)^{-1}$ and noting that

$$ \frac{1+Q}{1+r}(q_s + r\cdot{}_{1/2}q_s) = (1+Q)\,{}_{1/2}q_s + \frac{1+Q}{1+r}(q_s - {}_{1/2}q_s) , $$

we obtain

$$ \begin{aligned} e\cdot{}_sA &= (1+e)\frac{(1+i)^{1/2}}{1 - {}_{1/2}q_s}\{F'_{s-1} + (1 - C_s)\pi - E_s - (1+Q)\,{}_{1/2}q_s\} \\ &\quad - \frac{(1+e)(1+i)^{1/2}}{(1 - {}_{1/2}q_s)(1+r)}\{F'_s(1 - q_s - w_s) + (1+Q)(q_s - {}_{1/2}q_s) \\ &\qquad + D_s(1 - {}_{1/2}q_s) + (CV_s + W)w_s\} \\ &= (1+e)\,{}^aF'_{s-1/2} - {}^bF'_{s-1/2} . \end{aligned} $$

Hence

$$ {}^aF'_{s-1/2} - {}^bF'_{s-1/2} = e({}_sA - {}^aF'_{s-1/2}) . $$

APPENDIX II

CALCULATION OF THE CONTINGENCY MARGIN

It is suggested (see pp. 123–24) that a margin for contingencies should, on pragmatical grounds, be based on the rate of restoration of the level of assets. For our present purpose the assets may be regarded as composed of the policy reserve, together with such additional funds as are necessary to protect the business against fluctuations in asset values, investment losses, and mortality losses. The funds needed to cover fluctuations in asset values and investment losses will fall in proportion to the asset level maintained. Those needed to cover mortality losses will fall in proportion to the amount at risk. The asset level for a given policy duration t may thus be represented as

$$_tA = {_t\mathrm{MV}} + (k_f + k_i)\,{_tA} + k_m(1 - {_t\mathrm{MV}}) \tag{6}$$

$$= \frac{k_m + (1 - k_m)\,{_t\mathrm{MV}}}{1 - k_f - k_i}, \tag{7}$$

where constants k_f, k_i, and k_m are based on the funds required to protect the business as a whole against, respectively, fluctuations in asset values, investment losses, and mortality losses.

The constants k_f, k_i, and k_m, not being all related to the same function, are difficult to estimate. A more convenient approach is to express first the average asset level per unit volume (A, say) as a proportion ($1 + S$, say) of the average reserve per unit volume (MV, say). Thus

$$A = (1 + S)\mathrm{MV}\,.$$

We may now divide S into constituent proportions of the reserve that reflect the relative significance to the company of the need to hold funds on account of fluctuations in asset values and of the probabilities of investment and mortality losses. Let these constituent proportions be, respectively, s_f, s_i, and s_m. Then

$$A = (1 + s_f + s_i + s_m)\mathrm{MV}\,.$$

However, from equation (6) it is also apparent that, for the business as a whole,

$$A = \mathrm{MV} + (k_f + k_i)A + k_m(1 - \mathrm{MV})\,.$$

Equating the pairs of expressions for the funds required for each protective role

$$s_f \cdot \mathrm{MV} = k_f \cdot A$$
$$= k_f(1+S)\mathrm{MV} \text{ and hence } k_f = \frac{s_f}{1+S};$$
$$s_i \cdot \mathrm{MV} = k_i \cdot A$$
$$= k_i(1+S)\mathrm{MV} \text{ and hence } k_i = \frac{s_i}{1+S};$$

and (8)

$$s_m \cdot \mathrm{MV} = k_m(1-\mathrm{MV}) \text{ and hence } k_m = \frac{s_m \cdot \mathrm{MV}}{1-\mathrm{MV}}.$$

It is convenient to express the average annual increase in the average asset level, $\Delta(A)$, in terms of a fraction, f, of the reserve. Thus

$$\Delta(A) = f \cdot \mathrm{MV}.$$

Since the function of the contingency margin is to reimburse the company for losses actually incurred, it should be levied in proportion to the size of the funds required to protect the business against such losses. Hence, since

$$\Delta(A) = \frac{f}{s_i + s_m}(s_i \cdot \mathrm{MV} + s_m \cdot \mathrm{MV})$$
$$= \frac{f}{s_i + s_m}\{k_i \cdot A + k_m(1-\mathrm{MV})\}$$

it follows that

$$\Delta({}_tA) = \frac{f}{s_i + s_m}\{k_i \cdot {}_tA + k_m(1 - {}_t\mathrm{MV})\}.$$

Equation (5) shows that, when e is not equal to zero,

$$ {}^e_tA = {}_tA + \frac{\Delta({}_tA)}{e}$$
$$= {}_tA + \frac{f}{e(s_i+s_m)}\{k_i \cdot {}_tA + k_m(1-{}_t\mathrm{MV})\}$$
$$= \left\{1 + \frac{f \cdot k_i}{e(s_i+s_m)}\right\} {}_tA + \frac{f \cdot k_m}{e(s_i+s_m)}(1 - {}_t\mathrm{MV}).$$

