

The *Modeling* Platform

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REAL-WORLD INTEREST RATE MODELS AND CURRENT PRACTICES

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As actuaries we often find ourselves focusing on the internal consistency of our models and examining whether they are well calibrated and produce output in line with observable experience and intended uses. However, we may neglect the risks assumed by relying on a particular algorithm or model structure (i.e., model risk). This is particularly critical for real-world interest rate scenario generators, which produce stochastic interest rates under a realistic probability measure. In this article we review some common uses of real-world interest rate scenario generators in the life insurance industry and explore three fundamentally different approaches to building such generators.

Common Uses of Real-World Interest Rate Models

The use of risk-neutral and real-world interest rate models has grown substantially in the last decade as life and annuity products have become more complex. The need for risk-adjusted management information has grown, and accounting and regulatory frameworks have become more sophisticated, demanding principle-based views of risk and valuation. Here are some common uses for real-world interest rate models:

- Financial reporting
 - **U.S. statutory valuation, Actuarial Guideline (AG) 43**—This is a valuation standard for variable annuities with guaranteed benefits. Some companies use stochastic interest rates for the valuation.

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WE'RE OFF! BUT WHERE ARE WE GOING?

By Trevor Howes

This is it! The very first newsletter from the Modeling Section! We are off and running!

It has been an amazing 18 months from the first discussions about the idea of a section, to forming an organizing committee, petitioning the Society of Actuaries (SOA) to create us, and then officially being formed in July 2014 as the 20th special interest section of the SOA. And the response has been wonderful and supportive, from the original core who helped get us launched and stayed to contribute their time on our interim council, to the SOA staff who enthusiastically guided us and helped us get the word out, to the tsunami wave of new members who have signed up and joined our ranks so far. Over 1,200 members and counting! Thank you all! Now what are we going to do? And what do you, our members, want us to do?

Our mandate is wide open and inviting: anything related to the design, development, operation, maintenance and control of models in any line of business or area of practice is in scope. But we can't do everything at once, so we will focus first on modeling interests of our founding group and broaden out over time according to the demands and areas of practice of those who join us and speak up.

Initially, we will be concerned mostly with life insurance and annuity modeling. Our hot topics will be model governance and control, model validation, model efficiency, economic modeling, longevity modeling and predictive modeling. We will help create and run sessions at the SOA meetings, put on webinars and seminars, publish this newsletter, and sponsor research. Action has already taken place in all of these forms, and more will come, especially if you send us your ideas and your feedback. But I want us to be innovative, too. What more can we do for you? Is there a resource or service related to modeling that would help you learn, develop your skills and productivity, and broaden your horizons?

Having asked you these questions, I wonder how we will communicate ideas and opinions within our section effectively. Is there a means to invite and stimulate broad but protected two-way conversations on topics of interest on a timely and convenient basis? You would think in this age of electronic communication that some discussion group or chat facility would be easy to form and regularly monitor and participate in without the annoyance of numerous commercial interest messages. The only option I have seen so far is the LinkedIn private discussion subgroup for our section that the SOA supports and restricts to section members only. So far this option has been virtually ignored by you, our members. I would be delighted if we could make this work, and actually see some meaningful and interesting discussions happen. So please consider joining the Modeling Section subgroup using the SOA Web page link (<https://www.linkedin.com/groups/Society-Actuaries-684897/about>) and watching for discussion topics. Or maybe you have other suggestions on how to make this happen?

Whatever your viewpoint, I look forward to hearing from you at my email: Trevor.Howes@ggy.com. Welcome to the Modeling Section and may all your simulations be useful!



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- **U.S. regulatory capital requirements**—C-3 Phase 1 prescribes an interest rate generator. C-3 Phase 2 requirements are similar to AG43.
- **U.S. GAAP, SOP 03-1**—Requires a valuation under a “range of scenarios” covering risks applicable to that business. For products with interest rate risk exposure, this may include the use of real-world interest rate scenarios.
- **Canadian Asset Liability Method**—May be done using stochastic real-world interest rate scenarios. The guidance specifies general requirements that would generally be covered using key-rate models or function-based models (discussed below), as well as calibration criteria to ensure that the scenarios are adequately adverse.
- **Asset adequacy testing**—This may be done in a variety of ways, and commonly includes a stochastic real-world valuation to determine whether assets are sufficient to support the in-force liabilities under moderately adverse economic conditions. This often supplements testing under the “New York 7” scenarios and other deterministic scenarios.
- Other applications that are often modeled using real-world scenarios include economic capital, pricing and embedded value.

A real-world interest rate stochastic model not only reflects a “best-estimate” assumption for future interest rates, but also a best-estimate view of their fluctuation. Best-estimate assumptions are also used for a variety of financial reporting and other purposes beyond those discussed above.

Constructing Real-World Interest Rate Models

With many approaches available to construct real-world interest rate models, it is easy to struggle trying to balance the different needs and select and calibrate a suitable model. In the discussion below we will walk through some basic categories of interest rate models and the considerations in selecting a model.

Most models will fall under one of the following categories:

- Short-rate models
- Key-rate models
- Function-based models.

SHORT-RATE MODELS

Single short-rate models refer to the commonly discussed equilibrium models, such as Vasicek, Cox-Ingersoll-Ross (CIR), Brennan-Schwartz and Black-Karasinski. These models define the instantaneous interest rate (i.e., the short rate) using stochastic differential equations:

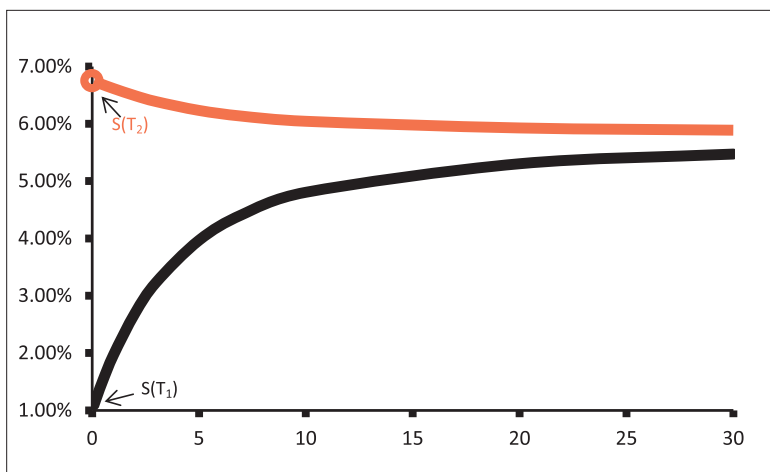
Model	Stochastic Differential Equation
Vasicek	$dr = a(b - r)dt + \sigma dZ$
CIR	$dr = a(b - r)dt + \sigma\sqrt{r}dZ$
Brennan-Schwartz	$dr = a(b - r)dt + \sigma rdZ$
Black-Karasinski	$d\ln(r) = a(b - \ln(r))dt + \sigma dZ$

These all follow a visible structure with a drift component (i.e., $a(b-r)dt$) and a stochastic component (σdZ), where Z is a Wiener process. The drift component includes a mean reversion target (b) and mean reversion speed (a). σ is a measure of the volatility of the short rate and can be applied in different ways.

A key advantage of most of these models (e.g., Vasicek or CIR) is that bond prices at any maturity have an analytical form (i.e., there is an explicit formula to define zero coupon bond prices at any time t) from which the yield curve can be derived. However, these models are based on the instantaneous spot rate, or the short rate, which is the rate an entity can borrow money for an infinitely small period of time. Also, the structure of these models is simplistic and could produce unintended term structures (e.g., inverted yield curves or negative rates in the United States).

