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G*a*-1951 MALE MORTALITY TABLE PROJECTED WITH SCALE C TO 1970—ACTUARIAL NOTE

G. B. SAKSENA

A CONSIDERABLE amount of time and effort is needed in the preparation of a mortality table from the experience of assured lives. Actuaries are thus led to various devices to keep pace with the improvement in mortality with lapse of time; for example, rating-down of ages, using a conservative rate of interest, and, quite recently, the use of projection scales.

The latest available mortality table of annuitants (G*a*-1951) was prepared by Mr. Ray M. Peterson. His tables were published in *TSA*, Volume IV, on pages 246–307. Mr. Peterson also provided in his paper two projection scales which could be used to determine rates of mortality at any future date after 1951.

Recently Mr. Gordon J. Munro prepared the (static) G*a*-1951 Mortality Table projected to 1960 (*TSA*, XII, 353–68), using the Projection Scale C as given by Mr. Peterson. The author has now prepared the (static) G*a*-1951 Male Mortality Table using Projection Scale C to 1970 in the hope that this table can be used for the next few years. The author has not prepared a corresponding female mortality table, as the male table could be used for females by rating down the age by five years, as in the case of the 1937 Standard Annuity Table.

The values of q_x were first derived by using the formula $q_x^{1970} = q_x^{1951} \cdot (1 - 0.01r_x)^{19}$, where r_x was taken from Table 1 of Mr. Munro's paper. The radix for age five was chosen 9999.9999 (the same as that used by Mr. Peterson for the G*a*-1951 Mortality Table) to prepare the G*a*-1951 Mortality Table Projection C to 1970. Monetary functions were computed at $3\frac{1}{2}$ per cent in order to facilitate comparisons with Mr. Munro's tables.

The values of q_x as computed above were found to be extremely irregular. It was, therefore, decided to "graduate" the projected table by a Gompertz formula similar to that used in the 1937 Standard Annuity Table (*TASA*, XXXIX, 8). The constants used were:

$$\log_{10} c = .042\,,$$

$$10^4 \operatorname{colog}_{10} p_x = A_x + 10^{.042(x-18.86)}\,,$$

where

$$A_x = 0.000014032\,(x^2 - 14x + 177)(x - 35)^2 \text{ for ages 5–35 ;}$$

$$= 0 \text{ for ages 35–105 ;}$$

$$= 152.1\,\frac{(x-105)^2}{110-x} \text{ for ages 105–110 .}$$

For the Seniority Table, the following formula was used:

$$w - x = \frac{\log_{10}[1 + (1/c)^{x-y}]}{\log_{10} c}.$$

Table 1 gives the "graduated" and the "ungraduated" values of q_x; Table 2 gives the "graduated" values of l_x, d_x, q_x, together with the values of D_x, N_x, and $\ddot{a}_x$ based on "graduated" table; and Table 3 provides the Table of Uniform Seniority based on "graduated" table.

