

COST OF VESTING IN PENSIONS

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INTRODUCTION

ONE of the major trends in retirement plans is toward the extension and development of vesting provisions. The trend has been given force by automation with its concomitant involuntary mobility of the labor force and has been given direction by collective bargaining, where the argument is heard that benefits are the equivalent of contributions which are the equivalent of wages and that therefore accrued benefits should be permanently available to the employee. This contention receives encouragement from government, partly by force of example in the OASDI system of the United States and the Old Age Security System in Canada; partly by direct pressure, such as legislation on portability in Canada; and partly by indirect pressure through the machinery for granting tax exemption for contributions and investment yield.

Public reports and published books have recently focused so much attention on vesting that a discussion of the mathematics of the determination of cost is timely. This paper presents a theoretical treatment of the determination of the cost of vesting by a method not hitherto demonstrated to the Society. It is followed by illustrative calculations of cost factors developed on the basis of certain actuarial cost determinants.

DEFINITION OF VESTING

Termination of employment arises from inability to continue to work, discharge for cause, or merely voluntary termination of employment. In employments where pensions are provided, in some or all of these circumstances, vesting may occur. A vested benefit, in pension plan parlance, is a benefit to which an employee has a right, contingent upon survival, but unaffected by his work history subsequent to his acquiring the right.

It may be noted that this definition differs from the legal definition of a vested right which indicates a nondefeatable right to current enjoyment of the benefits or the object to which the rights attach. For example, in a single-employer pension plan a vested right may be acquired upon attainment of 50 years of age and the accumulation of 15 years of credited service. To such an employee his vested right is not defeated by subse-

quent termination of employment, but it would be defeated by death prior to the normal retirement age since his vested pension becomes payable only when he survives to the normal retirement age.

In a multi-employer pension plan, accumulation of pension credits may be related to hours of work credited under the plan and may result from hours of employment by more than one employer. The accumulation of pension credits is not terminated by change of employer since the plan specifically provides for this type of vesting. However, accumulated pension credits could be canceled by failure to work minimum hours in employment covered by the plan, thus making this type of vesting contingent upon preserving a current working status. At some age and accumulated total of service credits, the vesting provision may assume a form similar to that seen in single-employer plans, where the accrued pension rights are then not defeatable by subsequent changes in employment patterns. In both types of plan, the vested right is usually a right to a deferred pension contingent upon survival to the earliest age at which a normal pension can be claimed and is thus defeatable by death.

When an employee retires upon pension, he acquires a vested right to his pension. This means that he can collect his pension each month if he is alive at the payment date and provided the plan has sufficient assets to make the payment. The vesting rights here also are seen to be contingent. In fact, the only way of satisfying the legal definition is to pay a lump sum in lieu of pension, and this procedure runs contrary to the purpose and the philosophy of the pension plan.

FACTORS AFFECTING THE COST OF VESTING

In a retirement plan covering a stated list of participants, the basic items in determining the theoretical cost to the employer of the plan benefits are the estimates of the number of survivors of the participants to retirement age, of the amount of the benefits provided for each retiring participant, and of the time interval until those benefits become due. The method described below uses the costs of nonvested benefits as the basis for determining the cost of vesting. Vested benefits mature at the same time as nonvested benefits. Vested benefits become "frozen" at an age and duration prior to maturity and mature on survival of the deferment period. Regular valuation functions can thus be adjusted by this additional survivorship to provide the value of accrued and prospective vesting benefits.

In establishing the survivorship, the use of mortality rates only is appropriate since termination of employment can no longer cancel entitlement to benefit. The deferment in time must involve the use of time

and the same rate of interest as that employed in the basic calculation of pension values, in order to make use of these relationships.

The value of prospectively vesting pension benefits will be calculated as a percentage of the normal contribution rate required or as a percentage of the value of accrued or prospective nonvested pensions for participants covered by the plan. These percentages will be referred to as "vesting cost ratios." The method will be found to be very flexible. The value of vesting in any segment of the benefits is directly related to the value of that segment of the nonvested benefits and, correspondingly, to the contributions required to provide it. In consequence, the cost of vesting may be incorporated into any of the recognized systems of funding prospective pensions.

The computed vesting cost ratios will vary (in size) according to (a) the annual rates of termination of employment used in the calculations, (b) the benefits of the plan, (c) the vesting formula, and (d) a vesting benefit claim rate where there are alternative benefits upon withdrawal.

It may be noted that there cannot be any theoretical additional cost of vesting unless the original calculation incorporated a reduction of cost on account of anticipated terminations of employment. If there are no anticipated terminations, the cost of vested rights is an implicit part of the calculated cost of the whole plan. An unanticipated termination in such circumstances produces an experience gain. A lesser gain develops if some part of the gain is earmarked for a vested benefit.

Since the vesting of benefits is usually effective at some stated age or duration of participation in the plan or when a combination of these criteria is satisfied, it will generally be found necessary to develop the calculations of actuarial liabilities by entry age and duration in order to be able to apply vesting costs ratios to the appropriate actuarial cost values. Whether the computation of cost is made by an accrued-benefit method or by a projected-benefit method, the dollar amount of the cost of the vesting benefit according to the rules of the plan will require determination under two heads: (1) the amount to be added to the annual cost and (2) the amount to be added to the estimate of pension liabilities in respect to (a) benefits already vested in inactive participants and (b) benefits expected to vest in the future in respect to participants now active but expected to terminate participation in the future.

For liabilities under 2(a), that is, terminated vested former participants, direct calculation is effected on the basis of the amount of vested pension and the age of the former participant. To determine liabilities under 1 and 2(b), the vesting cost ratios method may be applied in each case.

DEVELOPMENT OF PRIMARY VESTING COST RATIOS

No mathematical problems arise in the treatment of a vested immediate annuity, but the treatment of vesting rights in a deferred pension requires some analysis. Consider now how a vested benefit changes the status of the participant. If he quits before vesting, his accrued pension lapses and may not be restored on re-employment. A vested participant who quits work, however, remains entitled to a pension at the normal retirement age, and the present value of such liabilities, with respect to all such pensions, remains a claim on the assets of the plan. The gross liabilities of the plan are thus increased by the value of the deferred pensions to former participants with vested rights.

There remains the question of determining the value of prospective vested rights which may be expected to arise from the current group of participants, some of whom may have to complete many years of employment before becoming entitled to vested rights. In brief, the vested benefit turns a termination of employment with complete severance of connection with the pension plan into a cessation of the employer-employee relationship(s) with a contingent liability in the pension plan covering such former participants. This concept of continuing relationship is the basis of the mathematics which follow.

Suppose the pension plan liabilities are determined on the basis of a service table exhibiting the decrements of death, withdrawal, and disablement. The last decrement is not essential and would be merged in whole or in part into the first two if a benefit on disability were not payable from the plan. In order that a cost of vesting may be separately determinable, it is necessary to show deaths separately from terminations of employment. If there are no termination decrements, then there is no separately projectable cost of vesting. This is so because, where death is the sole decrement, termination of employment occurs but the survival of any deferment period does not and so the benefit and its present value do not exist. The equation determining the service table figures is

$$l_x - l_{x+1} = d_x + w_x + ir_x. \quad (1)$$

Summing from x to r (the normal retirement age), we have

$$l_x - l_r = \sum_{t=0}^{r-1} d_{x+t} + \sum_{t=0}^{r-1} w_{x+t} + \sum_{t=0}^{r-1} ir_{x+t}. \quad (2)$$

In words, the difference between the number of persons who start at age x and survive to age r is the sum of the deaths at the intervening ages, the terminations at the same ages, and the disablements before the

final age, and equation (2) holds even if values of any decrements are 0 at any age or series of ages.

In order to isolate the cost of the vesting benefit, it is necessary to assume that the rates of mortality and disablement remain unchanged, and, with that assumption, the use of rates of termination has the effect of reducing the value of l_r to a figure lower than it would otherwise be. High rates of termination reduce l_r more than low rates. The effect of vesting may be considered, from one point of view, to keep some of these terminations in the l_x column and so increase the value of l_r to a figure greater than it would otherwise have been. The expression for the increase in l_r is developed below.

From the w_{x+t} terminations in the service at age $x+t$, the survivors at age r are

$$w_{x+t} \cdot r-x-t-1/2 \dot{p}'_{x+t+1/2} \tag{3a}$$

(prime indicates mortality decrement only) or

$$w_{x+t} \cdot l'_r / l'_{x+t+1/2} . \tag{3b}$$

The sum of such survivors is

$$\sum_{t=0}^{r-x-1} w_{x+t} \cdot r-x-t-1/2 \dot{p}'_{x+t+1/2} \tag{4a}$$

or

$$\sum_{t=0}^{r-x-1} w_{x+t} \cdot l'_r / l'_{x+t+1/2} . \tag{4b}$$

The number of survivors at age r adjusted for the vesting provision is now

$$l_r + \sum_{t=0}^{r-x-1} w_{x+t} \cdot r-x-t-1/2 \dot{p}'_{x+t+1/2} , \tag{5}$$

which becomes

$$l_r \left(1 + \frac{1}{l_r} \sum_{t=0}^{r-x-1} w_{x+t} \cdot r-x-t-1/2 \dot{p}'_{x+t+1/2} \right) . \tag{6}$$

The primary vesting cost ratio is thus

$$\frac{1}{l_r} \sum_{t=0}^{r-x-1} w_{x+t} \cdot r-x-t-1/2 \dot{p}'_{x+t+1/2} \tag{7}$$

or

$$\frac{1}{l_r} \sum_{t=0}^{r-x-1} w_{x+t} \cdot l'_r / l'_{x+t+1/2} . \tag{8}$$

Formulas (7) and (8) show the proportionate increase in l_r from projected survivors from those who withdraw, which is used in combinations with vesting benefit provisions to develop the fully adjusted vesting cost ratios. A table of $r_{-x-t-1/2} p'_{x+t+1/2}$ (or $l'_r/l'_{x+t+1/2}$) can readily be calculated on a single decrement (mortality) table so that the operation of developing the number of survivors is a series of multiplications followed by a summation. The value of taking the ratio of the survivors of terminations to the value of l_r lies in the fact that l_r is a principal component in the formula for valuing the pension payable at the normal retirement age.

To obtain the addition to the normal cost arising from, for instance, the vesting of a portion of the normal retirement benefit, the vesting cost ratios, formulas (7) and (8) are applicable to the normal cost for normal retirement pension. They should not be applied to any other part of the normal cost.

To obtain the addition to the pension liabilities, it might have been sufficient to apply the appropriate primary vesting cost ratio to the present value of normal pension at attained ages had not the vagaries of the qualifying requirements for vesting intervened. If the qualification for 100 per cent vesting is solely the attained age, then application of the vesting cost ratios to present value of normal pension by attained age could suffice; but if, as appears to be most usual, the vesting qualifications include both age and service components, then a development of present values of normal pension by entry age and duration or some equivalent is necessary for the proper application of the ratios.

ADJUSTMENTS TO PRIMARY VESTING COST RATIOS

Before they can be applied to provide cost estimates applicable to a specific plan, the primary ratios as developed above will require adjustment under either or both of two heads: (1) the vesting qualifications and (2) the vested pension.

1. Let us assume that the entry age is x , the credited service is t , so that the attained age is $x + t$. Further, let the vesting qualification be attainment of age y and completion of n years of credited service. The formulas for the various cases which occur are as follows:

a) If (i) $y \geq x + n \geq x + t$ or if (ii) $y \geq x + t \geq x + n$, vesting occurs at age y and the primary vesting cost ratio is

$$\frac{1}{l_r} \sum_{s=0}^{r-y-1} w_{y+s} \cdot r_{-y-s-1/2} p'_{y+s+1/2} = \frac{1}{l_r} \sum_{s=0}^{r-y-1} w_{y+s} \cdot l'_r/l'_{y+s+1/2}. \quad (9)$$

The ratio at age y is applicable at all ages preceding y , and the ratios thereafter change with attained age $y + s$.

b) If $x + t \geq y \geq x + n$, let $x + t = y + k$. Then the above summation in (i) is made for values of s from k to $r - y - 1$, and the ratio is reduced accordingly.

c) If (i) $x + n \geq y \geq x + t$ or (ii) $x + n \geq x + t \geq y$, then vesting occurs at $x + n$ and the summation is adjusted accordingly by substituting $x + n$ for y .

d) If $x + t \geq x + n \geq y$, then the summation in (i) is made after substituting $x + t$ for y .

Numerical examples of the cases cited are given below:

Case	Entry Age x	Entry Age plus Duration $x+t$	Vesting Age y	Entry Age plus Vesting Minimum Duration $x+n$	Summation from:
a) (i).....	25	30	50	35}	50 (=y)
a) (ii).....	25	37	50	35}	
b).....	25	52	50	35	52 (=x+t=y+s)
c) (i).....	45	47	50	55}	55 (=x+n)
c) (ii).....	45	52	50	55}	
d).....	35	52	50	52	52 (=x+t)

2. The adjustment for the reduced vested pension has now to be introduced. It arises from the fact that the effect of applying the ratio to the value of normal pension is to provide on retirement at age r the full amount of pension accruing with respect to all service between x and r . This may not be consistent with the vested pension benefit to be provided under the plan, and, where it is not, adjustment is necessary. Consider the simple case in which a participant is credited with a pension of \$1 a year for each year of credited service. The entrant at age x has accrued \$ t a year at attained age $(x+t)$ in which he vests on termination at that age and could accrue a total of $\$(r-x)$ if he remained in employment until age r . By applying the primary vesting cost ratios to the value of the normal retirement pension, he would be credited, on vesting, with a pension of $\$(r-x)$ to commence at age r . To be consistent with the facts, this has to be cut down to \$ t by application of the ratio $t/(r-x)$. The effective vesting cost ratio, formula (9), then becomes

$$\frac{1}{l_r} \sum_{s=0}^{r-y-1} \frac{y+s-x}{r-x} \cdot w_{y+s} \cdot r^{-y-s-1/2} p'_{y+s+1/2} \tag{10a}$$

or

$$\frac{1}{l_r} \sum_{s=0}^{r-y-1} \frac{y+s-x}{r-x} \cdot w_{y+s} \cdot l'_r / l'_{y+s+1/2} \tag{10b}$$

In Appendix I will be found the calculation of the vesting cost ratios on the basis of the formula and on certain assumptions regarding termination rates. The cost of a number of vesting benefit formulas can be derived from this table, as is shown in the accompanying table.

