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A PROPOSED METHOD OF VALUING VARIABLE BENEFIT RETIREMENT PLANS

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IN THIS paper the authors present a proposed solution to some difficult problems which have plagued those who have designed variable benefit plans. A variable benefit retirement plan is a pension program in which the retirement income is stated in nonmonetary units whose dollar value may and usually does fluctuate. For example, a variable benefit plan could provide that the monthly benefit be paid in bushels of wheat, barrels of oil, or pounds of butter.

It must be emphasized that in a variable benefit plan the term "variable" does not apply to the number of units paid each month. Once the number of units in each participant's retirement benefit has been determined, this number remains fixed. However, it is the usual practice to convert the units into dollars before remitting the payment. Since the relationship between the unit and the dollar will fluctuate, the number of dollars in the payments will vary. For example, if the participant were entitled to 200 bushels of wheat per month, the size of his check each month would depend upon the price of wheat in the open market.

Since the number of units is fixed, the problem of valuing the liabilities of the plan becomes the same as for a conventional fixed dollar plan. If N dollars are needed in reserve at age x to provide \$1.00 of monthly income starting at age y, then it must also be true that N barrels of oil are needed in reserve at age x to provide a monthly income of one barrel of oil starting at age y.

The type of unit most commonly used for variable retirement plans in the United States is a participation unit in a portfolio of invested assets, largely or entirely composed of common stocks. This type of unit has a dollar value which fluctuates with the dollar value of the portfolio. The basic definition of the dollar value of a benefit unit of this type may be stated by means of the following formula:

$$\text{Unit Value} = \frac{\text{Value of Assets of Fund (Dollars)}}{\text{Reserve Liabilities of Plan (Units)}}. \qquad (1)$$

This type of unit removes the investment risk from the shoulders of the guarantor of the plan—the employer or the insurer—and places it on the participants in the plan. It is possible also to shift the mortality risk from the guarantor to the participants by having the value of the benefit unit

reflect the mortality experience of the group. Except for very large groups, however, the possible random fluctuations in the pensions of the retired participants arising from this type of valuation are large enough to render the method inappropriate. In addition, there are many who seriously question the propriety of ever shifting the mortality risk to the participants. Therefore, the remainder of the paper will be restricted to the situation in which the investment risk is borne by the participants and the mortality risk by the guarantor.

There are many different attitudes on the question of the proper investment philosophy for a variable benefit plan of the type under discussion. Probably most variable benefit plans now in existence are operated in conjunction with a fixed income plan as part of a pension package. When this arrangement exists, the variable benefit plan is usually supported by a portfolio invested completely in equities. On the other hand, there are a number of situations in which retirement benefits are provided by one variable plan. Although in such cases the entire retirement benefit is variable, the fluctuations in the unit value are dampened by a more conservative investment policy in which some major portion of the fund is invested in fixed income securities. A proper valuation method for such a fund uses the market value for equity investments and cost (or amortized value) for fixed income securities. A single plan funded and valued in this way will achieve results similar to the two-plan package described above. This single plan form will be treated in this paper.

Certain technical aspects of a variable benefit plan have presented difficult problems, particularly during the early years of the plan. In particular, when handling credits for past service, some method must be devised which prevents market conditions at the time of any particular contribution from having a disproportionate effect upon a given employee's ultimate total past-service benefit and which prevents a change in the employer's obligations as a result of market fluctuation. The employer would take on a serious risk if the entire past-service credit were given at one time and the value of the units allowed to fluctuate freely with the securities market. The magnitude of this risk can best be shown by an example.

Assume: (1) One participant aged (x);
(2) Past service annuity benefit of \$200 per month commencing at age 65;
(3) The reserve needed at age (x) and also at age $(x + 1)$ to provide an annuity of \$1 per month commencing at age 65 is \$50;
(4) The initial value of a unit is 1.00;
(5) The assumed interest rate is zero.

The past-service benefit of $200 converts to 200 units. The lump-sum past-service cost is 50 × $200, or $10,000.

Suppose that the employer contributes $1,000 to the plan. Suppose, also, that this money is invested in stocks which increase in market value to $1,200 by the end of the year. This development increases the unit value to 1.20 at the end of the year. Since the unit value has increased by 20 per cent, the annuity benefit has increased by 20 per cent to $240, and the required reserve by 20 per cent to $12,000. This means that the unfunded liability has gone up from $9,000 ($10,000 − $1,000) at the beginning of the year to $10,800 ($12,000 − $1,200) at the end of the year. In other words, an increase of $200 in the value of the assets has produced a ninefold increase of $1,800 in the obligations of the employer.