In more sophisticated models, practitioners can add more conditions in short-rate models such as embedding stochastic processes such as volatility or mean-reverting targets (e.g., two-factor Vasicek or Brennan-Schwartz models).

Figure 1: Yield Curve Model Derived from Short Rate at Two Different Time Periods (T1 and T2)—Illustrative Only



KEY-RATE MODELS

The stochastic equations used in single short-rate models can also be adopted to generate observable measures such as forward rates or yields. Under this approach, multiple stochastic processes are used to project the rate at each maturity term in the yield curve. These processes are then made codependent using an explicit correlation matrix or a copula. The following formulas provide a general definition of key-rate models:

$$dr_1 = \mu_1(r_1)dt + \sigma_1(r_1)dZ_1$$

$$dr_2 = \mu_2(r_2)dt + \sigma_2(r_2)dZ_2$$

...

$$dr_n = \mu_n(r_n)dt + \sigma_n(r_n)dZ_n$$

$$f(Z_1, Z_2, \dots, Z_n) = C[f(Z_1), f(Z_2), \dots, f(Z_n)],$$

Where $\{r_1, r_2, \dots, r_n\}$ are the modeled key rates, $\{\sigma_1(r_1), \sigma_2(r_2), \dots, \sigma_n(r_n)\}$ the volatility structure for each rate, and $\{Z_1, Z_2, \dots, Z_n\}$ are the associated Wiener processes.

The joint distribution of the Wiener processes, $f(Z_1, Z_2, \dots, Z_n)$ is defined using the copula C.

CONTINUED ON **PAGE 6**

Since each key rate is modeled under a separate stochastic process, the model can be defined to capture all possible (or desirable) curve movements (e.g., parallel shifts, twists, butterfly shifts), providing more flexibility and control to the user.

Figure 2 below illustrates the process of three rates of different maturities under a single scenario.

Figure 2: Key Rate Projection under a Sample Stochastic Scenario

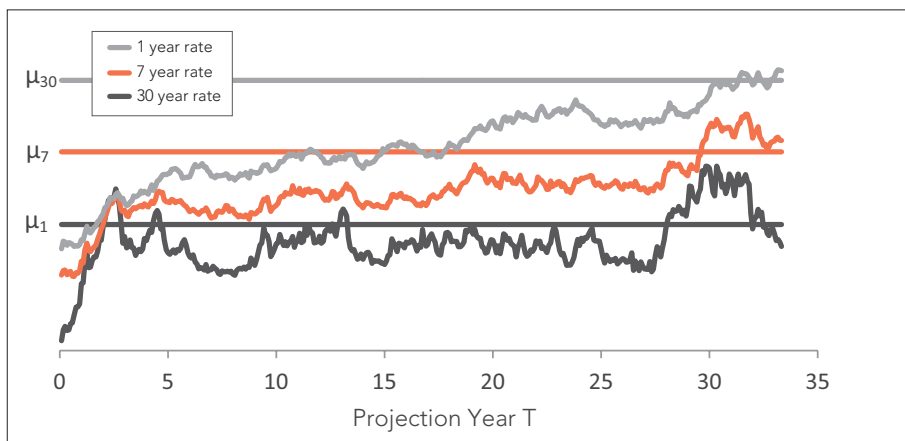
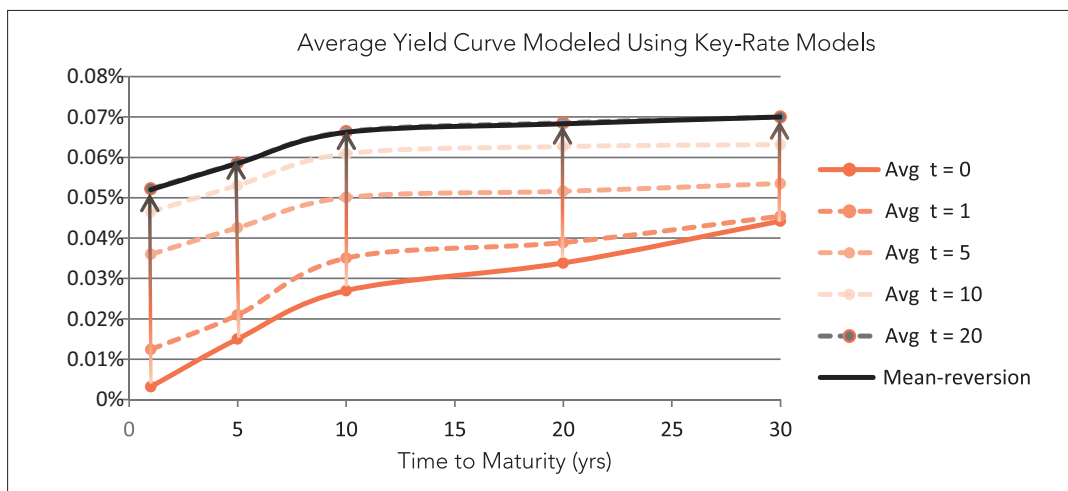


Figure 3 below illustrates the evolution of the average yield curve at different points in time across all simulated scenarios.

Figure 3: Average Evolution of the Yield Curve across Multiple Scenarios



However, the larger number of stochastic variables (and required parameters) significantly increases the difficulties—and risks—in calibrating the model, which should be considered when weighing the benefits of modeling each key rate.

FUNCTION-BASED MODELS

Since modeling each key rate may be unfeasible and introduce unwanted parameter risk, practitioners can achieve a more parsimonious modeling of yield curves by studying the functional properties of the curve itself. Instead of modeling specific points of the yield curve, function-based models focus on key latent features underlying the yield curve. Empirical studies (Pooter, 2007) have shown that changes in the level, slope and curvature of the yield curve explain most of its behavior. Changes in the level of the curve lead to parallel shifts (see Figure 4.1), changes in slope lead to flattening or steepening of the curve (see Figure 4.2), and changes in curvature lead to butterfly shifts in the yield curve (see Figure 4.3).

Figure 4: Illustration of Level, Slope and Curvature Effects

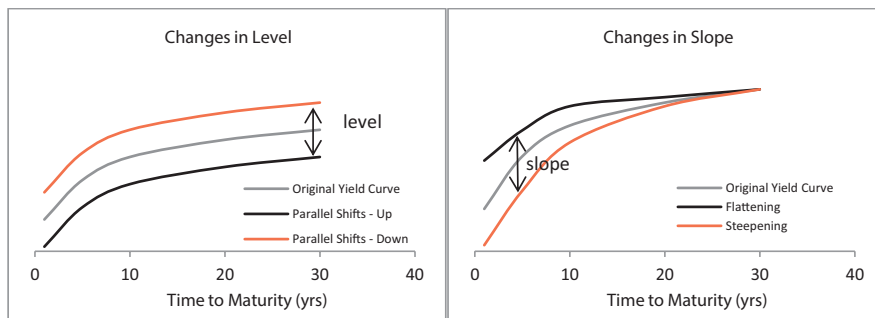


Figure 4.1

Figure 4.2

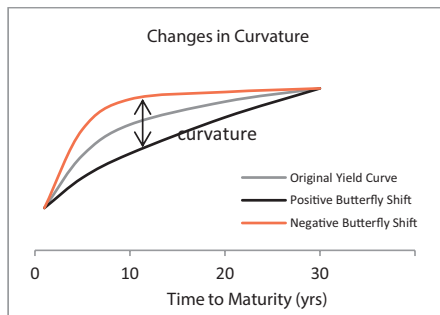


Figure 4.3

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Function-based models project the term structure of interest rates directly from the stochastic projection of these components. A common function-based definition of yield rates is provided by the Nelson-Siegel framework.

$$y_t(T) = L_t + S_t \left(\frac{1 - e^{-\lambda_t T}}{\lambda_t T} \right) + C_t \left(\frac{1 - e^{-\lambda_t T}}{\lambda_t T} - e^{-\lambda_t T} \right), \text{ where}$$

$y_t(T)$ represents the interest rate at time t with term to maturity T ,
 L_t, S_t, C_t represent factors associated with level, slope and curvature, and
 λ_t governs the exponential decay rate for the factor loadings applied to the slope and curvature factors.