BENEFIT VESTS 100 PER CENT AT:	VESTING COST RATIOS	
	Entry Age 20	Entry Age 25
10 years of credited service.....	+45.6%	+30.2%
20 years of credited service.....	+24.4	+14.7
Age 45 and 20 years of credited service.....	+15.7	+14.7
Age 50 and 15 years of credited service.....	+ 8.4	+ 8.1
Age 55 and 15 years of credited service.....	+ 3.1	+ 3.0

It is to be emphasized that these percentage increases only apply to normal cost rates calculated upon the table described, if a pro rata vesting of pension benefits occurs and on the assumption of 100 per cent vesting in the accrued benefit.

GRADED VESTING

Not infrequently vesting may be on a graded basis, such as, for instance, 50 per cent on reaching a qualifying age or period of service (or a combination of the two) and increasing by 10 per cent for each subsequent year of credited service up to 100 per cent. This can be incorporated into the calculations by including a function f_{y+s} , where f_y is the starting percentage and f_{y+s} increases according to the scale required up to unity. The ratio then is:

$$\frac{1}{l_r} \sum_{s=0}^{r-y-1} f_{y+s} \cdot \frac{y+s-x}{r-x} \cdot w_{y+s} \cdot r^{-y-s-1/2} p'_{y+s+1/2} \quad (11a)$$

or

$$\frac{1}{l_r} \sum_{s=0}^{r-y-1} f_{y+s} \cdot \frac{y+s-x}{r-x} \cdot w_{y+s} \cdot l'_r / l'_{y+s+1/2} \quad (11b)$$

In Appendix II is shown the calculations adjusting the values in Appendix I to a graded vesting formula. The results are compared with the previously quoted vesting cost ratios in the tabulation on page 285.

The cost ratios applicable under other grading formulas can be calculated in a similar manner. The grading adjustment procedure has to be followed for each entry age to develop vesting cost ratios applicable to entry age normal cost rates. For additions to pension liabilities, vesting cost ratios at attained ages are required. If the attained age is within the grading range, the vesting cost ratio can be taken from that table;

if the attained age exceeds the last age of the grading range, the value of the vesting cost ratio is not affected and can be taken from the ungraded table.

ADJUSTMENTS FOR BENEFITS RELATED TO SALARY

The adjustment factor allowing for benefit accruals to the date of termination assumes different forms according to the type of benefit provided by the plan. It has been shown so far on a uniform benefit accrual basis. Other forms have now to be considered.

COMPARISON OF VESTING COST RATIOS

Entry Age	Vesting 100 Per Cent at 10 Years of Credited Service	Vesting on Graded Scale 50 Per Cent at 10 Years of Credited Service Increasing by 10 Per Cent per Annum to 100 Per Cent
20.....	+45.6%	+42.3%
25.....	+30.2	+27.8

1. If the plan offers a benefit based on career average salary, and the valuation cost determinants included a salary scale S_x , then the form is

$$\sum_{s=0}^{t-1} S_{x+s} / \sum_{s=0}^{r-x-1} S_{x+s} \tag{12}$$

When $S_x = S_{x+1} = S_{x+2} = \dots = S_{x+t} = \dots = S_{r-1}$, the expression reduces to $t/(r - x)$.

Further, because S_x can be assumed to be a function which does not decline with an increase in x , then

$$\sum_{s=0}^{t-1} S_{x+s} / \sum_{s=0}^{r-x-1} S_{x+s} \leq t/(r - x), \tag{13}$$

and consequently the value $t/(r - x)$ is the upper limit of the adjustment factor.

2. For a "final average" benefit based on the average salary of, say, the five years preceding retirement, the adjustment factor at attained age $(x + t)$ becomes

$$\frac{t}{r - x} \cdot \frac{\sum_{s=0}^4 (S_{x+t+s} + S_{x+t+s-1})}{\sum_{s=0}^4 (S_{r-s} + S_{r-s-1})} \tag{14}$$

Again, unless S_x declines in value with an increase in x , the upper limit of formula (14) is $t/(r - x)$.

Values of the adjustment factors on two salary scales are shown in Appendix IV. Again it has to be stated that these adjustment factors only apply numerically to cost estimates calculated on the actuarial cost determinants indicated. It may seem from the vesting cost ratios in Appendix IV that the cost of vesting would be less in salary plans than it is in others and less still the steeper the salary scale. However, the vesting cost ratios may be less but they are applied to higher normal costs and thus produce higher absolute costs of vesting.

3. The method of dealing with a split-level benefit formula such as 1 per cent of salary under \$4,800 plus 2 per cent of the excess of salary over \$4,800 requires a combination of two sets of vesting cost ratios. The formula can be expressed as 2 per cent of salary less 1 per cent of \$4,800, so that the adjustment in formula (12) is required for the costs of the pension related to salary and the adjustment in formula (10) is applied to the deductive term. As a practical measure, a set of compromise values can be obtained and employed having regard to the relative weight of the two terms in the benefit formula.

CALCULATION OF VESTING COST RATIOS BY PRECEDING FORMULAS

The formulas set out above are more formidable in appearance than they are in calculation. No adjustments are required in the work sheet when select withdrawals are used. In use with a select table, at the appropriate age w_{x+t} will become $w_{[x]+t}$ in the formulas given earlier and calculation will proceed as before. In fact, whether select withdrawal rates are used or not, the calculations for many plans have to be made in select form, that is, by entry age and duration.

In Appendix I is set out the type of work sheet required for the calculation of a vesting cost ratio at every fifth entry age assuming the use of our aggregate or ultimate service table. The full calculations are shown for entry age 20 and entry age 25. The adjustments to give effect to a graded vesting formula are shown in Appendix II. The vesting cost ratios are different for service tables constructed with different rates of termination. For the same vesting formulas the vesting cost ratios are higher for heavier withdrawal rates. This is illustrated in Appendix III, where at every fifth age the vesting cost ratios are shown on the table of withdrawal rates used for Appendixes I and II together with vesting cost ratios calculated with respect to withdrawal rates of two-thirds and one-third the original withdrawal rates. The vesting cost

ratios show almost pro rata reductions because of the early 100 per cent vesting.

In Appendix IV salary scale adjustment factors are shown at fifth ages on two salary scales and are compared with the ratios without salary scale.

VESTING IN CONTRIBUTORY PLANS

Vested benefits in contributory plans are not infrequently made conditional upon the participant's leaving his contributions in the fund. If this is the case, vesting will not occur automatically upon termination of employment under conditions satisfying the vesting formula. It appears probable that a high proportion of such terminated participants will take the refund of contributions and allow their vested benefits to be voided. In such cases the cost of vesting will not emerge at the levels calculated earlier.

An estimate of the lower level of costs can be obtained by incorporating in the calculations still another adjustment factor—this time a graduated claim rate with respect to the vested benefit. Examples of graduated claim rates are to be found in Appendix XII to the *Report on the Canadian Civil Service Superannuation Fund* as of December 31, 1957. It would be a service if members of the Society would draw attention to other sources of statistics on this point.

Immediate total and permanent vesting in participant contributions occurs in most contributory pension plans, and, consequently, an increase in cost with respect to a vesting formula is only incurred on that portion of the benefit financed by employer contributions. A practical approach to the determination of this vesting cost may be made on the basis of the vesting cost ratios indicated earlier in this paper coupled with an estimate (for each entry age) of the proportion of the benefit financed by the employer. If a more sophisticated approach is warranted or desired, the full cost of the vesting benefit obtained by means of the vesting cost ratios or some other equivalent method has to be offset by the projected value at the normal retirement age of the participant contributions at termination. A valuation procedure by entry age and duration enables an estimate of the accumulated contributions of participants at termination to be derived in much the same way as the value of future pension accruals in a career average plan. If a claim rate were appropriate, this would again have to be incorporated in the calculations. The projected increase in expected survivors to normal retirement age as calculated earlier would complete all requirements for the calculation.

In a plan which is partly contributory, for example, where a basic noncontributory benefit is provided together with a supplementary benefit conditional on contributions, a combination situation occurs. The cost of vesting will depend upon the vesting benefit provided, and the actuary may find himself using one set of vesting cost ratios on the noncontributory benefit and another (incorporating, *inter alia*, a claim rate) for the contributory benefit.

EXPERIENCE GAINS AND LOSSES ON VESTING

Benefits to vested terminated participants are valued on an accrued benefit basis. They may be a source of gain if claims for benefit at the normal retirement age fail to be made. In order to avoid a mounting list of unclaimed benefits and potential escheat to a state exchequer, it may be desirable to adopt a rule of the plan voiding the benefit if it is not claimed within a definite period after the normal retirement age.

There is also the question of gains and losses on the experience of the plan. If terminations do not occur or occur in lesser numbers than expected, operating losses in respect to the major benefits will occur and the unused contributions in respect to vesting benefits will help offset the loss. If terminations occur in greater numbers than expected, gains will emerge. Some part of those gains may be absorbed in providing for the vesting benefit. In fact, the contributions with respect to the vesting benefit and more may be required to provide for the vested benefits. This should not involve the plan in over-all loss since, in order to develop vested benefits in excess of expected amounts, terminations in excess of expected numbers must first have occurred. Thus, the release of reserves on excess terminations must precede the re-absorption of those reserves on excess vesting.

COST ESTIMATES WITH MODEL PENSION FUNDS

No attempt has been made to present as part of this paper any over-all estimates of the cost of vesting as related to age and service distributions of participants in a model pension fund. For such an estimate to have relevance, it would be necessary to develop an age and service distribution which could have been the result of termination of employment experience for which the service table withdrawal rates would have been appropriate. For example, the results of applying vesting cost ratios derived from a high withdrawal rate to an age and service distribution developed from a low withdrawal experience would produce incorrect estimates of the over-all cost of vesting. A step in the direction of an over-all cost estimate would consist of assuming an age distribution

proportionate to the numbers of l_x in the service table and applying the appropriate vesting cost ratios like to like, that is, high withdrawal rate cost ratios to the l_x derived from a high withdrawal rate, medium to medium, and low to low. However, even these values of l_x stem from a number of entrants at a single entry age, and that not only is a rare feature in a pension plan but also returns us to the theoretical vesting cost ratios appropriate to that age.

A more elaborate statistical device would establish relative proportions of new entrants at selected entry ages. This expands the necessary arithmetic to the stage where access to a computer becomes desirable. Furthermore, the result is a somewhat stilted model of the mature population type, which is rarely met in practice. The estimated cost of the introduction of a vesting benefit or the change from one vesting schedule to another would in such circumstance not be applicable to any existing pension plan and would probably not be sufficiently accurate to warrant publication. The application of the appropriate vesting cost ratio to existing plans and the tabulation of results for examination would also require the statement of so many ancillary details if a sound judgment is to be made by any person other than the actuary responsible for the calculations that the project appears to be larger than can be accommodated in this paper.

CONCLUDING REMARKS

While the mathematics of determining the cost of vesting in benefits forms the major portion of this paper, the philosophical or managerial aspects must not be overlooked. The original pensions were granted on retirement from active employment, and persons who failed to stay the course failed to obtain the prize. No second thoughts were wasted on them. This is a far cry from the current concept that staying half the course might be worth a half-prize! This would have been a logical approach on the pension-equivalent-to-wage argument, but the probability is that it may have arisen from a general fair-treatment idea applied, first of all, retroactively in a case of hardship and then gradually as a basic concept applicable to all active employees.

The fact is that, as with many other things, concepts either stagnate and wither or else develop and flourish. The pension concept started on what would now be termed a "subsistence basis," developed, possibly thousands of years ago, into a cash transaction, and became consolidated in a flourishing condition under the pension fund practices of the past two or three centuries. It is now extending the philosophy of *retirement* allowances to cover early retirement, whether on account of

age or disability, and vested benefits for terminated employees not preparing to retire immediately from employment. It almost appears as if an attempt is made to establish a social right to accrued pension benefits. This "right" is entrenched in governmental systems of social security and appears, under the influence of writers on the social aspects of pensions, to be creeping over toward private pension plans.

The granting of vested rights to pensions is not favorable to management aspirations. In general, management desires to retain its skilled labor, white-collar or blue-collar, and provides a pension plan as part of the incentive to stay. The freezing of accrued benefits, in a form which can be collected later, favors the employee who wishes to break away from his employer. Occasionally, the employer concurs in the break and, in that case, the plan provisions on vesting help to forward the joint desires of the parties. In the main, however, vesting benefits aid mobility of labor and may be against the interests of management. The assistance afforded by a vested benefit to mobility of labor explains why governments are in favor of vesting, since economists have long preached that full employment rests at least in part upon mobility of labor.

Let us consider for a moment what could happen if vesting in benefits were carried to a final logical conclusion. This would seem to be a condition of affairs in which every employee vested in a pension benefit for every hour worked. A little thought suggests that, before this could be carried out or enforced, a system of uniform pension plans might have to be set up, one for *every* employer regardless of his financial or managerial capacity. In fact, rather than accomplish this, by such individual pension plans, it is probable that a central pension plan would be established and a staff set up to manage it. It needs only a short step from this to arrive at a situation in which all pension plans have been absorbed into a central government retirement system, with a gigantic financial trust to hold and invest the funds accumulating and a vast bureaucracy to manage it.

