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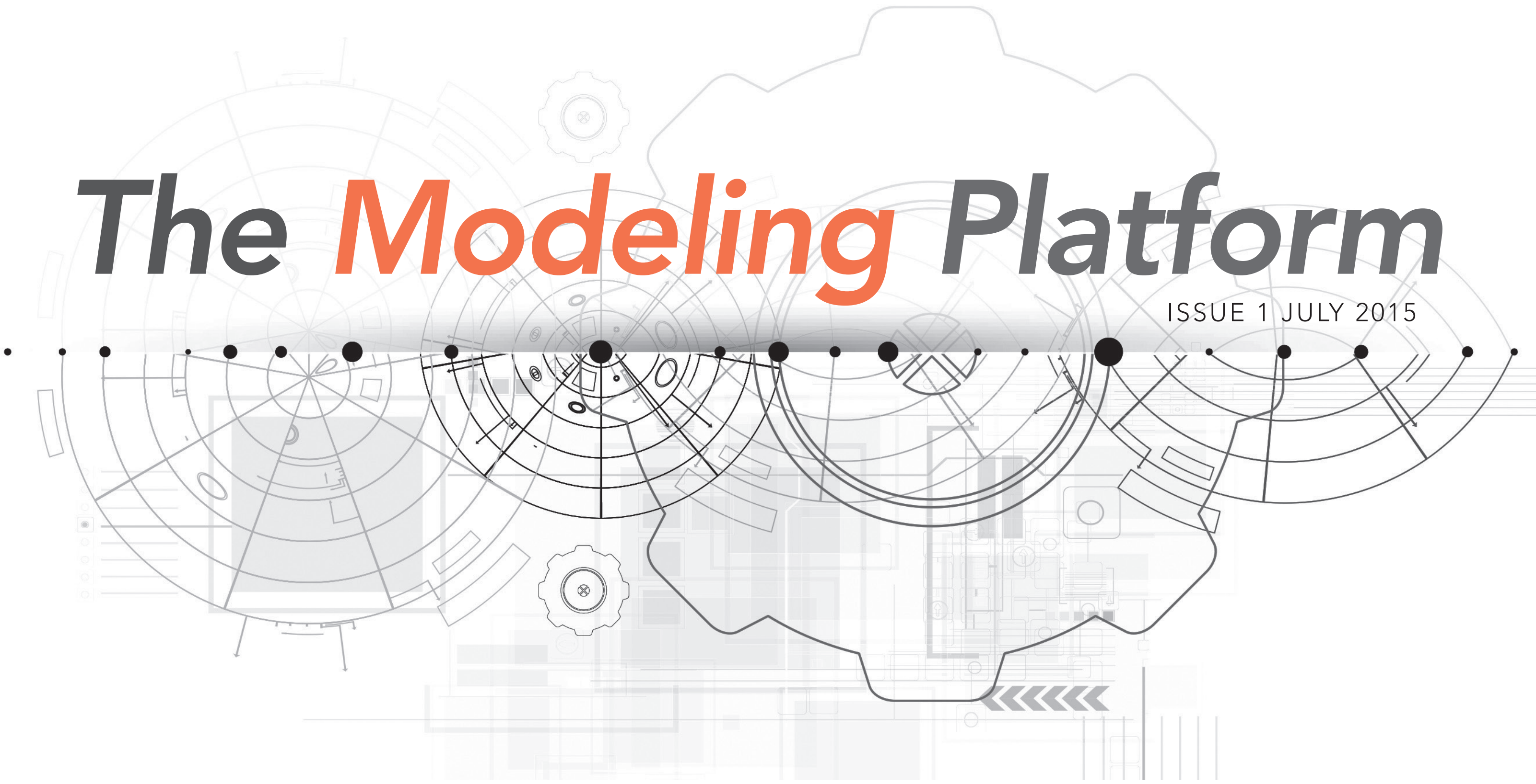

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REAL-WORLD INTEREST RATE MODELS AND CURRENT PRACTICES

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As actuaries we often find ourselves focusing on the internal consistency of our models and examining whether they are well calibrated and produce output in line with observable experience and intended uses. However, we may neglect the risks assumed by relying on a particular algorithm or model structure (i.e., model risk). This is particularly critical for real-world interest rate scenario generators, which produce stochastic interest rates under a realistic probability measure. In this article we review some common uses of real-world interest rate scenario generators in the life insurance industry and explore three fundamentally different approaches to building such generators.

