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VALUATION OF NONVESTED RENEWAL COMMISSIONS

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THE most thorough and authoritative investigation of the cost of paying a renewal commission or service fee that is contingent upon continuance in service (or perhaps also upon death while in service) of the full-time agent who produced the business is that presented to the Actuarial Society in 1942 by Mr. Edmund M. McConney and the late Mr. Richard C. Guest (*TASA*, XLIII, 287). Since that study was based on experience of a pre–World War II era it may be agreed that a fresh look at this actuarial problem is desirable.

In the McConney-Guest paper the method used was to decide first upon a survival table of agents, then to postulate production curves for those agents, and then to compute probabilities that business produced in a particular contract year would be with an active agent at each renewal policy duration. Valuation factors were obtained by combining these probabilities with persistency of business and interest factors. Mr. Daton Gilbert in his discussion of that paper pointed out (*TASA*, XLIV, 102) that a different "alternate approach" was available, i.e., to obtain from company records the proportions of active and orphaned business in force at each policy duration. This alternate approach develops valuation factors more directly and avoids necessity for estimating agent production patterns. Also, perhaps more important, it avoids the need for assessing differences in persistency between business produced by terminating and continuing agents. On the other hand, it does not have the collateral value of producing an agents' survival table that is useful for other purposes.

Current Investigation by "Alternate Approach"

During 1962 the actuaries of seventeen companies pooled samples of their data to permit the method suggested by Mr. Gilbert to be used to create the tables contained in this paper. These companies are all among the fifty largest United States companies in terms of ordinary life insurance in force. To help maintain as much homogeneity as possible in a study that inevitably involves a troublesomely low degree of homogeneity, the companies invited to contribute were only those with home offices in the United States that do not write weekly premium business and that do not write other than life and accident and sickness business. (Not all

the invited companies accepted the invitation.) Each company was asked to contribute data on a sample of policies that were in force at the date of investigation (late summer, 1962) and that were issued in the years 1960, 1959, 1958, 1956, 1951, 1946, and 1941, giving for each policy the information shown in the Appendix to this paper. Some companies contributed to only a portion of this study, the companies and the extent of data being as indicated in the following table:

CONTRIBUTORS TO THIS STUDY

Company	Years of Issue	Agents' Contract Years at Issue	Policies in Each Year of Issue	Total Sample (Policies)
Bankers, Iowa	All	All	200	1,400
Connecticut Mutual	"	"	200	1,400
Equitable, Iowa	"	"	100	700
Guardian	"	"	100	700
Home, N.Y.	"	"	100	700
Massachusetts Mutual	"	"	200	1,400
New England	"	"	200	1,400
Occidental, Calif.	"	"	200	1,400
Ohio National	"	"	100	700
Penn Mutual	"	"	200	1,400
Phoenix Mutual	"	"	100	700
Provident Mutual	"	"	100	700
Union Central	"	"	100	700
Continental Assurance	1958, '59, '60	1–4	200	600
Lincoln National	1951 and later	All	100	500*
Mutual, N.Y.	1951 and later	All	200	1,000
National, Vt.	1959 and 1960	1–4	100	200
				15,600

* Included in tests of results, but not in developing c-factors.

Before each company's contribution was used, efforts were made to test its reasonableness. These tests showed that difficulty was sometimes experienced in obtaining accurately the information desired, particularly in designating whether a departed agent's contract was terminated by death or for other reasons. Fortunately, this separation by mode of termination does not affect the major part of this study, but the undiscovered errors do affect results that involve renewal commissions payable after death. It also appears that at long contract durations where retirement becomes a significant possibility, differences in practice between companies that treat retired agents as active and those that consider them as terminated have some impact upon the results. Our preference has been to treat them as active and therefore as qualifying for nonvested commissions.

It should be emphasized that the purpose of this study is only to give a general picture of ranges and patterns of nonvested commission values that can be useful to an actuary studying the experience in his own company. By no means should the figures in this paper be regarded as either industry averages or as values that can safely be applied in any single company. It should also be recognized that in this study the sole criterion for payment of nonvested commissions was continuance in service (or prior death). This produces larger nonvested values than apply when, as usually happens in practice, payment of nonvested commissions depends upon meeting contractual production requirements.

Notation

We use the same notation as in the McConney-Guest paper, but with a change in definition made necessary by the facts that (1) the data analysed are dollars of premium, not dollars of insurance amount, and (2) our persistency criterion is payment of the premium in a policy year rather than entering the year. (This latter difference would disappear if there were no fractional premium business.)

In the notation of the McConney-Guest paper, P_t represents the proportion of original new business that has persisted to enter policy year t; $P_t(1 - 0_{[n],t})$ and $P_t \cdot 0_{[n],t}$ signify the respective segments of P_t that are with active agents and are in the orphaned class at the beginning of policy year t. The symbol n indicates business produced in the agent's nth contract year.

In this paper, on the other hand, $P_{[n],t}$ represents the proportion of original premium that is paid in the tth policy year; $P_{[n],t}(1 - 0_{[n],t})$ and $P_{[n],t}\, 0_{[n],t}$ separate this premium into its active and orphaned parts. We introduce the symbols c and c' with the meanings now to be described.

Rationale of Method Used

Our method presupposes that there exists a function c such that

$$\frac{c_{n+t-1}}{c_n} = 1 - 0_{[n],t}.$$

The usefulness of such a relationship, if it can be substantiated, is that select values of $1 - 0_{[n],t}$ can be obtained from an aggregate table of c-values. The fraction c_{n+t-1}/c_n expresses the probability that a unit of renewal commission that is conditional upon continuance of the agent's full-time status will be payable in the tth policy year on business produced in the agent's nth contract year, not discounted for policy termination or interest.

Let us first consider the premise upon which the above-stated relationship must rest. Since

$$\frac{c_{n+t-1}}{c_n} \equiv \frac{c_{n+t-1}}{c_1} \cdot \frac{c_1}{c_n} \equiv \frac{c_{n+t-1}}{c_2} \cdot \frac{c_2}{c_n} \equiv \frac{c_{n+t-1}}{c_3} \cdot \frac{c_3}{c_n}$$

$$\equiv \ldots \equiv \frac{c_{n+t-1}}{c_{n+t-2}} \cdot \frac{c_{n+t-2}}{c_n},$$

the relationship holds true only if all the following ratios are identical:

$$\frac{1 - 0_{[1],\, n+t-1}}{1 - 0_{[1],\, n}}, \frac{1 - 0_{[2],\, n+t-2}}{1 - 0_{[2],\, n-1}}, \frac{1 - 0_{[3],\, n+t-3}}{1 - 0_{[3],\, n-2}}, \ldots, \frac{1 - 0_{[n+t-2],\, 2}}{1 - 0_{[n+t-2],\, 3-t}}.$$

In words, the probability that a unit of premium on a continuing policy for which the agent was active at contract year h will still be with an active agent at contract year $h + k$ must be independent of the range of policy durations that embraces the k-year period. This supposition may not qualify as a self-evident truth, but it is convenient and is adequately supported as a reasonable approximation by the results of this investigation. The eight independent values of c_{10} given later in this paper indicate, for that attained contract year as an example, the reasonableness of our hypothesis.

It is also presumed that there exists a companion function c' which is similar in character to c, but which is derived from statistics that treat orphanage by withdrawal of the agent only, not by death. Thus c'_{n+t-1}/c'_n expresses the probability that a unit of renewal commission that is conditional upon continuance of the agent's full-time status or death will be payable in the tth policy year on business produced in the agent's nth contract year, not discounted for policy termination or interest.

Investigation Procedures and Results

From each company's material, summary cards were prepared containing the following information:

(1) Year of issue.
(2) Policy duration, taken as 1962 less the year of issue. Thus there were seven policy durations: 2, 3, 4, 6, 11, 16, and 21.
(3) Agent's contract year at issue. Each of the first seven contract years was recorded individually; the following were grouped: years 8–10 (treated as year 9), years 11–15 (13), years 16–20 (18), years 21–25 (23), and years 26 and above (30). Thus there were twelve contract years at issue studied.
(4) Total premiums on policies with active agents (status 1).
(5) Total premiums on policies with active or deceased agents (status 1 + 2).
(6) Total premiums on all policies (status 1 + 2 + 3).
(7) Premium ratio: (status 1)/(status 1 + 2 + 3).
(8) Premium ratio: (status 1 + 2)/(status 1 + 2 + 3).

Item (7) is of course the value of $1 - 0_{[n],t}$ arising from orphanage by withdrawals and deaths; item (8) recognizes orphanage by withdrawal only.