This picture cannot be contemplated with equanimity, and one wonders where the point of resistance should be established. There are many points at which resistance can be offered. The vesting benefit is clearly one of the most important points of resistance, and serious consideration should be given both by employer and employee to the consequences of unfettered development of the benefit.

This paper provides the means of determining the cost of vesting in benefits, or some part of them, accrued at termination of employment. Acceptance of these costs as a necessary burden by the employer and

enjoyment of the benefit by employees (or rather by *former* employees) should not be permitted to form a platform for continuous extension of the benefit under the guise of "liberalization." A reasonable vesting in benefits should be regarded as the mark of a well-designed plan and an enlightened employer but overdevelopment of the concept might lead to unpalatable consequences. It is hoped that in the discussion which this paper will open, some consideration may be given to this philosophical aspect of the subject and that suggestions will be made with regard to the reasonable limitations to be placed on the benefits.

APPENDIX I

COST OF VESTING IN PENSIONS

(Vesting: 100 Per Cent at 10 Years of Credited Service; Service Table: $q_x^{\frac{1}{2}}$, $a-49$; $q_x^{\frac{2}{3}}$, as Shown; $q_x^{\frac{1}{2}} = 0$ for All Ages; $S_x = 1$)

x	$q_x^{\frac{1}{2}}$	w_x	$1 - 1/2 p_x + 1/2$	Survivors to 65 (2) × (3)	Benefit Proportion (x-20)/45	(4) × (5)	Σ(6)	Vesting Cost Ratio (7) ÷ 1.05	Benefit Proportion (x-25)/40	(4) × (8)	Σ(10)	Vesting Cost Ratio (11) ÷ 1.05
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
						Entry Age 20				Entry Age 25		
30.....	0.0555	2,143	0.7822	1,676	0.2222	372.4	7,118.6	0.4594				
31.....	.0525	1,912	.7830	1,497	.2444	365.9	6,746.2	.4354				
32.....	.0495	1,706	.7839	1,337	.2667	356.6	6,380.3	.4118				
33.....	.0480	1,570	.7848	1,232	.2889	355.9	6,023.7	.3888				
34.....	.0450	1,401	.7858	1,101	.3111	342.5	5,667.8	.3658				
35.....	.0420	1,247	.7869	981	.3333	327.0	5,325.3	.3437	0.250	245.2	4,671.8	0.3015
36.....	.0405	1,149	.7880	905	.3556	321.8	4,998.3	.3226	.275	248.9	4,426.6	.2857
37.....	.0375	1,020	.7892	805	.3778	304.1	4,676.5	.3018	.300	241.5	4,177.7	.2696
38.....	.0360	941	.7905	744	.4000	297.6	4,372.4	.2822	.325	241.8	3,936.2	.2540
39.....	.0345	868	.7920	687	.4222	290.0	4,074.8	.2630	.350	240.4	3,694.4	.2384
40.....	.0330	799	.7935	634	.4444	281.7	3,784.8	.2442	.375	237.7	3,454.0	.2229
41.....	.0315	737	.7951	586	.4667	273.5	3,503.1	.2261	.400	234.4	3,216.3	.2076
42.....	.0315	711	.7970	567	.4889	277.2	3,229.6	.2084	.425	241.0	2,981.9	.1924
43.....	.0300	655	.7991	523	.5111	267.3	2,952.4	.1905	.450	235.4	2,740.9	.1769
44.....	.0285	601	.8015	482	.5333	257.1	2,685.1	.1733	.475	228.9	2,505.5	.1617
45.....	.0270	552	.8044	444	.5556	246.7	2,428.0	.1567	.500	222.0	2,276.6	.1469
46.....	.0255	505	.8074	408	.5778	235.7	2,181.3	.1408	.525	214.2	2,054.6	.1326
47.....	.0240	462	.8109	375	.6000	225.0	1,945.6	.1256	.550	206.3	1,840.4	.1188
48.....	.0225	420	.8150	342	.6222	212.8	1,720.6	.1110	.575	196.6	1,634.1	.1054
49.....	.0210	381	.8196	312	.6444	201.1	1,507.8	.0973	.600	187.2	1,437.5	.0928
50.....	.0195	345	.8247	284	.6667	189.3	1,306.7	.0843	.625	177.5	1,250.3	.0807
51.....	.0180	309	.8305	257	.6889	177.0	1,117.4	.0721	.650	167.1	1,072.8	.0692
52.....	.0165	277	.8369	232	.7111	165.0	940.4	.0607	.675	156.6	905.7	.0585
53.....	.0150	246	.8440	208	.7333	152.5	775.4	.0500	.700	145.6	749.1	.0483
54.....	.0135	215	.8519	183	.7556	138.3	622.9	.0402	.725	132.7	603.5	.0389
55.....	.0120	188	.8606	162	.7778	126.0	484.6	.0313	.750	121.5	470.8	.0304
56.....	.0105	160	.8702	139	.8000	111.2	358.6	.0231	.775	107.7	349.3	.0225
57.....	.0090	134	.8807	118	.8222	97.0	247.4	.0160	.800	94.4	241.6	.0156
58.....	.0075	109	.8923	97	.8444	81.9	150.4	.0097	.825	80.0	147.2	.0095
59.....	0.0060	87	0.9050	79	0.8667	68.5	68.5	0.0044	0.850	67.2	67.2	0.0043
60 and over.....												

APPENDIX II

COST OF VESTING IN PENSIONS

(Vesting at 10 Years 50 Per Cent + 10 Per Cent per Year to 100 Per Cent at 15 Years;
Service Table: $q_{\frac{1}{2}}^a$, $a-49$; $q_{\frac{1}{2}}^z$, as Appendix I; $q_{\frac{1}{2}}^{z'} = 0$ at All Ages)

x	SURVIVORS × BENEFIT PROPORTION	VESTING PROPORTION	(1)×(2)	Σ(3)	VESTING COST RATIO	SURVIVORS × BENEFIT PROPORTION	VESTING PROPORTION	(6)×(7)	Σ(8)	VESTING COST RATIO
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Entry Age 20					Entry Age 25				
	Appendix I, Col. 4									
30.....	372.4	0.5	186.2	6,573.5	0.4242					
31.....	365.9	0.6	219.5	6,387.3	.4122					
32.....	356.6	0.7	249.6	6,167.8	.3980					
33.....	355.9	0.8	284.7	5,918.2	.3819					
34.....	342.5	0.9	308.2	5,633.5	.3636					
35.....	327.0	1.0	327.0	5,325.3	0.3437	245.2	0.5	122.6	4,305.1	0.2778
36.....						248.9	0.6	149.3	4,182.5	.2699
37.....			Other Values as			241.5	0.7	169.0	4,033.2	.2603
38.....			Appendix I			241.8	0.8	193.4	3,864.2	.2494
39.....						240.4	0.9	216.4	3,670.8	.2369
40.....			Col. 6	Col. 7	Col. 8	237.7	1.0	237.7 etc.	3,454.4	0.2229

APPENDIX III

COST OF VESTING IN PENSIONS

(Vesting Cost Ratios: 100 Per Cent Vesting at 10 Years of Service;
Service Table: q_x^a , $a-49$; q_x^s , as Shown; $q_x^r = 0$; $S_x = 1$)

x	TERMINATION TABLE No. 1			TERMINATION TABLE No. 2			TERMINATION TABLE No. 3		
	q_x^s	Vesting Cost Ratios		q_x^s	Vesting Cost Ratios		q_x^s	Vesting Cost Ratios	
		Entry Age x	Entry Age 20, Attained Age x		Entry Age x	Entry Age 20, Attained Age x		Entry Age x	Entry Age 20, Attained Age x
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
20.....	0.4565	0.4565	0.3248	0.3248	0.1447	0.1447
25.....3015	.45652218	.32481036	.1447
30.....	0.0555	.1954	.4565	0.0370	.1484	.3248	0.0185	.0697	.1447
35.....	.0420	.1176	.3437	.0280	.0914	.2521	.0140	.0439	.1147
40.....	.0330	.0610	.2442	.0220	.0483	.1846	.0110	.0236	.0863
45.....	.0270	0.0224	.1567	.0180	0.0180	.1214	.0090	0.0088	.0582
50.....	.01950973	.01300666	.00650325
55.....	.01200313	.00800251	.00400124
59.....	0.0060	0.0044	0.0040	0.0036	0.0020	0.0017
60.....

Vested benefit: $1/(65 - x)$ of benefit at normal retirement age.

APPENDIX IV
COST OF VESTING IN PENSIONS
SALARY SCALE ADJUSTMENTS

*	ENTRY AGE 20							ENTRY AGE 25				
	No. 1: $S_x = 1$ $(x-20)/$ $(65-20)$	No. 2: $S_x = v^{65-x}$, 3 Per Cent			No. 3: $S_x = S_3^*$			No. 1: $S_x = 1$ $(x-25)/$ $(65-25)$	No. 2: $S_x = v^{65-x}$, 3 Per Cent		No. 3: $S_x = S_3^*$	
		S_x	Career Av. Adjustment Formula (8)	Final Av. Adjustment Formula (10)	S_x	Career Av. Adjustment Formula (8)	Final Av. Adjustment Formula (10)		Career Av. Adjustment Formula (8)	Final Av. Adjustment Formula (10)	Career Av. Adjustment Formula (8)	Final Av. Adjustment Formula (10)
30.....	0.2222	3.554	0.1236	0.0790	0.2890	0.0847	0.0571
35.....	.3333	4.120	.2006	.1373	.3770	.1504	.1180	0.2500	0.1520	0.1030	0.1192	0.0853
40.....	.4444	4.776	.2898	.2122	.4758	.2349	.1943	.3750	.2467	.1791	.2068	.1640
45.....	.5556	5.537	.3932	.3085	.5838	.3403	.3019	.5000	.3564	.2768	.3160	.2717
50.....	.6667	6.419	.5131	.4280	.7051	.4685	.4403	.6250	.4835	.4012	.4490	.4128
55.....	.7778	7.441	.6521	.5788	.8403	.6228	.6170	.7500	.6310	.5581	.6089	.5950
60.....	0.8889	8.626	0.8132	0.7668	0.9450	0.8032	0.8200	0.8750	0.8019	0.7550	0.7960	0.8072

* *Actuary's Pension Handbook*, by T. F. Crocker, H. M. Sarason, and B. W. Straight (published by the authors).

DISCUSSION OF PRECEDING PAPER

ROBERT F. LINK:

This discussion takes off from Mr. Marples' "Concluding Remarks," particularly the invitation in his last sentence to consider philosophical aspects.

People are quite excited these days about vesting (among other things). Some of them are dissatisfied with the current state of vesting provisions, or lack thereof, in pension plans. They hope to see significant or even drastic improvement.

Thinking of unilateral plans, the primary agency for improvement is employers. Why should employers introduce or enlarge vesting? Perhaps they have some "social" aims. For example, they may just think vesting is "good." Or they may think that enlarged vesting will inhibit increases in social security, and they consider this good.

Aside from social aims, an employer has two major objectives in setting up or continuing a pension plan. These are (1) to help attract and hold desirable employees and (2) to facilitate the removal of employees whose usefulness (either in fact or by adopted criteria) has substantially ended. We would expect employers to include vesting provisions in their plans that they regard as appropriate to further both these objectives. Such vesting provisions are likely to be measured against the norms in the labor markets in which an employer competes. If he gives too little vesting, an employer must balance this by other attractions. If he gives too much, he must justify the resulting cost by some alleged or demonstrated advantage.

If employers do not increase vesting of their own accord fast enough to appease critics of the present situation, we should not be surprised. Are there other forces to accelerate progress toward greater vesting? Employees covered under unilateral plans are probably an ineffective force. They choose an employer for a variety of reasons, and the pension plan is usually a minor factor. Most employees have no voice in the terms or management of unilateral plans.

Union employees, of course, have a very effective voice. Vesting under collectively bargained pension plans will increase about as fast as it is demanded. Any such demand, of course, reflects a priority for vesting as against alternatives such as higher benefits for age retirement.

The other major potential influence is government. The states have in general not been heard from. It seems that the federal government

wants earlier vesting. There are several levers that may be pushed to encourage the desired result. These include (1) specific vesting provisions as a requirement for plan qualification; (2) the "second layer" idea; (3) the threat of increased social security; and (4) possible tax incentives.

Mr. Marples' paper reflects a widely held view that private pensions are a private matter and that governmental requirements on such subjects as vesting represent unwarranted interference. This is a difficult and confusing subject. In the interest of balanced consideration, I want to list several points supporting the opposite view, with brief comments on each:

1. *Mobility of labor.*—It is alleged that absence of vesting ties employees to unwanted jobs and prevents the most effective employment of the labor force. There is little statistical evidence. Of course, many of us are acquainted with situations in which an employer and an employee would both be quite happy to part company, except for the destruction of a valuable pension expectation. It is a question whether such cases justify a major change.

2. *Frustrated pension expectations.*—There is concern, expressed by Bernstein and others, that too many persons covered by private pensions will receive no pension or a much smaller pension than they had hoped for. Serious questions have been raised about the over-all effectiveness of the private pension industry's leg of the three-legged stool. It would be pleasant to reply with facts.

3. *Pensions as compensation.*—A critic could say that the compensation afforded by many pension plans is quite strange. Suppose a plan provides vesting after fifteen years. This plan could be described as providing nothing for the first fifteen years of service, a whopping deferred-benefit bonus at the end of that period, and reasonable deferred benefits for service thereafter. These benefits bear little relationship to the value of the services rendered by the employee in expectation of a pension.

