Ryan Labs RL PPA Index Definitions and Methodology

Bond Selection

The bonds eligible for PPA are selected from a general universe of Semi-annual Coupon bearing bonds with term values 0.5 and 30.0. The selection criteria implemented in this process is as follows.

1. Par Amount/Amount outstanding>= 250 Million
2. Rating Criteria: A-AAA
3. No variable Coupon Bonds
4. Bonds with Coupon frequencies other than 2 are excluded
5. Put or Sink option bonds are eliminated
6. Zero Coupon Bonds are eliminated

With these bonds that satisfy the entire above criterion, a PPA Portfolio is created. This PPA portfolio assigns each rating cell A, AA, AAA one third of total weight of the portfolio (33.333 each). Within each cell, the 33.33 percent is distributed according to outstanding balance. Once, the fixed weighting is assigned, the PPA defined duration adjustment is applied. This adjustment lowers the weight for all position with Duration greater than 1.If the Duration is greater than one; the weight is divided by the square root of the duration. All weights are then adjusted such that the total weight = 100%. Now, this portfolio is divided into the following term cells as suggested by IRS.

Cell Name	Cell Begin	Cell End
01YR	0.5	1.5
03YR	1.5	3
07YR	3	7
15YR	7	15
30YR	15	30

Yield Curve Construction: The yield curve is constructed by taking the Term and Yield of each cell (1, 3, 5, 7, 15 and 30)

Basic PPA Curve

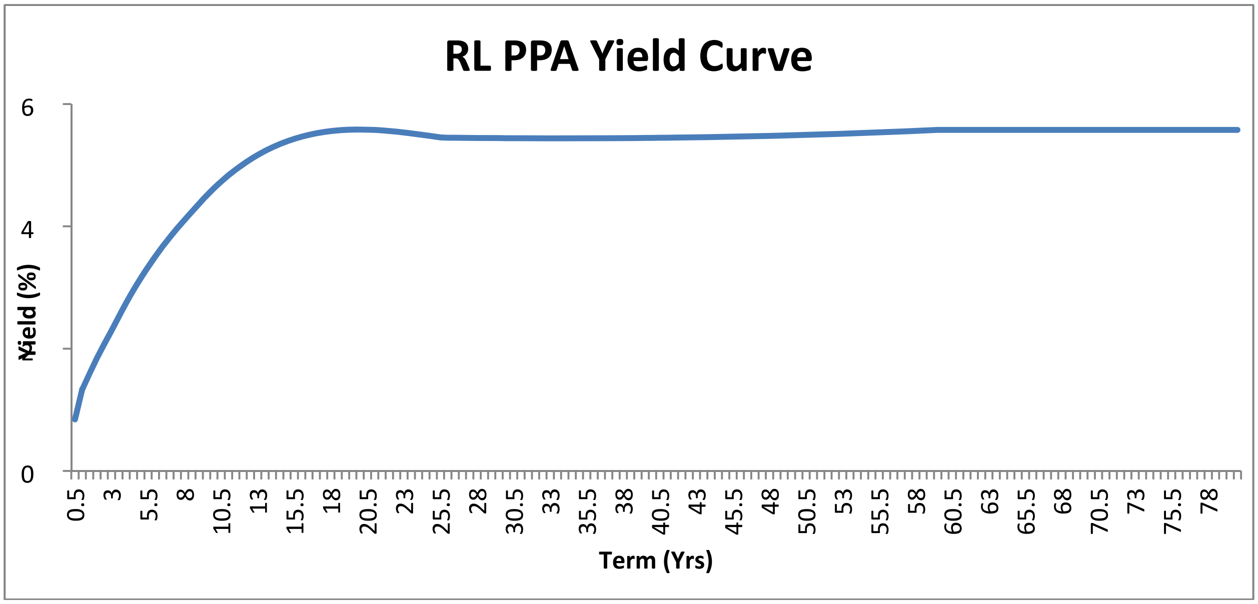
Cell Name	**Term**	**Yield**
01YR	0.9182	1.5115
03YR	2.2555	1.9678
07YR	4.452	2.9418
15YR	8.3827	4.2038
30YR	25.5942	5.4501

IRS uses commercial paper rates for the beginning point of the curve but we use 3, 6, and 12 month Libor rates. First, a 0.25 term point using 3 Month Libor rate is added to the curve. Then we calculate the average of the 6 and 12 month Libor Rates. Next, the 1 Year Cell values (Term and Yield) are equal weighted with a term value of 0.75 and the average of the 6 and 12 Month Libor rates.