Substituting first the expression for ${}_tA$ given by equation (7), and then the values of k_f, k_i, and k_m given by equation (8), we obtain

$$ {}^eA = \left\{1 + \frac{f \cdot k_i}{e(s_i+s_m)}\right\} \frac{k_m + (1-k_m)\,{}_t\mathrm{MV}}{1 - k_f - k_i} + \frac{f \cdot k_m}{e(s_i+s_m)}(1 - {}_t\mathrm{MV})$$
$$= \left\{1 + S + \frac{f \cdot s_i}{e(s_i+s_m)}\right\} \frac{\frac{s_m \cdot \mathrm{MV}}{1-\mathrm{MV}} + \left(1 - \frac{s_m \cdot \mathrm{MV}}{1-\mathrm{MV}}\right){}_t\mathrm{MV}}{1+s_m}$$
$$+ \frac{f \cdot s_m \cdot \mathrm{MV}(1 - {}_t\mathrm{MV})}{e(s_i+s_m)(1-\mathrm{MV})}$$
$$= {}^cK_1 + {}^cK_2 \cdot {}_t\mathrm{MV}$$

where

$$ {}^{e}K_1 = \frac{s_m \cdot \mathrm{MV}}{(1+s_m)(1-\mathrm{MV})}\left\{1+S+\frac{f\cdot s_i}{e(s_i+s_m)}+\frac{f(1+s_m)}{e(s_i+s_m)}\right\} $$

$$ = \frac{s_m \cdot \mathrm{MV}}{(1+s_m)(1-\mathrm{MV})}\left\{1+S+\frac{f(1+s_i+s_m)}{e(s_i+s_m)}\right\} $$

and

$$ {}^{e}K_2 = \left\{1+S+\frac{f\cdot s_i}{e(s_i+s_m)}\right\}\frac{1-\mathrm{MV}-s_m\cdot \mathrm{MV}}{(1+s_m)(1-\mathrm{MV})} - \frac{f\cdot s_m \cdot \mathrm{MV}}{e(s_i+s_m)(1-\mathrm{MV})}. $$

If equation (2) is generalized to take into account interest rate varying by policy duration, as well as a margin for contingencies, the premium becomes

$$ \pi = {}^{r}\pi' + \frac{\sum_{1}^{m}\{f_1(i_t)+f_2(i_t)\cdot {}_t\mathrm{MV}\}(1-{}_{1/2}q_t)\,a_t}{\sum_{1}^{m}(1-C_t)\,a_t}, \qquad (9) $$

where $f_1(i_t)$ and $f_2(i_t)$ are the respective values of

$$ \frac{v^{1/2}e}{1+e}\cdot {}^{e}K_1; \qquad \frac{v^{1/2}e}{1+e}\cdot {}^{e}K_2 $$

when calculated on the interest rate appropriate to policy year t.

Regarding the case when $e = 0$ as a limit, it is evident that a margin for contingencies may, for this case, be injected by substituting in equation (4) (and hence also in equations [1]-[3]) $\Delta({}_tA)$ in place of $e\cdot {}_tA$.

APPENDIX III

WORKING APPROXIMATIONS

The general expression for the premium given by equation (9) of Appendix II may be written

$$ \pi = {}^{r}\pi' + \frac{\sum_{0}^{m-1}\{f_1(i_{t+1})+f_2(i_{t+1})\cdot {}_{t+1}\mathrm{MV}\}(1-{}_{1/2}q_{t+1})\,a_{t+1}}{\sum_{0}^{m-1}(1-C_{t+1})\,a_{t+}}. $$

Since, with issue age x and commutation functions computed at an interest rate equal to r,

$$a_{t+1}=\frac{D_{[x]+t}}{D_{[x]}}\left(\text{or } \frac{D_{x+t}}{D_{[x]}}\right),$$

the proposed method extends the conventional gross premium calculation routine for each combination of plan and issue age by the summation

$$\sum_{t=0}^{j-1} f_1(i_{t+1})\cdot(1-{}_{1/2}q_{[x]+t})D_{[x]+t}+\sum_{t=j}^{m-1} f_1(i_{t+1})\cdot(1-{}_{1/2}q_{x+t})D_{x+t}$$

$$\sum_{t=0}^{j-1} f_2(i_{t+1})\cdot(1-{}_{1/2}q_{[x]+t})\,{}_{t+1}\text{MV}\cdot D_{[x]+t}$$

$$+\sum_{t=j}^{m-1} f_2(i_{t+1})\cdot(1-{}_{1/2}q_{x+t})\,{}_{t+1}\text{MV}\cdot D_{x+t},$$

where j is the select period, assumed to be less than m years. The first two of the four terms are comparatively small, and their constituents $(1-{}_{1/2}q_{[x]+t})$ and $(1-{}_{1/2}q_{x+t})$ may be taken as 1. These terms then enter the premium formula through the addition of $f_1(i_t)$ to the expense factor E_t.

Let

$$D'_{x+t}=(1-\tfrac{1}{2}\cdot q_{x+t})D_{x+t}$$

and

$$D'_{[x]+t}=(1-\tfrac{1}{2}\cdot q_{[x]+t})D_{[x]+t} \qquad \text{or} \qquad (1-\tfrac{1}{2}\cdot q_{x+t})D_{[x]+t},$$

the second value applying with a double decrement table after the select mortality period but before ultimate withdrawal rates emerge. The term D'_{x+t} is computed for all ultimate attained ages covered by the investigation; $D'_{[x]+t}$ is computed only for those ages and durations for which values are required.