The above problems have led those who have designed variable benefit plans to employ one or the other of the following devices:

1. Eliminate past service from the variable plan. Frequently a fixed dollar past-service plan is adopted along with a future service variable plan. This leaves the variable plan in the position of a more or less fixed-contribution profit-sharing plan.
2. Allocate specific contributions to the purchase of past-service benefits for specified individuals. In this type of plan each contribution toward past service is usually distributed among those nearest retirement. There is normally a restriction that no more than a stated percentage of any one employee's past-service benefit may be purchased in any year. Although this device alleviates the problem in question, it does not provide a complete solution, since the amount of any individual's retirement income becomes a function of the market condition at the time his past service is purchased.

The authors propose that in place of these devices the unfunded liability of the plan be regarded as an asset of the fund which earns exactly the rate of return assumed for purposes of valuation. In other words, the unfunded liability is treated as though it were an unsecured note of the employer which bears interest at the valuation rate.

If the unfunded liability is considered as an asset of the plan, it no longer becomes necessary to allocate specific contributions to the purchase of past-service benefits of specific employees. Moreover, since the unfunded liability is treated as a fixed asset which earns the assumed rate of return, the market conditions at the inception of the plan when the entire past-service credit is given do not have a disproportionate effect on any employee's ultimate past-service benefit.

To construct a method of computing the value of a benefit unit, let us assume for the sake of simplicity that one contribution C is made, one benefit payment B is due, and one disbursement D (including any benefit

payments) is made, all at the beginning of the year. In addition, other items will be denoted as follows (all amounts in dollars):

V_1 = Value of a benefit unit, beginning of year;
V_2 = Value of a benefit unit, end of year;
U_1 = Unfunded liability, end of previous year;
P = Current service (or normal) cost;
A_1 = Value of assets (other than unfunded liability), beginning of year;
A_2 = Value of assets (other than unfunded liability), end of year;
i = Valuation rate of return;
i' = Actual rate of return, including market fluctuation.

When the mortality risk is assumed by the employer, then

$$\frac{V_2}{V_1} = \frac{1+i'}{1+i}. \tag{2}$$

Since the addition of the current service cost increases the reserve liability, it also increases the unfunded liability. On the other hand, the passing of the date a benefit payment is due decreases the reserve liability and, therefore, the unfunded liability. Any contribution increases the monetary assets and decreases the unfunded liability while any disbursement has the opposite effect. Thus, after all these items have been taken into account at the beginning of the year,

$$\begin{aligned} \text{Unfunded liability} &= U_1 + P - B - C + D\,; \\ \text{Other assets} &= A_1 + C - D\,; \\ \text{Total assets} &= U_1 + P + A_1 - B\,. \end{aligned} \tag{3}$$

Therefore,

$$\begin{aligned} \text{Earnings on unfunded liability} &= (U_1 + P - B - C + D)i; \\ \text{Earnings on other assets} &= A_2 - (A_1 + C - D) \\ &= A_2 - A_1 - C + D\,; \end{aligned} \tag{4}$$

$$\text{Total earnings} = (U_1 + P - B)i - (C - D)(1 + i) + A_2 - A_1\,.$$

Since the rate of return is the ratio of earnings to assets at the beginning of the year,

$$i' = \frac{(U_1+P-B)i-(C-D)(1+i)+A_2-A_1}{U_1+P+A_1-B}, \tag{5}$$

and

$$\frac{V_2}{V_1} = \frac{1+i'}{1+i} = \frac{(U_1+P-B-C+D)(1+i)+A_2}{(U_1+P+A_1-B)(1+i)}. \tag{6}$$

It should be noted that formula (6) is independent of all valuation results at the end of the year except the performance of the investment portfolio. This means that the unit value may be calculated at the beginning of the new year independently of the results of the valuation. None of the financial results of the year's operation of the plan except the earnings on the investment portfolio and the change in its value have affected the unit value. This is the result which was desired.

