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Nested Stochastic Pricing: The Time Has Come

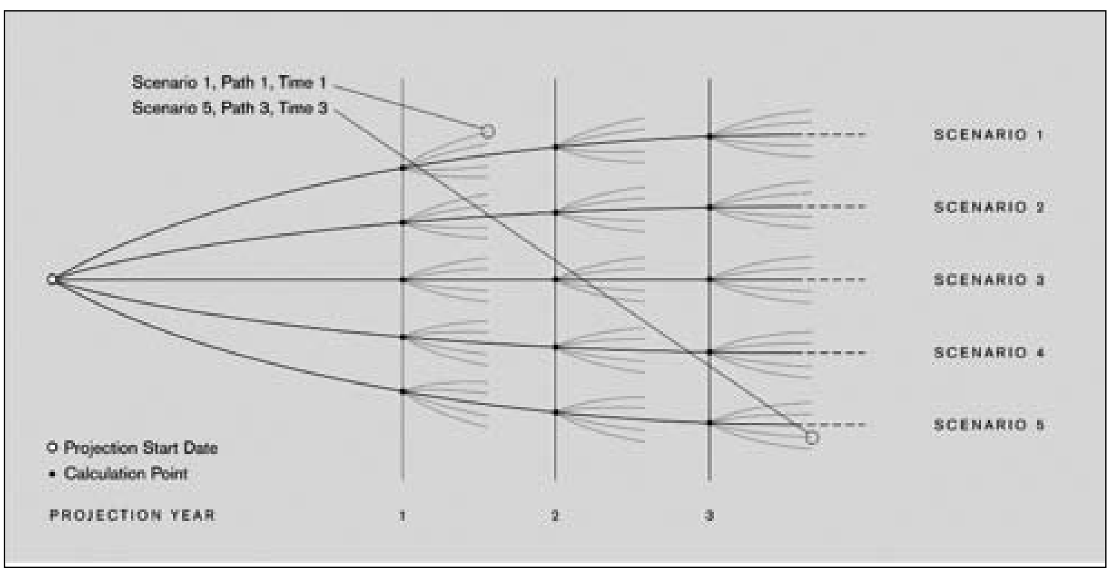
by Craig Reynolds and Sai Man

The last few years have seen a dramatic change in the way that insurance products are priced. If we traveled back in time 15 to 20 years, we would see a world where most insurance products were priced on the basis of a few static scenarios, perhaps including selected stress tests for changes in interest rates, policyholder behavior, mortality deterioration and/or expense changes. In the last several years, pricing on a stochastic basis has become more mainstream—at least for contracts with embedded guarantees.

The New World Order

What has caused the paradigm shift? We believe it is driven primarily by two factors. First, product designs have evolved that cannot be effectively priced using deterministic scenarios because of the options embedded in the designs. Two prime examples include secondary guarantee universal life (SGUL) and the alphabet soup of GMxBs now offered on variable annuity contracts. Second, technological advancements are acting as a catalyst for this paradigm shift. Twenty years ago, hardware and software limitations would have made stochastic pricing of insurance contracts for a large number of scenarios almost prohibitively impractical.

We are now on the verge of another similar paradigm shift. This time, the jump is to what we call nested stochastic pricing. Nested stochastic pricing refers to stochastic pricing where, at each node in the projection, a nested stochastic projection is used to determine reserve, capital or deferred acquisition cost (DAC) levels. Visually, nested stochastic pricing can be represented as in the diagram below, where four paths are initiated each year along five scenarios. (Note that in this article we use the term "scenario" to represent outer loops and "path" to represent inner loops.). The actual number of scenarios and paths to use will, of course, vary from application to application and will generally be much larger than is shown here. We have used five and four, respectively, for illustrative purposes only.



Again, we are the beneficiaries of advances in hardware and software that make the jump to nested stochastic pricing possible. Nested stochastic pricing causes a geometric increase in runtime. Suppose, for example, that we want to use 100 paths for 30 years at each annual node of a 30-year projection over 1,000 scenarios. Effectively, this is approximately equivalent to a stochastic projection of 1,000 x 30 x 100 = 3 million scenarios. Clearly, this requires a very efficient hardware and software package—one that would have been unthinkable more than a few years ago.

This is the first article of a two-part series. In this article we comment on some of the factors driving the change to stochastics and nested stochastics, and we discuss some of the issues an actuary must consider in moving to a world of nested stochastic analysis. In part two, which we hope to publish in the October issue of *Product Matters!*, we will shift the focus to a case study to illustrate the impact of nested stochastic analysis on an illustrative product.

Why Nested Stochastic?