The cards for all companies were then sorted together by year of issue and contract year at issue. Those comprising each of these cells were then ranked from the highest ratio of status 1/(status 1 + 2 + 3) down to the lowest; the two cards with the highest and the two cards with the lowest ratios were rejected; the card with the middle ratio was rejected if the number of cards was an uneven number; and the cards still remaining were divided into the 50 per cent with above-median ratios and the 50 per cent with below-median ratios. For each of these groups within each cell, a simple average was taken by dividing the sum of the ratios by the number of companies involved, thus producing a representatively high value of $1 - 0_{[n],t}$ and a representatively low value of $1 - 0_{[n],t}$ for each of seven values of t within each of twelve values of n.

Next, for the high H-values and the low L-values independently, the values of $1 - 0_{[n],t}$ for the missing values of t were filled in by first-difference interpolation. We thus arrived at complete ungraduated tables of $1 - 0_{[n],t}$ for twenty policy durations (t) within each of twelve contract years at issue (n).

We then proceeded to calculate the values of the desired function c, starting with the radix $c_1 = 1{,}000$. The value of c_2 emerges immediately, because $c_2/c_1 = 1 - 0_{[1],2}$.

However, two different values of c_3 are created because by definition

$$\frac{c_3}{c_2} = \frac{c_2}{c_1} \cdot \frac{c_1}{c_2} = \frac{1 - 0_{[1],3}}{1 - 0_{[1],2}}$$

and also

$$\frac{c_3}{c_2} = 1 - 0_{[2]\ 2}.$$

The ungraded value of c_3 adopted was the simple average of these two emerging values. Similarly, three values of c_4 emerged, and a simple average of these was taken.

The same process was repeated for successive contract years at issue, in each case the final unsmoothed value of c being the simple average of the independent values of c corresponding to the same attained contract year. For example, at attained contract year 10 the H-values of c_{10} corresponding to contract years at issue one through seven and nine were respectively 467, 378, 470, 395, 489, 517, 497, 471, and these were averaged to produce 460.

These average values of c were then smoothed by two summations in

threes, remaining irregularities being removed by inspection, and were extended by an arbitrary process for durations beyond 40 to a zero terminal value at the sixty-first year for Table H, and at the fifty-first year for Table L.

The next operation was to produce a third set of values of c (M-values), equal to the means of corresponding H-values and L-values. Table 1 gives the values of c obtained for these three tables, H, M, and L.

Consideration was then given to construction of the corresponding set of tables of c' that could be used to evaluate renewal commissions payable in event of either continuance in service or prior death. In this case the

TABLE 1

VALUES OF c_r

r	Table H	Table M	Table L	r	Table H	Table M	Table L
1	1,000	1,000	1,000	31	304	194	83
2	655	570	485	32	299	189	79
3	603	510	416	33	295	185	75
4	563	461	359	34	291	181	71
5	542	428	314	35	287	177	67
6	523	401	279	36	281	172	63
7	506	379	252	37	272	165	58
8	490	360	231	38	261	157	53
9	475	345	215	39	249	148	48
10	461	332	202	40	236	140	44
11	449	320	192	41	223	131	39
12	438	311	184	42	210	122	35
13	429	302	176	43	197	114	31
14	420	294	168	44	183	105	27
15	412	286	160	45	169	96	23
16	404	278	152	46	155	87	19
17	396	270	144	47	141	78	16
18	388	262	136	48	128	70	13
19	380	254	128	49	115	62	10
20	373	247	121	50	102	54	5
21	366	240	114	51	89	45	0
22	359	234	109	52	77	38	.....
23	352	228	104	53	66	33	.....
24	345	222	100	54	56	28	.....
25	338	218	97	55	47	24	.....
26	332	214	95	56	39	20	.....
27	326	210	93	57	32	16	.....
28	320	206	91	58	26	13	.....
29	314	202	89	59	20	10	.....
30	309	198	86	60	10	5	.....
				61	0	0	.....

The symbol r denotes attained contract year, i.e., contract year at issue plus policy duration.

procedure was the same as stated above up to the point at which crude H- and L-values of c' emerged. Tests were then made to determine what age at contract date (x) could be used in conjunction with mortality rates from the 1958 CSO Basic Table to produce from the values of c a pattern of c'-values consistent with the crude values obtained, using the formula

$$c'_r - c'_{r+1} = c_r(1 - q_{x+r-1}) - c_{r+1} .$$

By trial it was determined that use of age 35 for Table H and age 40 for Table L produced results that corresponded satisfactorily to the experience. Table 2 gives the values of c' obtained. Again the M-values are midway between the H- and L-values.

TABLE 2

VALUES OF c'_r

r	Table H	Table M	Table L	r	Table H	Table M	Table L
1.....	1,000	1,000	1,000	16.....	426	298	170
2.....	656	572	487	17.....	421	292	164
3.....	605	512	419	18.....	416	287	158
4.....	566	464	363	19.....	411	282	152
5.....	546	432	319	20.....	407	277	147
6.....	528	406	285	21.....	404	273	142
7.....	512	386	259	22.....	401	270	139
8.....	497	368	239	23.....	398	267	136
9.....	483	354	224	24.....	396	265	134
10.....	471	342	212	25.....	394	264	133
11.....	461	332	203	26 and			
12.....	452	324	196	higher.	393	263	133
13.....	445	317	189	.....	.....	.....	.....
14.....	438	310	182	.....	.....	.....	.....
15.....	432	304	176	.....	.....	.....	.....

The symbol r denotes attained contract year, i.e., contract year at issue plus policy duration.

Tests of Results

The c-factors and the c'-factors were tested against the original company data to demonstrate that first, the variation between the H-values and the L-values reasonably measures the dispersion that can occur in practice and second, that the patterns of these tables bear resemblances to actual conditions in particular companies. As illustrations of their applicability, Table 3 shows the c-factor results found in three companies whose sample results showed marked differences in agent survival rates. One of these results is close to Table H, another to Table M, and the third to Table L.

TABLE 3

CONTRIBUTIONS TO STUDY BY THREE COMPANIES
ACTUAL PREMIUMS WITH ACTIVE AGENTS COMPARED WITH EXPECTED PREMIUMS BY TABLES INDICATED

	Company A		Company B		Company C	
	Actual Status 1 Premium	Expected by Table H	Actual Status 1 Premium	Expected by Table M	Actual Status 1 Premium	Expected by Table L
	By Year of Issue					
1941	$ 8,046	$ 9,827	$ 3,382	$ 4,497	$ 1,752	$ 2,363
1946	17,174	16,382	7,138	8,788	4,609	5,509
1951	17,339	21,482	11,430	12,070	10,265	8,185
1956	36,677	35,595	29,287	28,365	12,413	11,256
1958	44,752	45,604	25,654	27,015	17,890	18,138
1959	67,453	67,209	28,645	30,942	19,246	21,271
1960	48,333	47,794	29,924	30,631	17,806	23,729
Total	$239,774	$243,893	$135,460	$142,308	$83,981	$90,451
	By Contract Year at Issue					
1	$ 19,434	$ 19,018	$ 9,772	$ 10,643	$ 4,464	$ 3,459
2	13,071	15,630	12,164	13,453	3,941	5,137
3	10,443	10,971	9,998	8,951	4,395	6,076
4	10,537	12,440	5,042	6,134	1,520	2,591
5	14,888	15,934	4,330	4,498	3,477	4,021
6	18,302	19,545	8,187	8,033	1,915	3,024
7	10,440	11,650	5,446	7,025	2,023	2,576
9	33,691	31,926	9,623	9,322	8,170	7,178
13	33,222	33,816	24,432	25,000	14,231	14,004
18	18,661	18,541	14,411	14,341	19,212	16,417
23	27,170	26,207	22,663	22,624	5,627	5,748
30	29,915	28,215	9,392	12,284	15,006	20,220
Total	$239,774	$243,893	$135,460	$142,308	$83,981	$90,451

RATIO, TOTAL ACTUAL TO TOTAL EXPECTED PREMIUM BY EACH OF THE THREE TABLES

Table H	98.3%	89.2%	77.1%
Table M	104.5	95.2	82.0
Table L	117.6	107.8	92.8

Development of Factors Reflecting Persistency of Business as Well as Agents' Survival

After the c-factors in Table 1 and the c'-factors in Table 2 had been used to construct values of $1 - 0_{[x],t}$ for the second-thirtieth policy years corresponding to contract years at issue 1–7, 9, 13, 18, 23, and 30, the next and final operation was to multiply these agents' survival factors by persistency of business factors discounted at 4 per cent interest. As indicated in Mr. Gilbert's discussion of the McConney-Guest paper, the question that arises is whether such persistency of business factors should reflect the persistency of business with active agents only or the total company persistency which is affected by the presumably higher lapse rate of orphan business. On the basis of the following analysis we conclude that the total company persistency is applicable.