A generic representation of level, slope and curvature and their associated factors is as follows:

$$\text{Level} = y_t(T_l)$$

$$\text{Slope} = y_t(T_l) - y_t(T_s)$$

$$\text{Curvature} = 2y_t(T_m) - y_t(T_l) - y_t(T_s)$$

Where $y_t(T_l)$ is the yield rate at longest maturity (e.g., 30-year); $y_t(T_m)$ is the yield rate at a medium maturity (e.g., 10-year); and $y_t(T_s)$ is the yield rate at the shortest maturity (e.g., three-month). These rates are modeled stochastically under a key rate model approach as discussed above.

This is the approach used in the American Academy of Actuaries' Interest Rate Generator (AIRG) and therefore implicitly adopted by many actuaries in the United States.

Figure 5: American Academy of Actuaries' AIRG (American Academy of Actuaries, 2010)

American Academy of Actuaries' Interest Rate Generator (AIRG)
 Probability measure: real-world

Yield interpolation method: Nelson-Siegel

L_t (level factor)—associated with the 20-yr rate

- Uses a stochastic log volatility model.
- Log long-term rate follows a mean-reverting Black-Karasinski (BK) process.
- Its mean-reversion strength varies with nominal spread.
- Log volatility of the long-term rate also follows a mean-reverting BK process.

S_t (slope factor)—associated with the difference between 20-year rate and one-year rate

- Follows an extension of the Vasicek process.
- Its volatility varies with long-term rate.
- Its mean-reversion strength varies with the log long-term rate.

C_t (curvature factor)—modeled with a constant factor

- Effectively removes any humps.
- Produces a “normal” nonlinear shape of the curve.

Note: AIRG does not model curvature stochastically and therefore does not introduce butterfly shifts of the yield curve in the simulated scenarios. The lack of these features can undermine the reliability of this model for purposes that require capturing all the plausible movements in the curve (e.g., economic capital or profit testing).

Model Types	Advantages	Disadvantages
Short-rate	<ul style="list-style-type: none"> • Simple to define and implement • Analytical tractability • Minimal computational demands 	<ul style="list-style-type: none"> • Overly simplistic view of interest rates • Can produce unintended yield rates and curve shapes
Key-rate	<ul style="list-style-type: none"> • Increased precision and control over desired outcomes • Most effective approach to achieve a refined view of tail risks for VaR and other tail metrics • Ability to flex parameters to achieve calibration criteria 	<ul style="list-style-type: none"> • Models become significantly more complex • Large number of parameters significantly intensifies data requirements and the dependency to the collected data • Requires significant judgment in setting parameters and interpreting the credibility of historical data
Function-based	<ul style="list-style-type: none"> • Focuses on the few components that explain the most (i.e., Pareto principle) • Reduced data consumption requirements in calibration 	<ul style="list-style-type: none"> • Inability to reconcile underlying dynamics with other models (e.g., arbitrage-free models) • There is a significant dependency on long-duration rate historical data, which is not always available (e.g., 30-year rates in the United States). • As noted with the Academy's generator, some variations will not be able to generate all possible forms of the yield curve.

Closing Remarks

Many actuarial liabilities show significant asymmetries with respect to interest rates; risks that are not apparent in traditional deterministic measurements. Their risk profile may be reflected not only with respect to the level of interest rates but also with the shape of the curve, the volatility and mean-reversion dynamics. Liabilities may also be long-term in nature, in which case modelers should understand the assumptions (and shortcomings) behind the projection of long-term interest rates. Policyholder behavior is commonly tied to the projected interest rates, which increases the relevance of real-world interest rate models.

Actuaries should have an understanding of the complexity and specificity of the interest rate models used given the intended purpose. The approaches discussed in this article, although not exhaustive, provide a starting point in understanding some of the primary options available.

Selecting a model is only the beginning of the process. Depending on the model selected, users will need to calibrate the parameters using a suitable set of historical data and exercise actuarial judgment in defining other model specifications.

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WHAT IS A MODEL?

By Trevor Howes

Modeling is fundamental to the work of most actuaries, and so almost every actuary sits up and takes notice when the word “modeling” appears in a meeting agenda or announcement. But it takes a bit more reading or listening to then determine if the context is really a familiar and relevant one or not. That is because there are many different types of models, with different uses, and many different meanings of the words “model” and “modeling.”

When the organizing committee for a Modeling Section formed last year and started discussing our common interests and the potential scope of activities and member recruiting for our planned section, we quickly realized that the section could well appeal to actuaries in all practice areas and even to non-actuaries such as academics researching in related fields. However, it was also clear that the actual modeling interests of all these actuaries might be quite different and thus require multiple streams of organizing action and deliverable benefits (research, webinars, articles, etc.) specific to each type of modeling.

As chairman of the new section, this presented me with two challenges to the Modeling Section. The first challenge was: Should we attempt to take on all potential areas of interest and invite all modelers to join us with the promise of immediate benefits?

When we petitioned the Society of Actuaries (SOA) to form a new section, we decided to propose a section open to all modeling interests, and that is reflected in our formal bylaws. However, we cannot promise to deliver services to every specific area of modeling interest immediately because we depend on volunteers to get anything done. So the actual delivery of benefits is up to you, our audience and our resource pool! Join us, help us to organize ourselves, contribute your experience and energy on behalf of all modelers like you, and the rest will follow. Kind of like “Field of Dreams,” right? “If you build it, he will come.” But no ghosts, please.

The second challenge arises from the first: How can we assist actuaries in communicating clearly with the section, and with each other, when discussing diverse modeling topics? This challenge prompted me to propose a project of creating a lexicon or glossary of terms related to modeling, thereby attempting to promote consistent usage of terms in the profession. This hasn’t been started but maybe we will do it and put it on our section home page on the SOA website.

Perhaps to kick-start that project, but mostly to stimulate interest from the modeling community, let me start by exploring the various meanings and usages of the word “model.”

What is a model? To quote the June 2013 Exposure Draft “Modeling” issued by the Actuarial Standards Board (ASB), a **model** is defined as “[a] representation of relationships among entities or events using statistical, financial, economic, or mathematical concepts and equations.” The Modeling Exposure Draft definition continues with the comment that “[m]odels are used to help explain a system, to study the effects of different components, and to derive estimates and guide decisions.”

A model is a representation of reality, an organization of concepts used to explain or simulate how something works; and from the actuary’s perspective, a model is a potential or actual tool used to make estimates or calculations, based on that assumed representation.

A famous quote by an English statistician, George Box, stated: “essentially, all models are wrong, but some are useful.” I have used this quote myself in a presentation some years ago and so I was amazed to hear it then attributed to me in a later presentation! No, no, it was Mr. Box in a 1987 book. And it’s still a good quote today.

A “representation of reality” cannot be 100 percent accurate and faithful to that reality, and does not need to be. But the

degree of wrongness that you can live with depends on your purpose for and your use of the model. The reason that the ASB is contemplating a formal standard of practice for modeling is that too often we lose sight of this underlying truth; when we use models to do work that others rely on, we need to be sure that those models are designed to be—and confirmed to be—well suited to their actual use, and that the degree of wrongness is acceptable.

The proposed ASB standards also discuss “**model risk**,” which is “risk of adverse consequences to output and decisions as a result of a flawed model, inappropriate inputs, or misapplication of a model,” so that we can properly assess, disclose and mitigate that risk.