TABLE 1

Ga-51 MALE MORTALITY TABLE PROJECTED WITH SCALE C TO 1970 UNGRADUATED AND GRADUATED RATES OF MORTALITY

AGE x	VALUES OF q_x		AGE x	VALUES OF q_x		AGE x	VALUES OF q_x	
	Ungraduated	Graduated		Ungraduated	Graduated		Ungraduated	Graduated
5...	0.000,440	0.000,444	41..	0.001,726	0.001,957	76..	0.057,193	0.056,182
6...	.000,409	.000,417	42..	.001,929	.002,156	77..	0.063,679	0.061,708
7...	.000,389	.000,397	43..	.002,180	.002,374	78..	0.070,984	0.067,756
8...	.000,379	.000,384	44..	.002,478	.002,615	79..	0.079,077	0.074,374
9...	.000,375	.000,377	45..	.002,819	.002,880	80..	0.087,783	0.081,609
10...	.000,376	.000,374	46..	.003,201	.003,165	81..	0.096,960	0.089,514
11...	.000,383	.000,375	47..	.003,621	.003,494	82..	0.106,580	0.098,143
12...	.000,391	.000,380	48..	.004,079	.003,848	83..	0.116,600	0.107,553
13...	.000,398	.000,387	49..	.004,573	.004,238	84..	0.127,022	0.117,805
14...	.000,407	.000,396	50..	.005,099	.004,667	85..	0.137,825	0.128,961
15...	.000,417	.000,407	51..	.005,659	.005,140	86..	0.149,077	0.141,088
16...	.000,428	.000,418	52..	.006,251	.005,661	87..	0.160,882	0.154,250
17...	.000,441	.000,431	53..	.006,875	.006,234	88..	0.173,336	0.168,515
18...	.000,454	.000,444	54..	.007,530	.006,864	89..	0.186,540	0.183,951
19...	.000.469	.000,458	55..	.008,217	.007,559	90..	0.200,594	0.200,622
20...	.000,485	.000,473	56..	.008,934	.008,323	91..	0.212,555	0.218,592
21...	.000,504	.000,488	57..	.009,684	.009,164	92..	0.225,161	0.237,920
22...	.000,524	.000,505	58..	.010,474	.010,090	93..	0.238,524	0.258,657
23...	.000,546	.000,522	59..	.011,322	.011,109	94..	0.252,765	0.280,848
24...	.000,570	.000,541	60..	.012,248	.012,230	95..	0.268,025	0.304,523
25...	.000,597	.000,563	61..	.013,281	.013,463	96..	0.284,455	0.329,702
26...	.000,627	.000,587	62..	.014,451	.014,820	97..	0.302,223	0.356,383
27...	.000,660	.000,615	63..	.015,802	.016,313	98..	0.321,515	0.384,546
28...	.000,697	.000,647	64..	.017,376	.017,954	99..	0.342,526	0.414,144
29...	.000,736	.000,685	65..	.019,227	.019,759	100..	0.365,462	0.445,103
30...	.000,780	.000,729	66..	.021,412	.021,744	101..	0.390,538	0.477,316
31...	.000,830	.000,781	67..	.023,711	.023,925	102..	0.417,979	0.510,639
32...	.000,883	.000,842	68..	.025,974	.026,322	103..	0.450,096	0.544,891
33...	.000,943	.000,914	69..	.028,302	.028,955	104..	0.489,201	0.579,854
34...	.001,009	.000,998	70..	.030,948	.031,848	105..	0.537,605	0.615,266
35...	.001,082	.001,096	71..	.034,332	.035,025	106..	0.597,619	0.653,873
36...	.001,161	.001,207	72..	.038,109	.038,512	107..	0.671,554	0.700,526
37...	.001,250	.001,330	73..	.042,212	.042,338	108..	0.761,722	0.761,723
38...	.001,347	.001,465	74..	.046,705	.046,536	109..	0.870,434	0.860,084
39...	.001,456	.001,613	75..	0.051,575	0.051,138	110..	1.000,000	1.000,000
40...	0.001,575	0.001,777						

TABLE 2

Ga-51 Male Mortality Table Projected with Scale C to 1970
Commutation Columns at 3½ Per Cent Interest
Based on "Graduated" Table