The probability that a unit of premium produced in any contract year will both persist and be with an active agent (i.e., will qualify for a nonvested commission) in the tth policy year is, of course,

$$\frac{\text{"Active" premiums in force at duration } t}{\text{Total premiums on new issues}}.$$

But this can be expressed as X times Y, where

$$X = 1 - 0_{[x],t} = \frac{\text{"Active" premiums in force at duration } t}{\text{Total premiums in force at duration } t}$$

and

$$Y = P_{[x],t} = \frac{\text{Total premiums in force at duration } t}{\text{Total premiums on new issues}}.$$

It is thus seen that the persistency of business factor (expression Y above) must reflect the persistency of all new business of the nth contract year, not just that on which the agent is still active in the tth policy year.

For our tables the persistency factors were taken from Tables R, S, and T, published in *TSA*, XII, 545, using Table T values for business of the first and second contract years, Table S for the third and fourth, and Table R for the fifth and later contract years.

Table 4 compares a few of the resulting values with those of the McConney-Guest Modified Table, using 4 per cent interest in each case. The McConney-Guest figures use Linton A persistency at all contract durations. Table 5 gives complete sets of valuation factors derived from c-factors (i.e., for payment contingent upon continuance of the agent in full-time service). Corresponding valuation factors with provision for vesting at death are available from the author on request.

TABLE 4

VALUE AT ISSUE OF RENEWAL COMMISSION OF ONE
(4 PER CENT INTEREST-PERSISTENCY INDICATED ABOVE)
Payment contingent upon continuance of agent in full-time service

Policy Year	Table H	Table M	Table L	McConney-Guest Modified
	Business of First Contract Year			
Second	.414	.360	.306	.687
Fifth	.249	.197	.144	.238
Tenth	.144	.104	.063	.077
Twentieth	.052	.034	.017	.022
	Business of Third Contract Year			
Second	.710	.688	.656	.768
Fifth	.497	.440	.359	.368
Tenth	.301	.253	.183	.145
Twentieth	.120	.092	.053	.047
	Business of Fifth Contract Year			
Second	.816	.792	.752	.809
Fifth	.592	.545	.463	.447
Tenth	.376	.334	.260	.203
Twentieth	.162	.132	.081	.071
	Business of Ninth Contract Year			
Second	.821	.813	.795	.837
Fifth	.610	.591	.554	.533
Tenth	.397	.369	.307	.296
Twentieth	.171	.152	.107	.105
	Business of Eighteenth Contract Year			
Second	.828	.819	.796	.856
Fifth	.625	.604	.541	.604
Tenth	.408	.389	.332	.354
Twentieth	.178	.160	.108	.104

TABLE 5

VALUE AT ISSUE OF RENEWAL COMMISSION OF ONE
4 PER CENT INTEREST

Payment contingent upon continuance of agent in full-time service

POLICY YEAR	CONTRACT YEAR 1			CONTRACT YEAR 2			CONTRACT YEAR 3			CONTRACT YEAR 4			CONTRACT YEAR 5			CONTRACT YEAR 6		
	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L
2.....	.414	.360	.306	.582	.565	.542	.710	.688	.656	.733	.706	.666	.816	.792	.752	.818	.799	.764
3.....	.331	.280	.228	.472	.444	.406	.620	.579	.521	.641	.600	.536	.729	.692	.627	.732	.701	.646
4.....	.282	.231	.180	.414	.376	.324	.552	.501	.428	.573	.524	.447	.656	.610	.534	.659	.624	.559
5.....	.249	.197	.144	.367	.324	.264	.497	.440	.359	.515	.462	.381	.592	.545	.463	.595	.560	.489
	1.276	1.068	.858	1.835	1.709	1.536	2.379	2.208	1.964	2.462	2.292	2.030	2.793	2.639	2.376	2.804	2.684	2.458
6.....	.222	.170	.118	.328	.282	.221	.448	.389	.306	.465	.412	.330	.537	.489	.406	.542	.503	.434
7.....	.199	.149	.099	.294	.249	.187	.404	.347	.265	.420	.369	.289	.488	.441	.360	.493	.457	.388
8.....	.179	.131	.084	.264	.221	.162	.366	.311	.232	.382	.332	.256	.446	.401	.323	.453	.416	.348
9.....	.161	.117	.073	.238	.197	.141	.332	.279	.206	.347	.301	.229	.410	.365	.290	.416	.379	.312
10.....	.144	.104	.063	.214	.176	.124	.301	.253	.183	.316	.272	.203	.376	.334	.260	.383	.346	.278
	2.181	1.739	1.295	3.173	2.834	2.371	4.230	3.787	3.156	4.392	3.978	3.337	5.050	4.669	4.015	5.091	4.785	4.218
11.....	.130	.093	.056	.194	.158	.110	.275	.229	.164	.288	.247	.181	.346	.304	.232	.352	.316	.248
12.....	.117	.083	.049	.175	.142	.097	.251	.207	.146	.264	.223	.161	.319	.278	.207	.324	.288	.221
13.....	.106	.075	.044	.159	.128	.086	.229	.188	.129	.241	.202	.142	.293	.253	.184	.298	.262	.195
14.....	.096	.067	.038	.144	.115	.075	.209	.170	.114	.219	.183	.125	.270	.230	.163	.274	.238	.173
15.....	.087	.060	.034	.130	.103	.066	.191	.154	.101	.200	.165	.110	.247	.209	.144	.252	.217	.153
	2.717	2.117	1.516	3.975	3.480	2.805	5.385	4.735	3.810	5.604	4.998	4.056	6.525	5.943	4.945	6.591	6.106	5.208

TABLE 5—*Continued*

Policy Year	Contract Year 1			Contract Year 2			Contract Year 3			Contract Year 4			Contract Year 5			Contract Year 6		
	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L
16.....	.079	.054	.030	.118	.092	.058	.174	.139	.088	.182	.149	.096	.228	.191	.127	.232	.198	.135
17.....	.071	.048	.026	.106	.082	.050	.158	.125	.077	.167	.135	.085	.209	.174	.113	.213	.181	.121
18.....	.064	.043	.022	.096	.073	.043	.145	.113	.068	.152	.122	.074	.192	.159	.101	.195	.165	.108
19.....	.057	.038	.019	.086	.066	.038	.132	.102	.059	.138	.110	.066	.176	.145	.090	.179	.150	.097
20.....	.052	.034	.017	.078	.058	.033	.120	.092	.053	.126	.100	.058	.162	.132	.081	.164	.138	.088
	3.040	2.334	1.630	4.459	3.851	3.027	6.114	5.306	4.155	6.369	5.614	4.435	7.492	6.744	5.457	7.574	6.938	5.757
21.....	.047	.031	.015	.070	.052	.029	.109	.084	.047	.115	.090	.052	.148	.121	.073	.151	.127	.081
22.....	.042	.027	.013	.063	.047	.025	.099	.076	.042	.104	.082	.047	.136	.111	.067	.138	.116	.074
23.....	.037	.024	.011	.056	.041	.022	.090	.069	.038	.095	.075	.043	.124	.101	.061	.126	.106	.067
24.....	.034	.022	.010	.050	.037	.019	.082	.063	.034	.087	.068	.039	.114	.093	.056	.115	.097	.061
25.....	.030	.019	.009	.045	.033	.017	.075	.057	.031	.079	.062	.035	.104	.084	.051	.106	.088	.055
	3.230	2.457	1.688	4.743	4.061	3.139	6.569	5.655	4.347	6.849	5.991	4.651	8.118	7.254	5.765	8.210	7.472	6.095
26.....	.027	.017	.008	.040	.030	.015	.068	.052	.028	.071	.056	.032	.095	.077	.046	.097	.080	.049
27.....	.024	.015	.007	.036	.026	.014	.061	.047	.025	.065	.051	.028	.087	.070	.041	.088	.073	.044
28.....	.021	.014	.006	.032	.023	.012	.055	.042	.022	.058	.046	.025	.079	.063	.036	.081	.066	.038
29.....	.019	.012	.005	.028	.021	.011	.050	.038	.020	.053	.041	.022	.072	.057	.032	.073	.059	.034
30.....	.017	.011	.005	.025	.018	.009	.045	.033	.017	.047	.036	.019	.065	.051	.027	.067	.054	.029
	3.338	2.526	1.719	4.904	4.179	3.200	6.848	5.867	4.459	7.143	6.221	4.777	8.516	7.572	5.947	8.616	7.804	6.289