The American ASB is not the only actuarial standards body to worry about models, what they are and how they are being used. The Board for Actuarial Standards (BAS) guiding the U.K. profession defined the word “model” in its Technical Actuarial Standard M—Modelling, issued in April 2010, as follows: “... a representation of some aspect of the world which is based on simplifying assumptions.” The British definition is certainly shorter and more general than the ASB version, and emphasizes the inherent approximation in a simulation.

Canadians are also thinking about including modeling specifically in standards of practice, but we haven’t released any official draft yet. In the 2014 Notice of Intent to address this issue, having studied both the U.K. and U.S. approaches, the Canadian Institute of Actuaries (CIA) proposed to define a model as “a practical representation of relationships among entities or events using statistical, financial, economic, or mathematical concepts. A model uses assumptions, data and algorithms that simplify a more complex system.” Again, there is agreement that simplification is inherent in modeling.

One commonality among the various approaches to defining modeling standards is the explicit distinction between

the three aspects of a typical model in actual use: namely, (1) a **model specification** that describes the model conceptually, using inputs and the relationships among them; (2) a **model implementation** that actually builds a working tool, usually a computer program, or platform; and (3) a **model realization**, i.e., a specific execution or run of that implementation, that produces a set of outputs based on chosen inputs. When we start talking about models in the abstract, we are usually most concerned about the specifications, the ideas and intent behind modeling we are doing. But when we concern ourselves with model governance, control, documentation, validation, etc., we need to worry about all three aspects of our models and take care to understand the implications on each and the consistency between the model specification and the other two forms.

These three aspects of a model are relevant for all types of models, whether they focus on a specific risk or are designed to contemplate products and entities actually in existence. A good example of a specific risk model would be an economic scenario generator that attempts to represent the potential future movement in yield curves, equity returns, credit spreads, prepayment rates, or any other economic variable that impacts our business. With this example we are concerned with the underlying theoretical constructs (e.g., Hull White Lognormal models), the software tools that are developed to actually generate a family of projected scenarios using that model, and the scenarios actually generated based on a given parameterization.

When we think of product, portfolio or company modeling, however, the models are typically more comprehensive, able to simulate multiple risks, driven by multiple inputs and producing a variety of outputs depending on the specific realization. Still the underlying specification is critical and must be validated along with the implementation in a software platform. In this type of model, both the specification and the implementation are almost certainly fluid and dynamic, being constantly expanded, improved, and likely corrected, to better serve changing needs and new demands

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of the entity itself. This changing reality introduces its own challenges including the need to be specific in your references: Which version of that model specification or implementation are you talking about?

Product/company models have been with us for decades, but these models are now becoming amazingly complex, detailed, powerful and flexible, exploiting the exponential pace of growth of underlying technology supporting the implementations and realizations of the models. While actuaries in the past only used these models for a deterministic, single scenario view of the entity being modeled, more and more modeling applications demand a stochastic, multiple scenario view of potential future paths.

Stochastic models have their own complications and implications on modeling actuaries that are causing new headaches. One of the interesting challenges with stochastic models is to reverse the historic trend toward more detail, accuracy and realism within the model specifications and realizations, which has been removing simplifications and

approximations wherever possible, and instead finding new simplifications and processing shortcuts that allow these stochastic models to be actually run in a reasonable time frame and at a practical, affordable cost in technology, energy consumption, human sweat, and yet still be useful. We have called this new field of research **model efficiency**.

Note that while scenario generation and sampling are perhaps the primary focus of model efficiency research for stochastic modeling, it is also useful to consider the business data input that comes into any model realization at a given starting date from the administrative system sources. The historic trend has been to start reflecting each individual policy contract and benefit within that contract in the model, individually and in great detail, due to the capabilities of modern IT infrastructure. However, new model efficiency methods are also offering the capability to dynamically and reliably construct a **compressed business model** that introduces a controllable level of approximation compared to full seriatim detail for a measurable and commensurate reduction in processing load when the purpose of the model and the resource costs of the total processing load warrant it.

Models and modeling tools are rapidly evolving, but in many cases the opportunity to take advantage of new capabilities and strengths in production applications requires an expensive and risky implementation project of uncertain duration, cost and benefit. Designing and selecting models and modeling tools, building and validating the implementations, controlling the realizations and the ongoing maintenance and enhancement of the entire modeling process are areas of challenge and growing professional interest which the Modeling Section hopes to support.

Model governance has emerged as a primary weakness in the way we actuaries have handled our modeling activities in the past. Willingly or not, we have to learn how to improve our practices and change our mindset toward our modeling tools. We need a better appreciation of model



risk that is consistent with the much more powerful, complex models being put in place and their importance to our stakeholders.

I have used the words “model” and “modeling” in the above paragraphs to refer to any actuarial application simulating company products and their risks. This includes valuation systems, both old and new, as well as those applications including asset-liability interaction that more commonly have been referred to as modeling by many actuaries.

While my comments on models above have been primarily focused on life and annuity models of insurance companies, I realize that a number of other types of modeling are currently of growing interest to the profession.

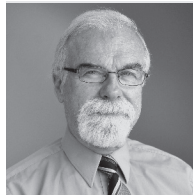
Predictive modeling is probably the best example of this, judging by the growing number of meeting sessions at professional conferences and articles on the topics in publications. Predictive modeling does not attempt to simulate a financial entity as much as explain a pattern of behavior or a risk experienced by the entity. It helps to find hidden relationships in observed experience, which can then be used to guide the development of assumptions used in projection models, or even to identify the possibility of claims fraud or ineffective underwriting and ratemaking processes.

I would classify predictive modeling as one in a number of advanced, nontraditional techniques that assist in understanding and simulating very specific modeling problems. Other examples may be **neural networks** and **agent-based modeling**. All of these nontraditional techniques have been subjects of interest by our colleagues in the Forecasting & Futurism Section, so it is not clear there is any urgent need for the Modeling Section to add to that discussion at this point in time.

I must apologize for the meandering nature of this journey through the world of actuarial models reflecting my own limited and personal perspective. However, it has allowed

me to list various meanings and uses of the word model, as well as introduce many of the possible interest areas we think the Modeling Section will be attempting to address. But not all! You readers are all potential members or already joined members, and in turn you have the power to drive our activities in any way you choose! And to hopefully to help us grow and help each other in these various modeling areas of interest.

We look forward to hearing from you. Tell us what you think a model is. And what it needs to be.



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LET'S NOT REINVENT THE WHEEL

By Brenda Perras



Actuarial modelers need to engage and leverage IT expertise in the design, maintenance, and control of actuarial models.

With the movement to principle-based reserve (PBR) models in the United States, there is increased emphasis on actuarial model risk and model governance. Not surprisingly, model risk and model governance are fast becoming hot topics in the actuarial industry as companies search for solutions to increase transparency and manage model risk for some of their key decision-making tools. Models are experiencing increased scrutiny by regulators, auditors and management.

Actuarial model systems now rival or surpass the largest pieces of commercial software in terms of complexity: vast, heterogeneous data sets are manipulated by multiple pieces of software code, both custom and off-the-shelf, and perform highly specialized calculations. Unfortunately, actuarial system design and development methodologies have not evolved at the same pace as the models themselves.

According to the 2012 SOA Actuarial Modeling Controls survey, life insurance firms do not rate themselves highly when self-reporting on most aspects of their model governance frameworks.

While there has been an increase in calls to adopt similar best practices to those already widely employed throughout the information technology (IT) field, only a few industry leaders are actually putting these practices in place. The simple answer for the rest of the industry: Rather than reinventing the wheel, reach out to the IT discipline to learn approaches developed over the past half century, and apply them in developing a rigorous systems development framework for the construction and maintenance of actuarial models.