An important disadvantage of the method described above is the fact that the choice of the assumed rate of return i will have an effect on the changes in the unit value. If the actual rate of return on the portfolio turns out to be consistently larger or smaller than the assumed rate of return, there is a built-in bias in the direction of movement of the unit values. It is generally accepted that there is a desirable correlation between the swings of the stock market and those of the cost of living. It is on this relationship that the rationale of the kind of variable benefit plan discussed in this paper rests. However, in general, no such favorable relationship exists between rates of return and the cost of living. In fact, rate of return and market price are inversely correlated.

For this reason, it would be desirable to eliminate the chosen value of i from the calculation of unit values and make the value of a unit dependent only upon fluctuations of the value of the portfolio. The only change necessary in the formulas above to accomplish this is the removal from the numerator of formula (6) of any gain or loss produced by a deviation of the earnings of the portfolio from the assumed rate of return. The earnings expected on the assets (other than the unfunded liability) are given by:

$$(A_1 + C - D)i\,. \tag{7}$$

Therefore, if we denote the actual earnings by I, the gain from this source is equal to

$$I - (A_1 + C - D)i\,. \tag{8}$$

When this quantity is subtracted from the numerator of (6), we obtain

$$\frac{V_2}{V_1} = \frac{(U_1 + P - B)(1+i) - C + D + A_2 + A_1 i - I}{(U_1 + P + A_1 - B)(1+i)}\,. \tag{9}$$

It should be noted that the use of this formula to determine unit values has shifted the "earnings risk" from the participants to the employer. In other words, the employer will benefit from yield in excess of required interest and will have to pick up any interest deficiencies.

For an actual plan, contributions and disbursements will occur and benefit payments will be expected at various times during the year.

Therefore, it is necessary to accumulate the amount of each such transaction from the date it occurs to the end of the year. Thus, formulas (6) and (9) are generalized to the forms:

$$\frac{V_2}{V_1} = \frac{(U_1+P)(1+i) - \sum_{j=1}^{n_1} B_j(1+i)^{t_{1j}} - \sum_{j=1}^{n_2} C_j(1+i)^{t_{2j}} + \sum_{j=1}^{n_3} D_j(1+i)^{t_{3j}}}{(U_1+P+A_1)(1+i) - \sum_{j=1}^{n_1} B_j(1+i)^{t_{1j}}}; \tag{10}$$

$$\frac{V_2}{V_1} = \frac{(U_1+P)(1+i) - \sum_{j=1}^{n_1} B_j(1+i)^{t_{1j}} - C + D + A_2 + A_1 i - I}{(U_1+P+A_1)(1+i) - \sum_{j=1}^{n_1} B_j(1+i)^{t_{1j}}}. \tag{11}$$

In actual practice, simple interest rather than compound interest would normally be used for fractions of a year.

We can show how the difficulties discussed earlier have been solved by the proposed valuation method by returning to the numerical example treated above.

Using formula (6) to obtain the unit value at the end of the year, we have:

$$\frac{V_2}{V_1} = \frac{(\$10{,}000 + 0 - 0 - \$1{,}000 + 0)(1.00 + 0) + \$1{,}200}{(\$10{,}000 + 0 + 0 - 0)(1.00 + 0)}$$

$$= \frac{\$9{,}000 + \$1{,}200}{\$10{,}000} = \frac{\$10{,}200}{\$10{,}000} = 1.02.$$

Since $V_1 = 1.00$, this means that $V_2 = 1.02$.

If the unit value at the end of the year is 1.02, the annuity benefit in our example has increased only to \$204, and the required reserve to \$10,200. The unfunded liability, which was \$9,000 at the beginning of the year, is still \$9,000 at the end. The obligations of the employer remain the same as if no change in unit value had taken place.

The same result as above would be obtained if formula (9) were used to compute the unit value, since we have ignored interest earnings in this example.

Since the initial past-service cost is usually a large sum in relation to the current-service cost and since the unfunded liability usually remains large

over a period of years, the method presented in this paper suggests that during the early years of the plan a large portion of or even all investments be made in equities. The point at which a shift in emphasis would be made would depend upon the ultimate investment philosophy of the fund. For example, the ultimate investment policy of the fund could be to invest half in equities and half in fixed assets. Under this policy, all investments during the early years would be made in equities. This practice would continue until the total cost of equities purchased to date and the remaining unfunded liability became approximately equal. For a time thereafter, new investments would be divided in such a way that the total cost of equities purchased to date would equal the remaining unfunded liability plus the total cost of fixed assets purchased to date. After the unfunded liability has been retired, new investments would be divided equally between equities and fixed assets.