Values of ${}_{t+1}\text{MV}\cdot D'_{[x]+t}$ and ${}_{t+1}\text{MV}\cdot D'_{x+t}$ need be calculated for sample durations only and may be summed using the simple but good approximation of the average of seven consecutive terms to the average of the second and sixth. When the seven values can be represented by a third-degree equation, no error arises. Interest rate changes may be assumed to take place after six, thirteen, and twenty years, a degree of flexibility adequate for practical purposes. With interest rates successively i_a, i_b, i_c, and i_d, for the whole life plan

$$\sum_{1}^{\infty} f_2(i_t)u_t \doteq (3.5u_2+3.5u_6-u_7)f_2(i_a)+3.5(u_8+u_{12})f_2(i_b)$$

$$+3.5(u_{15}+u_{19})f_2(i_c)+3.5f_2(i_d)\sum_{0}^{\infty}(u_{22+7s}+u_{26+7s}),$$

where

$$u_t = {}_t\text{MV}\cdot \text{D}'_{[x]+t-1} \qquad \text{when} \qquad t \leq j$$

and

$$u_t = {}_t\text{MV}\cdot \text{D}'_{x+t-1} \qquad \text{when} \qquad t > j\,.$$

With gross premiums calculated at ten-year age intervals, summations of ultimate values for paid-up life are best made first over corresponding intervals

$$\sum_{y+1}^{y+10} u_t \doteq u_{y+1} + u_{y+2} + u_{y+3} + 3.5\,(u_{y+5} + u_{y+9})\,,$$

with t here representing attained age.

Using the above approximation formula (for the whole life plan), transition from a double decrement to a single decrement table, ignoring withdrawals, may be made at durations thirteen or twenty. If it is desired to change from double decrement to single decrement tables at a shorter duration, interest changes may, for convenience, be assumed to take place after seven, fourteen, and twenty years (with corresponding modifications in the approximation formulas used), and the transition from double decrement to single decrement table made at duration seven.

APPENDIX IV

ILLUSTRATIVE ASSUMPTIONS[8]

Mortality

Mortality Table X_{18} with Mr. James C. H. Anderson's select modification.[9]

Interest

	Nonparticipating Assurance				Participating Assurance
	Policy Year				
	1–7	8–14	15–21	Subs.	All
Interest rate......	$4\frac{1}{4}\%$	$3\frac{3}{4}\%$	$3\frac{1}{2}\%$	$3\frac{1}{4}\%$	4%

[8] Apply to both participating and nonparticipating assurance unless otherwise stated.

[9] *TSA* XI, 393.

Dollar Expenses[10] *(per $1,000 face amount, including distributed per policy expenses)*

Item	Policy Size			
	$3,000	$6,000	$12,000	$30,000
a) First year	$16.99	$10.84	$7.81	$5.36
b) Renewal years	1.33	0.83	0.58	0.43
c) Claim expense	8.33	5.00	3.33	2.33
d) Other termination expenses	1.67	1.00	0.67	0.47

Percentage Expenses[11]

a) Commissions (including expense reimbursement allowance of 30 per cent of first-year commission):

Plan	Commission Rate for Policy Year			
	1	2–10	11–15	Subs.
Whole life	78%	7½%	5%	2%

b) Other percentage expenses: 2 per cent for premium taxes; all policy years and policy sizes.

Mode of Payment

Annual.

Valuation Reserves

Nonparticipating assurance: 1958 CSO 3 per cent CRVM.

Participating assurance: Models I and II, 1958 CSO 2½ per cent NLP; Model III, 1958 CSO 2 per cent CRVM.

Average Asset Level

Nonparticipating assurance: 115 per cent of reserves.

Participating assurance: Models I and II: 108 per cent of reserves; Model III: 110 per cent of reserves.

Allocation of Assets

Of the excess of assets over reserves, the amounts required to protect the business against fluctuations in asset values, investment losses, and

[10] These expense factors are equivalent to those used by Mr. Anderson (*ibid.*, p. 391).

[11] *Ibid.*

mortality losses are presumed weighted 40:35:25, respectively, in an office with an average reserve of $150 per $1,000 sum assured.

Contingency Margin

To increase the asset level by ¼ per cent of reserves per annum.

Persistency

Linton A termination rates used as probabilities of voluntary withdrawal.

Cash Values for Nonparticipating Assurance

1958 CSO 3 per cent minimum values.

Premiums, Cash Values, and Dividends for Participating Assurance

In what follows, a company expanding at the rate of interest (4 per cent) is assumed. The cash values and dividends derived as described below on the conventional basis are then used to determine gross premiums appropriate to rates of expansion 8 per cent and 12 per cent.

Model III.—Cash values are the full reserve. No dividends are payable until the end of the fifth policy year. The gross premium is set at the amount that would accumulate the allocable assets at the end of the fourth year. At the end of the fifth and later years a terminal dividend equal to the excess of the allocable assets over the reserve, less termination expense, becomes payable on withdrawal or death. Annual dividends are the excess each year of the amount accumulated over the allocable assets.

