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Target Volatility Fund: **An Effective Risk Management Tool for VA?**

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In the past few years, many institutional investors and hedge and mutual funds managers have embraced an investment strategy known as volatility targeting, aimed at maintaining a stable level of volatility for the whole portfolio. Compared to the traditional fixed-allocation strategy such as the popular 70 percent equity and 30 percent bond strategy (70/30), a target volatility fund moves money from risky assets (i.e., stocks) to safer assets (i.e., bonds), or vice versa, to achieve the right level of volatility for the investment.

This concept is behind many investment buzz words related to controlling risk such as risk allocation, risk budgeting, or risk parity. It relies on two basic empirical facts about the market: 1) market volatility and return have strong negative correlation; and, 2) high or low volatility tends to cluster together for a sustained period of time. Recent market history has reinforced these empirical facts such as with the highly volatile market crash of late 2008 and the calm period of double-digit returns of the late 1990s. During these two periods, we can clearly observe that spans of high or low volatility tend to persist for a sustained stretch of time.

Over the past year, many insurance companies have added target volatility funds to their variable annuity (VA) fund lineup. Unlike typical mutual fund investors, VA policyholders have a very long investment horizon. Is volatility targeting a better strategy than the traditional fixed-allocation strategy over the long term? Unfortunately, the history of these funds is too short to answer this question. A literature search of academic and industry research does, however, provide positive answers.

In research published by the EDHEC Risk Institute, Stoyanov (2011) used the Heston model calibrated to long-term equity market data to show that volatility targeting reduces downside exposure and improves upside potential compared to a fixed-allocation strategy for long-term investors. Busse (1999) empirically found positive correlation between mutual fund returns and their volatility timing activity. Other academic research papers based on various volatility forecasting models have also shown positive economic value for volatility timing.

However, there are a few important pieces missing from the existing research:

1. The reseachers assume the instantaneous volatility of the market is known, and they use the continuous market assumption. In reality, fund managers will make allocation decisions based on a combination of historical realized volatility and market-observed implied volatility. In other words, fund managers react to the market with a lag. Thus, these funds are vulnerable to a sudden market movement, or jump risk. The market has experienced sudden jumps, such as the 1987 crash, the 9/11 terrorist attack, or the more recent S&P downgrade of the U.S. government. With the looming debt crisis in the eurozone and ever-increasing geopolitical risks, sudden market jumps look more likely than ever. How well will volatility targeting hold in this environment? Existing research has not provided an answer.
2. Some funds use leverage to enhance returns, but leveraging can greatly amplify the jump risk. Yet existing studies have not been focusing on the impact of leveraging on the risk profile.
3. Finally, the existing research has been focusing on

the ultimate wealth accumulation for the investors. However, most VAs offer step-ups, roll-ups, or other bonuses. The increased wealth of the policyholders may not reduce the risk of the VA writers because the guarantees may also increase. Is volatility targeting effective in reducing the risk for VA writers as well as investors?

Contrary to existing research that favors volatility targeting over fixed allocation, this article will show that because of the existence of market jump risk, target volatility strategy does not necessarily offer an improved risk profile for VA writers. Investors may favor one strategy over the other based on their own evaluation of the jump risk. Further, target volatility coupled with leveraging can significantly increase tail risk for insurance companies.

The Approach

The analysis uses a model of one VA policy with a lifetime guaranteed minimum withdrawal benefit (GMWB) rider, which will start withdrawing the guaranteed amount after 10 years. The policy assumptions are shown in the chart below.

There are only two asset classes, equity and cash. Equity returns are modeled stochastically. The cash is assumed to return 2 percent per annum with zero volatility.

Two investment strategies are modeled. The fixed allocation strategy invests 70 percent in equity and 30 percent in cash. The investment is adjusted monthly so that it always maintains a 70 percent weighting in equity. The target volatility strategy rebalances monthly so that the trailing six months realized volatility of the whole portfolio is as close to a long-term target as possible.

The long-term volatility target is set to be equal to the long-term volatility of the 70/30 strategy so that the strategies can be compared directly.

Both account value (AV) and the guaranteed withdrawal balance (GWB) are projected at the end of years 10 and 15. We will simulate 1,000 equity paths and compare the distribution of AV and in the moneyness (ITM), defined as AV/GWB of the two strategies. The AV represents the accumulated wealth of the policyholder while the ITM reflects the risk of the VA writer.