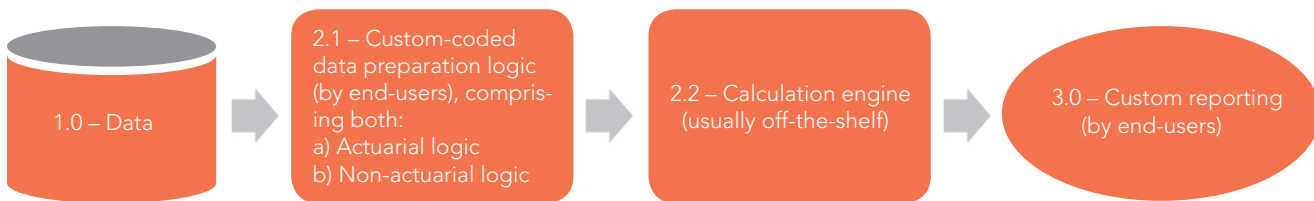
Actuarial Models Have Become IT Systems

Fifteen years ago, available computing power typically only allowed annual projections of quinquennial ages on large blocks of business, and only a few scenarios. Today, computing power is such that we can run thousands of monthly stochastic scenarios, rarely needing to use age groupings on

seriatim data for principle-based valuations. The result is much more complex actuarial models that have become IT systems unto themselves.

All but the simplest actuarial models can be classified as heterogeneous systems containing:

1. A **data component** (the input to the model);
2. Code to perform calculations:
 - 2.1. A **custom-coded component** developed by the modelers (end-users) to prepare the data for the calculation engine;
 - 2.2. The **calculation engine** itself. This is either an off-the-shelf component provided with the modeling software package, or end-user code that produces the model output;
3. **Reports** produced by custom code or custom external spreadsheets based on the model output.



While it is ideal to minimize end-user coding by actuarial modelers, in practice it is never 100 percent avoidable. The end-user component (2.1 above) is itself often complex in design and large in terms of code quantity, including things such as:

- Data manipulation and translation, e.g., consolidation, sampling, approximation and enrichment techniques;
- Product features and methodology coding;
- Assumptions setting.

The efforts to build and integrate an end-user data preparation piece (2.1), calculation engine (2.2), and custom reporting (3) have reached the scale of software development projects in their own right. Companies can therefore benefit from applying IT best practices to actuarial modeling (2.1a and 3) and shifting end-user coding of non-actuarial logic (2.1b) presently performed by actuarial professionals to software development professionals.

IT Best Practices Should Be Used Throughout the Actuarial Model Development Life Cycle

The past few years have seen extensive literature highlighting key issues with the current actuarial modeling governance landscape. For instance, over half of life insurance respondents to the SOA Actuarial Modeling Controls survey did not have a formal process to implement code changes, a way to detect unintentional model changes, or a formal code integration process.²

In addition, most life insurance companies rate their own model governance and change control practices poorly according to the SOA survey.¹

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The opportunities for applying IT approaches to model development include minimizing effort of actuarial staff in areas where non-actuarial staff would be better employed. They also include ensuring actuarial model developers follow a prescribed software development life cycle with IT guidance—just as IT practitioners benefit from well-established best practices under the “systems development life cycle” (SDLC).³

A COMMON REFRAIN: “BUT THE IT DEPT. IS TOO SLOW”

A common argument made by actuaries against getting greater IT involvement is “but whenever we request something from IT, it takes months. Their principles and governance have made them too slow.” The perception is that IT takes significantly longer to get things done than actuarial modelers would, and that in many situations, business decisions need to be rapidly analyzed and taken. The actuarial modeling industry’s shoot-from-the-hip approach is often justified by the stance that “our work is based on judgment and approximations” or the conviction that adding process will decrease speed resulting from unfamiliarity with the benefits of a controlled systems framework.

In fact, IT principles and governance do not result in lengthy turnaround times;⁴ rather, it is typically cases where accepted IT best practices have been deviated from—or a legacy of past implementations that deviated, cutting corners in the name of “saving time”—that result in slow delivery. Further, the fact that model approximations are used does not address the risk of material errors going undetected as a result of undocumented and untracked changes made to an opaque system.

One material error may simply have been offset by another in the scenario output, or not have been triggered in the particular scenario tested.

A properly designed, well-understood, well-documented, thoroughly tested, and properly maintained system will allow for rapid analysis in the form of quick configuration or input changes—at the same time mitigating the risks presently posed by models that lack transparency in their original design, testing practices and maintenance.

NO ONE-SIZE-FITS-ALL SOLUTIONS

Organizations must balance their appetite for risk and the cost of additional controls and training when selecting a modeling framework most suitable to their business needs. They should create a framework of principles and adapt them to the models and situations at hand based on the regulatory environment and company leadership’s objectives. Companies have different categories of models that require varying degrees of adherence to the principles.

Ensure Staff Have Appropriate Training and Experience

This is the low-hanging fruit: The staff coding end-user components should have sufficient systems design and programming training and skills, and likewise for staff documenting the system; presently this is often not the case. The costs associated with having these tasks completed by someone without the right skill set include:

- Increased maintenance costs;
- Increased change control risk;
- Increased model risk.

Work with IT to Identify Best Practices Most Suited to the Organization and Its Models

There is no need to develop the elements of a governance policy from scratch—model stewards can work with IT subject matter experts to get a full understanding of each, and the benefits and applicability of each to the organization’s situation. The key to efficiency with framework elements such as these is having adequately trained staff and management that support them. Building a model with

IT IS NOT ENOUGH TO SELECT AN ACTUARIAL STAFF MEMBER WHO HAS YEARS OF CODING EXPERIENCE. DO THEY HAVE APPROPRIATE TRAINING AND EXPERIENCE IN FOLLOWING THE DESIRED APPROACH FOR YOUR COMPANY'S MODEL DEVELOPMENT FRAMEWORK?

thorough documentation at the outset will make all future maintenance and changes to the model much easier and reduce cost in the medium and long terms.

The following is a non-exhaustive list of best practices that should be considered in the actuarial model development framework:

Design:

- Completing high-level and detailed designs before “jumping in and coding”;
- Ensuring algorithmic and computational efficiency;
- Designing for modularity and “white box” transparency, avoiding “spaghetti code” and hard-coded “magic numbers.” Today’s models often have data and business assumptions buried in code without documentation, resulting in a “black box” that even insiders only vaguely understand.

Coding:

- Ensuring proper parametrization—allowing rapid testing of different inputs without needing to change code in multiple places;
- Following standard coding practices—function, object and variable naming conventions; proper levels of commenting;
- Holding code reviews—while the focus often tends to be on results during model reviews, having multiple sets of eyes review code can quickly identify errors in logic, design, etc., and is a critical protocol with code changes.

Testing:

Proper testing is a critical piece of a system development framework. It is presently common practice to rely on a review of the results under a single scenario to identify actuarial model errors, and this is often focused on the change from one period to the next. This is insufficient for a number of reasons. There is much to learn from the IT field’s evolution over the past several decades here, including:

- Using a dedicated testing team—it is notoriously difficult for coders to uncover defects in their own code;
- Creating and following a test strategy, with a test

HAVING MULTIPLE SETS OF EYES REVIEW CODE CAN QUICKLY IDENTIFY ERRORS IN LOGIC, DESIGN, ETC., AND IS A CRITICAL PROTOCOL WITH CODE CHANGES.