TABLE 5—*Continued*

Policy Year	Contract Year 7			Contract Years 8–10			Contract Years 11–15			Contract Years 16–20			Contract Years 21–25			Contract Years 26 and Above		
	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L
2.....	.819	.803	.775	.821	.813	.795	.828	.824	.808	.828	.819	.796	.829	.824	.813	.832	.829	.816
3.....	.733	.710	.666	.738	.724	.697	.749	.739	.710	.750	.736	.695	.749	.746	.728	.756	.746	.717
4.....	.661	.636	.582	.669	.654	.621	.684	.668	.627	.684	.665	.608	.684	.681	.662	.693	.678	.633
5.....	.600	.570	.515	.610	.591	.554	.624	.604	.553	.625	.604	.541	.626	.623	.604	.637	.618	.558
	2.813	2.719	2.538	2.838	2.782	2.667	2.885	2.835	2.698	2.887	2.824	2.640	2.888	2.874	2.807	2.918	2.871	2.724
6.....	.546	.518	.460	.558	.537	.493	.570	.547	.488	.572	.549	.482	.573	.570	.552	.586	.564	.491
7.....	.500	.470	.411	.511	.489	.438	.522	.496	.428	.524	.499	.433	.526	.522	.504	.536	.512	.432
8.....	.458	.428	.368	.470	.445	.390	.480	.451	.380	.481	.459	.394	.485	.479	.456	.486	.460	.372
9.....	.421	.391	.329	.432	.405	.347	.441	.411	.335	.443	.423	.362	.447	.440	.413	.437	.410	.319
10.....	.388	.356	.293	.397	.369	.307	.406	.376	.301	.408	.389	.332	.412	.403	.369	.391	.363	.271
	5.126	4.882	4.399	5.206	5.027	4.642	5.304	5.116	4.630	5.315	5.143	4.643	5.331	5.288	5.101	5.354	5.180	4.609
11.....	.357	.325	.260	.365	.335	.271	.374	.344	.269	.376	.358	.305	.382	.370	.329	.348	.322	.233
12.....	.328	.296	.231	.336	.306	.241	.344	.314	.243	.346	.330	.280	.354	.340	.292	.309	.283	.194
13.....	.301	.269	.204	.309	.279	.213	.316	.290	.221	.320	.303	.254	.327	.311	.259	.273	.247	.163
14.....	.277	.245	.181	.285	.255	.191	.291	.267	.203	.295	.279	.230	.300	.284	.228	.240	.217	.136
15.....	.255	.223	.160	.262	.233	.171	.268	.245	.186	.272	.255	.205	.273	.256	.197	.209	.187	.111
	6.644	6.240	5.435	6.763	6.435	5.729	6.897	6.576	5.752	6.924	6.668	5.917	6.967	6.849	6.406	6.733	6.436	5.446

TABLE 5—*Continued*

Policy Year	Contract Year 7			Contract Years 8–10			Contract Years 11–15			Contract Years 16–20			Contract Years 21–25			Contract Years 26 and Above		
	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L	H	M	L
16.....	.235	.204	.143	.240	.213	.154	.247	.226	.171	.251	.234	.182	.245	.228	.169	.181	.160	.088
17.....	.216	.187	.128	.221	.196	.140	.227	.207	.157	.233	.214	.162	.219	.201	.143	.156	.136	.069
18.....	.198	.170	.115	.203	.180	.128	.209	.190	.142	.215	.196	.143	.194	.178	.123	.132	.114	.054
19.....	.181	.156	.105	.186	.165	.118	.193	.174	.128	.197	.178	.126	.172	.156	.102	.112	.096	.041
20.....	.167	.143	.096	.171	.152	.107	.177	.159	.114	.178	.160	.108	.152	.136	.086	.094	.079	.029
	7.641	7.100	6.022	7.784	7.341	6.376	7.950	7.532	6.464	7.998	7.650	6.638	7.949	7.748	7.029	7.408	7.021	5.727
21.....	.153	.131	.088	.157	.139	.098	.163	.145	.101	.160	.142	.093	.133	.119	.071	.078	.065	.014
22.....	.140	.120	.080	.144	.127	.089	.150	.133	.089	.142	.125	.078	.115	.102	.058	.064	.050	0
23.....	.128	.110	.073	.132	.116	.080	.138	.121	.079	.126	.110	.067	.099	.087	.046	.051	.040	0
24.....	.118	.100	.066	.121	.105	.071	.126	.110	.069	.111	.096	.055	.085	.073	.035	.041	.032	0
25.....	.108	.092	.059	.111	.096	.062	.113	.098	.059	.097	.083	.046	.072	.061	.028	.032	.025	0
	8.288	7.653	6.388	8.449	7.924	6.776	8.640	8.139	6.861	8.634	8.206	6.977	8.453	8.190	7.267	7.674	7.233	5.741
26.....	.098	.083	.052	.102	.087	.055	.101	.086	.050	.084	.072	.038	.061	.051	.021	.025	.020	0
27.....	.090	.075	.046	.093	.079	.048	.089	.076	.042	.073	.062	.031	.050	.042	.015	.019	.016	0
28.....	.082	.068	.040	.085	.071	.042	.079	.066	.036	.062	.052	.024	.041	.034	.007	.015	.012	0
29.....	.075	.062	.035	.076	.063	.036	.069	.057	.029	.053	.044	.018	.033	.026	0	.011	.009	0
30.....	.067	.055	.030	.067	.055	.030	.060	.049	.024	.044	.036	.014	.027	.020	0	.008	.006	0
	8.700	7.996	6.591	8.872	8.279	6.987	9.038	8.473	7.042	8.950	8.472	7.102	8.665	8.363	7.310	7.752	7.296	5.741

Values of $v^{t-1}(1 - 0_t)P_t$ *(Mixtures of Contract Years)*

As indicated in the McConney-Guest paper, there is some usefulness in a set of aggregate nonvested valuation factors computed for a mixture of various contract years of issue, such as might be produced in a given year by an established agency force. The McConney-Guest model company tables were based upon the distribution of agents by length of service in a level producing force. That of course was just one of many aggregate distributions that could have been used.

This present study does not produce the information that would be needed to postulate a level producing agency force, but the individual company samples do indicate in a general way the actual weights by contract year of business produced in a calendar year. Inspection of the premiums for issue years 1960 and 1959 suggests that the following weights may be reasonably typical:

Contract year.....	1	2	3	4	5	6	7	8–10 (9)	11–15 (13)	16–20 (18)	21–25 (23)	26 and Above (30)	Total
Per cent of total...	17	13	7	6	5	4	4	8	12	6	6	12	100%

Table 6 gives the nonvested valuation factors on the same assumptions as for Table 5 with this distribution by contract years.

Conclusion

The results shown by the contributions to this study confirm the view widely held by actuaries that the range of nonvested values between companies is large. Clearly it is hazardous to base calculations on any published table without making sure that the underlying assumptions are applicable. The combined experience of this particular group of companies suggests patterns by duration that do not run parallel to the McConney-Guest rates, and that more often than not show higher costs of nonvested commissions than the McConney-Guest values.

Appreciation is due to the contributors whose cheerful co-operation made this study possible. It appears worthwhile to repeat it from time to time as a means for measuring the prevailing patterns in different eras. The major contributions to this paper by Herbert S. Gardner, F.S.A., are gratefully acknowledged.

TABLE 6

VALUE AT ISSUE OF RENEWAL COMMISSION OF ONE WEIGHTED AVERAGE OF VARIOUS CONTRACT YEARS—4 PER CENT INTEREST

Payment contingent upon continuance of agent in full-time service

Policy Year	Table H	Table M	Table L
2.....	.712	.689	.664
3.....	.622	.600	.557
4.....	.561	.529	.479
5.....	.510	.472	.415
	2.405	2.290	2.115
6.....	.462	.427	.364
7.....	.420	.383	.321
8.....	.383	.349	.284
9.....	.351	.314	.251
10.....	.319	.286	.224
	4.340	4.049	3.559
11.....	.290	.259	.198
12.....	.266	.233	.175
13.....	.243	.213	.156
14.....	.222	.193	.137
15.....	.199	.172	.121
	5.560	5.119	4.346
16.....	.181	.157	.107
17.....	.166	.141	.094
18.....	.150	.127	.082
19.....	.134	.114	.072
20.....	.123	.103	.063
	6.314	5.761	4.764
21.....	.111	.091	.056
22.....	.101	.083	.046
23.....	.089	.073	.042
24.....	.082	.065	.035
25.....	.071	.059	.032
	6.768	6.132	4.975
26.....	.064	.050	.026
27.....	.056	.046	.025
28.....	.049	.040	.021
29.....	.044	.035	.015
30.....	.039	.029	.013
	7.020	6.332	5.075

APPENDIX

INFORMATION REQUESTED FROM CONTRIBUTORS TO INTER-COMPANY ANALYSIS OF BUSINESS ACCORDING TO STATUS OF WRITING AGENT

(Three postscripts have been added to record lessons learned during the study.)