Issue age	Gender	Initial Premium	Step up	Roll up bonus	Guaranteed withdrawal rate at year 11	Rider charge (bps)	M&E fee (bps)	Fund expenses (bps)
60	Female	$100,000	Annual	7%	5%	90	130	90

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> “ Volatility tends to cluster together. It tends to remain high or low for a sustained period of time. ”

The Equity Model

There has been strong empirical evidence that suggests against the normality hypothesis used in traditional market theory. In fact, market participants have observed:

- Volatility tends to cluster together. It tends to remain high or low for a sustained period of time;
- Return distributions are fat-tailed and skewed; and
- Current price changes depend on past price changes.

A model that reflects these characteristics in continuous time was proposed by Heston (1993). Its volatility follows a square root stochastic process with mean reversion.

However, equity market returns can experience jumps, a phenomenon that challenges the continuous assumption of the Heston model and other models based on a smooth market. An extension to the Heston model introduces a jump term to the stock returns. It is often called the stochastic volatility jump diffusion (SVJD) model, or the Heston model with jumps. The jump's occurrence is controlled by a Poisson process and its size is log-normally distributed with a downward bias. The stock distributions modeled by SVJD are not only fat-tailed with clustering volatility, but they are also more skewed with strong asymmetry in the upside and downside potential. SVJD is therefore used widely in studying dynamic asset allocation for long-term investors.

Due to the above considerations, this exercise uses SVJD to model equity returns. However, instead of directly calibrating the model to historical data, the analysis uses a phased approach. First, the Heston model calibrated to historical S&P monthly total returns over the past 10 years was used to validate the conclusions of some of the existing research on target volatility. Then a jump term of varying size and frequency was added to the return to study the impact of jumps on the risk profile. Finally, the model allowed a certain amount of leverage in the fund, to study the impact of jumps on leveraged investment.

Phase 1: The Calibrated Heston Model

In the Heston model, the equity return dynamic is described by the following stochastic differential equations (SDEs):

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_s$$

$$dv_t = \pi(\theta - v_t)dt + \sigma\sqrt{v_t} dW_v$$

$$dW_s dW_v = \rho dt$$

where ρ is the correlation coefficient of equity return and volatility, $\pi > 0$ is the speed of mean reversion, $\theta > 0$ is the long-run level and the unconditional mean of v_t, and $\sigma > 0$ is the volatility of volatility.

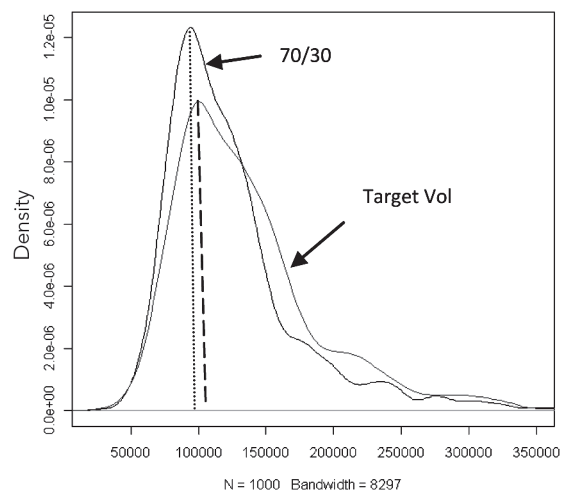
Stoyanov (2011) calibrated the Heston model to the monthly S&P 500 return for February 2002 to June 2010. The following are the parameter estimates:

π	θ	σ	ρ	μ
0.2145	0.0314	0.0955	-0.65	0.0072

The model simulated 1,000 equity paths and projected the AV along each path under the 70/30 and target volatility strategies. The equity weighting is capped at 90 percent in the target volatility case to prevent leveraging at this phase. It will be increased in phase 3 when impact of leveraging will be tested. Similar to Stoyanov, the target volatility strategy results in a better risk profile than the fixed 70/30 strategy. Figure 1 shows the comparison of the densities of the two distributions. The AV and ITM distribution produced by the target volatility strategy tilt to the right compared to those produced by the 70/30 strategy. It limits the left tail but at the same time increases the upside potential on the right side. And the target volatility distribution always peaks at a point (on the X-axis) bigger than the

70/30 does. This suggests that under the Heston model, target volatility is a better strategy not only for the investors (AV) but also for the VA writers (ITM). But will this conclusion hold under jump risk?

