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Nested Stochastic Pricing: A Case Study

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In the last issue of *Product Matters!* we discussed the environmental factors that motivate the development of stochastic and nested stochastic pricing, as well as the associated logistical complications. In this issue, we illustrate the practical and financial implications of applying nested stochastic techniques to a sample product.

You might choose to think of this as a pricing exercise in a world with a Principle-Based Approach (PBA) to reserves and capital, but we have made only minor attempts at implementing the precise U.S. PBA requirements as they now stand. To do so would be of limited value, as the landscape is shifting as we speak, and will likely shift again as we write this and the time this article is in your hands. Instead, view this as an illustrative exercise for one hypothetical PBA regime where nested stochastic functionality is required to project future reserves and capital along the scenario at each year-end. Detailed results will no doubt vary as PBA requirements develop. In particular—for the sake of simplifying the models and the presentation—we have ignored the impact of the deterministic scenarios.

An SGUL Example

Secondary Guarantee Universal Life (SGUL) is one product that has been the subject of considerable reserve controversy in recent years. As such, it seems appropriate to develop an initial case study to analyze the impact of one hypothetical PBA regime on a SGUL plan. To keep the analysis simple, we will focus on one cell for a hypothetical plan with the following key attributes:

- Male, preferred non-smoker, issue age 45 with a $250,000 face amount.
- Shadow account design with premiums set to fund the shadow account to maturity. Premiums and shadow account provisions are set to be competitive in the current market.
- AG38 valuation mortality: 2001 CSO Table.
- Anticipated mortality experience equal to 50 percent of the 2001 CSO table.
- Anticipated lapse rates by policy year of 6 percent in years one through three, 3 percent in years four through ten, and 2 percent thereafter.
- Anticipated expenses of $200 per policy for acquisition expense, and $40 per year for maintenance expense.
- First year commissions of 70 percent, with 3 percent renewal commissions.
- No reinsurance.
- “Interim Solution” reserves at 4 percent interest, and interim solution lapse rates.
- New money invested at 200 basis points over the 10-year Treasury rate in 10-year AA callable corporate bonds.

- Market based crediting strategy of 50 basis points over the 7-year Treasury rate.

- Required capital of 5 percent of reserves, 5 percent of premium, and 0.15 percent of net amount at risk;

- Tax rate on operating gain of 35 percent.

- Nested stochastic reserves calculated at the end of each year in a 50-year projection using 25 inner paths along 200 outer scenarios;

In a base case level scenario with reserves and capital set non-stochastically as described above, the secondary guarantee comes into the money only at the tail, and the plan is somewhat profitable, with an after-tax, after-cost of capital internal rate of return (IRR) of 10.0 percent.

We can then extend the analysis to capture the cost of the guarantee by modeling interest rates stochastically. For this purpose, we use a real-world mean reverting three-factor (short rate, slope and curvature) lognormal interest rate model, with parameters that are largely consistent with historical experience over the last 30 years.

In addition, we expand the model to include a simple lapse function that is sensitive to "in-the-moneyness" by applying the following logic:

- Set lapse rate to zero when policy is in-the-money.

- In-the-money is defined as the situation where the current account value drops below zero while the shadow account remains positive.

Given this, we can rerun the above model through 200 scenarios and obtain a mean IRR of 19.2 percent. The fact that this is higher than the base case may seem counter-intuitive, but it appears to be an inevitable result of the mean reversion assumption in our scenario generator which trends towards a long-term mean for the short-term rate of 4.5 percent. In fact, the post-1960 average of the short-term Treasury rate is 6.0 percent, so the 4.5 percent rate is somewhat conservative for this product relative to the long-term experience. In recent years, the rate has been much lower. Clearly this assumption will have a large impact on product profitability and should be subject to sensitivity testing.

Table 1 below shows the associated distributions of Present Value of Profits (PV Profits) at a 10 percent discount rate. The average PV Profits under the AG38 Interim Solution Reserving methodology is $7.0 with a sigma of $2.94.

Table 1
Percentile Distribution of PV Profits
10% Discount Rate
AG38 Interim Solution Reserves

Percentile	Adj Aft Tax
0%	(0.8)
10%	3.3
20%	4.2
30%	5.7
40%	6.4
50%	7.1
60%	7.9
70%	8.5
80%	9.9
90%	10.9
100%	13.8
Average	7.0
Sigma	2.94

SGUL with PBA

Now we would like to enhance our model to see how a PBA approach to reserves and capital might impact the expected profits and the distribution of profits. For this purpose—though PBA requirements are clearly still evolving—we will calculate PBA reserves using CTE 70 and capital using CTE 90. Both will be based on the greatest present value of accumulated deficiencies. Reserve calculations ignore income tax and capital calculations will reflect income taxes. As mentioned earlier, for the sake of simplicity, we will ignore the impact of the deterministic reserve, though this would appropriately be reflected in actual pricing.