Common Uses of Real-World Interest Rate Models

The use of risk-neutral and real-world interest rate models has grown substantially in the last decade as life and annuity products have become more complex. The need for risk-adjusted management information has grown, and accounting and regulatory frameworks have become more sophisticated, demanding principle-based views of risk and valuation. Here are some common uses for real-world interest rate models:

- Financial reporting
    - **U.S. statutory valuation, Actuarial Guideline (AG) 43**—This is a valuation standard for variable annuities with guaranteed benefits. Some companies use stochastic interest rates for the valuation.

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- **U.S. regulatory capital requirements**—C-3 Phase 1 prescribes an interest rate generator. C-3 Phase 2 requirements are similar to AG43.
- **U.S. GAAP, SOP 03-1**—Requires a valuation under a "range of scenarios" covering risks applicable to that business. For products with interest rate risk exposure, this may include the use of real-world interest rate scenarios.
- **Canadian Asset Liability Method**—May be done using stochastic real-world interest rate scenarios. The guidance specifies general requirements that would generally be covered using key-rate models or function-based models (discussed below), as well as calibration criteria to ensure that the scenarios are adequately adverse.
- **Asset adequacy testing**—This may be done in a variety of ways, and commonly includes a stochastic real-world valuation to determine whether assets are sufficient to support the in-force liabilities under moderately adverse economic conditions. This often supplements testing under the "New York 7" scenarios and other deterministic scenarios.

• Other applications that are often modeled using real-world scenarios include economic capital, pricing and embedded value.

A real-world interest rate stochastic model not only reflects a "best-estimate" assumption for future interest rates, but also a best-estimate view of their fluctuation. Best-estimate assumptions are also used for a variety of financial reporting and other purposes beyond those discussed above.

Constructing Real-World Interest Rate Models

With many approaches available to construct real-world interest rate models, it is easy to struggle trying to balance the different needs and select and calibrate a suitable model. In the discussion below we will walk through some basic categories of interest rate models and the considerations in selecting a model.

Most models will fall under one of the following categories:

- Short-rate models
- Key-rate models
- Function-based models.

SHORT-RATE MODELS

Single short-rate models refer to the commonly discussed equilibrium models, such as Vasicek, Cox-Ingersoll-Ross (CIR), Brennan-Schwartz and Black-Karasinski. These models define the instantaneous interest rate (i.e., the short rate) using stochastic differential equations:

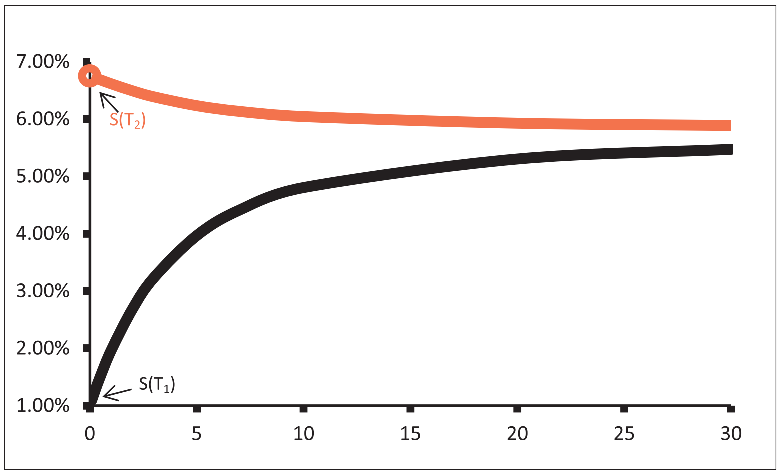
Model	Stochastic Differential Equation
Vasicek	$dr = a(b - r)dt + \sigma dZ$
CIR	$dr = a(b - r)dt + \sigma\sqrt{r}dZ$
Brennan-Schwartz	$dr = a(b - r)dt + \sigma r dZ$
Black-Karasinski	$dln(r) = a(b - \ln(r))dt + \sigma dZ$

These all follow a visible structure with a drift component (i.e., $a(b\text{-}r)dt$) and a stochastic component (σdZ), where Z is a Wiener process. The drift component includes a mean reversion target (b) and mean reversion speed (a). σ is a measure of the volatility of the short rate and can be applied in different ways.