Model I.—Using the gross premium derived for Model III, the asset share at the end of the first year is calculated. Then an "adjusted" premium is calculated that, when accumulated on the valuation table and interest rate from the first-year asset share less termination expense, would amount at the end of the twentieth policy year to the then terminal reserve. Cash values at intermediate durations are taken as the accumulation, on the valuation table and interest rate, of the adjusted premium from the first-year asset share less termination expense. Annual dividends are the excess each year of the amount accumulated over the cash value, less termination expense.

Model II.—Adjusted premium and total sums available on withdrawal are calculated as for Model I, except that at the end of twenty years the allocable assets are accumulated. Any excess over reserve of the total sum available on withdrawal is treated as terminal dividend. Annual dividends are the excess each year of the amount accumulated over the total sum available on withdrawal, less termination expense.

DISCUSSION OF PRECEDING PAPER

BERT A. WINTER:

Mr. Ryall has attacked a problem of great current interest to those of us concerned with participating individual life insurance policies—that of calculating premium rates and cash surrender values for a new series of such policies which, together with annual and possibly terminal dividends consistent with those currently being paid on in-force policies, will produce, if the experience on which such dividends are based continues, a contribution to the company's assets that bears an acceptable relationship to the policy reserves planned to be held at each future policy duration, no matter what the variation, within reasonably probable limits, of the aggregate amount and composition by issue age, plan, size band and premium mode of the policies subsequently issued at these rates.

In formulating his mathematical solution to this problem, Mr. Ryall ignores—perhaps naturally, but nevertheless unfortunately for the general usefulness of his method of approach—significant restraints on the discretion of the management of mutual companies in the United States. Interestingly enough, these restraints found their way into the regulatory statutes of New York and many other states in the first decade of this century precisely because, in the so-called Armstrong investigation, the then managements of some mutual companies had been found to be devoting to "expansion"—to an excessive degree in public opinion as reflected in the judgment of the legislatures—funds accumulated out of the surplus earnings of previously issued participating policies (particularly deferred dividend policies).

Historically older than these statutes, and nowadays perhaps even more fundamental a restraint after decades of interpretation by the supervisory authorities and the courts, is the contribution method, which asserts that the dividends apportioned to each class of policyholders should be in accordance with the contribution of that class to aggregate divisible surplus. The contribution *method* is of course much more extensively used than the three-factor contribution formula, which has been found inappropriate in various special situations. Indeed, over the decades, it has been found on occasion that significant differential contributions to divisible surplus arise from factors other than mortality, interest and expense—such as the experienced disability claim cost, or settlement guarantees which prove liberal in relation to experienced interest rates and annuitant mortality. However, it is at least an unfamiliar concept that the aggregate volume of insurance issued in a dividend class is such a factor, and even more difficult to justify as such a factor the aggregate volume of insurance in other dividend classes, whether concurrently or subsequently issued. Yet, this would seem to be contemplated by Mr. Ryall, not only as explicitly stated in his Conclusion, but from his assumption that the same dividends can be declared for each model company, regardless of the level of premium rates charged because of "expansion loading."

From another point of view, the "expansion" for which provision is being attempted is not going to be achieved unless a correspondingly large section of the insuring public can be persuaded that the excess of the premium charged by one's own company over that charged for the same benefits by other companies is going to be offset, if current earnings experience continues, by the dividends projected as payable at future durations. Apart from legal restraints, such persuasion is likely to prove difficult unless the company is now paying, on policies now at those durations, dividends which are obviously consistent when adjusted for any differences in premium rate and benefits. Moreover, apparent differences in projected net outlay in favor of new policies create dangers of replacement of presently in-force policies. If they materialize, replacements, in turn, act against the expansion of assets—both in the short run, since inevitably some of the cash value released will not find its way immediately into new premium payments to the same company and, more importantly, in the long run, as policyholders come to realize the disservice to their interests wrought by the replacement transaction.

These general considerations can be illustrated by the figures shown in Mr. Ryall's paper for a $12,000 Whole Life policy issued at age 35. Estimating the sum of 20 years' annual dividends from Table 3B, apparently a Model I company providing for 12 per cent expansion by Mr. Ryall's premium and dividend formulas would exhibit an average annual net outlay for this policy $3.36 per thousand greater, and an average annual net cost if surrendered $1.37 per thousand greater, than would be exhibited for a nonparticipating policy by a company with exactly the same expense, interest and mortality experience, but providing for 4 per cent expansion. (For durations less than twenty years, the comparison is even more favorable to the latter.) It is difficult to see how the Model I company could, in fact, achieve a 12 per cent expansion in the face of such competitive handicaps.

If the CRVM of Model III is used, the effect on net cost of providing for expansion by the author's formulas is not as marked. However, if the company has been using net level premium valuation for earlier issues, problems arise. Mr. Ryall's suggestions for a moving transition do not seem attractive as I understand them.