- plan and test cases;
 - Testing too narrowly tends to miss side effects, and ad hoc testing tends to be not easily repeatable or verifiable;
- Creating and using a “test suite”—a collection of test cases that can be automatically run and re-run, with an automatic process built in to review results for correctness;
- Using regression testing: ensuring changes did not have unintended side effects, “breaking” something else in the model;
 - Testing focused only on the expected changes will often miss side-effect defects introduced—for instance, an error in a valuation model on a small but growing block—this may not be caught if the focus is on reviewing results for period-to-period changes;
- Maintaining distinct test environments separate from development and production environments to ensure the right version of code is tested and promoted when appropriate.

Change control and version control:

The need for change and version control depends on factors such as the number of modifiers or users of the model. A model created by a single developer may not need such formal controls if the developer is disciplined enough to track changes, archive each version of the model, and store the present “production” version in a specific location.

However, as soon as multiple developers are involved in a model’s creation or maintenance, and always for business-critical models, change and version controls should be employed.

CONTINUED ON PAGE 20

"REGRESSION AND STRESS TESTING SEPARATE THE PROS FROM THE AMATEURS."

—BOB LEWIS

- Change control: ensuring modifications to the production model are strictly controlled, and are only put in production after having been deployed and validated in testing environments;
- Version control: ensuring every change is tracked and reversible—including what was changed, who made the change, and when—with the ability to revert back to prior versions of the model in case of defects being uncovered after testing.

Documentation:

Documentation extends beyond simply commenting code. Much as a consumer or industrial product comes with instructions and specifications, professionally developed models require multiple layers of documentation. If documentation is skipped or cursory, the delivery of the initial model may be “faster” but the time taken to maintain it in the future increases significantly. For example, years down the road a new team may be called on to convert a valuation model. If documentation is insufficient, the new team must first spend time figuring out “how does it work today?” (akin to archaeology), before even considering how it can be converted tomorrow. The same is true when knowledge walks out the door due to staff turnover or rotation.

- High-level documentation: “box” level description and diagrams of the model’s key components and data flows;
- Process documentation: how to run the model and how to change the model;
- Detailed description of all approximations used in the model; for instance, how product features and regulatory requirements are accounted for;

- Input/output specifications: detailed descriptions of how each input data field is transformed by the end-user code to produce each output field, and the significance of each output field.

Conclusion

It is widely accepted that actuarial modeling development and governance need improvement to catch up to advances in modeling complexity and today’s regulatory environment. Rather than try to concoct best practices in isolation, the most efficient approach is to leverage expertise from the IT discipline’s long head start, ensure staff doing actuarial coding have appropriate levels of training and a framework to follow, and shift development of non-actuarial model portions to IT.

Tip:

These articles have additional tips and strategies that can help you get started:

- 1) <http://www.actuaries.org.uk/research-and-resources/documents/report-actuarial-processes-and-controls-best-practice-working-party>
- 2) <http://www.louisepryor.com/wp-content/uploads/2011/09/managing.pdf>

ENDNOTES

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- ³ "Systems Development Life Cycle." Wikipedia. Wikimedia Foundation, Inc. Retrieved March 16, 2015. http://en.wikipedia.org/wiki/Systems_development_life_cycle
- ⁴ Selig, Gad J. 2008. Implementing IT Governance: A Practical Guide to World Class IT Management Using Current & Emerging Best Practices. GPS Group, Inc. Retrieved March 16, 2015. <http://www.atilim.edu.tr/~mrehan/ISE511-Text.pdf>
- ⁵ Lewis, Bob. 2013. 10 Old-School Principles That Still Rule. Oct. 17. InfoWorld, Inc. Retrieved March 23, 2015. <http://www.infoworld.com/article/2606417/it-management/123467-10-old-school-IT-principles-that-still-apply-today.html#slide5>



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A THOUGHT ON FERMI PROBLEMS FOR ACTUARIES

By Runhuan Feng

In physics and engineering education, Fermi problems are named after the physicist Enrico Fermi who was known for his ability to make good approximate calculations with little or no actual data, involving making justified guesses and deducing bounds without using sophisticated tools. As an illustration, we can use the following example:

How many McDonald's restaurants operate in the United States?

There are 10 McDonald's in Champaign-Urbana area, which has a population of about 200,000. ***Assume the number of McDonald's scales with population.*** Since the population of the United States is 300 million, a "back-of-the-envelope" calculation estimates the number of McDonald's at 15,000. The actual number is 14,267 as of 2014.

Using a simplifying assumption is a classic feature of Fermi problems. Although the uniform assumption italicized above is not perfectly correct, the focus of the problem is to produce good and fast approximation, when exact answers are either too time-consuming to determine or too difficult to carry out.

Why Are Fermi Problems of Relevance to Actuaries?

With the increasing complexity of equity-linked products, insurance companies are facing unprecedented exposure to financial risks in addition to traditional insurance risks. Furthermore, the financial risks embedded in equity-linked insurance are often complicated by policyholder behaviors, such as in the investment fund choice/allocation, the use of reset features, and options of withdrawal and surrender benefits, etc.

The current market practice is to develop very complex models and to rely almost exclusively on Monte Carlo simulations. As the computational burden of simulation grows exponentially, many companies simply respond by adding

on more and more computing facilities. However, this is an unsustainable solution as the industry practice continues to move toward more detailed modeling and financial reporting involving more stochastic components. In the foreseeable future, the costs of such nested simulations will be prohibitive even for the most resourceful companies. The running time of simulations can still be too long for results to be delivered in a timely manner.

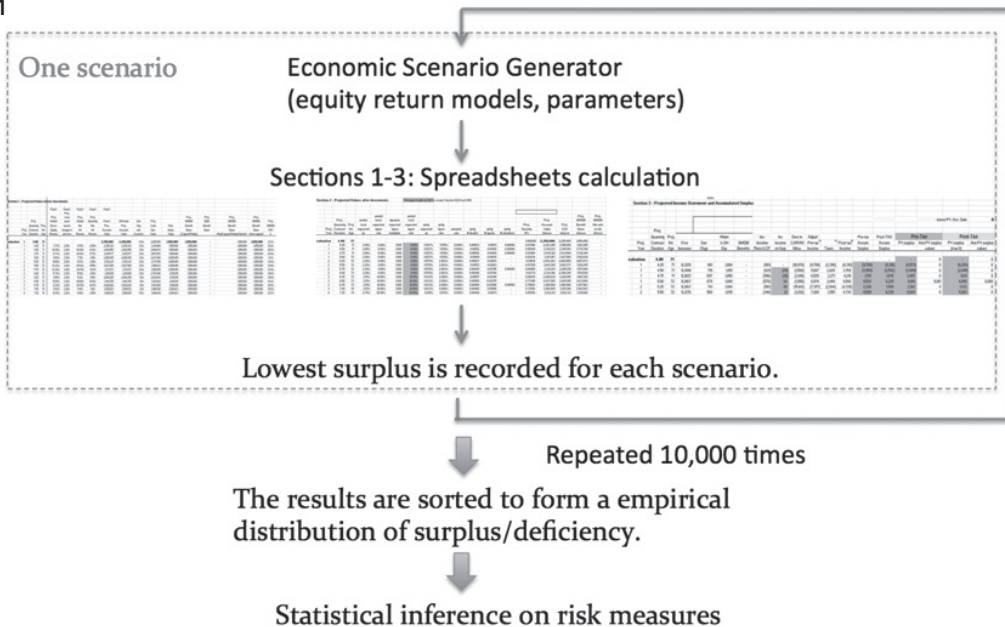
Due to the lack of alternative methods in the current practice, unscientific compromises on the scale of simulations may have to be made in order to save costs and cut running time. However, these "hash" end results may no longer be trusted or may even mislead management to wrong strategic decisions, exposing the industry to substantial model risks and systemic risks in the long term.