A key advantage of most of these models (e.g., Vasicek or CIR) is that bond prices at any maturity have an analytical form (i.e., there is an explicit formula to define zero coupon bond prices at any time t) from which the yield curve can be derived. However, these models are based on the instantaneous spot rate, or the short rate, which is the rate an entity can borrow money for an infinitely small period of time. Also, the structure of these models is simplistic and could produce unintended term structures (e.g., inverted yield curves or negative rates in the United States).

In more sophisticated models, practitioners can add more conditions in short-rate models such as embedding stochastic processes such as volatility or mean-reverting targets (e.g., two-factor Vasicek or Brennan-Schwartz models).

Figure 1: Yield Curve Model Derived from Short Rate at Two Different Time Periods (T1 and T2)—Illustrative Only



KEY-RATE MODELS

The stochastic equations used in single short-rate models can also be adopted to generate observable measures such as forward rates or yields. Under this approach, multiple stochastic processes are used to project the rate at each maturity term in the yield curve. These processes are then made codependent using an explicit correlation matrix or a copula. The following formulas provide a general definition of key-rate models:

$$dr_1 = \mu_1(r_1)dt + \sigma_1(r_1)dZ_1$$

$$dr_2 = \mu_2(r_2)dt + \sigma_2(r_2)dZ_2$$

...

$$dr_n = \mu_n(r_n)dt + \sigma_n(r_n)dZ_n$$

$$f(Z_1, Z_2, \dots, Z_n) = C[f(Z_1), f(Z_2), \dots, f(Z_n)],$$

Where $\{r_1, r_2, \dots, r_n\}$ are the modeled key rates, $\{\sigma_1(r_1), \sigma_2(r_2), \dots, \sigma_n(r_n)\}$ the volatility structure for each rate, and $\{Z_1, Z_2, \dots, Z_n\}$ are the associated Wiener processes.

The joint distribution of the Wiener processes, $f(Z_1, Z_2, \dots, Z_n)$ is defined using the copula C.

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Since each key rate is modeled under a separate stochastic process, the model can be defined to capture all possible (or desirable) curve movements (e.g., parallel shifts, twists, butterfly shifts), providing more flexibility and control to the user.

Figure 2 below illustrates the process of three rates of different maturities under a single scenario.

Figure 2: Key Rate Projection under a Sample Stochastic Scenario

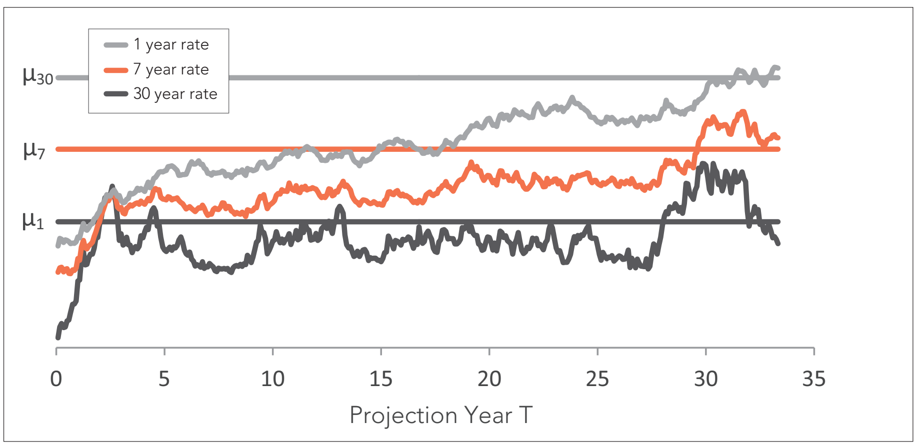
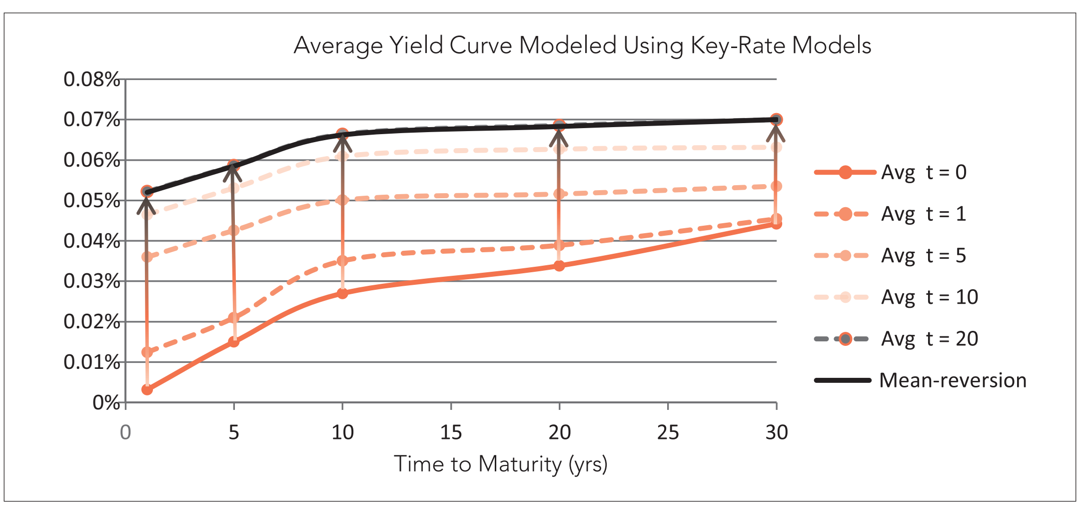