4. *Individual financial planning for retirement.*—An employee covered by the plan described above is considering what he should personally save for retirement. If he were sure that he would be employed for fifteen years, he might save at one level. The uncertainty on this point may cause him to save at a higher rate (or, more likely, to lie awake nights worrying because he has not). In short, it is difficult to make rational allowance for some typical pension plans in establishing one's personal savings program.

5. *Federal income tax treatment of private pensions.*—This one is perhaps more of an excuse than a reason. It is alleged that federal government intervention in private pensions is justified by the existence of a substantial tax subsidy. We in the private pension industry should have a better articulated position on this complicated subject. In a rather convincing article, entitled "The Myth of Special Tax Concessions for Qualified Pension Plans" (*Iowa Law Review* [Spring 1966]), Raymond Goetz argues that special tax concessions for qualified pension plans are a myth.

Most of the arguments for greater vesting derive more or less from the concept of pensions as compensation. The arguments against are associated with the concept of pensions as a reward for faithful service. An interesting socioeconomic study could be made of (a) how the present characteristics of pensions derive from the conditions of earlier times, (b) how these two concepts of pensions relate to the historical background, and (c) which concept better serves the needs of society today.

The actuary has a problem. He is inclined by nature to be conservative and to support freedom of contract and so forth. He is concerned about the interests of his pension clients and wary of threatened governmental action that is potentially adverse to those interests. On the other hand, he sees that governmental action might take a more palatable form if he were to contribute constructively in the planning stages. We should not sacrifice any principles. However, I wonder whether we would do better in some cases to help show how something can be done rather than concentrating on why it cannot or should not be done.

By way of illustration, some have suggested that any mandatory vesting should apply only to future-service benefits. For this purpose, future-service benefits would be those accruing for service after a plan is introduced or after a plan is amended to introduce the mandated vesting. Such an approach would greatly reduce vesting costs. It seems to go as far as necessary to meet the current criticism. However, many plans do not directly associate benefit accruals with periods of service. Pension experts have pointed out the difficulties of defining a benefit for vesting purposes under such plans. An effort to split an accrued pension into past and future service would compound these difficulties.

If anyone can solve these problems, it is the actuaries. Maybe we should give it a serious try. There is scope here to substitute new facts for old impressions, whether or not we advocate mandatory vesting.

WILLIAM K. WHITE:

I am certainly in full accord with the introductory comment in Mr. Marples' interesting paper to the effect that a discussion of vesting is most timely. This seems to be an area in which there is a dearth of literature in the *Transactions*, and thus we are especially fortunate to have two papers at this time presenting somewhat different mathematical techniques for estimating the possible cost of vesting. I am sure that the mathematical concepts presented in both these papers will be useful, timesaving tools to many actuaries for many of their pension plan valuations.

The comments in this discussion will be confined primarily to what

I shall term the practical, rather than the theoretical, aspects of the subject and perhaps to a considerable extent to what these papers did not say rather than to what they did say.

While many of the following comments will apply, to a considerable extent, equally to Mr. Marples' paper and the one by Mr. McGinn, they are being appended to Mr. Marples' primarily because it seems to delve into the practical considerations of vesting cost somewhat more than Mr. McGinn's paper does.

It is well known that many unions have become increasingly interested in having vesting provisions in the pension plans that they are negotiating, and many employers with unilateral plans have been adding a vesting provision when none existed or liberalizing an existing one. More recently, great interest in the subject of vesting has been generated by the "Report to the President on Private Employee Retirement Plans" by the President's Committee on Corporate Pension Funds (hereinafter referred to as the President's Committee's Report) issued in January, 1965. The Committee concluded that some form of mandatory vesting should be legislated, but it reached this conclusion on the basis of certain pardonable misunderstandings. I use the term "pardonable" because they were based on quite popular misconceptions which actuaries should be able to clear up but seem to have done very little about heretofore. For example, the President's Committee's Report opens the section on vesting with the following statement:

A vesting provision, as usually defined, *guarantees* to pension plan participants whose employment is terminated before becoming eligible for a retirement benefit the right to all or part of their accrued pension benefits at retirement age, regardless of their employment status at that time [emphasis added].

Mr. Marples is to be commended for having exploded this myth when he points out near the end of the second section that the guarantee exists only "provided the plan has sufficient assets to make the payments." When people of the stature of those comprising the President's Committee have such a basic misunderstanding, it is not surprising that it is very widespread. Because of the importance of this point, it might well have been stressed considerably more in the paper.

Perhaps the area of greatest public misunderstanding on the subject of vesting is the very title of this paper "Cost of Vesting in Pensions." There is far too prevalent an impression among laymen that it is only necessary to develop a reasonable set of turnover or termination rate assumptions, apply an actuarial formula, and out comes the cost of vesting. While Mr. Marples does refer to "theoretical" or "estimated" costs in several instances, it is my impression that the paper does so in such a

way as to lend further credence to this popular misconception rather than to scotch it. Perhaps this can best be illustrated by the section headed "Factors Affecting the *Cost* of Vesting" (emphasis added). The first sentence states that, among the items needed to determine the "theoretical cost," are *estimates* "of the number of survivors" and "of the amount of benefits." With such tools, we find in the next sentence a statement that we are now ready to develop a method "for determining the cost of vesting." While an actuary would subconsciously read into all this that only estimates are being considered throughout, the layman could hardly be blamed for taking these statements as authoritative confirmation that the impression described above is correct—that is, that, given estimates of turnover or survivors, one can readily determine costs, *real* costs.

It is unfortunate that this very important difference between estimated and true costs was not brought out clearly, and it could have been emphasized by a more careful choice of words. In my opinion it is vitally important that this be made eminently clear lest employers and unions adopt vesting provisions with an erroneous understanding of what their cost really is. Of perhaps even greater importance is the avoidance of governmentally mandated vesting requirements based on such a misunderstanding of costs. This is by no means remote, when one considers the following statement from the President's Committee's Report:

On the basis of available information and actuarial estimates, the Committee is convinced that a modest vesting requirement, with rare exceptions, would not have a substantial impact on pension costs. Precise cost estimates will, of course, depend on the final detailed specifications of the vesting requirement.

Although a precise estimate is a somewhat unrealistic concept, at least reference is made to "estimates." However, when one considers the following summary quotation, it seems evident that the Committee suffered from this common misunderstanding:

The most complete available information indicates that the added cost of adopting the basic features of the Committee's recommendation would be quite modest; that is, under 5 percent for a large majority of pension plans. Rarely would the added cost exceed 10 percent of present expenditure, and special provision should be made available for any plan where the cost of the vesting recommendation . . . would have this effect.

It is hard to believe that in making this statement the Committee was thinking of anything but the estimates as representing precise costs. It is equally hard to believe that they could possibly have been contemplating

a situation in which, say, ten years hence, it is found that the proposed vesting provision has actually cost perhaps 35 per cent instead of the anticipated, let us say, 8 per cent.

The misunderstanding is equally apparent in the following question taken from the agenda of the Federal Interagency Task Force on Private Pension Plans for its meetings with several private groups: "How significant are the differences in cost between 5-, 10-, and 20-year vesting standards?"

Since this question was drafted, the Task Force has had several meetings, and I am confident that it would not ask the same question in the same form today. This group is to be commended for the earnestness with which it is seeking information on pension matters prior to considering possible legislation and for its interest in implementing the Society's motto of substituting facts for appearances. It seems evident, however, that initially it had the common misconception that actuarial estimates are real cost figures.

In reading Mr. Marples' paper, I was looking forward to the possibility of the section entitled "Experience Gains and Losses on Vesting" clearing up the misunderstanding between actuarial estimates and true costs. Instead, however, about a third of this brief section that might have been the most important in the paper relates to the "Enoch Arden" clause used almost universally—in this country at least—to provide for canceling benefits if they are not claimed within a period of time stated in the plan.

The remainder of this section gives a very broad-brush statement of the impact of gains and losses, which is correct only in limited situations. Consider, for example, the following statement: "In order to develop vested *benefits* in excess of the *expected amounts*, terminations in excess of the expected *numbers* must first have occurred" (emphasis added). On the contrary, it is well within the realm of possibility that the *number* of terminations may be equal to or less than expected over a period of time, but the total vested *benefits* could be greater than expected because the average amount per individual is greater than expected, perhaps because the terminations occur at a later age than anticipated. The fact that the individuals who were expected to terminate stayed to retirement and thereby receive the full benefit makes the resulting loss all the greater.

To further emphasize that it is not just the relation of actual to expected number of terminations that determines whether there will be a gain or loss, consider a case with the experience of vested terminations being heavier than expected and at the same time the average vested

benefits also being greater than expected. Is there an experience gain or loss? Actually, the answer can go either way, depending upon the degree of deviation in each item in the particular case at hand.

All this points up the fact that the average amount of vested benefits compared with the expected is just as important as the number of terminations compared with the expected. In fact, to make a complete analysis, the number of actual to expected terminations before and after attaining the vesting requirements should be considered separately.

A more careful and detailed explanation of the impact of gains and losses on vesting costs, together with an analysis in layman's language of the variety of consequences, would be a valuable addition to actuarial literature.

If, then, actuarial cost estimates do not represent the real cost of vesting, the question might be asked whether they represent what might reasonably be expected on the average. The answer to this is a qualified "Yes." The reasons for qualification are several. First, experience can deviate radically from expectations, no matter how carefully and scientifically they are developed. Second, the actuary has little to go on in this area except past experience, and there is a considerable degree of uncertainty as to whether historical experience is going to be appropriate to the future in any given situation. For example, the mere addition of a vesting provision to a plan that did not have one previously may well change the pattern of terminations, and the more liberal the plan is the more dramatic the change is likely to be. This is based on the premise that those who might be thinking of changing jobs just prior to completing the vesting requirement will often tend to stay on their present job, just long enough to complete the vesting requirement.

As another consideration, there is nearly an endless list of factors which will result in variations in the cost of vesting. Without going into detail, it is probably fair to say that nearly all the factors that would affect the true cost of vesting would fall into one or more of the following broad categories: (a) characteristics and composition of the employee group; (b) economic environment; and (c) characteristics of the pension plan.

It is important to note that factors in all three of these categories can and do change from time to time and correspondingly influence the cost of any particular vesting provision over the years. Furthermore, changes in turnover patterns may be so dynamic as to make the cost of vesting one of the most difficult phases of pension costs to estimate with any sense of security.

With all these potential elements of uncertainty, then, one may well ask, "Why bother with actuarial estimates of vesting costs?" The answer to this is simply that, if a vesting provision is included in a pension plan, realistically its cost must be recognized as appropriately as possible. The actuarial estimate may very well recognize differences in turnover pattern by industry and perhaps by company within the industry; perhaps by locale within the company or within the industry, or both; and perhaps by sex. But once again we must not lose sight of the fact that this is based on historical information, and changing conditions in one or more of the three categories mentioned above can have an important effect on the appropriateness of the assumptions used.

In the final analysis, however, the actuary has little foundation for any basis of anticipating the future except (perhaps the most unlikely) that past trends will continue. Under such circumstances and particularly for smaller groups in which chance fluctuations are likely to be the greatest, it is understandable that the actuary will quite appropriately recognize the unreality of extreme refinement in vesting cost estimates and pay closer attention to how experience is developing under the plan and making appropriate changes in the assumptions as circumstances dictate. Once again, this all points up the importance of making it clear to the layman that actuarial cost estimates are just that, although they may and undoubtedly do turn out to be reasonably accurate the majority of the time. Conditions can and do develop in particular situations, however, where they turn out to be meaningless.

Mention was made earlier in this discussion of the propriety of the use of averages or of something mentioned in Mr. Marples' paper, which is closely related, model groups. To illustrate how misleading averages can be, an analysis was made recently of actual turnover experience of some ten pension plans by both life insurance company and consulting actuaries. Unfortunately, it was not feasible to relate the turnover experience to the expected cost; instead it was related to the normal cost. It was found that for a five-year-service vesting requirement the cost ranged from 3 to 93 per cent of normal cost, depending on case and year involved. For these same cases, had the vesting provision been twenty years of service, the added cost would have ranged from 0 to 24 per cent. It is interesting to note that the 0 per cent related to a group of nearly 500 employees with more than twenty years' service in a company whose over-all turnover experience would be classified as heavy. In any event, with such wide ranges developing, it is clear that it is of little solace to the company whose actual turnover experience deviates sharply in the

wrong direction from what is expected so that it averages out with another company that has gained a windfall because its termination experience is more favorable than expected.

Every actuary knows that his pension vesting cost estimates are no more than estimates and that only experience over the years will determine the true cost of any given vesting provision. He cannot, for example, prognosticate that ten years, or even one year, hence a major plant or operation will be closed down. This sort of thing happens all the time and, depending on the relative size of the employee group involved, can have a major impact on true vesting cost, either favorably or unfavorably, depending on the circumstances. With these considerations in mind, it is probably more important than ever now, with so much interest in vesting in private pension plans on the part of various branches of the federal government, employers, unions, and others, for actuaries to lift the shroud of mystery on the full cost implications of vesting provisions and to do so in such a way that it will be clearly understandable to the layman.

At the conclusion of his paper Mr. Marples invited discussion of the philosophical aspects of the subject. I am not sure whether some of the foregoing falls in that category, but, if it does, I suspect that it is not the type of philosophical commentary Mr. Marples contemplated.

WILMER A. JENKINS:

This paper includes either one too many or one too few sections. The final section, headed "Concluding Remarks" and concerned with the philosophy of vesting, should either have been omitted or made complete.

The paper develops quite fully a general statement that it makes that management desires to retain its skilled labor and, therefore, designs its pension plan to provide an incentive for employees to stay put in their jobs. The paper does not, however, mention a number of other important considerations in plan design that today are recognized by informed employers. These considerations raise the question of the validity of the author's statement that "the granting of vested rights to pensions is not favorable to management aspirations." Management aspirations are much broader than the lowest possible turnover. They also indicate that vesting is not solely for the benefit of ex-employees, as stated by the author.