The need for nested stochastics is driven by a number of changes in the regulatory and accounting world. In the last few years, we have seen the advent of:

- Statement of Position 03-1 (SOP 03-1)
- Fair value option (FAS 157/ FAS 159)
- FAS 133
- C-3 Phase II
- Economic capital

In the next few years, we are also likely to see:

- VA CARVM
- Principle-based approach for capital (C-3 Phase III)
- International Financial Reporting Standards (IFRS)
- Principle-based approach (PBA) for reserves

All of these calculations require a multi-scenario (usually stochastic) projection. Reflecting them fully in a deterministic projection like a business plan forecast requires stochastic projections nested inside a deterministic projection. Reflecting them fully in a stochastic pricing exercise requires stochastics nested within a stochastic projection. While the details of the mechanics of the above applications vary, all share some common attributes:

- They are based on actuarial models.
- They require multiple scenario projections (some real-world, some risk-neutral).
- They are dependent on actuarial judgment for many assumptions.
- They often require modeling dynamic policyholder and company behavior.

Fundamentally, there is no reason why the same basic model platform could not be used for any combination of these applications. The key is to build a model that is suitable for nested stochastic functionality; then it should be usable for most or all of these applications with only minor adjustments.

Characteristics of a Good Nested Stochastic Model Platform

Many of the traits that define a good nested stochastic platform are common to all good actuarial models. Others are unique to this particular problem. A few of these traits are as follows:

- **Flexibility:** There must be sufficient flexibility in the model to allow specification of relevant product features and policyholder/company behavior both on the scenarios and the paths.

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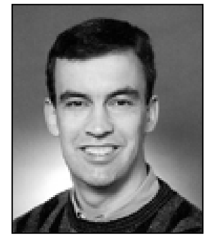

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- **Interaction:** It will be helpful in many applications to be able to model company behavior (e.g., crediting rates) and policyholder behavior (e.g., lapse rates or premium patterns) as a function of both the economic environment and the portfolio yield.

- **Internal consistency:** All aspects of liability and asset behavior need to be consistent with the economic environment and portfolio performance.

- **Controls:** Particularly because these models are very complex to create and maintain, it is important to have structures and processes to control and validate model changes to ensure that such changes do not endanger the integrity of the results.

- **Auditability:** As a nested stochastic run can easily generate millions of paths, tools and processes to audit paths and verify that they are working as intended are critical.

- **Reproducibility:** Random number generation and seed control must work in such a way that the same results are produced if the same model is run twice in a row. As many such runs may be distributed across grids of machines, it is important that random number generation is not dependent upon which machine is used for which path.

- **Speed:** Of course, the model must run exceptionally fast. Realistically, for large models or large numbers of paths-scenarios, grid-enabled applications will often be required.

- **Dial-a-granularity:** Nested stochastic pricing may often be run with just a few cells, but for other applications, it may be necessary to use varying levels of granularity that are appropriate for the purpose at hand. A well-constructed model can be used for any level of granularity simply by associating it with a more or less granular file of sales or in-force profits.

- **Appropriate scenarios and paths:** Scenarios and paths must be appropriate for the purpose at hand. In some cases, realistic scenarios or paths are appropriate. In other cases, risk-neutral scenarios or paths—or perhaps realistic scenarios with deflators—are needed. Most systems likely allow the import of scenarios from your favorite generator. But this is likely impractical for the paths. Thus, it is important that the actuary be comfortable knowing that the path generator has the appropriate characteristics for the application.

- **Support for non-economic stochastics:** For some applications, it may be desirable to make mortality rates, claim costs or other parameters stochastic instead of or in addition to economic conditions.

The Move from Stochastic to Nested Stochastic

Most systems and most actuaries can handle stochastic projections. When are nested stochastic projections appropriate? A few examples are as follows:

- **Asset valuation:** In any application where credible market values are required for assets with embedded options, such as callable bonds, multiple paths may be required to accurately estimate the market value of the assets. Market value may not be critical unless assets are being sold, but if any assets are marked as "available for sale" or "trading," such market values may be required at each projected reporting date in order to properly project GAAP or IFRS earnings.

- **Dynamic hedging:** In C-3 Phase II modeling or related applications, the ability to model dynamic hedging might be useful. Typically, this may require running hundreds of risk-neutral paths before and after various shocks in interest rates, equity indices, or volatility in order to calculate delta, rho, vega or other Greeks of the assets and liabilities.

- **SOP 03-1:** With the advent of SOP 03-1, even a deterministic scenario forecast of GAAP results will require nested stochastic projections to calculate the SOP 03-1 reserve at the end of each projection reporting cycle. When doing stochastic pricing of a related benefit, such as a GMDB, nested stochastics might be required in order to appropriately capture the distributions of GAAP earnings patterns.

- **FAS 133:** As with SOP 03-1, inner path calculations—this time on a risk-neutral basis—are required to project GAAP earnings.