Purpose

This study aims to determine at seven specified policy durations the proportions of business (and premiums thereon) for which at the time the 1961 renewal premiums became due the original writing agent was

1. "Present," i.e., either (*a*) still under a full-time contract with your company, or (*b*) in a field management or other active position with your company;
2. "Deceased," i.e., whose full-time contract with your company has been terminated by death;
3. "Absent," i.e., status other than (1) or (2). Included among "Absent" would be agents who have changed to part-time or brokerage contracts.

The general criterion distinguishing (1) from (3) is whether or not the agent would have been entitled to a nonvested commission or service fee if such were offered on the 1961 renewal premium. In making this distinction please ignore, if possible, any special qualification requirement (such as a minimum production rule) that your company may impose for payment of a commission or fee.

Material To Be Analyzed

Information is to be assembled on an equal sample (200 or 100 according to choice of each contributor) of insurance policies issued in each of the years 1941, 1946, 1951, 1956, 1958, 1959, and 1960.

Postscript 1.—If the *c*-factor method is valid, it is not necessary to go so far back in years of issue. Better results would be obtained by abstracting larger samples of policies from recent years of issue.

Include only policies that were in force on a premium-paying basis as of the 1961 premium anniversary. Since the sample is being taken in 1962, it will be assumed that the 1961 premium has been paid, but this need not be positively checked. (No special treatment need be given to pro rata premium policies that happen to have premium anniversaries running into the year following the valuation year of issue.)

Postscript 2.—The purpose of taking a sample in 1962 but recording agent's status at the 1961 anniversary was to insure that most policies terminating before payment of the 1961 premium would be out of the

sample. The effect is to show too high a proportion of business with active agents if the termination rate of orphaned business is heavier than that of active business, but the difference appears to be unimportant unless a long period of lag is used.

The sample of 200 or 100 policies should be net after all rejected policies. The policies to be rejected include:

1. Policies written by brokers, part-time agents, agents of other companies, general agents, agency managers, and others whose duties are primarily supervisory. It is not necessary to reject business written by agency supervisors who are engaged to a major extent in personal production.
2. Policies written jointly by two or more agents. However, a policy may, if desired, be included and treated as a single-agent policy if one agent had at least a 75 per cent commission interest.
3. Annuities, term conversions, and group conversions (if convenient).

Information To Be Recorded for Each Policy

Column	
1–2	Company code.
3–9	Policy number.
10	0—regular business. 1—use, if possible, to distinguish pension business or other mass sales which you consider might be nonhomogeneous with your regular business.
11–12	Premium anniversary month.
13–14	Policy issue month. Enter both if available or whichever one of these you can conveniently furnish. If both are given, we shall use policy issue month in determining the contract year of the business and attained age of the agent.
15–16	Policy issue year (41, 46, 51, 56, 58, 59 or 60).
17–22	Policy amount. Enter whatever "amount" is most conveniently available, e.g., including or excluding rider credits. The major emphasis in this study will be on the premium rather than the policy amount.
23–27	Annual premium in dollars. On a fractional premium policy enter either the equivalent annual premium or the product of the fractional premium and the frequency. If convenient, include premiums for supplemental benefits.
28–29	Agent's birth date—month.
30–31	Agent's birth date—year.

Postscript 3.—It was hoped that recording of agent's birth date would permit an analysis of orphanage in terms of age of the writing agent, but the material is inadequate in size and too diverse in content to produce useful results.

32–33 Original date of agent's full-time contract—month.

34–35 Original date of agent's full-time contract—year.
In cases involving such situations as transfers of agents from one location or from one contract to another, please be sure that you record the date the agent originally started with your company on a full-time basis.

36 Agent status at 1961 anniversary.
Present—1.
Deceased—2.
Absent—3.

DISCUSSION OF PRECEDING PAPER

DATON GILBERT:

As mentioned by Mr. Moorhead, my discussion of the 1942 McConney-Guest paper commented briefly on "an *alternate approach*" which would determine "the proportions of active and of orphaned business at the various policy durations" from a large, well-established company's insurance-in-force records. However, my thoughts at that time contemplated a rather simple determination by any such company of aggregate nonvested valuation factors for internal use. This paper has broadened the original concept to an intercompany framework and has refined it by investigating results according to agents' individual contract years.

This refinement was made practical by the development of the "*c*-factor method" described in the paper. The key "supposition" on which this method rests, as described by Mr. Moorhead, seems acceptable under circumstances where influences directly related to agents' contract durations appear by general reasoning from other data to be of controlling importance in following changes over time in the proportions of business in force with active agents. However, I do have some doubt as to the homogeneity and extent of the data and as to the general statistical treatment used.

As the author recognizes, the data have a "low degree of homogeneity." In spite of restrictions imposed, the companies included in the study seem to cover a rather wide range of operation types, with substantial differences on such major factors as (*a*) existence and relationship of a parallel brokerage organization; (*b*) full-time contract administrative practices at retirement and on drifting away from full-time activity; (*c*) nature and handling of any production requirements for contract continuance; (*d*) availability and use of any company-sponsored financing plans, etc. Moreover, after ratios of premiums on active agents' business to total premiums had been determined for each company at each data point, the resulting ratios were processed together in a manner which did not attempt directly to reveal individual company patterns. Figures for each of three companies are shown comparing actual and expected premiums with active agents by use of the tables developed. Owing to the relatively small samples furnished by each company, this technique is more practical than to attempt the paper's detailed approach by individual company. However, substantially larger amounts of data with a less detailed form of analysis carried out by individual company might show significantly different structural patterns. Then a review of such indi-

vidual company results should suggest logical company groupings, leading to "high," "low," or other valuation tables, presumably on a less detailed basis than given in the paper. Incidentally, the electronic data-processing systems now being adopted by many companies may ultimately make the necessary underlying information more readily available on a broad basis.

Mr. Moorhead's presentation brings to mind the need for defining intended uses for nonvested commission valuation tables and related figures. Important applications include at least the following:

1. *Tests for compliance with legal requirements.*—Here, there must be available a reasonable and *fixed* "full-time agent" standard to be used by all companies in valuing nonvested elements of compensation plans for such agents. Seemingly, the familiar "modified" basis developed by Messrs. McConney and Guest continues to provide a satisfactory standard for this purpose. The current paper and other data clearly show the relatively wide variations among individual companies in agent turnover and other underlying factors. But the major efficiencies stemming from good agent survival and the heavy general costs associated with poor survival effectively counterbalance the impacts of such survivals on nonvested commission costs. These considerations and the practical need for a stable basis lend strong support to the continued use by insurance department authorities of the "modified" nonvested valuation standard, without regard for individual company experience or for fluctuations in industry experience.

2. *Internal company estimates of compensation costs, etc.*—A variety of circumstances may require a company to estimate the cost of certain nonvested compensation elements, as, for example, when a new or revised agents' contract is under consideration. Ideally, nonvested valuation factors based on the company's own experience should be used for this purpose, possibly modified appropriately for the probable effect of the change on such experience. Alternately, a published table (such as one of those presented in this paper) could be used if tests show it to be a reasonable approximation to the company's expected results.

3. *Statistics to follow industry trends and relationship to own company.*—As Mr. Moorhead suggests, studies in this area seem worthwhile repeating from time to time. However, trends and company position are best followed by a continuous survey, and much light is shed on key related factors by LIAMA's Survey of Current Recruiting Activity and Early Performance. This survey gives for individual company and for companies grouped by size: the number of full-time agents hired each month, their performance in their six months and in their first year, etc. If this survey could be extended to cover several more contract years, it would become even more valuable.

Mr. Moorhead has provided useful new reference points when dealing with the important problem of valuing nonvested commissions. It is to be hoped that studies in depth of individual company experiences will verify the reasonableness of his several tables.

EDWARD A. GREEN:

I particularly welcome Mr. Moorhead's paper. As one who was privileged to participate in the preparation of the tables included in Mr. McConney's and Mr. Guest's 1942 paper, it gives me an opportunity to make further calculations and comparisons.

While Mr. Moorhead's method did not bring forth an agents' survival table, his Table 6 is comparable to the McConney-Guest Model Company Table using 4 per cent interest. The values in these tables depend upon the persistency of business and mixture of production by agent's contract year as well as the survival of agents. Since the two papers used different assumptions for these two items, I have adjusted Mr. Moorhead's figures to bring them approximately to what they would have been if he had used the Linton A persistency table and the same mixture of production, as did Mr. McConney and Mr. Guest. The adjusted distribution of production by contract year is as follows:

	Contract Year											
	1	2	3	4	5	6	7	8–10	11–15	16–20	21–25	26–30
Per cent of total...	15	11	8	6	5	4	4	10	12	10	9	6

The following tabulation shows the weighted average value at issue of a renewal commission of one contingent upon continuance of the agent in full-time service using the previously mentioned contract year weights, Linton A persistency of business, and 4 per cent interest.