The considerations to which I refer are mainly these:

1. An employer cannot keep an employee that he cannot attract to his establishment in the first place. One major purpose of a pension plan is to attract prospective employees, especially superior people, and for this purpose a good

vesting provision can be important. The smart young man may be unwilling to select the employer whose pension plan pays nothing if the man's selection proves to be the wrong one and should be changed later. The older, much-needed, experienced man can be taken on by the employer without incurring a substantial pension liability if the man already has a vested benefit from a previous employer.

2. Independence and self-respect, though intangibles, are very important to employees and, consequently, to employers. Throughout his working career—in the early, middle, and late years alike—an employee does a better job if he knows that, if his present job does not work out well for him or for his employer, he will reach retirement age with retirement income based on his years at his present job. Nonvested benefits encourage a middle-aged or older man to "rest on his oars" and merely to keep out of trouble until he reaches retirement age. They also encourage the employer to put up with such actions.

3. Paradoxically, nonvested benefits have the least holding power on the very people that the employer wants to keep the most—the highly trained, skillful, key people. A new employer can afford to compensate such a person for his loss, especially if he will be very valuable. The holding power of nonvested benefits is strongest for the mediocre and inefficient, and creates difficult problems when the unsatisfactory have to be dismissed. Therefore, the objective of management stated by the author tends to backfire, downgrading the quality of the employer's staff.

4. The author's fear of a central, governmental pension plan resulting from a requirement of universal vesting is in opposition to the fact that many proponents of federal social security cite, as their principal argument, that, because private pensions are nonvested and therefore "do not do the job," the federal vested plan should be expanded more and more, even to the point of replacing private plans. We are already seeing some of this process. Unfortunately for the private plans, the record of retirement benefits actually paid lends weight to this argument.

While the author does not mention these and other considerations, he seems to acknowledge their importance on his final conclusion that "a reasonable vesting in benefits should be regarded as the mark of a well-designed plan and an enlightened employer. . . ." One cannot disagree with this conclusion, although the word "reasonable" can have many meanings. No doubt, its meaning is in practice determined in the light of prevailing practices and the degree to which they are fulfilling the fundamental purpose of providing retirement income where needed, as modified by current informed opinion among the public, labor unions, and government officials. It is these broad forces, and not only the calculated normal cost of the employer, that have determined the recent and current trend, which has been gradually to more vesting. The prevailing vesting age is no longer 65; ages of 45 or 40 or less have become

more common. The prevailing nonvested period, likewise, has shortened to fifteen years or ten years or less. Most authorities agree with the author that vesting could reach a point at which it is overdeveloped, but there are many opinions as to where this point lies. How long and how far this trend will continue, and whether the point of maximum development has been approached or reached, are questions dependent on so many broad intangibles that they are largely unanswerable.

FRANK L. GRIFFIN, JR.:

Mr. Marples has made another interesting contribution to the actuarial literature, not the least important part of which is to be found in the "Concluding Remarks" section of his paper, which deals with the philosophical aspects of vesting. My comments will be confined to this subject.

Mr. Marples has pointed out the difference between the underlying philosophies that originally led to the adoption of private pension plans and the concept of pensions as deferred compensation, which has evolved in recent years. The idea that pension contributions "belong to the employee because he would otherwise have received larger cash compensation" may be open to question, but it is nonetheless one of the mainstays of the new philosophy that is developing. Strictly speaking, this latter concept logically would result only in money-purchase plans providing fully vested benefits for future service, but proponents of the idea are not deterred by logic from applying the concept to all types of plans. I agree with Mr. Marples' observations relative to the ultimate state of affairs if the principle of vesting were carried to its final conclusion.

With the present increasing tempo of governmental intrusion into private pension plans, one may well predict the following sequence of events:

1. If we begin by requiring private plans, as a condition of favorable tax treatment, to confer vesting after a certain period of time, would this not lead to a gradual shortening of the period mandated for vesting and the institution of a relatively short period over which past-service liabilities must be amortized?
2. Would not the developments mentioned in item 1 lead some employers to drop their funded plans in favor of unqualified pay-as-you-go plans, which provide even less security to employees?
3. Would not the planners then feel it necessary to legislate compulsory funding of pension plans and to outlaw pay-as-you-go plans?
4. Would not this, in turn, lead some employers to substitute profit-sharing plans for pensions?
5. Eventually, would not the planners therefore feel compelled to legislate *compulsory private* pension plans? (A conflict in terms.)

Thus, by yielding what may appear to be a relatively small point initially, we may eventually run the full gamut of regulation.

We have come to a point at which we should examine more closely the basic objectives and philosophy of private pensions. Have we indeed reached the point where many of the provisions of such plans must be uniform, without regard to the legitimate personnel and financial objectives of private employers? Is there any greater justification for the dictation of pension provisions than there is for the dictation of wages to be paid by all employers regardless of differences between industries or within the same industry?

Who, for example, can say with assurance that immediate or near-immediate vesting is socially desirable or that employers should be denied the right to incorporate provisions designed to hold down turnover and the costs of recruiting and training? This is a far more important cost factor in some occupations than in others, as we all know.

In 1757, Benjamin Franklin wrote in *Poor Richard's Almanac*:

For the want of a nail the shoe was lost,
For the want of a shoe the horse was lost,
For the want of a horse the rider was lost,
For the want of a rider the battle was lost,
For the want of a battle the kingdom was lost—
And all for the want of a horseshoe nail.

What may seem to be innocuous pension proposals constitute a new twist to the loss of a kingdom for want of a horseshoe nail. Here the kingdom is freedom of choice under private pension plans, all of which may be lost because we were willing to give up a relatively small initial point—the horseshoe nail.

HARRY M. SARASON:

Mr. Marples has shown how to shorten some actuarial calculations for vested benefits in retirement plans by using intermediate mathematical factors for more than one purpose. The discovery and use of commutation columns are the best-known actuarial example of such a mathematical shortcut.

More actuarial formulas of this "multiple use of factors family" probably have already been made effective by those who program our computations for electronic calculating machines—at least one to my personal knowledge. Still more formulas of this general family are likely to be discovered in developing future electronic calculation actuarial programs.

We should be alert to recognize such formulas, to generalize such discoveries, to publish the discoveries, and, eventually, to incorporate them in our textbooks. But our students should always be prepared to

derive such formulas from basic functions such as q_x and i . Many formulas are derived from basic functions in the first place; this is the way programmers do much of their thinking, and, incidentally, this is the way many difficult questions are developed for our actuarial examinations.

In proving an actuarial formula, as elsewhere, "The proof of the pudding is in the eating." Actuarial formulas should be tested numerically—not tested in one instance only (it takes two to test a formula based on averages or aggregates) or even in two instances. (I speak from a sad experience. My formula worked on both ends of a series but not on the middle terms and I had to redo two weeks' work—and an assistant had warned me!)

To strike a brighter note, a method that works numerically in one or two instances may prove to be the keynote to a general formula. (I speak from several happy experiences—well, fairly happy experiences. Half an hour of numerical testing was always followed by up to five days of spare-time algebraic drudgery.)

CRAWFORD E. LAING:*

These comments arise from the "Concluding Remarks" in Mr. Marples' paper. I am addressing myself mainly to the philosophy of the subject rather than to the detailed calculations.

I agree with Mr. Marples that the philosophy of retirement provision has developed over the years from what he refers to as "a subsistence basis" to what is commonly referred to in Britain as the concept of "deferred remuneration." As soon as this latter concept makes its appearance, the case for vesting is greatly strengthened; this is particularly so when the employer is seeking to take credit, in his presentation to the employees, for the hidden fringe benefits that accrue as a result of employment by the company. Further, as stated by Mr. Marples, it is generally recognized by economists that full employment to some extent rests on mobility of labor, and vesting of pension benefits is definitely an aid to such mobility. It seems to me that management would have to take a particularly narrow and selfish view to regard mobility of labor as being against its interests. Efficient, forward-looking, and aggressive managements will, in fact, benefit from mobility of labor, in that they will be able to recruit the best brains available, when they are required for their companies' further development, without inhibitions as to pension costs if some degree of vesting has been achieved generally.

It seems to me that the important aspect to consider is the degree of

* Mr. Laing, not a member of the Society, is a Fellow of the Faculty of Actuaries.

vesting or, to put it in other words, the level of benefits that ought to vest on termination of employment. The present trend in Canada which insists, under the Pension Benefits Acts of Ontario, Quebec and Alberta, that the whole of the benefit provided under a private plan should vest at age 45 with ten years' service, no matter how generous the private plan, is a philosophy that is open to question.

I am, of course, used to the concept in Britain whereby vesting is mandatory in respect to a level of benefits equal to the maximum benefit that can be obtained by the employee under the Government Graduated Scheme, but it seems to me quite logical that in any legislation it is only open to the government to insist on a minimum level of vesting in benefits and that this should be independent of the total benefits provided under the private plan. The course that has been adopted in Canada so far seems to me to be inimical to the sound development of pension provision. This seems to be contrary to the intention of the legislation which, for example, in the case of Alberta, declares the function of the superintendent to be "to promote the establishment, extension and improvement of pension plans throughout Alberta." The legislation may have, eventually, quite the opposite effect when employers begin to recognize that a large part of the money spent on providing benefits under a formal plan is in fact going to benefit employees who will have left them before reaching retirement age.

As Mr. G. N. Calvert has stated elsewhere recently, this may undo all the good work of recent years in the building-up of private plans, funded—as they should be—over the working lifetime of employees. The trend may well revert to the informal, unregulated, and haphazard method of granting retirement gratuities or, alternatively, of leaving all the pension provision to the state.

If the alternative of a minimum level of benefit vesting at certain ages and lengths of service is adopted, the possibility of 100 per cent state pension provision envisaged above, and by Mr. Marples, should be avoided. I quite agree with Mr. Marples' penultimate paragraph, particularly with the point that the vesting provision is one of the most important points of resistance to further state encroachment. This is a "pass" which must be defended, and, if it is "sold" or allowed to go by default, it is not clear whether there is a defensible position to be taken up at a later stage.

The pension industry in Britain has been through similar vicissitudes, and it was careless enough to "sell the pass" that was so carefully prepared by Lord Beveridge in his historic report. This occurred when they allowed the state to provide graduated benefits related to earnings up

to the modest level of originally £15 per week and recently £18 per week. Although, there, the option of "contracting out" of the graduated scheme was granted originally, the conditions became progressively more onerous as the ceiling was raised, and, if the precedents set by the Conservative governments in the past are followed by the present Labour government, they could make "contracting out" a dead letter overnight. Canada has already reached a higher level of state provision, but without granting any option to contract out, and is in the course of dealing a mortal blow to the remainder of the private pension industry by the current Pension Benefits Acts. It may be too late to stem the flow, but if these views have any merit, maybe the provincial governments should have this situation brought to their notice.

It is, of course, recognized that our personal interests are probably served better by allowing the trend toward vesting. I personally have suffered considerably from the lack of any vesting requirements, having lost twenty-five years' past-service benefits (except for a minimum vested amount of £16 per annum!) when I left the insurance industry in Britain. Nevertheless, I think that it is our duty as actuaries to look at the long-term problems of the industry generally and advise accordingly, even when this conflicts with our own personal best interests.

DANIEL F. MCGINN:

Mr. Marples' paper demonstrates that he has a very intuitive grasp of the effect of vesting schedules upon the cost of pension plans. Mr. Marples has followed the classic "service table" approach in developing a method of measuring the costs of vesting pension benefits. This approach is probably typical of the method that many consulting actuaries use in valuing large pension plans. However, this approach is not very adaptable to measuring the cost of vested pension benefits for smaller pension plans that are often underwritten by an insurance company.

Some of the more interesting points that Mr. Marples' paper covers are:

1. The vesting cost ratios essentially measure, in the form of a percentage increase, the increase in the number of retirees under a pension plan which results from adoption of a specific vesting schedule.
2. The approach basically is limited to measuring the increase in normal cost on account of adopting a vesting schedule.
3. The introduction of salary scales effectively reduces the relative cost of vesting (i.e., the vesting cost ratio) even though the absolute cost of the plan, including the cost of vesting, increases as a result of introducing a salary scale. Conversely, vesting cost ratios developed on the assumption

of a level unit of pension benefit accrual would always produce higher vesting cost ratios than those developed by incorporation of a salary scale in the calculation of the ratios.

4. Vesting costs depend upon the level of withdrawal rates (rates of employment termination); that is, as turnover rates increase, the cost of vesting increases both relatively and absolutely. However, the total cost of the pension plan for a specified vesting schedule will decrease with an increase in turnover rates.
5. There is no measurable cost of vesting unless withdrawal rates (or turnover rates) are assumed in the actuary's cost calculations.

Although Mr. Marples' paper does not include a large number of vesting cost ratios, I note that the table of withdrawal rates included in his paper is quite similar to that shown as Employment Turnover Table IV of my paper. A comparison of my "Entry Age Level Annual Cost Indices" for Employment Turnover Table IV with Mr. Marples' vesting cost ratios shows a reasonable correlation. For example, for entry age 20 and 100 per cent vesting at age 30, his ratio is 45.6 per cent, while the vesting factor included in my index is 43.74 per cent. An examination of factors for other entry age-vesting age combinations with the factors in my paper for Table IV show a reasonable correlation, but the relative cost of vesting in my paper is lower because the employment termination rates stop at a lower attained age.

Basically, the formula that I have used in my paper to derive level annual cost indices is the same as shown in Mr. Marples' paper except that his formula is based on a service table and mine is based on probabilities. Although he has presented a very complete picture of the concepts underlying vesting costs, Mr. Marples could have made a stronger and more readily understood presentation if he had demonstrated in some manner how these factors would have been used in deriving numerical results for an actuarial valuation. I believe that this paper is a significant contribution to actuarial knowledge of vesting costs and should give pension actuaries an insight which will be very practical in their everyday work.