DISCUSSION OF PRECEDING PAPER

CECIL J. NESBITT AND DONALD A. JONES:

In the development of their formula (9), the authors introduce I, the actual earnings on the assets (other than the unfunded liability). We believe the authors intend that I shall include only dividend and interest income and *not* any changes in value due to market fluctuations. Earlier in the paper, "earnings on other assets" have included both elements, and distinction should be made when only dividend and interest income is implied. For this purpose, and also to provide a set of formulas ranging from (6) to (9), we suggest the following additional notations:

Δ = change in value of assets due to market fluctuations;
i'' = actual rate of dividend and interest earnings on the portfolio, such earnings not including Δ.

Then, dividend and interest income I on the portfolio is given by

$$I = (A_1 + C - D)i'' ,$$

and total earnings on the portfolio $= I + \Delta = (A_1 + C - D)(1 + i'') + \Delta - (A_1 + C - D) = A_2 - (A_1 + C - D)$.

Let us now define distributable earnings on the portfolio to be

$$(A_1 + C - D)[(1 - k)i + ki''] + \Delta, (0 \leq k \leq 1) . \qquad (4a)$$

For $k = 1$, this expression equals the total earnings on the portfolio, and, for $k = 0$, equals the expected dividend and interest income on the portfolio plus the change in market value. If expression (4a) is used in place of the authors' formula (4) for earnings on assets (other than the unfunded liability), and calculations parallel to those for their formulas (5) and (6) are made, there results

$$\frac{V_2}{V_1} = \frac{(U_1+P-B)(1+i)+(C-D)k(i''-i)+A_1[1+(1-k)i+ki'']+\Delta}{(U_1+P+A_1-B)(1+i)} . \qquad (6a)$$

For $k = 1$, formula (6a) yields their formula (6), as may be seen by substituting $A_2 - (A_1 + C - D)(1 + i'')$ for Δ. For $k = 0$, formula (6a) reduces to

$$\frac{V_2}{V_1} = 1 + \frac{\Delta}{(U_1+P+A_1-B)(1+i)} , \qquad (9a)$$

which is equivalent to the authors' formula (9). Formula (9a) exhibits that if only expected interest is allowed on the portfolio, then the unit

value changes in accordance with the ratio of the change of value of assets to the expected assets at end of the year.

The authors' inclusion of the unfunded liability among the assets is reminiscent of the "frozen initial liability" funding method. This raises the question as to what funding methods can be used for a variable benefit retirement plan, and perhaps the authors might give us the benefit of their experience by commenting on the possibilities.

DONALD S. GRUBBS, JR.:

The authors refer to two methods which may be used to give employees a balance between fixed income and variable income: (1) establishing two separate plans and (2) establishing one plan invested partly in equities and partly in fixed-dollar investments. Two differences should be noted. First, under a single plan the desired level of balance can be maintained, while under two plans benefits which were intended to be provided on a 50-50 basis may actually be on a 70-30 basis or a 30-70 basis as the result of changes in asset values in the equity plan. Second, if two separate plans are maintained, the fixed-dollar plan may also be funded substantially in equity investments, which may have a major effect on the employer's cost.

The paper advocates a method to prevent "market conditions at the time of any particular contribution from having a disproportionate effect upon given employee's ultimate past-service benefit." To accomplish this, the authors propose to treat an unfunded liability as if it were an asset. Basically, the treatment of an unfunded liability for the purpose it is suggested is equivalent to the process which is usually used treating it as an unfunded liability. The liability normally increases with interest at the assumed rate plus or minus a gain or loss in mortality. If the employer absorbs the mortality gain or loss, then the liability increases only by the assumed interest. It does not have to be an asset either as a secured or as an unsecured note to have this characteristic of normally increasing at an assumed rate of investment return.

The proposed treatment as to past-service liability is no more or less than allocating any contributions actually made for the past-service liability in proportion to the liability for each member with such benefit. This, in effect, funds the past service for all employees covered by the plan proportionately. No employee has his past service fully funded before any other. In this it may be argued that it is more equitable than other methods of allocation. In general, this process of funding will take a relatively long period—twelve years or more.

Use of the unfunded liability as an asset will result in the assets con-

sisting entirely of fixed dollars at the outset, and it may be a considerable period before the proportion as assets invested in equities reaches a satisfactory level. During this period of time the equity plan could not be expected to meet fully its objective of keeping up with inflation.