If the new premium rates are adopted for 1963 issues and at the end of that year these issues are valued 90 per cent net level and 10 per cent CRVM, how will those issues be valued at the end of 1968, when 1968 issues are being valued 40 per cent net level and 60 per cent CRVM? If the 90–10 blend is retained for 1963 issues, the actuary will be certifying, as adequate and proper provision at the same time for the future obligations of issues at the same premium rates, to a rather curious mixture of reserve factors. If 1963 issues are then valued on the 40–60 blend, the resulting weakening of reserves on those issues would probably be unacceptable to many regulatory jurisdictions.

In the replacement area, there would also be problems. Presumably, the company issued Whole Life policies at age 34 in 1962, being valued on, say, 1941 CSO $2\frac{1}{2}$ per cent net level premium reserves and with a projected dividend scale similar to the Model I scale of Table 3B. There might be a certain difficulty in justifying, on the basis of the same experience assumptions as the new CRVM policies, a dividend scale for the 1962 policies which started off with a second-year dividend as originally illustrated and produced a projected ledger cost from the policy anniversary in 1963 to that in 1983 more favorable than that projected for the first twenty years of the new CRVM policy. But unless this is true, it will be hard to persuade holders of the old policies not to replace them with new ones.

Mr. Ryall is to be complimented on his courage in attacking this interesting and formidable problem. By failing to deal with some of its practical and legal complexities, however, the formulas he has developed, while ingenious, do not, in my opinion, offer a useful approach to its solution.

J. BRUCE MACDONALD:

As the actuary of a small rapidly expanding stock company, I looked forward to reading Mr. Ryall's paper after noting the title in the program for the meeting. On reading it, I found it interesting, ingenious, and provocative, but not of as much direct use to me as I hoped, although certain modifications of his techniques to suit my situation are fairly obvious.

In nonparticipating insurance, the stress is usually upon low guaranteed cost with smaller emphasis on guaranteed values, and naturally none at all upon such items as dividends—either annual or terminal—which our agents decry as rather nebulous. Thus competition sets an upper limit upon the premium which we may charge. By rather unscientific (but not necessarily unactuarial) methods, we may thus determine what we may charge to finance the expansion program. (Depending upon circumstances, this may include the normal profit and contingency margins.) Then by rearranging Mr. Ryall's equations we may determine what rate of expansion can be afforded. If this is not sufficient for our ambitions, it is then up to our shareholders to make a contribution, either by way of diminished surplus or additional capital, all of which they expect to receive back many times over.

The paper provoked me to consider the broader implications of expansion. In a stock company the benefits to the shareholders are fairly obvious, providing we actuaries are doing our job properly. The policyholders have no interest one way or another as long as the company honors its guarantees.

In a mutual company, I am not clear as to what benefit the policyholders obtain from a high rate of expansion, apart from the one that an expanding company probably is better run than a contracting one. If premiums must be increased to finance expansion, the policyholder is obviously getting less for his money immediately than he might with a competitor. Presumably in the fullness of time and the equity of the dividend scale everything will even out satisfactorily. But for those policyholders who for one reason or another do not achieve fullness of time, the cost would be higher than in a less rapidly expanding competitor.

There are several statements in the paper with which I disagree, particularly in the paragraph on slowly expanding companies. A charge to a man who purchases a policy in a rapidly expanding company is justified on the grounds that he is contributing to the surplus strain. Perhaps if this were explained to him he would decline to make the purchase! We must remember that insurance is sold—not bought. This is a nice point, especially if you cannot explain to him how he will benefit from the expansion.

The paper seems to imply that present policyholders have no equity in the surplus emerging in a slowly expanding company. Presumably this is because this has been built up by a different generation of policyholders, who would be entitled to it if they were still around. (Of course, I might suggest that those of us who resolutely refuse to buy insurance from such a company should share in the surplus, just as those who buy from an expanding company pay for the surplus strain. Perhaps we even might if it embarks on a promotional campaign, and gives away pens, or notebooks or sponsors a television program.)

But if present policyholders are not allowed to benefit from surplus built up by past policyholders, how may they expect to benefit from future expansion which involves future policyholders. They are paying for this expansion, and presumably will receive this payment back, but this is far from guaranteed. I fail to see how a policyholder in a rapidly expanding mutual company is any better off than in a less rapidly expanding mutual company, assuming equally capable management.

Perhaps the leaders of our mutual companies have similar doubts, and this accounts for their less rapid expansion than the stock companies.

MOHAMED F. AMER:

This paper is a very good exposition of the difficulties facing a rapidly expanding small mutual life insurance company. It also suggests a method of coping with these problems by introducing a fourth factor in the traditional three factor dividend formula. The author, I am sure, did not expect that his suggested method will get 100 per cent approval of many Actuaries. When it comes to equity between classes of policyholders, honest opinions will certainly differ. There is nothing absolute about the concept of equity in dividend distribution.

In what follows I present comments on some specific points:

(1) The author mentions that the growth of the corporate share account may take an erratic course and demonstrates how to correct this presumed error mathematically. While it is good mathematics, it should be used only as a guide and not as a substitute for management judgment. It is up to the management to see that the corporate share (or the surplus) is adequate irrespective of the method of surplus distribution.