MAX BLOCH* AND CLARK T. FOSTER:

How timely the subject of vesting, its cost, and its philosophy really is may be seen from the fact that not one but two papers on it have been put before this meeting of the Society. Our discussion of Mr. Marples' paper will often make reference to Mr. McGinn's paper as well. Both

* Mr. Bloch is not a member of the Society. He holds a degree in actuarial science from the Technical University in Prague in prewar Czechoslovakia and has been in the pension field in this country for over fifteen years.

gentlemen have given us a wealth of material, much new insight, and some valuable practical help with respect to a subject which, to our knowledge, has never before received systematic treatment in the *Transactions*.

The two gentlemen, however, are not the only ones who have been wrestling with these problems over the years. We have been doing some work and have obtained results which may be of general interest.

We started where our fathers left off—with the well-known commutation function:

$$C_{x+k}^{aw} = v^{1/2} \cdot D_{x+k}^t \cdot q_{x+k}^{(w)} \cdot r^{-1/2-x-k} / \ddot{a}_{x+k+1/2}^{(12)} \quad (1)$$

In it, D_{x+k}^t stands for

$$\frac{D_{x+k}^t}{t_{x+k}} = D_{x+k} / \prod_{t=0}^{r-x-k-1} [1 - q_{x+k+t}^{(w)}] \quad (2)$$

It may be debatable that the $-q^{(w)}$ in equation (1) should be a dependent probability. We have used an independent probability for reasons that will soon become clear. At worst, this practice results in an overstatement which is truly negligible at the young ages, where the withdrawals really count, and not significant at the higher ages, where assumed withdrawal rates tend to be low. (At age 50, where many turnover tables end, the overstatement is $\frac{3}{10}$ of 1 per cent on the unprojected 1951 Group Annuity Table and less on projected tables.) After all, Mr. McGinn has used independent q 's which, even for moderate turnover rates, differ from the dependent q 's by much larger percentages; moreover, he implicitly assumes that terminations will occur at the end of the year, while Mr. Marples, like us, assumes that they will occur at the middle of the year.

For $r^{-1/2-x-k} / \ddot{a}_{x+k+1/2}^{(12)}$ we used $N_r^{(12)} / \frac{1}{2}(D_{x+k} + D_{x+k+1})$. We then re-wrote formula (1) as follows:

$$C_{x+k}^{aw} = \frac{q_{x+k}^{(w)}}{t_{x+k}} \cdot N_r^{(12)} \cdot \frac{2 \cdot v^{1/2} \cdot D_{x+k}}{D_{x+k} + D_{x+k+1}} \quad (3)$$

If the fraction at the end is developed in terms of i and $-q_{x+k}$ and the series for $(1+i)^{1/2}$ is broken off after the fourth term, the fraction becomes

$$\frac{2+i - \frac{1}{4}i^2 + \frac{1}{8}i^3}{2+i - q_{x+k}^{(d)}} \quad (4)$$

It is immediately apparent from this form that its value is very close to 1. As Table 1 indicates, for all practical purposes its value can be

assumed to be 1. We thus recognize that our original commutation function is very close to

$$C_{x+k}^{aw} \doteq \frac{q_{x+k}^{(w)}}{t_{x+k}} \cdot N_r^{(12)} \tag{5}$$

The fraction C_{x+k}^{aw}/D_x^t represents the value at age x of an annual benefit of 1 that becomes vested at the midpoint between $x + k$ and $x + k + 1$.

TABLE 1
VALUES OF THE FACTOR $2 \cdot v^{1/2} \cdot D_{x+k}/(D_{x+k} + D_{x+k+1})$

AGE	INTEREST RATE			
	3%	3½%	4%	5%
1937 Standard Annuity Table				
20.....	1.000547	1.000507	1.000461	1.000352
30.....	1.000909	1.000868	1.000821	1.000711
40.....	1.002041	1.001997	1.001947	1.001831
50.....	1.004487	1.004437	1.004381	1.004253
1951 Group Annuity Table				
20.....	1.000194	1.000155	1.000110	1.000003
30.....	1.000379	1.000339	1.000294	1.000186
40.....	1.000877	1.000836	1.000789	1.000679
50.....	1.003090	1.003044	1.002991	1.002870

Dividing it by $r_{-z}/\ddot{a}_z^{(12)}$ leaves us with the "basic vesting ratio" $(BVR)_{x+k}$.

$$(BVR)_{x+k} = \frac{q_{x+k}^{(w)}}{t_{x+k}} \tag{6}$$

The meaning of this function is very similar to that of Mr. Marples' "primary vesting cost ratio," but it has two important advantages. It is easier to compute, and it is entirely independent of the interest and mortality assumptions. It also readily demonstrates a few general propositions:

- a) It furnishes the theoretical background for Mr. McGinn's empirical determination of the negligible influence of variations in the interest and mortality assumptions on his level cost indices. (Mr. Marples quite correctly

- never suggests any connection between vesting cost ratios and the assumed rate of interest.)
- b) The basic vesting ratio is higher at any age than the withdrawal rate at that age.
 - c) The basic vesting ratio is greatest at the lowest age and decreases steadily.
 - d) Basic vesting ratios increase (decrease) with the use of higher (lower) withdrawal rates.
 - e) The basic vesting ratio becomes zero when $q_x^{(w)}$ becomes zero.

Our $(BVR)_{x+k}$ function can be tabulated once and forever as an adjunct to each employment turnover table and can then be readily combined with any other set of actuarial assumptions.

Mr. Marples notes in passing that his cost ratios apply equally to normal cost and to the total cost of the vested benefit; he does not mention the "accrued liability" at all. Mr. McGinn furnishes the formal proof of the influence on this accrued liability (i.e., the accumulation of past normal costs) of changes in the interest rate and gives us ample tables that should be of great help in preparing cost estimates. However, two questions come to mind: Are we, apart from cost estimates, very much in need of accrued liability indices? Do we want to compute the normal cost always in the manner taken for granted by both authors?

Where the funding method is the frozen initial liability method, the accrued liability indices can be useful in the initial valuation and again at the time the liability is "unfrozen" in view of major plan amendments. Otherwise, it is the total present value of future vested benefits that is of principal interest, and only the normal cost indices are then needed. The same is true for applications of the attained age normal and aggregate cost methods.

More serious problems are raised if, for one reason or another, a plan is to be valued on the "orthodox" entry age normal cost method. Mr. McGinn's accrued liability cost indices fall below 1 at the higher ages, and necessarily so, because the total liability for vested benefits decreases toward zero prior to age r while normal cost payments continue to age r ; as a consequence, the accrued liability for vested benefits becomes negative at those ages. Such negative liabilities are a nuisance to the actuary who understands them, and they are almost incomprehensible to the layman who has the opportunity to look at detailed valuation results.

We have found that a very convenient way out is to compute a different kind of normal cost. Instead of dividing the lump-sum cost at entry age of an employee's potential vested benefits by a temporary annuity from entry age to retirement, we divide by one that runs only

to the last age for which the rate of turnover is not yet zero. If the first age at which the turnover rate disappears is z , then no normal cost and no accrued liability exist for ages from z up. This does not render our (*BVR*) function useless. It can be applied as before, provided that the normal cost so computed is then multiplied by the fraction $(N_x - N_r)/(N_x - N_z)$.

The practical application of the (*BVR*) function, while laborious in desk calculations, presents no particular difficulties when working with a computer. Suppose that a projected pension benefit payable at retirement has been computed (with or without a salary scale) and that the present lump-sum value has been determined. It is then necessary to project the benefit to the midpoint between every two ages from the "vesting age" on (and to reduce the result on account of any vesting percentage stipulated by the plan). Each such projected benefit is multiplied by the appropriate (*BVR*) factor, and the products for all ages included in the process are summed up. Their total is divided by the pension benefit at retirement, and the ratio so obtained is applied to the present lump-sum value of the pension benefit. The result is the present value of potential future vested benefits. The year-by-year projection is not carried beyond age z . Where the present age x is equal to or higher than the "vesting age," the first age to which the pension benefit is projected is $x + \frac{1}{2}$. This description applies to career average plans of all types. Modifications for a final pay plan are not difficult, provided that the concept of the "vested benefit" under such a plan has been clearly defined.

Some plans are valued on the accrued benefit cost method, and the addition of one-year term costs of vested benefits to the future service cost is then usually considered sufficient. The customary approach is to multiply the accrued benefit of each employee who already meets the vesting conditions by a "term cost factor."

$$v^{1/2} \cdot q_{x+r-1/2-x}^{(w)} / \ddot{a}_{x+1/2}^{(12)} \quad (7)$$

Where exact work is desired, this factor should be applied to the benefit that is expected to have accrued at age $x + \frac{1}{2}$. The result is the "term cost," that is, the reserve required at age x to fund in full the potential vested benefit resulting from a possible termination at $x + \frac{1}{2}$. If we multiply this term cost by the fraction

$$\frac{N_r^{(12)} / D_x^t}{N_r^{(12)} / (D_x / t_x)} \quad (8)$$

the value of which is, of course, 1, we obtain a new form:

$$\frac{q_x^{(w)}}{l_x} \cdot \frac{2 \cdot v^{1/2} \cdot D_x}{D_x + D_{x+1}} \left[B_{x+1/2} \cdot \frac{N_r^{(12)}}{D_x^t} \right]. \quad (9)$$

We see immediately that the first fraction is $(BVR)_x$ and that the second fraction can be "omitted," that is, assumed to be 1. The product in brackets is the sum of the accrued liability at x and one-half of the future service cost for the year beginning at x . Thus the labor usually devoted to computing "term cost factors" in accordance with formula (7) is unnecessary. The calculation of basic vesting factors instead provides an easy way of obtaining the desired results, since these factors are applicable to liabilities and costs which must be calculated in any event.

One-year term costs which, at least in theory, do not serve to build up reserves for future vested benefits may be considered to be inadequate and, therefore, undesirable. To some people, their simplicity is appealing; these people might contend that vesting represents only a "fringe benefit" when measured against the principal benefits of a pension plan, so that the inclusion of a term cost for vesting might be sufficient even where the principal benefits are valued on a level cost method. It would also be possible to take the opposite view: where principal benefits are valued on the accrued benefit method, vested benefits should still be valued on level cost in order to provide reserves.

While the practical significance of term costs may be debatable, their theoretical meaning is substantial and deserves a brief discussion. For an employee who has reached "vesting age," the accrued liability for vested benefits under a level cost method decreases from year to year until it reaches zero at age z . Without developing all the symbols that would be needed for a formal proof, we may say that this is so even though another annual normal cost is added to the accrued liability at age x , as well as interest and an expected release of reserves. What causes the reserve at age $x + 1$ to be smaller is the reduction of the reserve at x by an annual term cost with interest. It should be no surprise that our (BVR) function appears in formula (9). The true state of affairs is that the (BVR) function is basically a term cost function and that it appears in the level premium formulas because the annual reduction of level premium reserves is equal to the term cost for that year (with interest).

In order to demonstrate that the (BVR) function is related to term cost, we make the artificial assumption that all terminations occur at the beginning of the year. At that point, a reserve of $N_r^{(12)}/D_z^t$ is held—

on an accrued benefit valuation basis—for each dollar of accrued pension benefits for employees whose age is x . Where there is no vesting, a reserve of $-q_x^{(w)} \cdot N_r^{(12)} / D_x^t$ is expected to be released. When vesting is introduced (meaning 100 per cent immediate vesting), two things happen: the expected release must be restored and the reserve for those whose benefits become vested must be raised to $N_r^{(12)} / D_x$, because their benefit rights have become immune to “turnover.” The cost of restoring the reserves and raising them to the higher level is the term cost of vesting. This is

$$q_x^{(w)} \cdot \frac{N_r^{(12)}}{D_x} = \frac{q_x^{(w)}}{t_x} \cdot \frac{N_r^{(12)}}{D_x^t} \quad (10)$$

Thus, the (*BVR*) function is indeed a term cost factor applicable to the lump-sum liability for accrued pension benefits.

Up to this point we have not questioned whether the results of calculations made in accordance with the helpful methods proposed by Messrs. Marples and McGinn—or, for that matter, with our own (*BVR*) method—really represent “the cost of vested benefits.” Mr. Marples may have had slight doubts when he wrote, “If there are no termination decrements, then there is no *separately projectable* cost of vesting” (emphasis added). This is unquestionably true, but it is not something a pension consultant can tell his client—and get away with it. Where assumed turnover rates are conservative (and most of them are), the vesting cost figures shown in valuations represent no more than a formal accounting for a portion of the total cost, an accounting that is fully consistent with the over-all valuation method. To tell a client or the public that they represent “the cost of vesting” could be quite misleading.

How can a better approximation of the true cost of vesting be obtained? The true cost of vesting, like the true cost of a pension plan, can be ascertained only in hindsight. It should be possible, however, to prepare, as a practical matter not as an integral part of actuarial valuations, current estimates which come closer than the theoretical, formally consistent figures produced by the methods that we have been discussing. While we have made “educated guesses” in various concrete situations, we do not have a universally applicable answer. It may be that some of our fellow actuaries will wish to contribute to the solution of this problem.

At the end of his article, Mr. Marples discusses the philosophy of vesting. After reading his remarks, one senses a serious problem for the conscientious pension consultant who shares his basic feelings. Should he discourage his client from including vesting provisions in his plan? Should he make any recommendation in favor of vesting only to the accom-

paniment of half-spirited comments on overwhelming pressures—pressures of “keeping up with the Joneses”?