One potential solution to this efficiency problem is to follow the spirit of Fermi's estimate, which is to find good approximation with reasonable simplifying assumptions. For complicated problems such as the modeling of guaranteed benefits, it is unrealistic to expect a simple one-size-fits-all rule of thumb. Nevertheless, there are many computational techniques developed in the academic literature that can be used to construct a modern-day "back-of-the-envelope" calculation, which requires only modest computational efforts. We take the guaranteed minimum death benefit (GMDB) as an example to show that some deterministic techniques can be used to reach fast and efficient results.

A Simplified Example of Stochastic Reserving

While the actuarial practice on reserve calculation can be highly complex and vary company by company, we shall take a minimalistic approach here to summarize the basic principles of reserving under Actuarial Guideline (AG) 43. Policyholders make purchase payments to buy variable annuity products, which offer a selection of fund allocations at the discretion of policyholders. Then the policyholders' account values are linked to the particular equity-index/fund allocation in which they invest. The GMDB is a type of ben-

Figure 1



enefit that offers a minimum account value guarantee at the time of policyholders' death. The insurer's gross liability is in essence a put option, which pays the amount by which the guarantee base exceeds the then-current account value at the time of death.

However, the rider is much more complicated than a conventional put option, as the option is funded not by an upfront fee, but rather by a stream of mortality and expenses (M&E) charges that are fixed percentages of account values deducted on a periodic basis. The financial risk is not only on the liability side from investment volatility interacting with mortality risk, but also present on the income side. For example, in adverse economic scenarios where the equity-index/fund values are persistently low over time, the insurer's liability is high, as the account value is expected to be lower than the guarantee base. Meanwhile, the problem is exacerbated by the fact that these lower account values also generate lower-than-expected fee income.

Valuation actuaries typically quantify and assess the above-mentioned losses by running spreadsheet calculations through simulated scenarios of fund performances, or using other software that performs much the same procedure. The calculations can be summarized as illustrated in figure 1.

1. A set of economic scenarios is generated to reflect a company or regulator's expectation of the variability of economic outcomes over the lifetime of the policies.
2. Under each scenario, policyholders' account values are projected according to certain accounting conventions and model assumptions. Spreadsheets are used to compute the outcomes of profitability measures such as the present value (PV) of accumulated surplus/deficits.
3. Combining all scenarios, the profitability measures are then ranked to form an empirical distribution.
4. The reserves/risk capitals are then determined by an estimation of a chosen risk metric, such as value at risk or conditional tail expectation (CTE), applied to the distribution of profitability measures.

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While the simulation procedure is easy to implement and works generally for all product designs, one should bear in mind that simulation-based techniques are statistical procedures for which estimation errors are unavoidable.

It is a well-known fact that the sampling error of Monte Carlo simulation for averages in general goes down by $1/\sqrt{n}$ as the sample size n increases. In other words, the sample size has to increase a hundredfold in order for the estimate to improve one significant digit. What makes it more challenging is that practitioners are typically interested in sensitivity measures, such as sensitivity of profitability to interest rate shocks, Greeks for hedging, etc. A small sampling error of the profitability measures can lead to a huge relative error of sensitivity measures.

Analytical Alternative Solutions

There is typically a trade-off between efficiency and generality of computational methods. In contrast with simulations, which can be used for all models despite their inefficiency at times, analytical approximations are only efficient under specific model assumptions, as less computation is achieved through careful use of analytical properties of the underlying model.

An example would be the celebrated Black-Scholes option pricing formula. It is developed under the normality assumption of log equity returns and flat term structure of interest rates, which were all proved wrong by empirical evidence. Yet it is widely accepted for pricing and hedging with some fine-tuning of the volatility parameter in the formula. Some practitioners have described this as “the wrong number in the wrong formula to get the right price.”

How is one formula based on seemingly unrealistic assumptions preferred more than simulations under more complex and realistic models on interest rates,

equity prices, volatilities, etc.? This goes back to the idea of Fermi’s problem, which is to provide good estimates with a minimal amount of computational effort. While trying not to get into mathematical details, we can point out some quantitative tools developed in the literature that have not yet gained much attention in the practitioners’ world.

Differential Equation Methods

The basic step of the above-mentioned spreadsheet calculation is to determine incremental changes in surplus for each valuation period.

$$\text{Changes in surplus} = \text{Fee income} + \text{Interest on surplus} - \text{Benefit payments} - \text{Expenses}$$

From a mathematical point of view, the spreadsheet calculations are essentially numerical algorithms based on difference equations. Each row in a spreadsheet corresponds to a recursive formula, aka a first-order difference equation. Such equations typically do have explicit solutions that can be represented in terms of the sample paths of account values.

Here is an example of the PV of accumulated deficiency for a GMDB rider, which was derived in Feng et al. (2015) based on a practitioner’s spreadsheet calculation. The policy lasts for T years and fees are collected n times each year as a fixed percentage m_d of account value $F_t (0 \leq t \leq T)$. The initial guarantee base is G with a rollup rate δ and the interest rate is r per annum. The symbols P and Q are standard actuarial notation for survival and mortality rates. In this formula for the PV of accumulated deficiency, the first term is the PV of all benefit payments (put options) over all periods up to maturity and the second term is the PV of all fee income. For simplicity, we ignore non-asset-value-based expenses.

$$L = \underbrace{\sum_{j=1}^{nT} e^{-rj/n} {}_{(j-1)/n}p_x {}_{1/n}q_{x+(j-1)/n} (Ge^{\delta j/n} - F_{j/n})_+}_{\text{Benefit Outgo (death benefit of put options)}} - \underbrace{\left(\frac{1}{n}\right) \sum_{j=0}^{nT-1} e^{-rj/n} {}_{j/n}p_x m_d F_{j/n}}_{\text{Fee Income (fixed percentages of acct values)}}$$

Bear in mind that the basic principle of AG43 reserving is to determine the 70%-CTE of the PV of accumulated deficiency.

$$\text{CTE}_p(L) := \mathbb{E}[L | L > \text{VaR}_p] \quad \text{VaR}_p(L) := \inf\{y : \mathbb{P}[L \leq y] \geq p\}$$

There are several advantages of using the above explicit solution over simply running spreadsheet calculations.

- (1) The cash flow structure becomes more apparent and it is easier to identify scenarios under which outgoes exceed incomes.
- (2) The above solution is an additive functional of the underlying fund values $F_t(0 \leq t \leq T)$. The tail distribution of an additive functional can be determined or approximated by analytical methods. See below for an example of comonotonic approximations.
- (3) The computational complexity of summation is typically less than that of (spreadsheet) recursion.

If we shrink the valuation period to zero, then the difference equation (corresponding to spreadsheet calculation) goes to its limit—a differential equation. In other words, it can be treated as if the fees are taken continuously and benefits are payable immediately upon death. Even though the assumptions are not realistic, the continuous-time approximation can be very close to the discrete-time true value, just as we often use continuously paying annuities to approximate monthly paying annuities for pricing annuities-certain. There are many well-established numerical methods for solving ordinary/partial differential equations, such as finite-difference, finite-elements, etc. An example of such approximations can be found in Feng (2014).

Furthermore, there are many cases of guaranteed benefits, for which analytical formulas for risk-neutral valuation and risk measures are available under certain model assumptions. For example, we recently worked out closed-form pricing formulas for guaranteed minimum withdrawal benefit (GMWB) and guaranteed lifetime withdrawal benefit (GLWB). They can be used to remove inner components of nested simulation for financial reporting on these complex riders

Comonotonic Approximations

Observe that the accumulated deficiency L is essentially determined by a weighted sum of fund values at various time points. The adverse scenarios of the accumulated deficiency are results of tail events of the weighted sum. Instead of going through all the trouble with simulating or analyzing the complex dependency among various fund values, one can study the extreme events by looking at a single random variable that characterizes the dependency.