x	l_x	d_x	q_x	D_x	N_x	$\ddot{a}_x$
5.....	9,999.9999	4.4400	0.000,444	8,419.7316	224,011.2535	26.6055
6.....	9,995.5599	4.1681	.000,417	8,131.3944	215,591.5219	26.5135
7.....	9,991.3918	3.9666	.000,397	7,853.1436	207,460.1275	26.4175
8.....	9,987.4252	3.8352	.000,384	7,584.5661	199,606.9839	26.3175
9.....	9,983.5900	3.7638	.000,377	7,325.2692	192,022.4178	26.2137
10.....	9,979.8262	3.7325	.000,374	7,074.8866	184,697.1486	26.1060
11.....	9,976.0937	3.7410	.000,375	6,833.0826	177,622.2620	25.9945
12.....	9,972.3527	3.7895	.000,380	6,599.5365	170,789.1794	25.8790
13.....	9,968.5632	3.8578	.000,387	6,373.9407	164,189.6429	25.7595
14.....	9,964.7054	3.9460	.000,396	6,156.0135	157,815.7022	25.6360
15.....	9,960.7594	4.0540	.000,407	5,945.4838	151,659.6887	25.5084
16.....	9,956.7054	4.1619	.000,418	5,742.0909	145,714.2049	25.3765
17.....	9,952.5435	4.2895	.000,431	5,545.5949	139,972.1140	25.2402
18.....	9,948.2540	4.4170	.000,444	5,355.7534	134,426.5191	25.0995
19.....	9,943.8370	4.5543	.000,458	5,172.3434	129,070.7657	24.9540
20.....	9,939.2827	4.7013	.000,473	4,995.1444	123,898.4223	24.8038
21.....	9,934.5814	4.8481	.000,488	4,823.9437	118,903.2779	24.6486
22.....	9,929.7333	5.0145	.000,505	4,658.5406	114,079.3342	24.4882
23.....	9,924.7188	5.1807	.000,522	4,498.7324	109,420.7936	24.3226
24.....	9,919.5381	5.3665	.000,541	4,344.3325	104,922.0612	24.1515
25.....	9,914.1716	5.5817	.000,563	4,195.1519	100,577.7287	23.9748
26.....	9,908.5899	5.8163	.000,587	4,051.0048	96,382.5768	23.7923
27.....	9,902.7736	6.0902	.000,615	3,911.7168	92,331.5720	23.6038
28.....	9,896.6834	6.4032	.000,647	3,777.1122	88,419.8552	23.4094
29.....	9,890.2802	6.7748	.000,685	3,647.0226	84,642.7430	23.2087
30.....	9,883.5054	7.2051	.000,729	3,521.2796	80,995.7204	23.0018
31.....	9,876.3003	7.7134	.000,781	3,399.7223	77,474.4408	22.7885
32.....	9,868.5869	8.3094	.000,842	3,282.1904	74,074.7185	22.5687
33.....	9,860.2775	9.0123	.000,914	3,168.5283	70,792.5281	22.3424
34.....	9,851.2652	9.8316	.000,998	3,058.5819	67,623.9998	22.1096
35.....	9,841.4336	10.7862	.001,096	2,952.2024	64,565.4179	21.8703
36.....	9,830.6474	11.8656	.001,207	2,849.2432	61,613.2155	21.6244
37.....	9,818.7818	13.0590	.001,330	2,749.5693	58,763.9723	21.3721
38.....	9,805.7228	14.3654	.001,465	2,653.0554	56,014.4030	21.1132
39.....	9,791.3574	15.7935	.001,613	2,559.5833	53,361.3476	20.8477
40.....	9,775.5639	17.3712	.001,777	2,469.0383	50,801.7643	20.5755
41.....	9,758.1927	19.0968	.001,957	2,381.3051	48,332.7260	20.2967
42.....	9,739.0959	20.9975	.002,156	2,296.2753	45,951.4209	20.0113
43.....	9,718.0984	23.0708	.002,374	2,213.8401	43,655.1456	19.7192
44.....	9,695.0276	25.3525	.002,615	2,133.8980	41,441.3055	19.4205
45.....	9,669.6751	27.8487	.002,880	2,056.3458	39,307.4075	19.1152
46.....	9,641.8264	30.5164	.003,165	1,981.0855	37,251.0617	18.8034
47.....	9,611.3100	33.5819	.003,494	1,908.0342	35,269.9762	18.4850
48.....	9,577.7281	36.8551	.003,848	1,837.0700	33,361.9420	18.1604
49.....	9,540.8730	40.4342	.004,238	1,768.1169	31,524.8720	17.8296
50.....	9,500.4388	44.3385	.004,667	1,701.0856	29,756.7551	17.4928
51.....	9,456.1003	48.6044	.005,140	1,635.8905	28,055.6695	17.1501
52.....	9,407.4959	53.2558	.005,661	1,572.4464	26,419.7790	16.8017
53.....	9,354.2401	58.3143	.006,234	1,510.6713	24,847.3326	16.4479
54.....	9,295.9258	63.8072	.006,864	1,450.4867	23,336.6613	16.0888
55.....	9,232.1186	69.7856	.007,559	1,391.8170	21,886.1746	15.7249
56.....	9,162.3330	76.2581	.008,323	1,334.5858	20,494.3576	15.3563
57.....	9,086.0749	83.2648	.009,164	1,278.7227	19,159.7718	14.9835
58.....	9,002.8101	90.8384	.010,090	1,224.1589	17,881.0491	14.6068
59.....	8,911.9717	99.0031	.011,109	1,170.8282	16,656.8902	14.2266
60.....	8,812.9686	107.7826	.012,230	1,118.6681	15,486.0620	13.8433
61.....	8,705.1860	117.1979	.013,463	1,067.6200	14,367.3939	13.4574
62.....	8,587.9881	127.2740	.014,820	1,017.6296	13,299.7739	13.0694
63.....	8,460.7141	138.0196	.016,313	968.6458	12,282.1443	12.6797
64.....	8,322.6945	149.4257	.017,954	920.6225	11,313.4985	12.2890
65.....	8,173.2688	161.4956	.019,759	873.5204	10,392.8760	11.8977
66.....	8,011.7732	174.2080	.021,744	827.3048	9,519.3556	11.5065
67.....	7,837.5652	187.5137	.023,925	781.9477	8,692.0508	11.1159
68.....	7,650.0515	201.3647	.026,322	737.4296	7,910.1031	10.7266
69.....	7,448.6868	215.6767	.028,955	693.7381	7,172.6735	10.3392
70.....	7,233.0101	230.3569	.031,848	650.8705	6,478.9354	9.9543
71.....	7,002.6532	245.2679	.035,025	608.8324	5,828.0649	9.5725
72.....	6,757.3853	260.2404	.038,512	567.6407	5,219.2325	9.1946
73.....	6,497.1449	275.0761	.042,338	527.3234	4,651.5918	8.8211
74.....	6,222.0688	289.5502	.046,536	487.9203	4,124.2684	8.4528
75.....	5,932.5186	303.3771	.051,138	449.4826	3,636.3481	8.0901
76.....	5,629.1415	316.2564	.056,182	412.0743	3,186.8655	7.7337
77.....	5,312.8851	327.8475	.061,708	375.7712	2,774.7912	7.3843
78.....	4,985.0376	337.7662	.067,756	340.6600	2,399.0200	7.0423
79.....	4,647.2714	345.6362	.074,374	306.8389	2,058.3600	6.7083
80.....	4,301.6352	351.0521	.081,609	274.4136	1,751.5211	6.3828
81.....	3,950.5831	353.6325	.089,514	243.4966	1,477.1075	6.0662
82.....	3,596.9506	353.0155	0.098,143	214.2031	1,233.6109	5.7591

