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Real-World Interest Rate Models in a Low Interest Rate Environment[1]

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United States Treasury rates have decreased significantly and stayed at historically low levels since the 2008 financial crisis. This has direct implications for interest-sensitive life insurance and annuity products. For instance:

- Sustained low interest rates make it difficult to earn the yields needed to support minimum crediting rate guarantees (interest rate and spread risk).
- Rapidly rising interest rates can lead to a substantial increase in surrenders, forcing the insurer to sell a significant volume of assets at a loss (disintermediation risk).

This article presents a case study that explores the use of real-world interest rate scenario generators with a block of fixed deferred annuities (DAs). We contrast cash flow testing (CFT) results based on New York 7 (NY7) deterministic scenarios to stochastic scenarios generated with the Academy Interest Rate Generator (AIRG), as well as an alternative economic scenario generator (ESG) designed explicitly to capture the risk of:

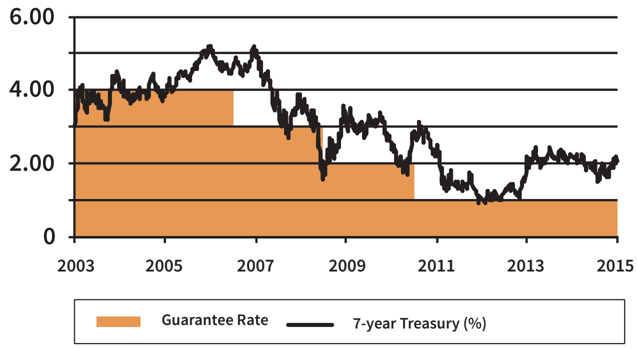
1. A persistently low interest rate environment, and
2. The transition to a rising interest rate environment.

CASE STUDY: PRODUCT OVERVIEW

The hypothetical inforce block in this case study consists of DA policies issued between 2003 and 2015. The following table summarizes the interest rate guarantees by issue year, along with the current weight by account value for each guaranteed rate.

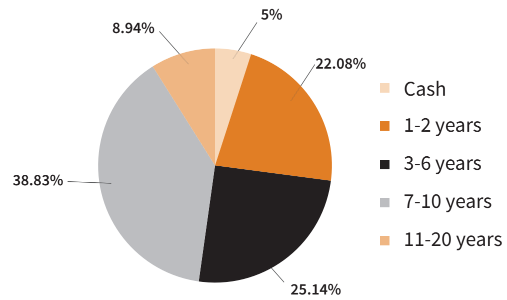
Deferred Annuity (DA) Block of Business		
Issue year	**Weight**	**Minimum guarantee**
2003–06	50%	4.0%
2007–08	28%	3.0%
2009–10	11%	2.0%
2011–15	11%	1.0%
Weighted average guarantee rate		**3.2%**

7-Year Treasury Spot Rate vs. DA Minimum Guarantee Rate (%)

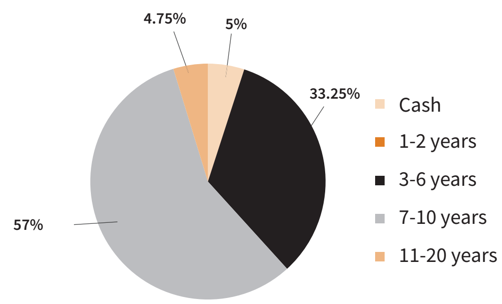


- Crediting rates are based on the current asset portfolio yield less a target spread of 1.0 percent, subject to the policy minimum credited rate. Lapse rates are set dynamically, based on the difference between market competitor rates and the crediting rate. Shock lapses at the end of the surrender charge period are also defined dynamically.
- At the June 30, 2015, valuation date, the starting portfolio consists of bonds across various maturities with 40 percent in NAIC Class 1 bonds and 60 percent in NAIC Class 2. The market-to-book value ratio of the starting portfolio is 106.5 percent.
- Positive cash flows are reinvested to meet the target allocation given below. Negative cash flows are first covered

Initial Allocation by Term to Maturity



Target Allocation by Term to Maturity



with available cash and then with asset sales that minimize incurred capital gains and losses.