Most equity variable annuity plans with which I am acquainted are either unit-credit career average plans or money-purchase plans. It is argued that the use of equity annuities under such plans can help solve the problems of inflation both before and after retirement. The desired appreciation in equity variable annuity plans results from the appreciation of the equity assets. To the extent that benefits are not funded in equities, there can be no appreciation. Therefore, the existence of unfunded liabilities, regardless of whether they are disguised as assets, limits the effectiveness of such plans.

A proposed better way is the use of a five-year final average pay variable annuity plan, administered similarly to an insured deposit administration contract in some respects. Employer contributions are not allocated to individual employees until they reach their fifth year prior to normal retirement. For example, consider an employee hired at age 25 under a plan providing 1 per cent per year of service. Since his projected benefit would be 40 per cent of final average pay, in each of the last five full years prior to his normal retirement date an allocation would be made to his account to fund a benefit at retirement equal to 8 per cent of his salary for that year. This will produce the final average benefit desired and spread his funding over five years so as to avoid the effect of funding the whole benefit at a high or low point of the market. For employees who retire during the first five years of the plan or who retire prior to their normal retirement date, 20 per cent of the total benefit would be allocated each year following retirement, and fixed-dollar payments would be made with respect to the portion not yet funded, as is presently done the first few years under some equity plans with an unfunded liability.

Career average equity annuity plans may provide benefits which are either larger or smaller than appropriate at the retirement date. The final-pay plan gives more assurance that the benefits will be at the appropriate level at retirement. It also eliminates the effect of the unfunded liability on the level of benefits within five years of the establishment of the plan, since the unfunded liability is allocated to the "deposit fund" and not to benefits of individual members.

Smoothing of equity variable annuities (*TSA*, XIV, 340–64) helps ameliorate the problems of crediting units at a high or low point in the market.

The authors assumed that dividends and market-value appreciation

are unrelated and propose that dividends should affect employer cost, while appreciation in market values should affect employee benefits. Actually, the trustee will often have to choose between high-yielding stocks with little prospect of appreciation and low-yielding stocks with substantial prospects for appreciation. Thus the proposal would leave the trustees to determine investment policy based on whether they want to reduce employer costs or increase employee benefits.

STUART J. KINGSTON:

Messrs. Smith and McKelvey present this definition of a "variable benefit retirement plan": "a pension program in which the retirement income is stated in nonmonetary units whose dollar value may and usually does fluctuate." The authors do not claim that this is the only correct definition. As a matter of fact, a statement which appears shortly after the above definition strongly suggests an alternate, and equally correct, definition. The statement in question is: "However, it is the usual practice to convert the units into dollars before remitting the payment."

In view of the fact that it is possible, or even usual, for all contributions to a variable benefit retirement plan to be in dollars and all benefit payments out of the plan to be in dollars, it must be possible to define this process without resort to hypothetical nonmonetary units. It is convenient, but not necessary, to convert incoming dollars to units and later convert the outgoing units to dollars. The principal convenience is that the use of units reduces the necessity of devising new calculation procedures for variable benefit plans. The usual procedures, which, in nonvariable plans are applied to dollars, are, instead, merely applied to units. The only new procedures needed are for the conversion of incoming dollar payments to units and outgoing unit payments to dollars.

Messrs. Smith and McKelvey have refined the procedure for converting outgoing units to dollars to reflect the well-known fact that nonexistent assets cannot experience investment gains. In particular, the unfunded portion of a past-service accrued liability cannot experience investment gains. The authors have developed the ingenious method of crediting this contingent asset with the valuation rate of interest, thereby "forcing" it to have no investment gain.

Would other definitions of a "variable benefit retirement plan" lead to different calculation procedures? What are these other definitions, and what calculation procedures flow from them?

Possibly there are many other definitions, but I know of only one. This alternate definition stems from this statement of Messrs. Smith and McKelvey: "This type of unit removes the investment risk from the

shoulders of the guarantor of the plan—the employer or the insurer—and places it on the participants of the plan."

Further discussion points out that the mortality risk is also capable of being shifted. However, the authors confined their analysis to a plan in which only the investment risk was shifted. There is no serious loss of generality in this, since analogous procedures may be devised for the shifting of any risk.

Therefore, in developing the alternate definition of a "variable benefit retirement plan," the same assumption was made by me that the authors used—namely, only two risks are involved, mortality and investment, and, only one, investment, is to be shifted. This assumption leads to this alternate definition of a "variable benefit retirement plan": "a pension program in which the incoming dollar contributions are adjusted to reflect mortality gains (or losses) and the outgoing dollar benefits are adjusted to reflect investment gains (or losses)."