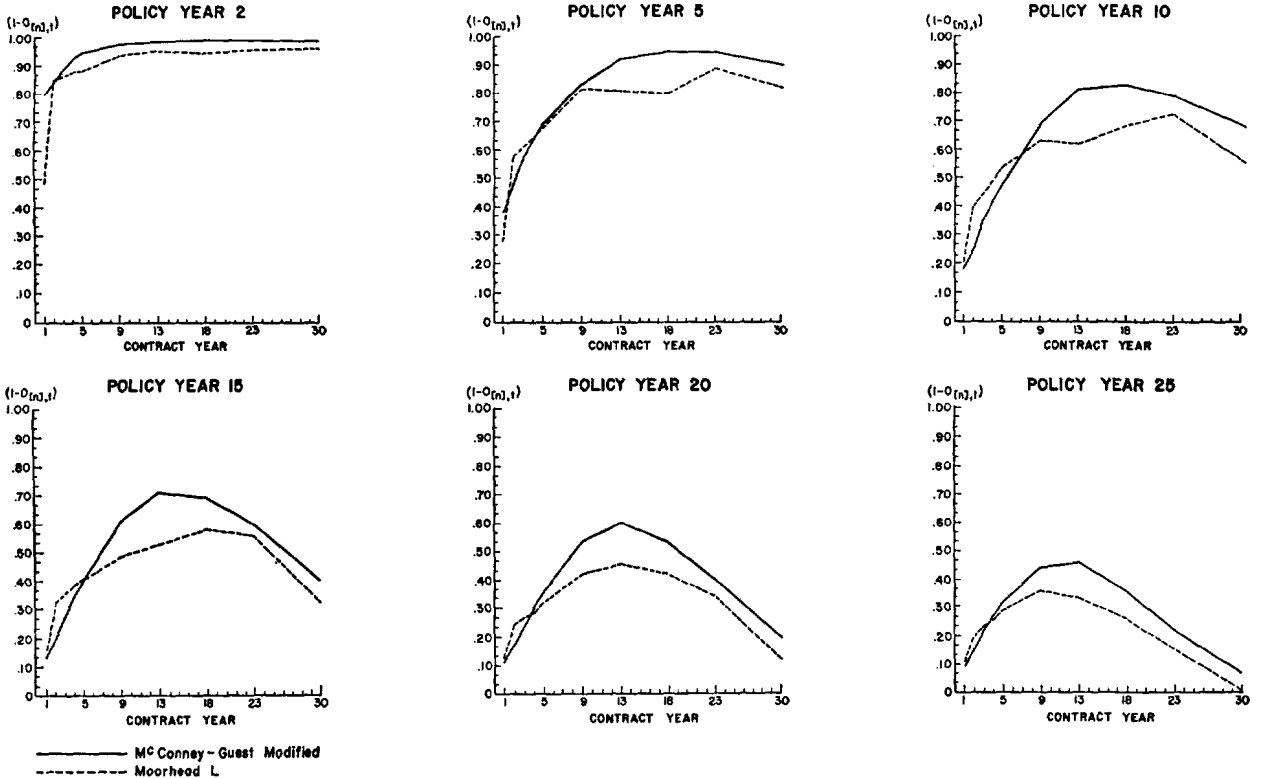
Policy Years	Moorhead			McConney-Guest	
	H	M	L	Mod.	Bureau 1938–41
1– 5	2.616	2.490	2.291	2.363	2.275
1–10	4.581	4.268	3.757	3.751	3.457
1–15	5.772	5.309	4.530	4.491	4.005
1–20	6.492	5.916	4.931	4.907	4.284
1–25	6.901	6.249	5.126	5.123	4.420
1–30	7.113	6.414	5.211	5.222	4.479

The values produced by the Moorhead L table and the McConney-Guest modified table are amazingly close. The L table is based on the average experience cell by cell of the half of the companies contributing to Mr. Moorhead's study whose proportion of orphan business fell above the median for the entire group. On the other hand, the McConney-Guest modified table used the Bureau 1938–41 agents' termination rates for the first five contract years graded uniformly to one-half of the 1938–41 rate at the end of fifteen years. The Bureau table in turn used for the first five contract years the median experience of five of the twelve companies contributing to the study whose termination rates were lower than the median of the whole group graded into the median of the twelve companies for contract years beyond five. Since the Moorhead L table represents survival experience lower than the average for his study and the McConney-Guest modified table represents survival experience higher than the average for the Bureau study, the closeness of values might be interpreted to indicate a substantial improvement in agents' survival experience during the last twenty years, more so than has been recognizable by those closely associated with agency management.

Looking for some other influence that might contribute to the similarity of results, I divided out the interest and persistency factors in the L figures in Mr. Moorhead's Table 5 to get an approximation to $(1 - 0_{[n],\,t})$. These were then compared with $(1 - 0_{[n],\,t})$ from the McConney-Guest modified table secured by dividing out the persistency factor from their no-interest table. The results are shown on Chart I.

It would appear from this chart that the weighted average values from the Moorhead L table would be lower than those from the McConney-Guest modified table if it were not for the first few contract years of policy years 10, 15, and 20. These policy years are the issues of 1951, 1946, and 1941, respectively, in Mr. Moorhead's original data. The relationship of these dates to World War II and the Korean War can be noted. It is possible that the effect of these wars on the economy and availability of manpower had an effect on persistency of business and agents' survival which is reflected in the H, M, and L tables fully as much as any improvement in agents' survival. Mr. Moorhead's ingenious use of his c-function has the effect of reducing any bulge present in the original data. Any value of $(1 - 0_{[n],\,t})$ derived from the table reflects to an extent the data for all lower values of n and t. Of the eight values shown for computing the H-values of c_{10}, the four related to contract years 1–4 have a sufficiently lower average than the four related to contract years 5–7 and 9 to raise the question as to the validity of the hypothesis back of the c-function for other than Mr. Moorhead's basic purpose of giving a general

CHART I


POLICY YEAR 2
POLICY YEAR 5
POLICY YEAR 10
POLICY YEAR 15
POLICY YEAR 20
POLICY YEAR 25
$(1-q_{[n]+t})$
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0
1
5
9
13
18
23
30
CONTRACT YEAR
Mc Conney - Guest Modified
Moorhead L

picture of ranges and patterns. It would be interesting to know if the original data shed any light on this matter.

I agree wholeheartedly with Mr. Moorhead that studies such as his and that of Mr. McConney and Mr. Guest are worthwhile to repeat from time to time. As he points out, they produce a general picture of ranges and patterns of nonvested commission values under varying circumstances that can be useful to an actuary studying the experience in his own company.

HARRY D. GARBER:

My company, the Equitable Society, did not contribute to Mr. Moorhead's study because, at the time his request was received, we were already involved in analyzing the persistency and the relative production by contract year, etc., of our agency force in the calendar years 1949 through 1960. My discussion of Mr. Moorhead's paper contains a summary of the results of our study and a comparison with Mr. Moorhead's figures. First, however, I would like to comment briefly on the methodology of Mr. Moorhead's study.

The basic approach used by Mr. Moorhead was to determine proportions of orphaned and nonorphaned business directly by reviewing the records of in-force policies and recording (i) whether the procuring agent is still under contract with the company, and (ii) whether, if the agent's contract has been terminated, the termination was a voluntary one or by death. Theoretically, this type of study is better than the approach used by Messrs. McConney and Guest, because it avoids the necessity of estimating differences in persistency between the business produced by continuing agents and that produced by agents who have terminated. Unfortunately, however, a study of this nature is a difficult one for most companies because individual policy records rarely contain information on the current status of the procuring agent or agents. To avoid putting an undue strain on any one company, Mr. Moorhead ingeniously split the burden among a number of companies. Unfortunately, his total sample was, in my opinion, too small to produce significant results for the number of cells involved. In addition, the combining of the results of different companies with different philosophies of agent recruitment, compensation, etc., diluted still further the validity of the study.

In all, Mr. Moorhead's study is based on about 16,000 policies drawn from seventeen different companies. Our study, which will be described later in this discussion, was based on over 120,000 agent-years of exposure, during which time the Equitable paid for over 2,000,000 individual insurance policies. While the relative significance of the two studies is not

proportionate to the number of policies involved, I believe that, by any measure, the data in our study were many times that included in Mr. Moorhead's.