(2) It is not necessary to introduce explicitly a fourth factor in the dividend formula. Many factors are implicitly considered in the already existing three factor formula such as: initial expenses, reserve strengthening, extra cost of settlement options, and sometimes realized capital gains and losses. The introduction of other explicit factors will complicate the dividend formula and the equity achieved might be more apparent than real.

(3) How rapidly should the business of a small mutual company expand? New York law, section 212, imposes limits on new business of domestic life insurance companies with more than 50 million dollars inforce on the theory that rapid expansion might be against the interest of existing policyholders.

(4) If, however, management finds that rapid expansion is permissible and necessary to provide service to a large segment of the population, deductive elements in the dividend formula might be needed. In this case, there must be a provision for returning back such borrowed amounts to the

contributing class of policyholders. One way might be through Termination dividends provided for by applying negative factors against the financed policies.

JAMES C. HICKMAN:

The Society of Actuaries owes a debt of gratitude to Mr. Ryall for directing our attention to some of the fundamental matters discussed in this paper. I will organize my comments under the paper's section headings.

"Surplus Strain: an Industry Cost Estimate." How was the rule derived that 7 per cent of assets plus one-fifth of gross premiums is an industry-wide approximation to the corporate share? Is there a table analogous to Table 1 which supports this estimate? My guess, based on a very small sample, is that the approximations are about right, but any supporting statistics would be of even greater interest than Table 1.

Table 1 is an interesting but not surprising summary of life insurance growth statistics. If these figures were deflated for population growth and monetary inflation, I doubt if the resulting growth rates would be as impressive as those shown. I assume that the primary purpose of including this table is to support the fact that the principal measures of life insurance growth have been greater than the interest yield rate. It certainly would be distressing if the contrary were true.

"Calculation of Gross Premiums and Policyholder Equities." My remarks on this section center around the statement that, when the rate of expansion equals the interest rate, no contribution to assets is required. This statement assumes that existing contingency funds, built up from previous generations of policyholders and/or stockholders, are adequate for the current generation of policyholders in the absence of rapid growth. Thus, the gross premium problem is reduced to an exercise in equating the present expected value of income to the present expected value of benefit payments. This result certainly follows from the author's concept as to the nature of contingency funds. However, it is at variance with the result obtained from adopting the alternative and competing principle that each block of business should be self-supporting, and therefore should contribute its own contingency funds. If this self-supporting principle were adopted, stripping the gross premiums of their contingency margins would be very risky indeed, for the probability that expected results will be exactly realized is very small. My own view is that contingency fund contributions are required from each significant block of business and that unexpended contingency funds should be reduced perhaps by terminal dividends as per an actuarially determined schedule.

Within the same section in the paragraph entitled "The Contingency Margin," the author discusses another fundamental question—one that I fear has not received the attention that it deserves. He is certainly correct in stating that the financial consequences of adverse contingencies and also surplus goals are usually thought of in terms of their relation to assets. I'm not sure, however, that this is an ideal technique. I'm greatly attracted to Anderson's idea that is quoted in this paragraph. To leave the multitude of risks and their financial consequences that an insurance fund must face unanalyzed strikes me as being too unscientific. I'm willing to grant that the mathematical models for measuring mortality risk are not perfect, that they are inadequate for studying asset fluctuation and that they are almost unavailable for the war (vaporization) hazard. However, it still seems that because of the fundamental nature of this decision it should be squarely faced and our best actuarial tools of analysis brought to bear on it. The various papers on collective risk theory as outlined in the recent paper by Kahn, and an individual risk theory in the older paper by Menge concerning mortality risk, and the Warters-Rae paper on the risk of equity investment of pension funds are examples of some of the actuarial techniques that I have in mind.

One final point concerning this problem. If the problem of determining the proper contingency fund is broken into various subproblems associated with the different risks that the fund faces, the variance of our estimate will be less than the variance of one global estimate. Crudely stated, this results from the fact that the errors in the subestimates may well be offsetting in nature. The same kind of advantage is achieved in stratified sampling. Of course, estimates of the financial impact of the interactions between certain of the possible contingencies must be made, but this additional complication does not reduce my enthusiasm for the analytic determination of surplus levels.

"The Slowly Expanding Company." It is in this section that the author's view of surplus as being made up of irretrievable contributions of past generations of policyholders is put to the test. I quite agree that a new policyholder in a contracting company has no equitable claim on the contingency funds generated by previous generations of policyholders. However, if a mutual company adopts policyholder equity as a guiding principle, isn't there a rather compelling argument for a different basic concept of surplus? Tagging surplus funds, as we are now attempting to do with various generations of investments, so that they grow and then ultimately are reduced as the risk associated with a closed block of business declines is an alternative concept. Obviously, initial contingency funds for any particular generation would have to be borrowed from general contingency funds, but this could be handled by fund accounting methods. It is too much to expect perfect equity and some risks are such that irretrievable contributions are the only practical type; yet the objective remains clear.

"A Policy for the Rapidly Expanding Mutual Company." I agree with the author's policy recommendations. I have seen instances where a shift to CRVM results in a marked and more realistic increase in the surplus growth rate.