It is apparent from this alternate definition that the usual pension plan calculation procedures must be altered, but there is no need to develop techniques for converting dollars into units or units into dollars.

This alternate definition lends itself to seriatim valuations, with contributions and payments allocated to individuals contrasted to unallocated valuations available if units are used.

It is necessary under the alternate definition to be able to compute the exact amount of a total contribution which pertains to each individual. The valuation assumptions (mortality and interest) must be used to determine these regular contributions. In addition, the additional negative contributions to absorb mortality gains (or additional positive contributions to absorb mortality losses) must be computed.

Once we have determined each individual's contributions, his asset share can be brought forward using the actual investment results.

There may be problems in obtaining the normal cost part of the regular contributions for each individual, if the normal cost of the plan is computed by an aggregate method of funding. (By "aggregate method" is meant an approximation of the exact cost, found by adding numerators and denominators separately and dividing the totals.) The exact cost can be found for each individual, and the excess (or deficiency) of the aggregate method cost can be considered as a positive (or negative) contribution toward past service.

The allocation of the past-service contribution to each individual is a little tricky if the past-service contribution is found by amortizing the initial past-service liability over a fixed period of years by dividing it by the present value of an annuity-certain.

If the initial past-service liability of each individual is obtained as of the effective date of the plan, the regular past-service contribution made for each individual on the effective date is easily determined by dividing his individual past-service liability by the common annuity-certain factor for the plan. The more difficult part of the problem is determining each individual's share of the total past-service contribution in a future year (consisting of the same amount as in the first year plus his share of the current year's past-service payment for all decedents).

This difficulty may be greatly reduced by use of the fact that an annuity-certain for n years of 1 per year is mathematically identical to a temporary life-annuity for n years of an increasing annual payment, starting at 1, and increasing inversely with the "Number Living" column in the mortality table.

Some actuaries explain the annuity-certain amortization of past-service liability by saying: "This is a level payment—but it must be made for n years for each life regardless of whether or not he lives for n years." I have always preferred to say, "This is an increasing payment for n years (or earlier death)—and the reason it increases is that there are fewer lives left each year for whom payments are due." My preference is based on my belief that the first method is more susceptible of leading to misunderstandings. Regardless of personal preferences as to semantics, the second explanation has the characteristic of being quite amenable to the determination of individual past-service contributions.

The present value of future past-service contributions for an individual can be computed from his current past-service contribution either by multiplying it by an increasing temporary life-annuity (increasing in the manner described above) for the balance of the amortization period or by multiplying it by the necessarily equal level annuity-certain for the balance of the amortization period. The second method is more convenient. The present value of future past-service contributions for an individual is a necessary ingredient for the computation of the amount of pension of a retired individual whose past service has not yet been completely funded (due to the amortization period ending later than his retirement date). This will be mentioned in greater detail later when payouts are discussed.

Once the regular normal cost contributions and regular past-service contributions have been computed for each individual, there remains only one more type of contribution to compute, namely, the adjustment for mortality.

The employer is presumably willing to pay the same contributions as in a nonvariable plan except the contribution to adjust for investment.

The contribution to adjust for mortality has to be computed (on the valuation assumptions as to mortality and interest).

This is equal to the excess of the terminal reserve (on the valuation assumptions as to mortality and interest) expected to be released by death (on the valuation assumption as to mortality) over the terminal reserve (again on the valuation assumptions as to mortality and interest) actually released by death.

Therefore, it is necessary to keep a record (or compute anew each year) the terminal reserves on the valuation assumptions in order to apply to these reserves the excess of the valuation mortality rate over the experience mortality rate.

In some plans the contribution to adjust for losses (or gains) is amortized one way or another rather than being set exactly equal to the loss (or exactly equal to but opposite in sign to the gain).

It is also necessary to reallocate the asset accounts of each decedent to the survivors in proportion to the expected mortality release for each survivor (on the valuation mortality and interest).

Now we have completed the determination for each individual of (1) the normal cost contribution; (2) the past-service contribution; (3) the mortality adjustment contribution; and (4) the transfer to him of all decedents' assets. These four facts, plus the actual investment results of the plan, determine the assets of each individual.