In order to obtain a full set of probabilities of orphanage, Mr. Moorhead assumes that there exists a function, c, such that

$$\frac{c_{n+t-1}}{c_n} = 1 - 0_{[n],\,t}.$$

(I assume that this approach was necessary because of the paucity of data in the study.) There exists such a function if, and only if, all the following ratios are identical:

$$\frac{1 - 0_{[1],\,n+t-1}}{1 - 0_{[1],\,n}}, \frac{1 - 0_{[2],\,n+t-2}}{1 - 0_{[2],\,n-1}}, \frac{1 - 0_{[3],\,n+t-3}}{1 - 0_{[3],\,n-2}}, \cdots, \frac{1 - 0_{[n+t-2],\,2}}{1 - 0_{[n+t-2],\,3-t}}.$$

Mr. Moorhead indicates that "this supposition . . . is convenient and is adequately supported as a reasonable approximation by the results of this investigation." As an example of such result, Mr. Moorhead cites the eight values ranging from 378 to 517 which were averaged to produce the H-value of c_{10} of 460.

From our own study, we were able to calculate a full set of $[1 - 0_{[n],\,t}]$'s. In order to test the validity of Mr. Moorhead's assumption, we also constructed a table of c's using the techniques described in the paper. Using this table of c's, we then calculated the resulting values of $1 - 0_{[n],t}$. If the technique is valid, it should reproduce, approximately, the original values. We found, however, that the values developed from the table of c's exceeded the actual values (i) for each policy year except the second on business produced in the agent's first contract year; (ii) for policy years 5 and over on business produced in the agent's second contract year; (iii) for policy years 8 and over on business produced in the agent's third contract year; (iv) for policy years 17 and over on business produced in the agent's fourth contract year; and (v), generally, for policy years 18 and over on business produced in the agent's twenty-first and later contract years. For all other points the actual values were higher than the values based on the table of c's.

Table 1 compares selected values of $1 - 0_{[n],\,t}$ as developed in our study with the corresponding values computed from the table of c's which was constructed by using Mr. Moorhead's techniques. The artificially calculated differ significantly from the actual experience values at many points.

On further investigation, we have concluded that Mr. Moorhead's assumption is not valid if the production of agents who will eventually

TABLE 1

VALUE OF $1 - 0_{[n],\,\epsilon}$

Policy Year	As Developed in Equitable Study	As Developed on Basis of Moorhead Techniques
	Business of First Contract Year	
Second	.7613	.7613
Fifth	.3604	.4271
Tenth	.2004	.2753
Twentieth	.1327	.1923
	Business of Third Contract Year	
Second	.8889	.8285
Fifth	.5969	.5666
Tenth	.4035	.4123
Twentieth	.2921	.3092
	Business of Fifth Contract Year	
Second	.9419	.8825
Fifth	.7503	.6865
Tenth	.5594	.5345
Twentieth	.4339	.4247
	Business of Ninth Contract Year	
Second	.9768	.9389
Fifth	.8672	.8100
Tenth	.7208	.6859
Twentieth	.6150	.5914
	Business of Eighteenth Contract Year	
Second	.9910	.9761
Fifth	.9503	.9244
Tenth	.8956	.8732
Twentieth	.7391	.7260

terminate varies relative to that of agents who continue under contract. However, we found in our study, and it has been found in previous analyses, that the relative production of terminating agents does decrease for five years or so before the termination. Therefore, while Mr. Moorhead's assumption is reasonably valid for ratios which involve only policy years 6 and over, it will not apply generally to ratios in which any of the first five policy years are involved.

As stated earlier, in our study we analyzed the persistency and first-year commission earnings of our agency force during the calendar years 1949 through 1960. The study covered all persons holding a soliciting agent's contract and therefore includes the experience on part-time agents, and on agency and district managers, all of whom hold agents' contracts.

The first step in the study was the determination of probabilities of voluntary termination according to agent's contract year and probabilities of death according to attained ages. From these probabilities, which were based on numbers of agents, we constructed an agents' survival table by contract year. The probabilities of death taken into account were those for an agent who was age 35 at the time he came under contract. The agents' survival table is presented in Table 2, along with the McConney-Guest table. Our agents' persistency experience may not be applicable to many other companies because of the relatively low content continuance requirements ($600 of first-year commissions for years after the first) contained in the Equitable's principal soliciting agents' agreement during this period. On the other hand, our managers, with this requirement in mind, may have hired more part-time agents.

The second step was the development, based on the first-year commission earnings data in our study, of relative production assumptions (i) by contract year, (ii) for agents terminating for reasons other than death, and (iii) for agents terminating by death. These assumptions are presented in Tables 3, 4, and 5, respectively. The relative production assumptions by contract year are also based on an assumed entry age of 35. Because our agents are permitted to submit new business after attaining age 65, we found that production at the higher ages grades off much more slowly than the pattern assumed by Messrs. McConney and Guest. The pattern of decreases in production levels in the five years preceding death or voluntary termination also differs somewhat from those used by McConney and Guest. The most important difference is our finding that, on the average, the agents who terminated during the study never reached the production level of those who survived. We assumed that the production of agents who, according to our survival table, will terminate for reasons other than death five or more years after the current

TABLE 2

COMPARISON OF EQUITABLE 1949–1960 AGENTS' SURVIVAL TABLE AND MCCONNEY-GUEST MODIFIED AGENTS' SURVIVAL TABLE

CONTRACT YEAR (n)	EQUITABLE 1949–1960 TABLE			MCCONNEY-GUEST TABLE		
	Number Entering (l_n)	Deaths (d_n)	Other Terminations (w_n)	Number Entering (l_n)	Deaths (d_n)	Other Terminations (w_n)
1	100,000	100	47,500	100,000	480	42,520
2	52,400	52	19,650	57,000	280	20,810
3	32,698	33	9,646	35,910	180	10,050
4	23,019	46	5,409	25,680	140	5,640
5	17,564	35	3,249	19,900	110	3,570
6	14,280	29	2,071	16,220	90	2,380
7	12,180	24	1,401	13,750	80	1,710
8	10,755	22	1,022	11,960	80	1,300
9	9,711	29	825	10,580	70	990
10	8,857	27	664	9,520	70	750
11	8,166	24	531	8,700	70	570
12	7,611	30	419	8,060	70	430
13	7,162	29	358	7,560	70	320
14	6,775	34	305	7,170	70	220
15	6,436	32	257	6,880	70	150
16	6,147	37	215	6,660	80	110
17	5,895	35	177	6,470	80	80
18	5,683	40	142	6,310	90	70
19	5,501	39	110	6,150	90	60
20	5,352	43	80	6,000	100	50
21	5,229	47	52	5,850	100	50
22	5,130	46	31	5,700	110	30
23	5,053	51	15	5,560	120	20
24	4,987	50	5	5,420	120	20
25	4,932	54	0	5,280	130	0
26	4,878	59	0	5,150	140	0
27	4,819	63	0	5,010	150	0
28	4,756	67	0	4,860	160	0
29	4,689	70	0	4,700	160	0
30	4,619	74	0	4,540	170	0
31	4,545	77	0	4,370	180	0
32	4,468	85	0	4,190	180	0
33	4,383	92	0	4,010	190	0
34	4,291	99	0	3,820	200	0
35	4,192	105	0	3,620	210	0
36	4,087	110	0	3,410	210	0
37	3,977	119	0	3,200	210	0
38	3,858	127	0	2,990	220	0
39	3,731	134	0	2,770	220	0
40	3,597	140	0	2,550	220	0
41	3,457	149	0	2,330	210	0
42	3,308	155	0	2,120	210	0
43	3,153	164	0	1,910	210	0
44	2,989	170	0	1,700	200	0
45	2,819	178	0	1,500	190	0
46	2,641	182	0	1,310	180	0
47	2,459	187	0	1,130	160	0
48	2,272	191	0	970	150	0
49	2,081	191	0	820	140	0
50	1,890	191	0	680	120	0

TABLE 3

RELATIVE PRODUCTION RATES
BY CONTRACT YEAR

Contract Year	Equitable 1949–1960 Experience	McConney-Guest
1	50%	80%
2	85	90
3	95	95
4	100	100
5–25	100	100
26	100	90
27	95	80
28	90	70
29	85	60
30	80	50
31	75	0
32	70	0
33	65	0
34	60	0
35	55	0
36	50	0
37	45	0
38	40	0
39	35	0
40	30	0
41	25	0
42	20	0
43	15	0
44	10	0
45	5	0
46 and over	0	0

TABLE 4

RELATIVE PRODUCTION RATES FOR AGENTS
TERMINATING FOR REASONS OTHER
THAN DEATH (a)

Contract Year	Equitable 1949–1960 Experience	McConney-Guest
Year of termination	20%	25%
Preceding year	40	50
Second preceding year	50	70
Third preceding year	60	85
Fourth preceding year	70	95
Fifth preceding and earlier years	75	100

contract year will amount to only 75 per cent of that for agents who will continue in service to death. It should be noted that the relative production ratios shown in Tables 3, 4, and 5 should be combined by multiplication. For example, the relative production rate in the first contract year of an agent who will terminate for reasons other than death in that year is 10 per cent (i.e., .50 of 20 per cent).