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Here is a simplified description of the mathematics behind this. For any continuous random vector (X_1, X_2, \dots, X_n) , the sum of the random vector dominates the sum of the vector conditioned on another random variable Λ in the sense of the so-called “convex order.”

$$S^l := \sum_{i=1}^n \mathbb{E}(X_i|\Lambda) \leq_{cx} S := \sum_{i=1}^n X_i$$

The left-hand-side single random variable is known as a comonotonic bound. Ignoring the technical definition of convex order, the most important consequence of such a relation is that the CTEs of the two sums are also ordered regardless of the choice of Λ .

$$\text{CTE}_p(S^l) \leq \text{CTE}_p(S)$$

Note, however, that the computation of risk measures for the single random variable S^l is much easier than that of the sum of highly dependent random variables, S . The goal of comonotonic approximation is to find the best choice (optimization) of Λ so that the CTE of S^l is as close as possible to S .

Perhaps the best way to visualize a comonotonic approximation for the reserving exercise is to think of the fund values at various time points as competing horses in a chariot. Even though the horses have different speeds individually, their average performance can be characterized by the speed of the chariot. Keeping track of the speed of the chariot is

much easier than averaging the individual measurements of the horses at all times. In a similar way, comonotonic bounds usually provide much more efficient solutions with only small compromises on accuracy, in the same spirit as in Fermi problems.

A recent work in Feng et al. (2015) gives several examples where comonotonic approximations can be used to estimate various risk measures of the PV of accumulated deficiencies. Here we reproduced the table for the comparison of efficiency among comonotonic approximations (labeled “optimizations”) and Monte Carlo simulations (standard deviations in brackets). If we only need accuracy up to three decimal places, then the approximations (labeled “75% reduced”) can be about 30 times faster than simulations based on a sample size of 1 million scenarios. To achieve accuracy up to four decimal places, the approximation (labeled “50% reduced”) runs roughly 750 times faster than simulations based on the sample size of 100 million.

Method	VaR _{0.9}	CTE _{0.9}	Time (secs)
Nonlinear optimization	0.03035	0.06126	69.97
Nonlinear optimization (50% reduced)	0.03031	0.06123	30.50
Nonlinear optimization (75% reduced)	0.03018	0.06111	7.83
Monte Carlo (1 million)	0.03059 (0.00013)	0.06137 (0.00010)	226.16
Monte Carlo (100 millions)	0.03035 (0.00002)	0.06128 (0.00002)	22602.80



Note

Computational Risk Management Lab is a research lab, housed in the Department of Mathematics at the University of Illinois, whose mission is to develop and implement efficient computational solutions for risk management problems. We invite practitioners to make use of our state-of-art research resources and to provide us with challenging research questions. Interested practitioners should contact rfeng@illinois.edu for further information.

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NOTEWORTHY READS

By Tim Cardinal

Noteworthy Reads has a simple aim—to inform our readers of good sources on models; specifically, to point out sources beyond what we can include in a newsletter. *Noteworthy Reads* will select a few and provide mini-reviews.

Models are a central component of the actuarial skill set, as they are foundational to risk management and principle-based reserves, capital and accounting. Models will become even more important in the future as they take a greater role in solvency and financial reporting.

As a member of Society of Actuaries (SOA) exam curriculum committees, I am exposed to and peruse a wide range of foundational and emerging topics related to models. Modeling topics are being added throughout the SOA exam syllabus—from model methodologies to assumptions, validation, governance, efficiencies, applications, and more.

So, onto our first installment of *Noteworthy Reads*.

Heavy Models, Light Models and Proxy Models

by *The Proxy Model Working Party*
Institute and Faculty of Actuaries, 2014

Link: <http://www.actuaries.org.uk/research-and-resources/documents/heavy-models-light-models-and-proxy-models-working-paper> (available for free)

I place this in the must-read category. A fundamental issue we are/will be facing is meeting increased demands placed on models in Own Risk and Solvency Assessment (ORSA), Solvency II, VM-20, and other principle-based approaches with deadlines—that is, achieving speed, fidelity and usability simultaneously.

This outstanding paper explores types of proxy models available to practitioners, the options available in the design and implementation of a model, and the potential impact of

the choices made. Four specific proxy models are discussed in greater detail: replicating polynomials, radial basis functions, replicating portfolios, and commutation functions. Two of these are the subject of a case study of a fairly simple life insurance liability.

Monte Carlo Methods and Models in Finance and Insurance

by *Ralf Korn, Elke Korn and Gerald Kroisandt*
CRC Press, 2010

Link: <http://www.crcpress.com/product/isbn/9781420076189>

Sometimes we take for granted basic model integrity or that one model works for everything. Are those random numbers really random? Are all Monte Carlo methods equal? Does the context matter? How do we go from the theoretical underpinnings of stochastic processes to practice—using and applying stochastic models in our daily work? I like this book as it provides “what’s, how’s and why’s.” It strikes a good balance between accessibility and rigor without going overboard on the theoretical mathematics.

Measurement and Modelling of Dependencies in Economic Capital

By *R.A. Shaw, A.D. Smith and G.S. Spivak*
British Actuarial Journal, Vol. 16, Issue 3, September 2011

Link: <http://www.actuaries.org.uk/research-and-resources/documents/measurement-and-modelling-dependencies-economic-capital> (available for free)

Understanding and modeling dependent relationships are a challenge. This paper explores different approaches to modeling dependencies ranging from basics to copulas to causal models with feedback loops. The authors posit that “the economic capital model can be seen as a combination of two key components, the marginal risk distribution of each risk and the aggregation methodology which combines these into a single aggregate distribution or capital number. This paper is concerned with the aggregation part, the

methods and assumptions employed and the issues arising.”

How to Measure Anything: Finding the Value of Intangibles in Business

By Douglas W. Hubbard

Wiley; 3rd edition, 2014

Link: <http://www.amazon.com/How-Measure-Anything-Intangibles-Business/dp/1118539273/>

Businesses rely on metrics. One of the central tenets of risk management is measurement. Sometimes our nose is too close to the picture, and it is good to take a step back and look from afar. Hubbard, through numerous examples, steps back to explore improving the value of decision-making information. Are you asking the right questions? Do you have the right data? Are you using the right metrics? Hubbard provides a guide on how to define, determine and use metrics in a world of risk and uncertainty. One of Hubbard’s examples includes Fermi problems as described in Runhuan Feng’s article, “A Thought on Fermi Problems for Actuaries,” in this issue of the newsletter.

SOA, ACLI, CIA and IFA Research

I suggest you bookmark and make a habit of window shopping these and other actuarial organization sites. There has been a proliferation of papers related to all aspects of models. For example, Canadian Institute of Actuaries (CIA) papers include *Risk Assessment Models* and *Use of Stochastic Techniques to Value Actuarial Liabilities under Canadian GAAP*. Just a few of the SOA papers published in 2014 related to experience and assumptions include:

- Modeling of Policyholder Behavior for Life and Annuity Products
- Report on the Lapse and Mortality Experience of Post-Level Premium Period Term Plans
- Variable Annuity Guaranteed Living Benefits Utilization.

These were all good reads but only a few of the many. *Noteworthy Reads* suggests checking out the Research & Publications pages at:

<https://soa.org>

<http://www.cia-ica.ca>

<https://www.acli.com>

<http://www.actuaries.org.uk>

Future Noteworthy Reads

Future *Reads* will come from our universe of readers. If you have a suggestion related to models, please email us at tcardinal@actuarialcompass.com. Send your nomination to *The Modeling Platform* editors. We will compile and select suggestions (with due acknowledgment).



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