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A Note Regarding “Risk Neutral” and “Real World” Scenarios—Dispelling a Common Misperception

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Circa 2000

A typical conversation between a veteran actuary and a youngster taking the investment track exams might have gone like this:

Vet: Explain to me again these “risk-neutral” scenarios.

Youngster: Well, they’re just like regular scenarios, except that stocks only get the risk-free rate.

Vet: That makes no sense, everyone knows that stocks outperform bonds given enough time.

Youngster: Yes, but since you discount everything at the risk-free rate, it all works out.

Vet: But it’s not realistic.

Youngster: It’s not meant to be. It’s just a trick to get the correct price. *The scenarios don’t mean anything individually—only on average.*

Vet: OK, I understand that using these scenarios correctly prices options, but it still doesn’t feel right. The idea that stocks don’t outperform bonds bugs me.

Youngster: Well, just remember that the scenarios *aren’t supposed to be realistic*; it’s the valuation that matters.

The youngster almost has it right—but not quite ...

Fast forward to 2008

We have made a lot of progress. Today, I would say that the majority of actuaries who have had to deal with either market consistent embedded value (MCEV) or FAS133-valued variable annuity guaranteed living benefits (VAGLBs) have had to grapple with risk neutral scenarios and have gotten themselves to an understanding at least as good as Youngster’s (who is not so young anymore). Let me characterize Youngster’s understanding (which I believe is fairly common among actuaries young and old):

Risk neutral pricing is just a mathematical trick to recover the price of options using Monte Carlo simulation.

The scenarios are not real world; they just get us to the correct price.

This understanding is mostly accurate, but is significantly flawed. This note is an attempt to address the subtle but important misconception embodied above. I should add that I think it is a great sign of progress that the above misconception seems worthy of addressing.

Individually, all the scenarios in a set of risk neutral scenarios are real world. Each scenario is just one path among many possible paths in the future. Each path is either possible or impossible. If it is possible, it may be part of a risk neutral scenario set and it may be part of a real world scenario set. If the path is impossible, it can be part of neither a risk neutral nor a real world scenario set. The difference between risk neutral scenarios and real world scenarios is **not** the individual scenarios themselves; it is the **probability** of those scenarios occurring.

Recall that the whole point of risk neutral pricing is to recover the price of traded options in a way that avoids arbitrage. As such, the probabilities of various paths are implied from the prices of various traded securities whose payoffs depend on those paths. Since investors are in aggregate risk averse, these prices imply higher probabilities to bad scenarios than they do to good scenarios. Hence, while everyone (almost!) agrees that stocks have a higher expected return than risk free bonds, the prices of stock and stock options imply the only difference between stocks and risk free bonds is that stocks are more volatile. Put another way, a risk neutral scenario set has many more really bad scenarios than a real world scenario set precisely because investors fear these scenarios. They therefore overweigh their probability when deciding how much a security is worth.

Implication

There is an important implication that has relevance for actuaries struggling with the MCEV concept. *Our models need to reflect real world behavior within a given scenario.* That does not mean that we create some kind of parallel universe companion scenarios from which we derive policyholder and management behavior. Rather, the models should be based on the assumption that, should the economic scenarios actually transpire (how-



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ever unlikely), the modeled cash flows are a best estimate of policyholder and management behavior.

“ Individually, all the scenarios in a set of risk neutral scenarios are real world. ”

If we are modeling a product with management determined interest crediting, then there is an important question: do we share credit losses with the policyholders? If the answer is no, then our model should show that the crediting rate depends on corporate bond yields, but not on bond total returns. That is to say, the model should assume that that crediting will depend on the credit risk premium that we are hoping to make. This means that, if corporate bond spreads are high, our products look more expensive. If they are low, our products look cheaper. Put another way, if we are issuing free credit default swaps to our policyholders, our MCEV models need to reflect that. On the other hand, if we allocate all default losses back to policyholders, then it is appropriate to reference crediting to the total return, and the expected risk premium is irrelevant.

How does this look in practice? In the former case, we could model the crediting as Risk Free Rate + Bond Spread – Targeted Earnings Spread. In the latter, it would make more sense to model the crediting as Risk Free Rate – Targeted Earnings Spread. This is a huge difference for many products.

At the same time, we must make an assumption of how our competitors will credit. If we decide to not issue free credit default swaps to policyholder, that won't necessarily prevent the competition from doing so. Our models need to reflect this.

Finally, a risk neutral scenario set will likely have many scenarios with rather extreme behavior. Interest rates may be very high or very low, or equity returns may be negative over a long horizon. We need to be sure that the modeled dynamic behavior (policyholder, management or competition) is real world within those scenarios. For example, if the normal crediting policy is to stay within

200 basis points of current rates; would that still hold true when current rates are over 20 percent?

Summary

When it comes to understanding risk neutral pricing, we've come a long way, but there remain some subtle misperceptions. I have attempted to address one of them here. It is the notion that the scenarios themselves are not real world. In fact, all of the scenarios in a risk neutral scenario set are real world. This point implies that our models need to reflect what we think would really happen, given a specific scenario. □