Distribution of AV end of year 10: Heston



Distribution of ITM end of year 10: Heston

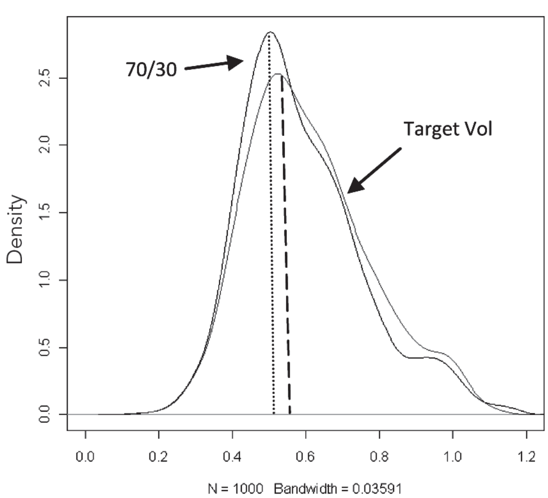


Figure 1: Distribution produced by Heston model. Density of distribution using target volatility compared to density of distribution of 70/30 strategy.

Phase 2: Layering on Jumps

The SDE of the SVDJ can be specified as follows:

$$dS_t = (\mu - \lambda g)S_t dt + \sqrt{v_t} S_t dt + (e^q - 1)dQ$$

$$dv_t = \pi(\theta - v_t)dt + \sigma\sqrt{v_t}dW_v$$

$$dW_s dW_v = \rho dt$$

Compared with the Heston model, the process of v_t is exactly the same. But the equity return process has an extra jump term. dQ is a Poisson process with intensity λ. The probability of having n jumps over the investment horizon τ is $e^{-\lambda\tau}\frac{(\lambda\tau)^n}{n!}$. The term $g = E(e^q - 1)$ captures the mean percentage jump conditional on the jump happening; q is assumed to be normally distributed with $N(\mu_q, {\sigma_q}^2)$. The drift term of the return process of the Heston model is adjusted by $\lambda.g$ so that the overall average return stays the same.

Wu (2003) has calibrated the jump parameters specified in Merton's model to the S&P 500 index using data from the period of 1962–97. Although the model used by Wu is different and the data used in the calibration is not recent, the jump size and intensity are nevertheless indicative, and the author will use them as a starting point. The size and intensity of the jump term will be gradually increased to study the impact of this risk. The following are Wu's jump parameters:

λ	μ_q	σ_q
0.45	-0.044	0.088

One thousand equity paths were simulated using the Monte Carlo method based on the SVJD model. The target volatility and 70/30 strategies were also simulated along each path. Figure 2 (see pg. 26, left) shows one equity path, the trailing six-month realized volatility and the equity weighting of the portfolio assuming a target volatility strategy. The equity weighting is capped at 90 percent to prevent leveraging at this time.