DETERMINISTIC SCENARIOS: NY7

Actuaries typically look at NY7 scenarios as an umbrella that covers a wide spectrum of interest rate movements. Using the June 30, 2015, yield curve and starting reserves of $12.4 billion, the NY7 results are summarized in the following table.[2]

CFT Results: 30-Year Projections		
NY7	Description	PV of ending surplus[a] ($ millions)
1	Level	397
2	Increasing	599
3	Up/down	451
4	Pop up	689
5	Decreasing	(280)
6	Down/up	269
7	Pop down	(320)

[a]Discounted at the pretax portfolio yield rate.

The decreasing scenarios (5, 6, and 7) present the most significant profitability/reserve adequacy risk because of the inability to support the guaranteed rates. For the increasing scenarios (2, 3, and 4), the benefit of higher portfolio yields is partially offset by realized capital losses due to higher lapses.

STOCHASTIC SCENARIOS: AIRG

We further tested the DA block under stochastic scenarios produced by the AIRG. We used the latest version available at the time of this study (7.1.201406), only updating the starting yield curve to the selected valuation date.[3]

The Value at Risk (VaR)[4] and conditional tail expectations (CTE)[5] of the present value of ending surplus are summarized in the table below.

PV (Ending Surplus) ($ Millions)		
Level	VaR	CTE
50.0%	475	364
70.0%	407	311
80.0%	370	272
90.0%	297	207
95.0%	230	146
99.0%	97	7
99.5%	47	(54)
99.9%	(23)	(277)

Commonly used ESGs may not fully address the unique challenges presented by the current low interest rate environment.

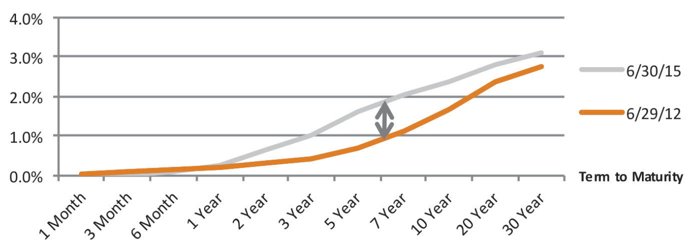
Because asset adequacy analysis is performed to test "moderately adverse" conditions, an appointed actuary might conclude that no additional reserves are required with the results shown above. However, certain risks exist that the appointed actuary needs to consider that are not explicitly covered by the AIRG scenarios:

1. Changes in the Curvature of the Yield Curve

As shown in the chart below, the short- and long-term rates have both remained relatively the same, but the medium-term rates (3 to 10 years) have increased between 2012 and 2015.

As noted in the prior article of this series, the AIRG does not model curvature stochastically and therefore does not introduce butterfly shifts to the yield curve in the simulated scenarios.

U.S. Treasury Yield Curve



2. Transition Out (or Prevalence) of the Current Low Interest Rate Environment

General consensus holds that interest rates will likely eventually revert to historical levels, but there are disagreements regarding when and how this will happen.

The AIRG model assumes that interest rates will revert to the selected mean reversion point (MRP) over approximately 50 years but does not allow users to explicitly define how long they believe low interest rates will persist and how the transition to higher rates will occur. For example, if users believe interest rates will remain low for an extended period of time before rapidly rising to the MRP, they will not be able to specify this path to MRP using AIRG's model parameters.

STOCHASTIC SCENARIOS: ALTERNATIVE ESG

To explicitly capture the view of extended low interest rates discussed above and evaluate the potential impact on CFT, we created an alternative ESG designed as follows:

- A key-rate model form[6] based on the Cox-Ingersoll-Ross (CIR) model to project each point on the Treasury yield curve
- A regime-switching process to model transition explicitly from the current low interest rate environment to a rising interest rate environment. The regime-switching process explicitly captures cases with sustained low-interest rates, as well as the transition to rising interest rates.