Libor Adjusted PPA Curve

Term	Yield
0.25	0.5389
0.8341	1.2424
2.2555	1.9678
4.452	2.9418
8.3827	4.2038
25.5942	5.4501

Using this curve, an enhanced curve is constructed by interpolating the yields for any give term going up to 50 Years which is use for discounting Liability cash flows.



RL PPA Generic Liability Index

RL PPA Generic Liability Index is constructed from an equally apportioned Liability schedule for 50 Years. The liability schedule, a listing of payment dates and amounts, is converted into a portfolio of Corporate Zero Coupon Bonds. Each scheduled payment is treated as a Zero Coupon Bond holding with a maturity date equal to the payment date and a Par Amount equal to the payment amount. The complete collection of these Zero Coupon Bond holdings becomes the PPA Generic Liability Portfolio.

Liability Pricing

The first step is the calculation of daily prices for each Zero Coupon Bond holding within the portfolio. Liability prices are calculated using PPA Yield Curve. A yield value is interpolated from the PPA Yield Curve for the term (or duration) of each holding within the liability portfolio. The Liability price is then calculated using Standard Industry Association (SIA) Formula 21 through SIA Formula 23.

Liability Return Calculation

The calculated Liability Prices are used to calculate the Current Market value for each liability holding. Current Market Value is calculated as Par Amount * (Price/100), where the Par Amount equals the future value dollar of the liability stream.

The daily return for each liability holding is calculated as the percentage change in market value for one day, or

100 * ((Today’s Market Value/Yesterday’s Market Value) - 1.0)

The daily return for the entire portfolio is simply the Market Value Weighted Average of all the individual daily returns within the portfolio.

The Liability Index

The Liability Index is constructed from the daily returns calculated for the Liability Portfolio. The Liability Index exists as a Daily Series of “Index Levels”, starting at 100.

Each day, the new Index Level is calculated as

Yesterday’s Level * (1.0 + (Today’s Percentage Return/100))

Periodic Return Calculation

The Series of Index Levels are then used to calculate the Percentage Return for any time period as

100 * ((End Date Level/Start Date Level) – 1.0)

RL PPA Investable Index Methodology

The main issue concerns the investability of the RL PPA curve used for a present value measurement of the liabilities. In terms of the universe of available single A to AAA corporate bonds, corporate issuance is available in the 1 to 7 year duration years (10 year maturity range) but is very light much further out on the duration spectrum. Corporate supply is very thin once you get beyond seven years in duration (10 years in maturity) and then picks up again in the Long Duration space (30-year maturity). There are significant gaps in necessary credit compared to the liability Term Structure being hedged. Therefore, the RL PPA curve has not been used to benchmark asset management performance because it is not investable in the 15 to 30 year duration cells. The “PPA Investable” (RL PPA Investable) benchmark that is available daily via the Ryan Labs manifold is transparent, investable, and more accurately reflects the investment strategy for liability-based benchmarks across the term structure of 1 to 30 years in duration. We believe this also provides a better economic measure of funding exposure than a 100% corporate IRS PPA curve because of the lack of issuance of long dated corporate bonds

The PPA yield curve has limitations and non-investability characteristics from a capital markets and asset management perspective. Without getting too technical, a sampling of these limitations are:

1. Limited supply of A rated securities
2. Fictional discount of cash flows with durations past 15 years
3. Downgraded issues reset and roll off monthly from the index
4. Longer duration securities are artificially shorten to give lees weights in the index
5. Cash flows updated annually remain the same even if the discount rate jumps or declines based on changes in the bonds supply (Reconstitution Risk).

    i. Downgrades can cause a higher yielding bond (which would generate a low present value) to fall out of the universe…but you still have to discount the same cash flow (the benefit payment didn’t get downgraded).