"Illustrative Assumptions." I'm fascinated by the 40:35:25 split of surplus into funds for fluctuations in asset values, investment losses, and mortality losses. Recognizing that they were taken only for illustrative purposes, I'm still interested in the motivation of their selection. I agree that for ordinary life insurance, asset fluctuation is currently the primary hazard. As a result of various government guarantees (e.g., FHA, VA insured home mortgages), and the various scheduled repayments written into most current loan agreements, the risk of asset loss is probably smaller than a few years ago.

My own view may be summarized by saying that I have a bias, perhaps emotional bias, toward a rolling-stone concept rather than a snowball concept of surplus. I feel that the buildup and then distribution of contingency funds involved in the rolling-stone concept is implied if the principle of equity is adopted in guiding a mutual company's operation. Second, I feel that a good hard look at exactly what risks we face and estimation of their possible financial consequences are in order, rather than application of an arbitrary rule of thumb.

(AUTHOR'S REVIEW OF DISCUSSION)

PETER L. J. RYALL:

I wish to thank those who have contributed to the discussion. There is evidently some misunderstanding as to the scope and intention of the paper, and I shall try first to correct this.

The paper does not, as Mr. Winter seems to think, advocate that a company should expand either rapidly or slowly. It simply says, in effect, "*If* a company maintains a long-term rate of expansion of its corporate share account of so much, then this is the cost the policyholder will have to bear on account of surplus strain." The question of the maximum rate of expansion that a company can or should plan and provide for is left open. Surplus strain is only one element in the cost of rapid expansion, and probably not the one of most concern to management in setting limits on an expansion program. The cost of increased advertising, the opening up of new agencies, and increased branch and home office expenses would be major considerations. It may be felt that the long-term rate of expansion at which these items of cost become overly burdensome is either less than or more than the long-term rate of interest earned. It would be sheer coincidence were this critical rate to be exactly the same as the long-term rate of interest. Doubtless companies vary widely in the maximum rate of expansion that they can prudently absorb. Only after study of all factors is it possible to suggest a maximum rate for a given company. It is not for either the author or Mr. Winter to lay down an arbitrary limit for all companies based on considerations pertaining to just one element in the cost of rapid expansion.

The relevance of Mr. Winter's reference to the Armstrong investigation to the thesis of the paper is not clear. The Armstrong Committee criticized the high rate of expansion of certain companies not on account of the surplus strain incurred but because "Extravagant commissions have been paid and these have been supplemented by liberal bonuses and prizes. Clubs have been formed, conventions held, and money lavishly expended for the entertainment of agents to excite them to their utmost endeavor."[1] Undoubtedly there is for any given company a certain rate above which it cannot expand without incurring such extravagance. This rate, however, is not determined by the rate of interest earned. The limits imposed on new business by section 212 of the New York law, to which Mr. Amer refers, do not prevent a company from maintaining a rate of expansion much higher than its earned interest rate.

The first paragraph of the conclusion makes no explicit reference to methods of calculating dividends, and none was intended. The "factors" mentioned have evidently been misinterpreted by Messrs. Winter and Amer as meaning "dividend factors." The body of the paper gives no ground for such an interpretation, an explicit dividend formula in terms of either three or four factors nowhere appearing. A formula linking asset shares at successive durations is derived, and this may be used to test a dividend scale obtained by any established method. I would expect, however, that, rather than charge the cost of sustained surplus strain to the policyholder, a company issuing participating assurance and maintaining a long-term rate of expansion of its business higher than the rate of interest earned would prefer to reduce this cost by taking action along the lines suggested under the heading "A Policy for the Rapidly Expanding Mutual Company." The paper advocates this course, not any change in the method of calculating dividends.

For reasons best known to himself, Mr. Winter reads my *illustrations*

[1] *Report of the Joint Committee of the Senate and Assembly of the State of New York, Appointed To Investigate the Affairs of Life Insurance Companies*, p. 389.

of the relative cost of differing rates of expansion in specified model companies as an *expression of opinion* of the feasibility of a certain rate of expansion being maintainable in these companies. Far from suggesting that the high charges shown for the rapidly expanding Models I and II companies can or should be passed on to the participating policyholder, I devote a whole section to explanation of how this cost may be obviated. To Mr. Winter these proposals are anathema. In attacking them on the ground that their implementation would result in replacement of presently in-force policies he ignores two of their most important features: (*a*) Any savings emerging would rebound to existing as well as to new policyholders. That this is so follows from the effect of the proposals which is to reduce the rate of expansion of the corporate share account, an attribute of the business as a whole and not just new issues, to the rate of interest earned. (*b*) It is explicitly stated that "the transition from one mode of operation to another [would be] smooth and gradual." It is difficult to see why the bogy of policy replacement should be raised when no sudden change is contemplated.

In suggesting an interpolation method of changing from net level premium to Commissioners values I state, "The interpolation factor would be varied only by calendar year of issue." Why, then, does Mr. Winter purport to find an ambiguity here and suggest that I might mean that the blend is intended to change with increasing policy duration? He does not argue that the method I clearly propose is actuarially unsound, not feasible, or not effective in reducing surplus strain. His only objection is that he feels it would be difficult for the actuary to certify that "adequate and proper provision" for future obligations had been made. The phrase "adequate and proper" when applied to a gross premium valuation would imply "adequate but not redundant." It would appear to be this meaning that Mr. Winter has in mind. However, the phrase can hardly be given this interpretation when applied to a net premium valuation which is in its nature rigid and to some extent arbitrary. Here the term "proper" must rather be interpreted to mean "in accordance with legal standards." The interpolation method I suggest meets this requirement. Furthermore no inconsistency arises from the certification of the various interpolated reserves as all "adequate."