We do not believe that vesting has harmful consequences. Our most advanced companies these days are in dire need of highly qualified personnel, both at the executive and the skilled-labor level. Experience and mature knowledge are largely matters of age and long service. Do we want to restrict the availability of the various talents by forcing employees to consider the loss of accrued pension rights first and a new challenge to their abilities only second? The company which puts its good people to the test every day has little to fear from the adoption of a vesting provision; the company which allows its staff to become stale will not be protected by the absence of such a clause. We live in a competitive society and we want to keep it so. Vesting will help.

THOMAS P. BLEAKNEY:

Mr. Marples asks for statistics regarding claim rates for the vesting benefit under contributory plans. The statistics below are very limited but may be of interest.

In a public employees' plan with which I am familiar, a great increase in the number of employees becoming eligible for vesting recently occurred because of liberalization of the plan. Specifically, the plan now provides for full vesting for persons terminating with more than fifteen years of service or, if over age 50, with ten years of service. During the period of eight months immediately following the effective date of the change in the plan, there were twenty-eight participants who terminated service while eligible for vesting. Although some had rather substantial benefits accrued that were forfeited upon withdrawal of their contributions, every person forfeited his vested benefits by taking cash.

As a matter of fact, under this particular plan, many participants could withdraw their contributions instead of taking current retirement benefits. Over a ten-year period, over 600 retiring participants, or about 7 per cent of all retirees, actually forfeited substantial amounts of immediately payable retirement benefits in order to withdraw contributions.

The importance of these statistics is emphasized by the fact that the plan is now about twenty years old and has, therefore, not reached maturity. Since a substantial past-service benefit is provided—a benefit, of course, forfeited by withdrawal of contributions—there is substantial reason now to take the benefit instead of cash. In the future most participants will have significantly higher accumulated contributions relative to accrued benefit, with a corresponding increased likelihood of forfeiting the benefit for cash.

This natural tendency to take the bird in hand leads me to feel that the cost of vesting will be of far less importance if contributory plans should become more prevalent. If the United States tax laws should ever be modified to allow for the deduction of employee contributions, as those in Canada do, this might come to pass.

LEE H. KEMPER:

I wish to congratulate Mr. Marples on his very fine paper, "Cost of Vesting in Pensions." Not only has Mr. Marples introduced a very interesting and practical approach to determining vesting costs, but, in addition, he has presented some philosophical views with respect to the vesting benefit. In his paper the author has asked that a discussion include consideration of the philosophical aspects of the subject with regard to reasonable limitations to be placed on vesting benefits.

In arriving at the reason for including a vesting provision in a retirement plan, it is important for the employer to determine at the outset whether his plan is a retirement plan or a deferred-compensation plan. If it is a deferred-compensation plan, then, as mentioned by Mr. Marples in his paper, the employee would be entitled to the full amount of the benefit accrued to his credit at the time of termination. If the plan is considered a retirement plan without a deferred-compensation feature, the primary reason, from a theoretical point of view, for including a vesting provision in the plan is to secure the employee's benefit against nonvoluntary termination. Mr. Marples touches on the question of nonvoluntary termination when he states, "The provision may have arisen from a general fair-treatment idea applied, first of all, retroactively in a case of hardship and then gradually as a basic concept applied to all active employees."

This statement implies that the provision is for the purpose of alleviating the situations in which the employee is forced to terminate because of health or other circumstances beyond his control. In addition, it should probably include the case in which the employee is discharged by the employer. There have been a number of situations in which an employer has terminated the services of an employee, who is approaching retirement, for the sole purpose of reducing the employer's retirement plan cost. In this connection, we cannot overlook the fact that a vesting provision is also included to compete with other plans with similar benefits. These are probably the most important reasons for including a vesting provision in a plan which has been designed primarily for the purpose of providing for superannuated employees.

The next point to consider is at what point the vesting benefit should

commence. During the early years of participation in a plan, the cost of deferred pension benefits for younger employees is small as a result of the future interest earnings to be earned on the funds and the fact that the employee must survive to normal retirement age to receive the benefit. Under a contributory plan, the contributions plus interest might be larger than the present value of the deferred benefit; this is particularly true in the case where the vesting allowed is less than 100 per cent of the accrued benefits. The vesting benefit at this point would have little or no value to the participant. The logical point for the vesting benefit to begin is sometime after the present value of the benefit accrued to the employee exceeds his contributions with interest.

Another point to consider in connection with the commencement of the vesting provision is that the company must maintain records for all terminating employees with a vested interest. To avoid carrying a large number of records involving insignificant amounts of benefits, the vesting benefit should begin at the point when the benefit is sufficiently large to warrant the cost of the additional record keeping.

The fact that vesting is much more costly to the company at the older than at the younger ages for participants with the same length of service suggests the amount of vesting should vary by age as well as by length of service if complete equity is to be achieved. For example, a \$100 per month normal retirement benefit at age 65 for an employee with fifteen years' service at age 35 would be worth \$4,100, while the same benefit for an employee with fifteen years' service at age 55 would be worth \$8,400. I realize that vesting varying by age and length of service would introduce a number of practical problems and might in some respects defeat the purpose of the benefit provided; however, the inequities of the situation have always bothered me.

The final point on which I wish to comment is the level of the vested benefit. Based on the reasoning that the vesting provisions should be included to secure the pensions for such nonvoluntary terminations and assuming that the plan is not a pension-equivalent-to-wage plan, it is reasonable that a vesting provision should gradually increase until it becomes equal to the retirement benefit at normal retirement age or at the age at which an early retirement benefit is available. The grading of the benefit in this way, combined with the point at which vesting begins, will, to some extent, determine the level of the benefit.

In my opinion the gradual vesting provision overcomes many of the disadvantages to the employer mentioned in Mr. Marples' paper. Under this type of vesting provision, the employee can always look forward to additional credits if he remains with the company.

Again I would like to congratulate Mr. Marples on his paper which, in my opinion, represents a fine addition to our actuarial literature.

WALTER RIESE:

It is a pleasure to follow through the development of the formulas and tables contained in Mr. Marples' paper, which should prove a useful addition to actuarial literature. The main body of the paper deals with facts and demonstrations relating to the cost of vesting, and it remains to be seen whether some readers discover grounds for argument in this area. However, under the heading "Concluding Remarks," the author expresses his opinions regarding the philosophical and managerial aspects of vesting, and it is difficult to resist his invitation to follow him into the more fertile, if somewhat treacherous, land of appearances and impressions. Although the following discussion will not be one to fulfill the author's desire for suggestions of "reasonable limitations to be placed on benefits," it may help to show how difficult it is for the Society of Actuaries to speak with one voice.

It is possible that in Canada, where most pension plans are contributory, employees are more likely to be aware of accrued pension benefits and to think of these benefits as deferred compensation "equities" than are employees in the United States. However, it is probably true in both countries that considerable publicity is given to the contributions employers make to pension plans, expressed as percentages of salaries or wages. Employees, not unnaturally, tend to think of these contributions as being allocated on an individual basis. With such mental conditioning, a simple statement of fact like Mr. Marples' "If he quits before vesting, his accrued pension lapses . . ." is therefore likely to lead to as unfavorable an emotional reaction as might the discovery that traditional nonforfeiture benefits had been abandoned and that insurance companies had adopted the policy of using gains from lapsed policyholders for greater dividends to continuing policyholders who were considered to be of more value to the companies. If such a view had been taken—and it could have been taken, perhaps, more easily in Canada and the United Kingdom, where no statutory nonforfeiture laws exist—the insurance industry might not have withstood repeated attacks as successfully as it has. By analogy, one would think that improvement in vesting and portability—far from bringing on the downfall of the private pension movement—is likely to further its growth and general acceptance.

In retrospect, the gradual development of plans emphasizing accrual of pension benefits related to length of service as well as earnings, such

as money-purchase and unit benefit plans, seems natural not only because such plans provide desirable incentives to continued employment but because it seems reasonable to have pension payments reflect the differences in the quality and length of service rendered by different employees. It might be rationalized that such plans also reflect the philosophy that responsibility for retirement income should reasonably be prorated among employers, giving some weight to the value of services rendered by the employee to each enterprise (in contrast to the obsolescent notion that the only employer who might have any pension obligation to a given individual is the one who has had the employee's service over long periods of years and over the later years of his working life). While such a philosophy seems to pose a problem in the case of service with an employer who has no pension plan, this problem appears to be resolved by the deferred-wage theory, since it may be rationalized that there is simply no deferred compensation arising out of such service (certainly such service was rendered without any expectation of deferred compensation).

While stating that "the granting of vested rights to pensions is not favorable to management aspirations," Mr. Marples concedes that "reasonable vesting in benefits should be regarded as the mark of a well-designed plan and an enlightened employer." However, the author clearly believes that no restrictions should be placed on the employer's freedom to determine the degree of his own enlightenment. The question seems to be whether the perpetuation of shortcomings of the private pension movement through such a laissez faire attitude will not be more inimical to its survival than limited restrictions on the employer's freedom of choice.

The vesting requirements enacted by the governments of the provinces of Ontario, Quebec, and Alberta are based on the premise that it is the decision of the employer (subject to the importunities of collective bargaining) whether or not retirement income for employees is provided in addition to benefits under government-operated programs. This seems only natural since, apart from minimum wage laws, compensation for service is determined by the employer (again subject only to the importunities of collective bargaining). However, the statutes of the three provinces provide that, if a private employer-employee pension plan is operated, the benefits must vest after ten years of service and attainment of age 45. Furthermore, such vesting is not conditional on the employee's leaving his contributions in the fund, since any employee's contributions become "locked in" at this point.

Of course, there is no assurance that at future points in time the various legislatures concerned may not decide to make vesting requirements more stringent. However, Mr. Marples' vision of a system of uniform pension benefits or the absorption of all pension plans into a central government retirement system seems rather gloomy. Such a development seems hardly conceivable, unless a system of uniform wages had become acceptable to society—a rather remote contingency.

If we could detach ourselves momentarily from the absorbing activities and controversies of the social organisms in which we operate and view our free-enterprise society from the perspective of outer space, would our myriads of decision-making bodies—be they management, labor, or government—not be clearly discernible as agents responsible ultimately to society as a whole? Also, regardless of the media, intermediaries, and intricacies of accounting, would not pensions—whether public, private, funded, or unfunded—be recognizable as no more and no less than a charge on future production which, though charged initially to a particular enterprise, must ultimately be borne by society through higher prices, lower wages, or higher taxes than would obtain if the deferred claims on production did not have to be honored? If so, it would seem inevitable that, even though society may be happy to leave the determination of levels of compensation largely to the market place and even though elected representatives of employees may have agreed to the pension conditions, sooner or later the employer's exclusive right to determine conditions of pension payment will be questioned. Indeed, how reasonable is it and how necessary from a managerial point of view that retirement income in respect to service rendered to a particular enterprise in the first instance, but presumably ultimately to society as a whole, should be conditional on the employee's attachment to the enterprise for long periods of years and until attainment of a relatively advanced age?

There is no doubt that management desires to retain its skilled labor, but are there not sufficient incentive devices in longevity pay, longer vacations, seniority rights, and so forth, not to mention increases in pension expectation produced by longer service and increases in earnings, without having to make the receipt of retirement income liable to full or partial forfeiture over long periods of years?

Mr. Marples expresses the view that "acceptance of these costs [of vesting] as a necessary burden by the employer . . . should not be permitted to form a platform for continuous extension of the benefit." While in noncontributory pension plans it would be difficult to obtain visible

contributions toward plan liberalization from employees, and while it may be equally difficult to offset the cost of vesting by a reduction in the unit benefit, it could well be argued that any cost of vesting borne by the employer is by necessity offset by the denial of some other form of employee benefit that the employer might otherwise have granted.

CHARLES V. SCHALLER-KELLY:

First of all, I would like to make a brief comment on the kind of investigation that Mr. Marples made. This comment also applies, to some extent, to Mr. McGinn's paper.

It seems to me that the paper requires, in effect, a calculation for each employee or group of employees divided into age and service classifications. This seems to me, in certain circumstances, to take more time than is practical, and sometimes it is more expensive than it need be if you deal with a small group. When I refer to its taking time, I am thinking of such occasions as negotiations. Moreover, for the very small group, if the deposit administration or trustee approach is to be used—and I think that this is the most sensible way of funding pensions for a number of reasons—then a method such as the one proposed, or the one proposed by Mr. McGinn, might be more elaborate than is really justified or than can be borne by a group of, say, twenty employees.

We must remember that we have so many assumptions in pension costs at any time that they are always approximate. We are really charting our way through unknown future currents of interests and turnover and goodness knows what. I wonder whether it would not be desirable—and I shall bring this up again in a later portion of the whole program—to try to do precisely what Mr. Marples suggests he cannot do, that is to say, to look at the effect on model distributions so that we have some rules of thumb for evaluating costs of various pension benefits.

In the second place, there is the philosophical issue. I would like to stress that the views I am going to express are not specifically those of the United Automobile Workers. I have been called by my colleagues anything from a "crypto-conservative" to "that foreign socialist," so I do not think that my ideas should be taken to represent UAW opinion, except insofar as they happen to coincide with statements which have been made by people who can speak on behalf of the United Automobile Workers.

It seems to me that employee benefits should be looked at more from the employees' view than they commonly are by insurance companies for two reasons. First, the intention of employee benefit plans is to satisfy the employee. This is management's reason. I think that insurance com-

panies should therefore also look at them from the point of view of satisfying employees than of satisfying management. The employee benefit program comes from pressure by employees, either organized or unorganized pressure at the time of hiring.

I think that insurance companies are really, in another way, missing the boat insofar as there are so many large groups which are not covered by pension plans. In Canada I recently did a little study on this. The UAW alone in one of its regions has approximately 4,500 members not covered by pension plans. Multiplying this by 16 or 17 regions, and I don't know how many unions, it would seem to me that it would be desirable for insurance companies to start thinking in a slightly different direction.