$$dr_{3M}(t) = a_{3M}^{I(t)}\left(b_{3M}^{I(t)} - r_{3M}(t)\right)dt + \mu_{3M}(t)dt + \sigma_{3M}^{I(t)}\sqrt{r_{3M}(t)}dZ_{3M}$$

$$dr_{6M}(t) = a_{6M}^{I(t)}\left(b_{6M}^{I(t)} - r_{6M}(t)\right)dt + \mu_{6M}(t)dt + \sigma_{6M}^{I(t)}\sqrt{r_{6M}(t)}dZ_{6M}$$

$$\ldots$$

$$dr_{30Y}(t) = a_{30Y}^{I(t)}\left(b_{30Y}^{I(t)} - r_{30Y}(t)\right)dt + \mu_{6M}(t)dt + \sigma_{30Y}^{I(t)}\sqrt{r_{30Y}(t)}dZ_{30Y}$$

$$f(dZ_{3M}, dZ_{6M}, \ldots, dZ_{30Y}) = C[f(dZ_{3M}), f(dZ_{6M}), \ldots, f(dZ_{30Y})]$$

$$\boldsymbol{P} = \begin{bmatrix} P(I(t)=1|I(t-1)=1) & P(I(t)=2|I(t-1)=1) \\ P(I(t)=1|I(t-1)=2) & P(I(t)=2|I(t-1)=2) \end{bmatrix}$$

where $\{r_{3M}, r_{6M}, \ldots, r_{30Y}\}$ are the modeled key rates,
$\left\{a_{3M}^{I(t)}, a_{6M}^{I(t)}, \ldots, a_{30Y}^{I(t)}\right\}$ the mean-reversion speed,
$\left\{b_{3M}^{I(t)}, b_{6M}^{I(t)}, \ldots, b_{30Y}^{I(t)}\right\}$ the mean-reversion level,
$\{\mu_{3M}(t), \mu_{6M}(t), \ldots, \mu_{30Y}(t)\}$ an additional drift component that reflects the impact of the level of each key rate relative to one another,
$\left\{\sigma_{3M}^{I(t)}, \sigma_{6M}^{I(t)}, \ldots, \sigma_{30Y}^{I(t)}\right\}$ the volatility factors, and
$\{Z_{3M}, Z_{6M}, \ldots, Z_{30Y}\}$ the associated Wiener processes for each rate.
The joint distribution of the instantaneous change in Wiener processes, $f(dZ_{3M}, dZ_{6M}, \ldots, dZ_{30Y})$, is defined using a correlation matrix.

$I(t)$ denotes the regime at time t, which follows a Markov chain process with transition matrix $\boldsymbol{P}$. Note that each set of parameters $\{a_T^I, b_T^I, \sigma_T^I\}$ is unique for a given term to maturity and each regime.

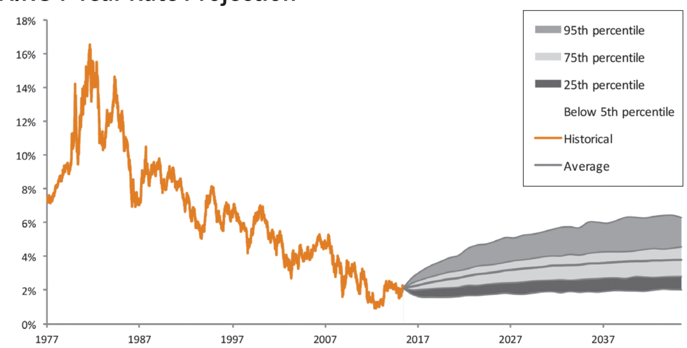
We calibrated the regime-switching CIR (RSCIR) model parameters with maximum likelihood estimates (MLEs) using historical treasury rates from 2009 to 2015 for the low interest rate environment (the "low regime") and from 1977 to 2008 for the interest rate environment observed before the 2008 financial crisis (the "high regime"). The transition probability between the low regime and the high regime is based on the user's explicit view of how long he or she expects the current (low) interest rate environment to persist. The parameters used in our runs are summarized in the following table.