- **FAS 97:** Even "normal" FAS 97 projections might require deterministic path projections to be nested at each node. For example, when running a single scenario along any given path, it will often be useful to project DAC unlocking occurring periodically along the way, with DAC balances at the end of a given projection period consistent with the actual emerging experience to date and what the prospective assumptions would be at that time.

- **IFRS:** International Financial Reporting Standards appear to be heading toward fair value as the basis for valuing liabilities. For most products, this will require inner risk-neutral paths to value the liabilities, similar to what we do for assets.

- **PBA:** The principle-based approach to reserves and capital will require mechanics very similar to those of IFRS to calculate reserves or required capital at each projection node.

Managing Nested Stochastic Runtime

The move to nested stochastics will require some thought as to how to manage the runtime. As noted above, geometric expansion in the calculations will require huge amounts of computation time if certain efficiency steps are not taken. As an example, consider an insurer who wants to perform the following calculations in a seriatim projection of a block of in-force business:

- 30-year projection with quarterly reporting cycles
- 1 million liability model points
- 1,000 scenarios
- Dynamic hedge rebalancing at the end of each quarter, with each rebalance requiring path projections to maturity with the following specifications:
  - 30-year projection
  - 100 paths for the base case and each of the 20 shocks (10 up and 10 down). This results in 2,100 paths jat each projection node.

The total number of liability cell projections for this liability portfolio is:

(30 years) * (4 quarters / year) * (1 million cells) * (100 paths) * (1 + 2* 10 shocks) * (1,000 scenarios)

= 252 trillion cell-path projections

Assume for the moment that you have an extraordinarily fast system that can project 10,000 cell-paths per second. This would still add up to about 25.2 billion seconds, or around 799 years. This is clearly not practical in the real world!

Fortunately, in a pricing context, we can usually run with far fewer cells than this, but the runtime might still be unacceptable. So what can we do to optimize these calculations? Fundamentally, there are a few basic options:

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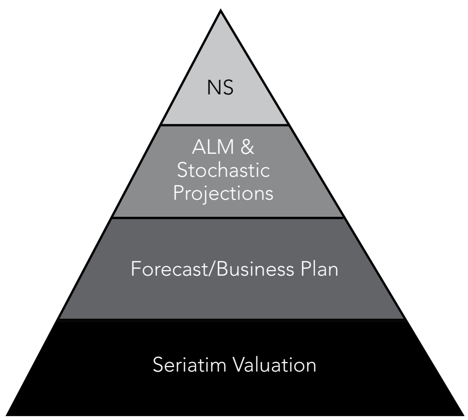
- Faster hardware and software
- More machines
- Fewer cells
- Fewer scenarios
- Fewer paths or shocks
- Less frequent rebalancing

In reality, a combination of these will likely be required. Fortunately, in a pricing context the cell count might shrink dramatically—perhaps to as few as 10 cells or so. We view the diagram below as representative of the levels of granularity that a well-designed model must be prepared to accept. Ideally, a user should be able to change the granularity, scenario count, or path count on the fly with minimal model changes.



* NS represents nested stochastic

Cell count alone will not be enough, however. No doubt, the science of scenario and path reduction techniques will need to continue to evolve. And it seems increasingly likely that more and more companies will acquire or rent large "grid farms" or 500 or more personal computer engines. In the example above, a 500-machine grid and a reduction in cell count from 1 million to 10 (assuming no loss in efficiency) would reduce the 800 years to eight minutes. That's more in the range of a viable answer!

Why Now?

One might logically ask why we should worry about this now. If your company is not GAAP-focused (so that FAS 97, SOP 03-1 and FAS 133 are not relevant) and not engaged in dynamic hedging, the immediate need may not be clear. But with IFRS and PBA coming soon, now may be the time to begin modeling your products to see how they behave under the new regime.

As an example for how this can come into play, consider C-3 Phase II. For several companies that we worked with when this first came out, the standard scenario was the driver for capital requirements on in-force profits. Given this, one might argue that it would be appropriate to price products using the standard scenario to measure required capital. This has a definite impact on product pricing, as, for example, a ratchet design for a guaranteed minimum death benefit on a variable annuity contract is likely to have little or no marginal contribution to required capital in the standard scenario. In contrast, a roll-up looks very expensive under the standard scenario. This certainly has implications for how these products should be priced and marketed.

Many actuaries have done some modeling as PBA and IFRS have evolved. But we have observed that few companies are using PBA or IFRS on a nested stochastic basis in their current pricing. It may be time for this to change. Even though we may be years away from a time when companies will need to be valued on a PBA or IFRS basis, products priced today may well be the products that are being sold when PBA comes into effect.

In the next issue we will provide a case study of nested stochastic pricing to show how principle-based reserves and capital impact product profitability when viewed on a nested stochastic basis. ☐