Of course, you have the disadvantage that your image with unions is of such a nature that, if we have the choice, we tend to go through trust companies, which probably are just as reactionary but do not make so much noise about it.

Second, I think that employee benefits should be looked at from the employees' point of view because there are more employee voters than employer voters. If you cannot manage to provide what is required, then the same employee, in his anonymous capacity of citizen, will provide it for himself through the government, which is what happened with that quite conservative government in Ontario which provided the vesting legislation.

Third, of course, management is composed of employees, too, in 90 per cent of the cases today.

Fourth, I think what is good for management in the short run may not be good for the country. In fact, it quite frequently is not, because every management inevitably will attempt to do whatever is good for it in the short run, and there may be a tendency to forget about the country; it is not necessarily good for the insurance companies. I think that it is assumed that, if it is good for management, it must be good for insurance companies. I would question that assumption.

Let me briefly say that, as I see it, employee benefit plans should have at least four characteristics: (1) they should be adequate when required, that is, when earnings stop—in old age, upon disablement, and for dependents at death; (2) they should provide security of those benefits; (3) they should be provided in as cheap a way as possible; and (4)—this is an item which I think you should like—they should be flexible.

Government social security can provide all these things except flexibility, and I think that, if the private industry wants to stress something, it should stress its flexibility. That is what it can sell.

In regard to the first characteristic, arguing against vesting in the way in which Mr. Marples did is basically arguing against one aspect of adequacy. This is the most basic need of all pension systems, and if the private pension system cannot provide adequacy, then let us get rid of it. I know that this is heresy here; but if the private pension system cannot provide adequacy, it is not fulfilling a function any more than a dinosaur is.

Second, there is security. Try to tell a Studebaker employee that the private system has done well in security! There are still too many cases—and I come across them, unfortunately, all too frequently—in which the private system turns out not to have provided the pensions that were promised. And then the insurance industry opposes the Hartke Bill. It may not be a perfect bill; if you can come up with something better, splendid. I am sure even Senator Hartke would be delighted. Otherwise, there is another gap.

Third, there is a question of the provision of employee benefit plans in the cheapest way. A nonfunded plan—or almost nonfunded plan, such as the government plans—is probably cheaper in many ways. However, this can be counteracted in the private system by investing in stocks which go up in capital gain and ultimately enable higher pensions to be paid. So, I am prepared to go along on that.

Finally, as I say, the private system has flexibility. I do appreciate it. Just recently I became involved in a situation where we had some problem because of inflexibility.

I believe that the words of the new Republican insurance commissioner at a recent Michigan Actuarial Society meeting are also applicable in this case:

The political process from time to time reallocates the risks borne by various parts of our society; for example, it shifted the risk of enterprise from the employee to the employer in the field of employee injuries. This risk allocation is a political and social decision based upon a determination of what segment of society should bear a particular risk at a particular point in time. The role of insurance is to spread these risks, once they are allocated, by social, political, and legal mechanisms.

There you have a Republican view of the same kind of thing.

The private system should therefore create mechanics to overcome its shortcomings, some of which I have mentioned, in the field of security and portability. In this connection I should add one further shortcoming in portability; that is that since vested rights do not increase in value once a person has left they tend to get left behind in much the same way as in a career average plan, even when, in fact, a man may have been

covered by a succession of final-average plans. This is another item which I think the insurance industry should put its best brains to.

(AUTHOR'S REVIEW OF DISCUSSION)

WILLIAM F. MARPLES:

As I expected, a major part of the discussion related to the philosophical aspect of vesting, and I am grateful to those who extended the scope of my remarks; in particular, to Robert Link for his contribution on mandatory vesting provisions, to William K. White and Wilmer Jenkins for their expansion of the subject, to Frank Griffin for his support of my views, and to Harry Sarason for his advice on procedures.

I have to confess that I am an unreconstructed member of the Do-It-Yourself Club, and as such I have a fundamental objection to government-imposed rules. I did think that I made it clear that I considered vesting to be a feature of a well-designed pension plan, and I believe that plans should assume a position in this matter sufficiently commanding to guard the "pass," in Crawford Laing's words. However, I confess to being disappointed grievously in the failure of any participant in the discussion to indicate where he thinks this position might be. The position has to be found—in some place where it commands substantial support—perhaps further forward than in some plans and possibly not so far forward as might be advocated by some extremists.

I appreciate the contribution of Dan McGinn particularly, since his time must have been almost totally absorbed in writing his own paper, and I regret that I could not repay the compliment by discussing that paper. However, I have to go on record that I disagree with some of his comments on my paper, lest his summary of points in my paper lead others astray.

My approach is fully adaptable to all circumstances except one, in which it produces a zero answer; that one is the case in which withdrawal rates are not employed in the cost calculations. Is this the "smaller pension plans" to which he refers? Second, my approach is not limited to measuring the increase in the normal cost alone, as will be seen from paragraph 5. Finally, I add in formula form, not the arithmetical form, the demonstration he requests as to the use of my form of ratios. In a formula similar to those for policy values, the reserve value for a pension is:

$$\begin{aligned} & (\text{Projected benefit}) \times (\text{Value of pension of 1 at retirement}) \\ & - (\text{Amount of normal cost}) \times (\text{Temporary} \\ & \quad \text{amount to retirement age}). \end{aligned}$$

In his symbols, for attained age x and entry age w ,

$$(AL)_x = B_r \cdot \frac{D_r}{D_x} \cdot \ddot{a}_r^{(12)} - P_w \cdot (\text{Actual salary})_x \cdot \ddot{a}_{x:r-x|}$$

Let the vesting cost ratios be $1 + k_x$ and $1 + k_w$, respectively. Then the reserve value, including the cost of vesting, is

$${}^v(AL)_x = (1 + k_x) \cdot B_r \cdot \frac{D_r}{D_x} \cdot \ddot{a}_r^{(12)} - (1 + k_w) P_w \cdot (\text{Actual salary})_x \cdot \ddot{a}_{x:r-x|},$$

from which the vesting reserve can be determined if necessary.

Mr. McGinn gives the ratio ${}^v(AL)_x / (AL)_x$, and it may be noted that over certain ages the ratio is less than unity. The explanation of this feature is that k_x becomes zero at the attained age x , at which age the withdrawal rates become zero, while k_w remains at its entry age value. In cases where P is obtained as the normal cost rate to balance liabilities, the adjustment by $1 + k_x$ produces a weighted increase in P , automatically including the effect of k_x equaling zero at certain ages.

The accrued benefit method would use the attained-age ratios. Incidentally, the total reserve, that is, the reserve for active participants plus the reserve for terminated vested participants, will theoretically always be greater than the unadjusted reserve for active participants. This offers an explanation which Mr. Bloch can give legitimately to those of his clients who examine his worksheets. On Mr. Bloch's formula, I present the following reconciliation with my own.

The expression inside the summation symbol in my formulas (7) and (8) can be written

$$\frac{w_{x+t}}{l_r} \cdot \frac{l'_r}{l'_{x+t+1/2}}$$

To proceed on Mr. Bloch's basis, we have to use the following formula:

$$l_{x+k} = l_x \prod_{s=0}^{k-1} [1 - q_{x+s}^{(d)}][1 - q_{x+s}^{(w)}],$$

from which

$$w_{x+t} = l_{x+t} [1 - q_{x+t}^{(d)}] q_{x+t}^{(w)},$$

assuming deaths subtracted first, and

$$l_r = l_{x+t} \prod_{s=0}^{r-x-t-1} [1 - q_{x+t+s}^{(d)}][1 - q_{x+t+s}^{(w)}],$$

and

$$\frac{l'_r}{l'_{x+t+1/2}} = [1 - q_{x+t}^{(d)}]^{-1/2} \prod_{s=0}^{r-x-t-1} [1 - q_{x+t+s}^{(d)}].$$

Then

$$\begin{aligned} \frac{w_{x+r}}{l_r} \cdot \frac{l'_r}{l'_{x+t+1/2}} &= \frac{l_{x+t} \cdot [1 - q_{x+t}^{(d)}] q_{x+t}^{(w)}}{l_{x+t} \prod_{s=0}^{r-x-t-1} [1 - q_{x+t+s}^{(d)}] [1 - q_{x+t+s}^{(w)}]} \\ &\times [1 - q_{x+t}^{(d)}]^{-1/2} \prod_{s=0}^{r-x-t-1} [1 - q_{x+t+s}^{(d)}] = \frac{q_{x+t}^{(w)} [1 - q_{x+t}^{(d)}]^{-1/2}}{\prod_{s=0}^{r-x-t-1} [1 - q_{x+t}^{(w)}]}, \end{aligned}$$

which differs from Mr. Bloch's formula only by the factor $[1 - q_{x+t}^{(d)}]^{-1/2}$.

It is now clear that Mr. Bloch has achieved identical results by different methods so far as the primary vesting ratio is concerned, and I assume that he has taken the further steps to allow for the accrued benefit and the salary-scale benefits.

I thank Mr. Bleakney for responding to my call for figures on the question of claim rates for vesting benefits in contributory plans, and I agree with his comment on the cost of vesting if contributory plans should become more fashionable. An alternative approach preventing contribution refunds is that of the Canadian government, which has prescribed a "locked in" vesting benefit in which the participant cannot retrieve his contributions and the employer must provide the vested pension benefit. The length to which this government regulation has gone may be thought on investigation to exceed reasonable limits and, if so, any osmosis south of the border needs to be resisted.

To refer now to the question of the difference between estimates and real costs raised by William K. White, the effect of the technique that I have set out in the paper is to develop, from age series of probabilities of death, withdrawal, and disability, the results which would have been obtained for each entry age separately had the appropriate rates been selected for each attained age and allowing for the provision of a vested benefit to the expected terminations after qualifying for such a benefit. It is usual to find in projected benefit actuarial valuation methods that the reserve value in respect to benefits accruing between the valuation date and the normal retirement date is positive. Hence, terminations tend to produce small valuation releases when the past-service benefit is vested,

so that differences in numbers and average benefits do not produce valuation strains. It is to be noted that the unfunded liabilities for these people still have to be paid up even though they are now not in the company's employment.

In the case of the accrued benefit valuation method, if liabilities for pensions accruing before the valuation date have been calculated on a mortality table basis only, then no valuation strain occurs. If withdrawal rates have been employed, there may be a valuation strain in 100 per cent vested cases equal to the difference in the single-premium costs without and with withdrawal rates. If the vesting percentage is less than 100 per cent, there should be some offset, because the reserve required for vested cases is reduced by the vesting percentage and the full reserve on the withdrawal rates is subtracted. These remarks apply at ages where withdrawal rates are used, and the effect of withdrawal rates should be substantially eliminated beyond the vesting qualification limits.

To go to the question of the increase in annual cost as the result of vesting, let us take the valuation methods under the two heads of accrued benefit method and projected benefit method. There are two elements in the additional cost, the theoretical increment, and the practical increment.

Accrued Benefit Method

If the cost calculations have been made on the basis of a mortality table only, there will be no theoretical or practical increase in the total liabilities for pension accrued prior to the valuation date. If withdrawal rates have been used with the mortality tables, the introduction of a vested benefit will involve an additional lump-sum supplemental cost. The theoretical increment for prior service is thus zero or positive. There will be no practical increment in respect to prior service.

In developing the company contributions for the new year, however, we have an amount equal to:

- a) the normal cost calculated on the old actuarial cost determinants, *plus*
- b) the theoretical increment for vesting in the current year's benefits, *plus*
- c) a payment to amortize the theoretical increment for vesting in respect to prior-service benefits, *less*
- d) reserves released by termination in the valuation year, *plus*
- e) the reserves required for vested benefits claimed and granted during the valuation year, *plus*
- f) a payment in respect to any unfunded liabilities.

Before the vesting benefit was incorporated into the plan benefits, the company contribution would have been calculated as $a - d + f$; with vesting, there has to be added $b + c + e$.

The theoretical increment is $b + c$, and the practical increment is e less the part of item d related to former participants who terminated and qualified for some vested benefit.

Projected Benefit Methods

These methods are divided into two main groups: Group A, in which unfunded liabilities are "frozen" and the whole of the experience of the plan in the year is reflected in the change in the normal cost rate, which is recalculated at the end of the year; and Group B, in which the normal cost rate is calculated theoretically at the end of each year and all experience of the plan in the year is reflected in the amount of the unfunded supplemental liability at the end of the year.

In Group A, we shall have the theoretical increment in respect to both *prior- and future-service* benefits. These may be separated and the amount for prior-service benefits added to the unfunded supplemental liability. The reserves for vested benefits to inactive participants would be set up during the valuation and would form part of the total liabilities. The new normal cost rate is now obtained as the rate required to balance assets and liabilities, that is, to meet the balance of the liabilities after deducting the fund and the unfunded liability. The normal cost thus is affected by all experience gains and losses during the year and, in this particular instance, will be increased on account of the theoretical increment for future-service benefits and may be for prior-service benefits also.

The adjustment for practical increment of reserves expected to be released by termination and reduced by reserves set up for vested inactive participants is also automatically included in the calculation. The analysis of theoretical increment and practical increment is the same as for the unit credit method, but the amounts are the equivalent normal cost rates.

In Group B, the theoretical increment for total benefits will be incorporated in the new normal cost calculation. The liabilities for prior- and future-service pensions will also be increased by the appropriate theoretical increment. The reserves for vested benefits to vested inactive participants will be included in the total liabilities of the plan. The residual unfunded supplement cost will thus be adjusted automatically by:

- a) the theoretical increment on pension liabilities, *less*
- b) the present value of the theoretical increment on the normal cost, *less*
- c) reserves released on terminations during the year, *plus*
- d) reserves set up for vested benefits to vested inactive participants.

The theoretical increment is $a - b$ and may be negative (see Dan McGinn's tables), and the practical increment is d less the part of c related to vested cases.