When retirement occurs, if there is no unfunded liability, the initial amount of pension is simply equal to the assets of the individual divided by the appropriate life-annuity factor (on the valuation mortality and interest).

His assets are brought forward just the same as before, except that the benefit payment is a negative contribution, there is no normal cost contribution, and there is no past-service contribution. There would be a mortality adjustment contribution and a transfer of all decedents' assets.

At the end of the year (or other period selected) his current asset value is divided by the current annuity factor (on the valuation mortality and interest) to obtain his new amount of pension until the next redetermination date.

If a retired life has an unfunded liability, usually due to the fact that past-service contributions are still due for him, it is necessary at each pension determination or redetermination to increase his asset value by the previously described "present value of future past-service contributions" before dividing by the annuity factor. (His assets, of course, also reflect the actual past-service contribution for him each year.)

This treatment of the unfunded liability is exactly the same as the

fundamental assumption of Messrs. Smith and McKelvey that the unfunded liability is an asset of the fund earning the valuation rate of interest. However, in this alternate approach, this concept is used explicitly only for retired lives with unfunded liabilities, which is generally a temporary situation. The same basic concept is implicit throughout all the aspects of this alternate method, because, by working with assets only (i.e., with funded reserves only), the unfunded liability is merely transposed to the other side of the equation and subtracted and hence has the same effect as the method of Messrs. Smith and McKelvey.

It remains for one more gifted in mathematics than myself to prove rigorously that the same benefit payments would occur from both methods if the same experience occurred.

To the extent that an aggregate method is used, which shifts the amount and incidence of contributions, the results will differ from what would be the case in a nonaggregate method. In fact, whether aggregate or nonaggregate, the results will be affected by the funding method (level, increasing unit credit method, etc.), which determines the incidence of regular contributions. Furthermore, the incidence and amount of mortality adjustment contributions will depend on the valuation mortality and interest. And the reallocation of decedents' assets also depends on these assumptions.

The amount of pension depends directly as well as indirectly on the valuation mortality and interest. The smaller the annuity value divided into a retired individual's assets, the larger the pension (and hence the smaller the next redetermined pension). By choosing a high valuation rate of interest, those who die early are favored, and vice versa.

These last considerations illustrate that, although investment is the only factor causing a benefit to change in this type of plan, the exact way in which the change occurs depends on arbitrary decisions as to funding method and as to actuarial assumptions.

Finally, I would like to point out that the calculations flowing from my alternate definition appear to be more laborious than the Smith-McKelvey method and yet produce the same result. The question then arises, "Why bother?"

My reason was a strong compulsion to prove that it is not necessary to invent a hypothetical nonmonetary unit in order to keep track of real dollars and to question the propriety of incorporating the concept of the nonmonetary unit in the definition of the "Variable Benefit Retirement Plan." Logical nicety indicates, at least to me, that extraneous concepts, however convenient, should be in derived theorems rather than in basic definitions.

Thus the door is not closed to avoiding the nonmonetary unit should

that prove to be more convenient in some future problems arising in the area of variable benefit retirement plans.

HENRY E. BLAGDEN:

Some six or seven years ago when we at the Prudential had the naïve idea that we were on the threshold of entering the variable annuity business, we gave a lot of consideration to this problem of past service. There is a conflict between two objectives. One is to limit the obligations of the employer. If you do not, the employer's position is similar to one in which he assumes a foreign-currency obligation but finances it in dollars; if the foreign currency appreciates, it increases his dollar obligations. Of course, in this respect it is no different from a final-salary plan, where essentially the same thing can happen.

The other objective, of course, is to maintain the general variable annuity approach, so that the result of the common-stock fund will serve as a hedge against inflation and also as a means of participating in the growth of the economy generally. The authors have presented a rather ingenious solution which represents a compromise between these two objectives. Like every other compromise, it has its limitations. It defeats to some extent the objectives of the variable benefit plan, and Mr. Grubbs has pointed that out in his discussion.

I also have a few other comments which I might make. Formula (1), which is given in the fourth paragraph of the paper, seems to me to have the effect of imposing the mortality risk upon the participants, although the comment in the fifth paragraph might suggest that the authors have a different opinion. Perhaps this can be explained in the reply to the discussion.

I am not at all sure that I agree with the authors that the use of an investment increment assumption is undesirable. It has always seemed to me that such an interest assumption is really a part of a pension planning program. When you are dealing with a variable annuity, I think it is worth having. In case this part of my comment is incomprehensible to some of you, I would mention that, the lower the investment increment assumption—if you are dealing with the same unit benefit—the more liberal the pension plan.