Figure 3 below illustrates the evolution of the average yield curve at different points in time across all simulated scenarios.

Figure 3: Average Evolution of the Yield Curve across Multiple Scenarios



However, the larger number of stochastic variables (and required parameters) significantly increases the difficulties—and risks—in calibrating the model, which should be considered when weighing the benefits of modeling each key rate.

FUNCTION-BASED MODELS

Since modeling each key rate may be unfeasible and introduce unwanted parameter risk, practitioners can achieve a more parsimonious modeling of yield curves by studying the functional properties of the curve itself. Instead of modeling specific points of the yield curve, function-based models focus on key latent features underlying the yield curve. Empirical studies (Pooter, 2007) have shown that changes in the level, slope and curvature of the yield curve explain most of its behavior. Changes in the level of the curve lead to parallel shifts (see Figure 4.1), changes in slope lead to flattening or steepening of the curve (see Figure 4.2), and changes in curvature lead to butterfly shifts in the yield curve (see Figure 4.3).

Figure 4: Illustration of Level, Slope and Curvature Effects

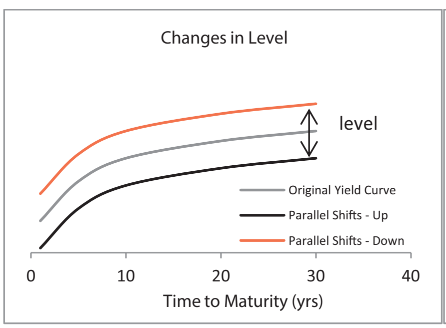


Figure 4.1

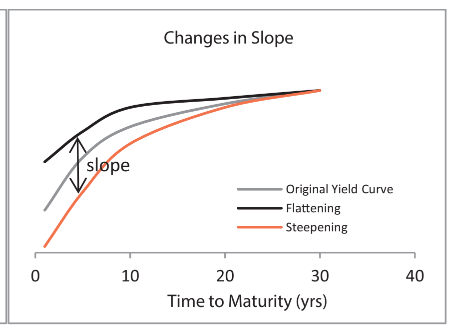


Figure 4.2

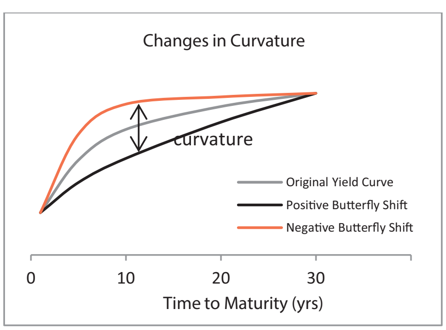


Figure 4.3

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Function-based models project the term structure of interest rates directly from the stochastic projection of these components. A common function-based definition of yield rates is provided by the Nelson-Siegel framework.

$$y_t(T) = L_t + S_t\left(\frac{1-e^{-\lambda_t T}}{\lambda_t T}\right) + C_t\left(\frac{1-e^{-\lambda_t T}}{\lambda_t T} - e^{-\lambda_t T}\right), where$$

$y_t(T)$ represents the interest rate at time t with term to maturity T,
L_t, S_t, C_t represent factors associated with level, slope and curvature, and
λ_t governs the exponential decay rate for the factor loadings applied to the slope and curvature factors.