Annual Transition Probability	To low regime	To high regime
From low regime	85.0%	15.0%
From high regime	2.1%	97.9%

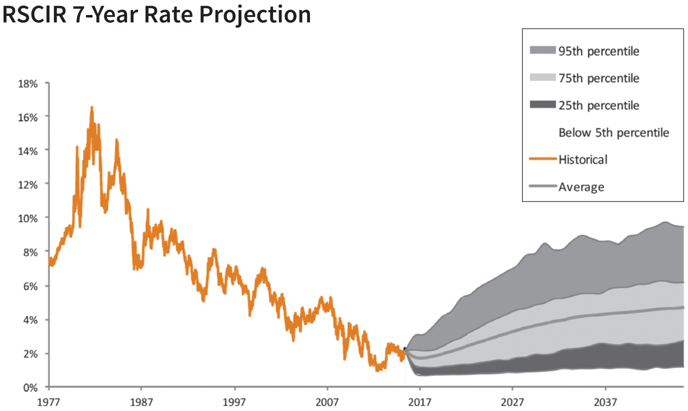
In other words, a 15 percent probability exists in a given year that there will be a transition from the low regime to the high regime. This translates to an average of 6.6 years to exit the low interest rate environment. Given the nature of a two-regime model, the convergence toward the weighted average MRP takes longer than 50 years.

We generated 1,000 scenarios using both the AIRG and RSCIR models.[7] The distribution of the 7-year rate under each model is shown in the following figures.

AIRG 7-Year Rate Projection



RSCIR 7-Year Rate Projection



The CFT results with 1,000 scenarios are summarized in the following table.

PV (Ending Surplus) ($ Millions)				
ESG	**AIRG**	**RSCIR**	**AIRG**	**RSCIR**
Level	**VaR**		**CTE**	
0%			455	336
50.00%	475	353	364	167
70.00%	407	235	311	84
80.00%	370	166	272	26
90.00%	297	55	207	(63)
95.00%	230	(28)	146	(147)
99.00%	97	(202)	7	(316)
99.50%	47	(275)	(54)	(398)
99.90%	(23)	(364)	(277)	(610)

The main drivers of the differences between AIRG and RSCIR are the following:

1. The RSCIR generates a higher number of scenarios that show low interest rates for an extended period of time (i.e., staying in low regime) and cases where interest rates had sharp increases after transitioning into the high regime.
2. The MRPs between RSCIR and AIRG are different. The RSCIR defines an MRP under each regime using historical data, placing the same weights on each historical rate. The AIRG places more weight on recent experience when defining the MRP.

Depending on how the appointed actuary defines moderately adverse conditions, using an alternative ESG may lead to a different conclusion on whether additional reserves are needed.

As this case study illustrates, capturing interest rate risk using stochastic models poses additional challenges to actuaries, but allows us to better understand the risks embedded in our portfolios. This analysis was focused on asset adequacy, but the choice of ESG is also relevant in other business applications, such as asset-liability management and risk management. Actuaries should understand both the explicit assumptions they make when calibrating an ESG and the implicit assumptions they make when selecting an ESG. ■

ENDNOTES

1 This discussion follows from the previous article in the July 2015 issue of *The Modeling Platform*, "Real-World Interest Rate Models and Current Practices," where we discussed common uses of real-world interest rate scenario generators in the life insurance industry and different approaches to building such generators.

2 Rates are subject to the proportional shift floor, in which the curve is never allowed to be lower than half of the initial curve at valuation date.

3 In practice, although some companies do recalibrate the AIRG, we often see companies update only the starting yield curve and the MRP of the long-term rate, based on the Academy's recommended formula ("MRP Formula and Seed Volatility 2007-09-30.xls," published on the Academy's website).

4 Value at Risk at level q—VaR(q)—in this context is the $(1-q)$ quantile of the empirical distribution of surplus.

5 Conditional tail expectation at level q—CTE(q)—is the average of the surplus values that are lower than VaR(q).

6 As described in our article in the previous issue of *The Modeling Platform.*

7 The stochastic scenarios used in this study passed AAA's calibration criteria.

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