TABLE 2—*Continued*

x	l_x	d_x	q_x	D_x	N_x	$\ddot{a}_x$
83.....	3,243.9351	348.8950	0.107,553	186.6479	1,019.4078	5.4617
84.....	2,895.0401	341.0502	0.117,805	160.9404	832.7599	5.1743
85.....	2,553.9899	329.3651	0.128,961	137.1796	671.8195	4.8974
86.....	2,224.6248	313.8679	0.141,088	115.4481	534.6399	4.6310
87.....	1,910.7569	294.7343	0.154,250	95.8065	419.1918	4.3754
88.....	1,616.0226	272.3240	0.168,515	78.2883	323.3853	4.1307
89.....	1,343.6986	247.1747	0.183,951	62.8942	245.0970	3.8970
90.....	1,096.5239	219.9868	0.200,622	49.5891	182.2028	3.6743
91.....	876.5371	191.6040	0.218,592	38.3000	132.6137	3.4625
92.....	684.9331	162.9593	0.237,920	28.9159	94.3137	3.2617
93.....	521.9738	135.0122	0.258,657	21.2910	65.3978	3.0716
94.....	386.9616	108.6774	0.280,848	15.2502	44.1068	2.8922
95.....	278.2842	84.7439	0.304,523	10.5963	28.8566	2.7233
96.....	193.5403	63.8106	0.329,702	7.1203	18.2603	2.5645
97.....	129.7297	46.2335	0.356,383	4.6113	11.1400	2.4158
98.....	83.4962	32.1081	0.384,546	2.8676	6.5287	2.2767
99.....	51.3881	21.2821	0.414,144	1.7052	3.6611	2.1470
100.....	30.106,027	13.400,283	0.445,103	0.965,202,56	1.955,897,02	2.0264
101.....	16.705,744	7.973,919	0.477,316	0.517,476,33	0.990,694,46	1.9145
102.....	8.731,825	4.458,810	0.510,639	0.261,330,05	0.473,218,13	1.8108
103.....	4.273,015	2.328,327	0.544,891	0.123,560,14	0.211,888,08	1.7149
104.....	1.944,688	1.127,635	0.579,854	0.054,331,73	0.088,327,94	1.6257
105.....	0.817,053	0.502,705	0.615,266	0.022,055,33	0.033,996,21	1.5414
106.....	0.314,348	0.205,544	0.653,873	0.008,198,49	0.011,940,88	1.4565
107.....	0.108,804	0.076,220	0.700,526	0.002,741,75	0.003,742,39	1.3650
108.....	0.032,584	0.024,820	0.761,723	0.000,793,32	0.001,000,64	1.2613
109.....	0.007,764	0.006,678	0,860,084	0.000,182,64	0.000,207,32	1.1351
110.....	0.001,086	0.001,086	1.000,000	0.000,024,68	0.000,024,68	1.0000

TABLE 3

G*a*-51 MALE MORTALITY TABLE PROJECTED WITH SCALE C TO 1970
TABLE OF UNIFORM SENIORITY BASED ON "GRADUATED" TABLE

Difference of Ages (Years)	Addition to Older Age (Years)	Difference of Ages (Years)	Addition to Older Age (Years)
0........	7.1674	21........	1.2749
1........	6.6795	22........	1.1638
2........	6.2157	23........	1.0618
3........	5.7758	24........	0.9684
4........	5.3596	25........	0.8828
5........	4.9667	26........	0.8045
6........	4.5966	27........	0.7329
7........	4.2487	28........	0.6675
8........	3.9225	29........	0.6077
9........	3.6171	30........	0.5532
10........	3.3319	31........	0.5034
11........	3.0659	32........	0.4580
12........	2.8184	33........	0.4166
13........	2.5885	34........	0.3789
14........	2.3752	35........	0.3446
15........	2.1777	36........	0.3133
16........	1.9951	37........	0.2848
17........	1.8265	38........	0.2589
18........	1.6710	39........	0.2353
19........	1.5278	40........	0.2138
20........	1.3960		