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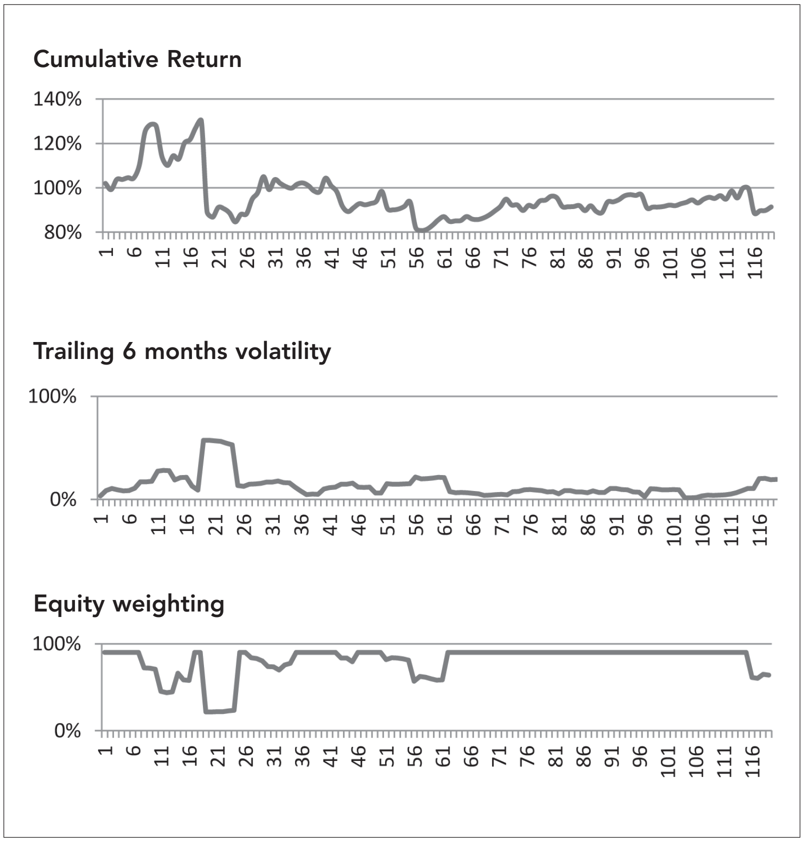


Figure 2: One simulated equity path based on the SVJD model for the first 120 months

Figure 3 (right) shows the projected AV and ITM at the end of 10 years along these 1,000 paths. Although target volatility thickens the right side of the distribution, unfortunately it also fattens the left side. The distributions of both strategies also peak at around the same point on the X axis. In fact, volatility targeting seems to simply flatten the distribution, which is an indication of increased risk.

Intuitively, because the rebalancing of funds is a reaction to market movement, it always lags behind in terms of adjusting to the right equity weighting. When the market is smooth, it allows the strategy to catch up and adjust to the right equity proportion. But when the market experiences a significant dislocation, the portfolio suffers a big loss or misses a big market run up before it has time to adjust. Depending on the size and intensity of the jumps, target volatility strategy may not produce a better risk-and-return relationship than the traditional fixed-allocation strategy.

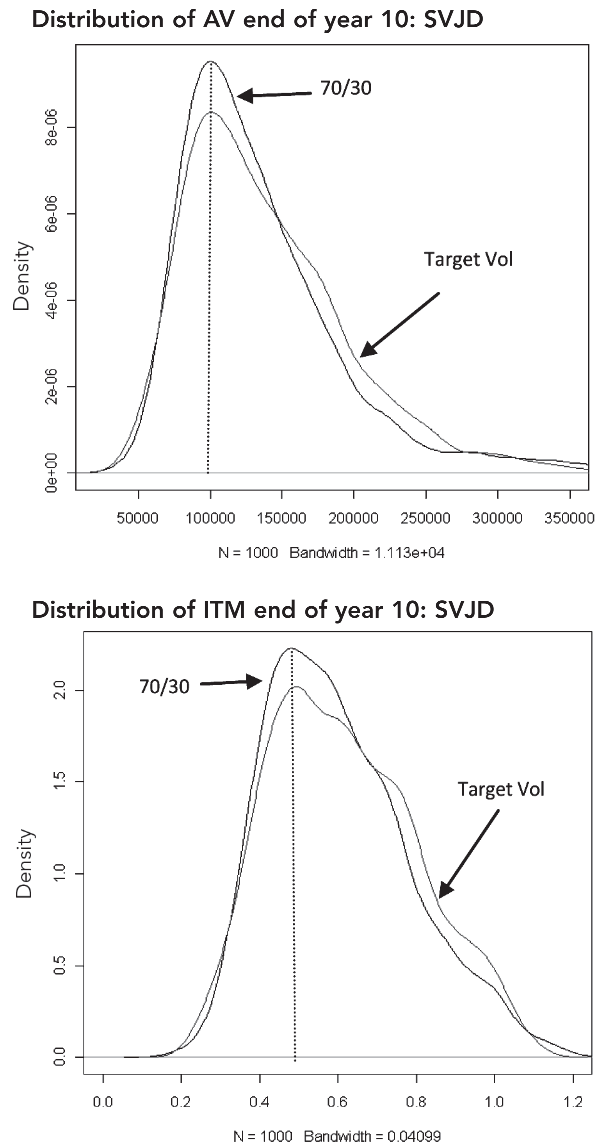


Figure 3: Distributions produced by SVJD model. Density of distribution using target volatility compared to density of distribution of 70/30 strategy.