Mr. MacDonald in his contribution says, "Presumably in the fullness of time and the equity of the dividend scale everything will even out satisfactorily." The paper, however, does not suggest that it is only those policyholders who fail to achieve "fullness of time" who bear the brunt of surplus strain. In the company continually expanding at an average rate higher than the average interest rate, policyholders are continually required to contribute to surplus strain, unless the company changes its financial structure as recommended in the paper. There is no refund of earlier charges at later durations as implied by Mr. MacDonald.

While, as noted by Mr. Amer, I mention that the growth of the corporate share account may take an erratic course I neither "presume" this to be an "error" nor "demonstrate how to correct [it] mathematically." This is not an error; it is a fact of business life. The theory of the paper is designed to show the effect of surplus strain over an extended period. For this purpose year-to-year fluctuations in the growth of the corporate share account are irrelevant and for analytical simplification are treated as leveled out. No implication as to the propriety of their existence is intended.

Mr. Hickman asks how my estimate of the industry corporate share was obtained. From tables on pages 62, 64, and 68 of the *Life Insurance Fact Book, 1962,* an estimate of the long-term average level of "surplus" was derived. I have no statistics to offer in support of my estimate of the excess of policyholder reserves over asset share account as 20 per cent of gross annual premiums. However the quality of this estimate is not crucial to the reliability of an estimate of surplus strain. My estimate, though very rough, is unlikely to be in error by more than 15 per cent of gross annual premiums. This is equivalent to an error of about $\frac{1}{2}$ per cent in the estimate of the rate of expansion. Few would expect to err by no more than $\frac{1}{2}$ per cent in projecting a long-term growth rate, either for an individual company or for the entire industry.

Mr. Hickman criticizes my statement "When the rate of expansion equals the interest rate, no contribution to assets is required" on the ground that it takes no account of the need for a contingency margin. The quotation is drawn from the subsection "Basic Gross Premium Theory." Subsequently in the subsection "The Contingency Margin" I show how the basic theory can be modified to incorporate a contingency margin. Clearly, then, statements made in the subsection "Basic Gross Premium Theory" are not intended to take account of the need for a contingency margin.

Mr. Hickman feels that my method of charging for adverse contingencies is not "an ideal technique." I agree. The difficulty, however, is that there are many factors of a practical nature that are apt to be overlooked in the more scientific approach favored by Messrs. Anderson and Hickman. For example, one company desiring to be in a position to maintain the level of its dividends (either stock- or policyholder) for as long as possible after the onset of adverse conditions will accumulate a high level of surplus and hope thereby to ride the storm. Another, willing to reduce its dividends much sooner in the face of threatening circumstances, will build its funds to a somewhat lower level. The impact of adverse experience will be met more from reduced dividend disbursements at the time in the latter case than in the former. While the effects of such differences in business philosophy and practice are automatically incorporated in the criteria by which the margins of my paper are determined, their expression as adjustments to the mortality, interest, and expense factors of the traditional premium formulas would present a problem, if indeed they would be taken into consideration at all. I am not suggesting that the actuary should ignore the underlying factors or apply "an arbitrary rule of thumb" but rather that he should use his business experience as a base against which the effect of likely changes in the impact of individual hazards can be assessed.

Mr. Hickman evidently feels that surplus should be regarded as a fund attached to a policy or group of like policies throughout their life and developing with them. Initial contingency funds for any particular generation have to be "borrowed" from general contingency funds. I regard surplus, or more precisely the corporate share account, as being a fund held to enable the insurer to guarantee performance. The relationship of a fund held by a guarantor to a fund owned by the protected party cannot be likened, even in a constructive sense, to the relationship of a fund lent to one already held by the borrower. In the latter case the funds are merged for the use of the borrower until the loan is repaid; in the former they retain their separate identity, unless the pertinent contingency occurs. It may be argued that in the case of a mutual company the policyholders have a proprietary interest in the whole company, and therefore the distinction between guarantor and protected party becomes blurred. However it is not the identity of the parties that is relevant here, but their roles. In relation to the asset share account the policyholders stand, in equity, as *owners*, whereas in relation to the corporate share account they stand as *trustees*.

Mr. Hickman's approach, besides obscuring the true nature of contingency funds, does not facilitate the equitable apportionment of the cost of long-term surplus strain. He tacitly assumes that the general contingency funds will be sufficient to finance new business. Whether or not this is so depends on the relationship between the amount of the funds required to finance new business and the amount concurrently released at later durations. If the company maintains a long-term rate of expansion such that the former amount exceeds the latter where are the additional funds to come from? My approach provides the answer. It shows how the cost may be equitably distributed among policyholders. Does Mr. Hickman's? If not, his claim that his approach results in greater policyholder equity would appear to be ill-founded.