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The selection of assumptions for a PBA calculation is worthy of a series of articles in and of itself, so for now we will test using the prudent anticipated experience assumptions in the model, with 10 percent margins on mortality and expenses, and a 50 percent reduction in lapse rates. In this case, there is no surplus strain and the IRR is thus undefined. Table 2 shows the resulting distribution of PV Profits.

Table 2
Percentile Distribution of PV Profits
10% Discount Rate
PBA Reserves and Capital

Percentile	Adj Aft Tax
0%	11.9
10%	14.7
20%	15.5
30%	15.9
40%	16.1
50%	16.2
60%	16.4
70%	16.6
80%	16.8
90%	17.2
100%	17.9
Average	16.1
Sigma	0.93

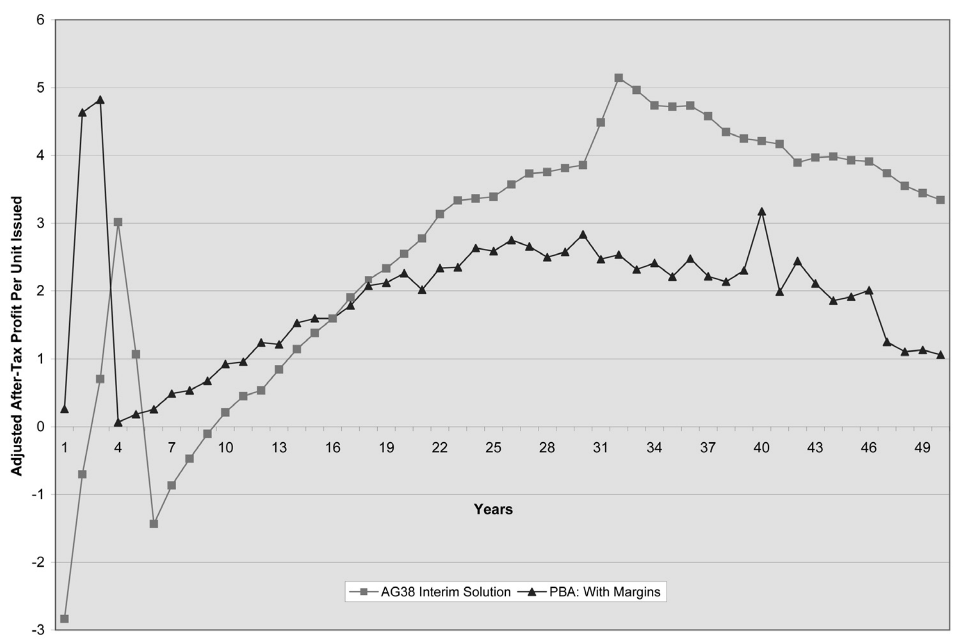
Interestingly, the PV Profits are now considerably higher, with an average PV Profits of $16.1. If one accepts the margin levels, experience assumptions and scenario parameterization as appropriate, this suggests—that for this product at least—the AG38 reserves may be unnecessarily conservative. Note also that the standard deviation of the margins is considerably smaller than with AG38 Interim Solution reserves.

Our understanding is that the current PBA proposals call for pre-packaged scenarios that are approved by insurance regulators, rather than proprietary scenarios or company-selected scenarios. If this proposal holds, the results may in fact be materially different.

Figure 1 below contrasts the mean adjusted after-tax earnings by year between AG38 interim solution and our PBA proxy results.

Note that the PBA earnings are positive, initially due to the reserves and capital having a zero value at the end of the first

Figure 1
Average Earnings by Year; AG38 Interim Solution versus PBA Proxy 200 Scenarios



year. First year profits are thus essentially equal to the excess of premiums over commissions and other acquisition expenses. In fact, they remain zero through year two. Since year two has materially less cash flow strain, year two earnings are materially positive. In later years, earnings level off and are relatively smooth as the reserves increase. Later year PBA profits essentially arise as a result of release of margins.

These results are interesting but should not be considered to be fully generizable to other product designs or assumption sets. Their main value comes in demonstrating the impact on earnings under the proposed PBA regime. Because of this, it is important to conduct PBA-based pricing now on products that may be sold once PBA comes into play.

Other Issues to Consider

There are clearly other issues to consider as well. For example, aggregation is one key issue. The analysis above measures reserve and capital as if this were the only product that the company issues. In reality, PBA reserves and capital are calculated on an aggregate basis. Thus, the marginal reserve and capital requirement for a new issue will differ from company to company and might in fact be zero. This will certainly create some interesting philosophical discussions in pricing.

Taxes are of course another potentially significant issue. For simplicity, our analysis above assumes that the tax reserves are always equal to the product surrender value. In reality of course, no one knows yet how PBA will impact tax reserves, and the eventual resolution of this issue might materially alter the landscape as presented above.

Conclusion

The actual impact of PBA on any given company or product will clearly vary dramatically depending on the product, the company and the assumptions used. But now is not too soon to begin this analysis. For many companies, this will require new hardware or software. For almost all companies, material changes will be required in processes or mindset. ◻

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