Phase 3: Impact of Fund Leveraging with Jump Risk

Some investment strategies based on target volatility rely on leveraging to enhance returns. Does it really work? The projection of AV and ITM for 10 years was repeated, but this time, the equity weighting was allowed to go as high as 200 percent of the portfolio, essentially borrowing cash to purchase equity when volatility is significantly below the target. The jump frequency and size were also doubled to mimic somewhat extreme conditions. Figure 4 (right) displays the result.

The distributions of the AV and ITM are significantly flattened by the target volatility strategy. Both the left and the right tails are undoubtedly thickened, more so in the case of the ITM which reflects the risk to the insurance company. This result clearly demonstrates the danger of leveraging in the presence of market jump risk. Although leveraging can sometimes produce a higher mean return, the distribution is significantly widened, suggesting greater variance and risk.

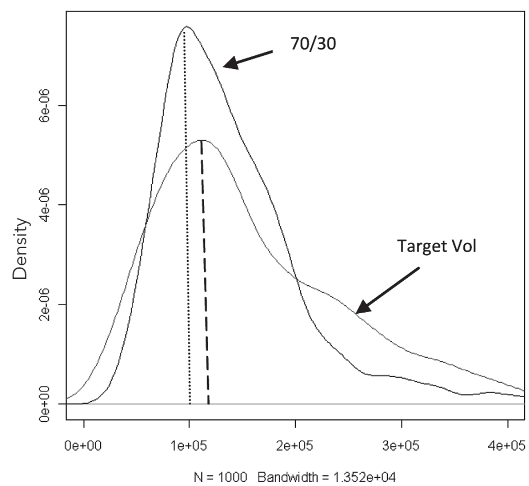
Final Thoughts

The results of this analysis, using the SVJD model, suggest that volatility targeting may not be a superior strategy to traditional fixed allocation in terms of risk-and-return profile, contrary to some existing research. The main reason is market jump risk. Big market surprises will sometimes lead to underperformance for a volatility targeting fund since it cannot react instantly. In addition, if leveraging is also allowed in such a fund, the jump risk will be amplified.

The VA writers should not automatically regard the volatility targeting strategy as a risk management tool. When offering funds with such a strategy under the GMxBs, companies should ensure that leveraging is not used and hedging strategies are in place within the fund to deal with the jump risk. More importantly, in analyzing the risks of any fund with some variation of the target volatility strategy, the equity scenarios generated by a process with the continuous market assumption may not be adequate any more. Companies should adopt a scenario-generation model that properly captures the jump risk, such as the SVJD.

The above analysis assumes a jump frequency and size consistent with the historical market data. One can have a different view with regard to these parameters and arrive at a different conclusion. Particularly, one can justify investing in a target volatility fund if he or she believes the equity market will have fewer surprises in the future.

Distribution of AV end of year 10: SVJD/Leveraged



Distribution of ITM end of year 10: SVJD/Leveraged

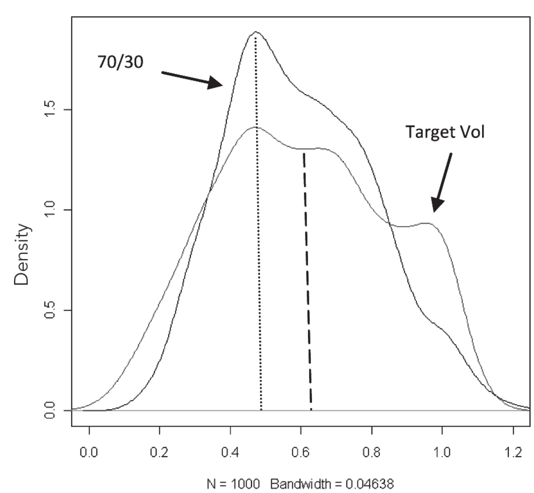


Figure 4: Distributions produced by SVJD model allowing fund leverage. Density of distribution using target volatility compared to density of distribution of 70/30 strategy.

Acknowledgement

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