I am not at all sure of the desirability of having the employer assume the income risk. It seems to me, again, that it imposes limitations upon the investment of the fund. Some people think that IBM, for example, is a very good investment, but certainly the income on IBM investments is pretty small, and this would be a case in which the provider of the pension was making up for the deficiencies in the income on IBM, whereas future appreciation is intended to do that.

(AUTHORS' REVIEW OF DISCUSSION)

FRANKLIN C. SMITH AND CHANDLER L. MCKELVEY:

We are very pleased that there has been some discussion of our paper and that some interesting sidelights and alternate points of view have been put forward. It seems to us that there are bound to be a considerable number of alternate approaches developed in a new field such as this.

Messrs. Nesbitt and Jones have presented a very interesting generalization of our formulas (6) and (9) which give some additional insight into the nature of these relationships. They are completely correct in their observation that the frozen initial liability method of funding seems to be in harmony with the methods developed in our paper. In fact, this is the valuation method which has been used for all the dozen or so variable benefit plans which have been set up based on the principles outlined in the paper. In this connection, it may be noted that, if the initial liability is set equal to zero—that is, if the "aggregate cost" funding method is used—the unfunded liability disappears from the formulas. This change in funding method eliminates some of the very real problems which are caused by the dampening effect of the unfunded liability on the change in unit values. These problems are discussed briefly by Mr. Blagden and Mr. Grubbs in their discussions of the paper. However, it will be frequently found that any advantages gained through elimination of the unfunded liability will be more than offset by the increased rigidity in financial planning which is always associated with the aggregate cost method.

We have found, however, that the proposal contained in our paper does work best when the unfunded liability is relatively small in comparison to the normal cost. If the unfunded liability for a particular group is larger than, say, ten or twelve times the annual cost, the dampening effect of so large a fixed asset tends to dampen the movement of unit values to an excessive degree and over a rather lengthy period of time.

Mr. Grubbs proposes a benefit formula which is of the "final-average salary" type as a superior method of insuring the proper relationship, at least at the beginning of the pension period, between pension and cost of living. There is, of course, considerable merit in this proposal—the arguments pro and con being the same as for a "final five-year average" formula in a nonvariable plan. It should be pointed out, however, that Mr. Grubbs resorts to one of the artificial devices illustrated in our paper in order to accomplish his goal—namely, to allocate specific contributions to the purchase of benefits for specified individuals. It is unfortunately true that any attempt to "improve" the fit between the cost of living and

the performance of the unit values in a variable benefit plan must lead to strained devices of one sort or another.

Mr. Kingston, in a valuable discussion, brings out several excellent points. His mathematical analysis demonstrates that it is not necessary to introduce the concept of a nonmonetary benefit unit in order to operate a variable benefit plan. However, it does seem to us that the very mathematics of his demonstration give a very good illustration of why such a unit is desirable. Certainly, the actuarial mathematics of valuation and determination of unit values would be considerably more cumbersome without the benefit unit. In addition, it is difficult to visualize how the operation of such a plan could be easily explained either to the client or to the participants in the plan.

Mr. Kingston is very correct when he points out that mortality and interest assumptions used in the valuation have a definite effect on the unit values over the life of the plan. There is a special responsibility facing the actuary in choosing, especially, the interest assumptions to be used in this type of a plan. A degree of conservatism which might be highly commendable in a traditional plan could lead to excessive bias in the amounts of benefits which actually turn out to be paid to the participants in the plan. It should be remembered that the purpose of a variable plan is presumed to be to provide payments which will give a reasonably constant purchasing power over a period of years. This purpose is defeated if an unreasonable bias either upward or downward is reflected in the actuarial assumptions used.

Mr. Blagden correctly points out that the method proposed in our paper represents a compromise between the basic objective of a variable retirement plan and the necessity of the employer's maintaining a sound fiscal structure. That some sort of compromise is necessary has been evident in the past to those who have watched the enormous increases in liability resulting from final-average salary plans where these same sorts of compromises have sometimes been lacking.

It is apparent that the amount of activity in the variable benefit retirement plan area will increase substantially over the next few years. It is our hope that the questions raised in our paper and the proposals contained in the paper and the discussions will serve both as a warning and as background for solutions for those who will be working in this field.