A generic representation of level, slope and curvature and their associated factors is as follows:

Level = $y_t(T_l)$
Slope = $y_t(T_l)$ - $y_t(T_s)$
Curvature = $2y_t(T_m)$- $y_t(T_l)$- $y_t(T_s)$

Where $y_t(T_l)$ is the yield rate at longest maturity (e.g., 30-year); $y_t(T_m)$is the yield rate at a medium maturity (e.g., 10-year); and $y_t(T_s)$is the yield rate at the shortest maturity (e.g., three-month). These rates are modeled stochastically under a key rate model approach as discussed above.

This is the approach used in the American Academy of Actuaries' Interest Rate Generator (AIRG) and therefore implicitly adopted by many actuaries in the United States.

Figure 5: American Academy of Actuaries' AIRG (American Academy of Actuaries, 2010)

American Academy of Actuaries' Interest Rate Generator (AIRG)
Probability measure: real-world

Yield interpolation method: Nelson-Siegel

L_t (level factor)—associated with the 20-yr rate
- Uses a stochastic log volatility model.
- Log long-term rate follows a mean-reverting Black-Karasinski (BK) process.
- Its mean-reversion strength varies with nominal spread.
- Log volatility of the long-term rate also follows a mean-reverting BK process.

S_t (slope factor)—associated with the difference between 20-year rate and one-year rate
- Follows an extension of the Vasicek process.
- Its volatility varies with long-term rate.
- Its mean-reversion strength varies with the log long-term rate.

C_t (curvature factor)—modeled with a constant factor
- Effectively removes any humps.
- Produces a "normal" nonlinear shape of the curve.

Note: AIRG does not model curvature stochastically and therefore does not introduce butterfly shifts of the yield curve in the simulated scenarios. The lack of these features can undermine the reliability of this model for purposes that require capturing all the plausible movements in the curve (e.g., economic capital or profit testing).

Model Types	Advantages	Disadvantages
Short-rate	• Simple to define and implement • Analytical tractability • Minimal computational demands	• Overly simplistic view of interest rates • Can produce unintended yield rates and curve shapes
Key-rate	• Increased precision and control over desired outcomes • Most effective approach to achieve a refined view of tail risks for VaR and other tail metrics • Ability to flex parameters to achieve calibration criteria	• Models become significantly more complex • Large number of parameters significantly intensifies data requirements and the dependency to the collected data • Requires significant judgment in setting parameters and interpreting the credibility of historical data
Function-based	• Focuses on the few components that explain the most (i.e., Pareto principle) • Reduced data consumption requirements in calibration	• Inability to reconcile underlying dynamics with other models (e.g., arbitrage-free models) • There is a significant dependency on long-duration rate historical data, which is not always available (e.g., 30-year rates in the United States). • As noted with the Academy's generator, some variations will not be able to generate all possible forms of the yield curve.

Closing Remarks

Many actuarial liabilities show significant asymmetries with respect to interest rates; risks that are not apparent in traditional deterministic measurements. Their risk profile may be reflected not only with respect to the level of interest rates but also with the shape of the curve, the volatility and mean-reversion dynamics. Liabilities may also be long-term in nature, in which case modelers should understand the assumptions (and shortcomings) behind the projection of long-term interest rates. Policyholder behavior is commonly tied to the projected interest rates, which increases the relevance of real-world interest rate models.

Actuaries should have an understanding of the complexity and specificity of the interest rate models used given the intended purpose. The approaches discussed in this article, although not exhaustive, provide a starting point in understanding some of the primary options available.

Selecting a model is only the beginning of the process. Depending on the model selected, users will need to calibrate the parameters using a suitable set of historical data and exercise actuarial judgment in defining other model specifications.

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