DISCUSSION OF PRECEDING PAPER

WILLIAM H. CROSSON:

It is quite useful to have presented before the Society of Actuaries the tables derived by projecting the Ga-1951 table to various years. Mr. Munro projected the male table to 1960 (*TSA*, XII, 353), and Mr. Saksena projects the male table to 1970.

Mr. Saksena has gone beyond Mr. Munro by graduating the resulting mortality rates. A Makeham graduation of the Ga-1951 table was considered and was rejected, and a modified Whittaker-Henderson Type B graduation was used instead. Now, after projecting nineteen years, we find that we can use a Gompertz graduation. This is certainly an unexpected, but fortunate, development. I hope that Mr. Saksena can accommodate us by describing how he arrived at the particular graduation in the note and by presenting a discussion of the closeness of fit of the graduated to the ungraduated projected mortality rates.

Since the table has been regraduated, the name of the table should suggest this fact, to distinguish the table from the corresponding ungraduated table. I would suggest "The Ga-1951 Male Mortality Table Projected with Scale C to 1970 (Regraduated)."

(AUTHOR'S REVIEW OF DISCUSSION)

G. B. SAKSENA:

I must thank Mr. William H. Crosson for his remarks regarding my actuarial note.

The values of q_x and u_x were plotted on several types of graph papers in order to determine whether or not a mathematical curve could be fitted to the projected table, and after several attempts it was found that a Gompertz curve could be satisfactorily fitted between the ages of 35 and 105 (the most essential range of the table). Mr. Crosson is surprised that a Gompertz curve could be fitted to the projected table, whereas the Ga-51 table could not be graduated by the Makeham or the Gompertz Law. On this point the author has the following comments.

A few years ago the author requested and obtained from Mr. Peterson the crude data from which he constructed the Ga-51 table with a view to fitting a Gompertz or a Makeham curve to the Ga-51 table. Because so many adjustments had been made to the crude data before arriving at the final Ga-51 table, the author found it extremely difficult to graduate it applying the various criteria of good graduation described in Mr. Miller's monograph on graduation and still preserve some resemblance to the published table. He had to abandon the project. In the author's opinion the "kink" in the rates of mortality at the higher ages is not an essential feature of the crude data. It is simply due to the paucity of the exposures and deaths at ages above 95 (see *TSA*, IV, 290–91). We have always been taught that rates of mortality progress smoothly from age to age, and this is exactly what the author has tried to provide by fitting the Gompertz curve from age 35 to age 105. As a matter of fact, if the rates of mortality for the males and females were compared in the Ga-51 table, the male mortality rates are lower than the female rates from ages 103 to 110, primarily due to the fact that both mortality tables have been forced to terminate at age 110. This feature could not be attributed to the crude data available to Mr. Peterson when the total number of deaths for males over age 95 were 16 and for females 0. As a test, the author used the rates of mortality at ages above 90 as given by Mr. Peterson, instead of the graduated rates, to study the effect on $\ddot{a}_x$, and the maximum difference at age 5 is 0.0015. The author agrees that there should be some kind of evidence to show the quality of the graduation, and would have done some form of comparison to prove this point. Unfortunately, it is not feasible to make any mathematical comparisons due to the fact that this is a projected table so that it is not possible to compare the actual with the expected deaths (which would be normally possible for a table compiled from actual exposures and deaths). The comparison that could be made would be a graphical one. Table 1 supplied with the actuarial note provides the ungraduated and the graduated rates of mortality. The author plotted the values of q's on various types of graph paper, and the changes in the curvature and the slopes of the tangents of the ungraduated rates are exceedingly irregular. The graduated rates of mortality, of course, proceed with mathematical smoothness and cross and recross the ungraduated rates as they should in order to preserve the over-all fidelity of the ungraduated rates. The greatest advantage of using the graduated rates of mortality is the facility with which one could compute joint life functions. If the ungraduated rates were used, this facility would be lost, and computation of joint life functions would be a monumental task. The author was not quite sure which of the two rates of mortality would be more acceptable to actuaries in general, and that is why in Table 1 he provided both rates of mortality. The author prefers to use the graduated rates due to the advantages enumerated above.