The formulas to be used in translating these assumptions into factors that can be used in evaluating the present value of nonvested commission payments are set forth in the following paragraphs.

TABLE 5

RELATIVE PRODUCTION RATES FOR AGENTS TERMINATING BY DEATH (β)

Contract Year	Equitable 1949–1960 Experience	McConney-Guest
Year of death	50%	25%
Preceding year	70	50
Second preceding year	80	70
Third preceding year	90	85
Fourth preceding year	95	95
Fifth preceding and earlier years	100	100

The probability that business produced in the agent's nth contract year which is in force at the beginning of the tth policy year is in a "nonorphaned" status at that time is

$$1-0_{[n]\ t}=1-\left[\sum_{i=0}^{t-2}(a_i\cdot w_{n+i}+\beta_i\cdot d_{n+i})\div\sum_{i=0}^{\infty}(a_i\cdot w_{n+i}+\beta_i\cdot d_{n+i})\right],$$

where d_{n+i} is the number of deaths for contract year $n+i$ from the Equitable 1949–1960 Agents' Survival Table (Table 2); w_{n+i} is the number of other terminations for contract year $n+i$ from the same table; a_i is the relative production rate for agents terminating for reasons other than death for the ith year preceding termination based on Equitable 1949–1960 experience (Table 4); and β_i is the relative production rate for agents terminating by death for the ith year preceding death based on Equitable 1949–1960 experience (Table 5).

The probability that business (produced in all contract years combined) in force at the beginning of the tth policy year is in a "nonorphaned" status at that time is

$$1-0_t=1-\sum_{n=1}^{\infty}\pi_n\cdot 0_{[n]\cdot\ t}.$$

In this expression, π_n represents the proportion of business (produced by the entire agency force) that is produced by agents in their nth contract year. A set of π_n's is shown in Table 6, which set has been developed from the present and projected composition of the Equitable's agency force, the experience table and the relative production assumptions presented in Tables 3, 4, and 5. Also shown, for comparative purposes, is the distribution of production by contract year contained in Mr. Moorhead's paper.

TABLE 6

ASSUMED DISTRIBUTION OF PRODUCTION BY AGENT'S CONTRACT YEAR (π)

Contract Year (n)	Based on Equitable Data (Per Cent)	As Presented in Paper (Per Cent)
1	11	17
2	11	13
3	9	7
4	4	6
5	4	5
6	4	4
7	3	4
8–10	9	8
11–15	15	12
16–20	13	6
21–25	5	6
26 and above	12	12
Total	100	100

To take into account vesting provisions under which commissions are paid to agents who have died but not to agents who have withdrawn for other reasons, it is necessary to compute probabilities that business in force at the beginning of policy year t is orphaned because of the voluntary withdrawal of the procuring agent. The formula for these probabilities is

$$0^{w}_{[n]\ t} = \sum_{i=0}^{t-2} (a_i \cdot w_{n+i}) \div \sum_{i=0}^{\infty} (a_i \cdot w_{n+i} + \beta_i \cdot d_{n+i}).$$

To obtain values at issue of a renewal commission of one (with payment contingent upon continuance of the agent in full-time service), the following expression is applicable

$$v^{t-1} \cdot P_t \cdot (1 - 0_{[n]\ t}),$$

where P_t is the probability that the policy will be in force at beginning of policy year t, and v^{t-1} is the standard interest discount factor.

In Table 7, we compare, for certain selected agent's contract years and

TABLE 7

VALUE AT ISSUE OF RENEWAL COMMISSION OF ONE
(4 PER CENT INTEREST)

(Payment Contingent upon Continuance of Agent in Full-Time Service)

Policy Year	Table H	Table M	Table L	McConney-Guest Modified	Equitable 1949–1960 Experience
	Business of First Contract Year				
Second	.414	.360	.306	.687	.656
Fifth	.249	.197	.144	.238	.231
Tenth	.144	.104	.063	.077	.086
Twentieth	.052	.034	.017	.022	.026
	Business of Third Contract Year				
Second	.710	.688	.656	.768	.766
Fifth	.497	.440	.359	.368	.383
Tenth	.301	.253	.183	.145	.172
Twentieth	.120	.092	.053	.047	.058
	Business of Fifth Contract Year				
Second	.816	.792	.752	.809	.812
Fifth	.592	.545	.463	.447	.482
Tenth	.376	.334	.260	.203	.239
Twentieth	.162	.132	.081	.071	.085
	Business of Ninth Contract Year				
Second	.821	.813	.795	.837	.842
Fifth	.610	.591	.554	.533	.557
Tenth	.397	.369	.307	.296	.308
Twentieth	.171	.152	.107	.105	.121
	Business of Eighteenth Contract Year				
Second	.828	.819	.796	.856	.854
Fifth	.625	.604	.541	.604	.610
Tenth	.408	.389	.332	.354	.382
Twentieth	.178	.160	.108	.104	.146

policy years, the value at issue of a renewal commission of one based on (i) the Equitable 1949–1960 experience; (ii) the McConney-Guest Modified Table; and (iii) Mr. Moorhead's Tables H, M, and L. All values have been discounted at 4 per cent interest. The Equitable figures have been based on Linton A persistency, which, at least in the earlier years, is slightly better than our actual experience and, hence, represents a measure of the persistency that might be experienced on the business of continuing agents. Both the McConney-Guest figures and the Equitable figures differ from those developed by Mr. Moorhead in the way in which the probabilities that a policy will be in force have been determined. The Equitable and McConney-Guest factors reflect the proportion of policies in force at the beginning of the policy year. The Moorhead factors reflect the proportion of original premiums paid during the year. These differences in the definition of the persistency factors are consistent with corresponding differences in the ways in which the $[1 - 0_{[n]}, {}_t]$ factors have been developed.

(AUTHOR'S REVIEW OF DISCUSSION)

ERNEST J. MOORHEAD:

It is valuable to have discussions from two actuaries who participated actively in the work that led to the McConney-Guest Tables and also to have from Mr. Garber the most complete study of nonvested values for a single company that to my knowledge has ever appeared in actuarial literature.

With the observations from Mr. Gilbert, all who have studied the subject will probably agree. It is perhaps worth observing that criticisms (*a*), (*b*), (*c*), and (*d*) of his third paragraph apply also to the data used in the McConney-Guest study. His valid objections to pooling of nonhomogeneous statistics make it desirable to follow the procedure of this study which was to create a family of tables rather than a single average of all the results at each point.

Mr. Green has developed a most enlightening comparison between the results of the present study and that of McConney-Guest. He has alertly noted the tendency of the values of c_{10} illustrated in the paper to be below average for the early contract years and above average for later contract years. This observation stimulated us to check the pattern of values of $c_{[n]+t-1}$ for increasing values of n but constant values of $n + t - 1$. We found that, when $n + t$ is small, the crude values tend consistently to decrease with increasing values of n. On Table H the reverse is true when $n + t$ exceeds 8; on Table L the phenomenon disappears when $n + t$ exceeds 11 but does not significantly reverse itself.

I think that Mr. Garber expresses rather too strongly the difficulties and shortcomings of the method I have used. It is important that it makes unnecessary all of the study that has produced his Tables 3, 4, and 5, all of which appear to reflect rather freehand treatment of the underlying data.

The considerable differences in factors exhibited in Mr. Garber's Table 7 are substantially due, at least in the early contract years, to different assumptions as to persistency of business rather than to differences in patterns of agents' survival. To me it seems that the use of a single persistency table for all contract years is difficult to justify in the light of our accumulated knowledge of this subject. If the statement in my paper that "the persistency of business factor must reflect the persistency of all new business, not just that on which the agent is still active" is true, then Mr. Garber seems to be on the wrong track when he employs persistency rates that are considered appropriate for the business of continuing agents.

Mr. Garber has made a useful exploration of the validity or otherwise of the c-factor method. I think, however, that each of the two contrasting methods is subject to some objections and computational difficulties. The validity of any method depends upon the fidelity of computed results to those observed. When we calculated for each contributing company the ratio of actual premiums with active agents to those expected by the various tables, I was rather encouraged by what we found.