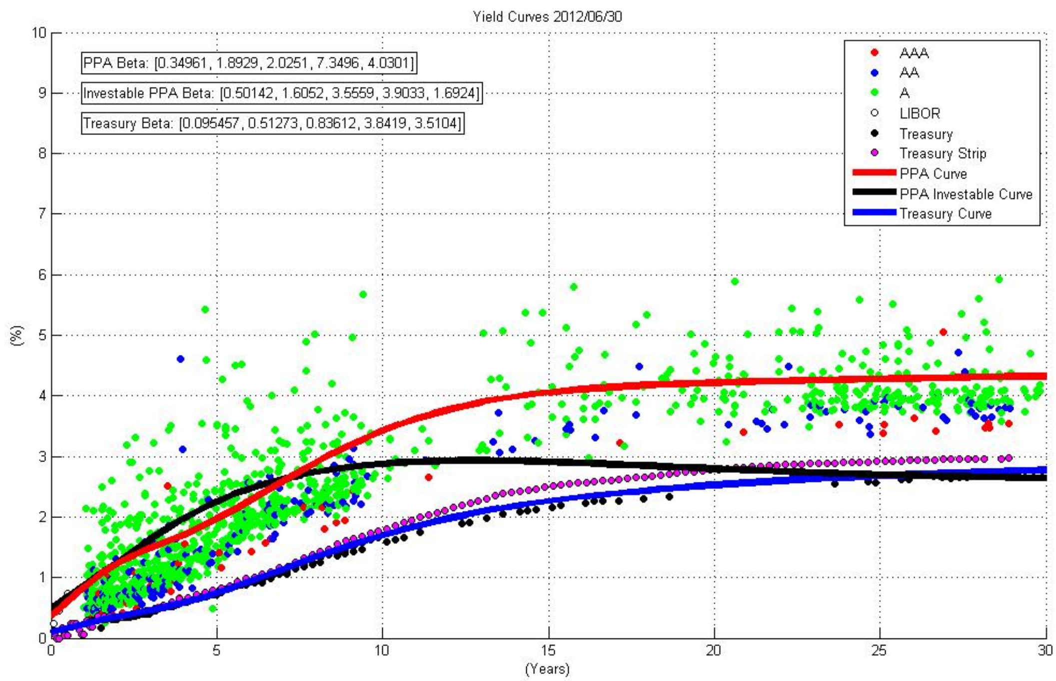
    ii. Bond supply rolling out of the universe can affect the return of the liability without affecting the return of the assets.

Ryan Labs Investable PPA Curve

The *RL Investable PPA* curve is constructed from a larger bond universe containing the US Treasury coupon bonds. Within the 10-to-20 maturity range, the total par amounts of the US Treasury and the Corporate are equally weighted. Within the 20-to-30 maturity range, the bond universe is all Treasury. At the short-maturity end, the LIBOR rates and the corporate bonds within 5-year maturity remains unchanged as in the RL PPA universe. Besides the change of the weighing, the RL Investable PPA curve is fitted by the same methodology as the PPA curve.

The exhibit below demonstrates the investable bond universe and the relative yield curves based off of this universe. The “dots” reflect actual bond issues available, color coded to match their credit ratings, and plotted according to their yield and maturity date. The black yield curve is the

RL PPA Investable. You can clearly see the plurality of corporate bond issues are available in the first 10 years while we use Treasuries further out due to limited availability of corporates.



1 Preliminaries

1.1 Yield Curve

Fixed-income analysis starts with the yield curve. For better understand of definition of yield curve, we define spot rate cuve $s(t)$ for a given security to be the benchmark of the return from now to time t. For example Treasury spot rate curve at $t = 0.5$ shows the coupon rate of a zero-coupon Treasury Bill maturing in six months from today. The forward rate curve $f(t)$ is the spot return at time t. It can be derived from spot rate curve by rolling over infinitely many times under "no-arbitrage" assumption. That is, in continuous time model

$$ts(t) = \int_{x=0}^{t} f(x)dx$$

or

$$f(t) = s(t) + ts'(t).$$

Discount function $d(t)$ represents the present value of future \$1 cash flow at time t. Therefore we can define it in mathematical form:

$$d(t) = e^{-ts(t)} = e^{\int_{x=0}^{t} f(x)dx}.$$

1.2 Corporate Bond

Corporate bonds are issued by corporations for the purpose of raising capital. Each bond has specific coupon rate and mature date. The present value of each bond is then calculated by the sum of the present value of cash flows. If a bond maturies at time t_n with par amount a and n coupon cash flows $c_1, \ldots, c_n$ at time $t_1, \ldots, t_n$ respectively. The current value p of this bond based on a discount function $d(t)$ is

$$p = d(t_n)a + \sum_{i=1}^{n} d(t_i)c_i$$

We refer corporate bonds as long-term debt instruments. For shorter term, corporations issue commercial papers. Commerical paper is an unsecured promissory note with a fixed maturity of 1 to 270 days. Usually corporate bonds and commerical papers are not backed by collateral, therefore only firms with excellent credit ratings from a recognized rating agency will be able to sell their bonds or commercial paper at a reasonable price. There are two primary rating classifications published by Moody's and Standard & Poor's:

Moody's	Standard & Poor's
Aaa	*AAA*
Aa	*AA*
A	*A*
Baa	*BBB*
Ba	*BB*
B	*B*
Caa	*CCC*
Ca	*CC*
C	*C*
	D

1.3 B-Splines

Splines are used to fit arbitrary curves. The approximation scheme usually provide us well-behaved functions to interpret unknowns. For example, interpolating or extrapolating a curve based on finite many points. General definition of B-spline was introduced by Cox-de Boor. Short introduction of B-spline is given below.

Definition 1.1. *Given a closed interval* $[0, T]$, *define a knot vector to be a finite sequence of real numbers*

$$0 = t_1 \leq t_2 \leq \cdots \leq t_{n-1} \leq t_n = T.$$

Each t_i *is called a knot. Each* $[t_i, t_{i+1})$ *is called a range.*

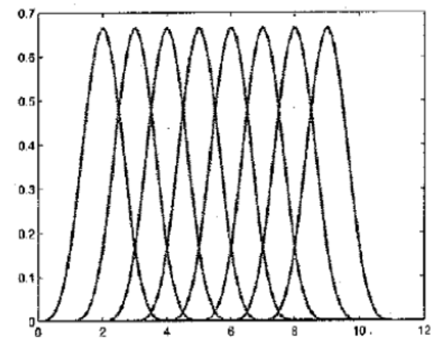


Figure 1: Uniform B-Spline on $\langle 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \rangle$

If $|t_i - t_{i+1}|$ are equal for any i, then the knot vector is uniform and the B-spline generated from the vector is called uniform B-Spline. See Figure (1.3). If it is not the case, the B-spline is non-uniform.

Definition 1.2. *The B-splines of degree n are denoted by B_i^n. For $n = 0$, define*

$$B_i^0(x) = \begin{cases} 1 & , t_i \le x < t_{i+1} \\ 0 & , otherwise \end{cases}$$

and

$$\hat{V}_i^k(x) = \frac{x - t_i}{t_{i+k} - t_i}.$$

The recurrence form of B_i^k can be written as

$$B_i^k = \hat{V}_i^k B_i^{k-1} + (1 - \hat{V}_{i+1}^k) B_{i+1}^{k-1}. \tag{1}$$

The only exception is when $i = n - 1$. We let $B_{n-1}^0 = 1$ for $x = t_n = T$. Since the definiteion accept duplicated knots, the convention that $\frac{0}{0} = 0$ is adopted. The following example shows the recursive tree to generate $B_{i_k}^3(x)$:

1.3.1 B-Spline Properties

We give some useful lemmas for further verifications and proofs.

Lemma 1.3. *For $k \ge 1$,*

$$B_i^k(x) = 0.$$

Lemma 1.4. *Given n knots, for any k*

$$\sum_{i=1}^{n-k-1} B_i^k(x) = 1.$$

Proof. We prove it by induction on k. For $k = 0$, it is obviously true. Assume the property holds for k. For $k + 1$,

$$\begin{aligned}
\sum_{i=1}^{n-k} B_i^{k+1}(x) &= \sum_{i=1}^{n-k} V_i^k B_i^k + (1 - V_{i+1}^k) B_{i+1}^k \\
&= V_1^k B_1^k + V_{n-k}^k B_{n-k}^k + \sum_{i=2}^{n-k-1} (V_i^k + (1 - V_{i+1}^k)) B_i^k \\
&= V_i^k + (1 - V_{i+1}^k) \\
&= 1.
\end{aligned}$$

Lemma 1.5. *For $k \geq 2$,*

$$\frac{d}{dx}B_i^k(x) = \left(\frac{k}{t_{i+k} - t_i}\right)B_i^{k-1}(x) - \left(\frac{k}{t_{i+k+1} - t_{i+1}}\right)B_{i+1}^{k-1}(x).$$

Lemma 1.6. *For $k \geq 1$,*

$$B_i^k \in C^{k-1}(\mathbb{R}).$$

Lemma 1.7. *The set $\{B_j^k, \ldots, B_{j+k}^k\}$ is linear independent on (t_{k+j}, t_{k+j+1}).*

Lemma 1.8. *For any k,*

$$\int_{\mathbb{R}} B_i^k(x) = \frac{t_{i+k+1} - t_i}{k+1}.$$