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# RESERVE FOR UNMATURED LIFE-INCOME OPTIONS 

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## INTRODUCTION

ANUMBER of companies have incurred an unmatured option liability under policies issued more than a decade ago. These are policies under which the cost of providing a guaranteed life-income option at the time of future settlement will be greater than the policy proceeds then payable. That cost may be 20,40 or even 60 per cent greater than those policy proceeds.

Life insurance companies in the United States and Canada now hold over a quarter of a billion dollars in reserves to meet these additional actuarial liabilities.

This paper traces the history of the reserve for unmatured options, describes one method by which it may be calculated, and discusses the problems that arise in reporting the reserve in the Convention Blank at the end of each year. We wish to make clear at this point that our discussion is limited to the life-income options, even though the reserve for unmatured options of some companies includes amounts for interest-only options as well.

Our objective is to stimulate discussion of this subject.

## HISTORY

Before 1900 few policies guaranteed any optional life-income settlements, and virtually all United States life insurance was settled by a single cash payment.

Between 1900 and 1920 life-income options were gradually introduced into the policy contract by one company after another. In this early exploratory era the option rates were expressed on an "annual income" basis and were usually on the "installment refund" or the "twenty-year-certain" plan.

Then-during the early 1920's-the technique of life insurance programming was developed. The life-income options began to play a more significant role in sales presentations. The rates were made more attractive. The "monthly life-income-10-year-certain" plan was introduced and immediately emphasized. Undoubtedly, the competitive forces of this period and the extremely favorable interest earnings of the 1920's were responsible for these developments. By 1930 all the major life insurance
companies guaranteed ten-year-certain life-income options in their policy contracts.

The range of these guarantees among twenty of the large companies in each of the years 1930-63 is shown in Chart 1 and Table 1 for a payee 65 years of age. The male and female rates are shown separately.

The rates guaranteed in those policy contracts that were issued in 1930


[^0]TABLE 1
Range of Monthly Life Income (Ten Years Certain) per \$1,000 of Proceeds Guaranteed by Twenty Life Insurance Companies*

| As of January 1 | Male Beneficiary Age 65 |  |  | Female Benefictary Age 65 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | High | Low | Average | High | Low | Average |
| 1930. | \$7.93 | \$6.76 | \$7.43 | \$7.93 | \$6.76 | \$7.36 |
| 1931. | 7.93 | 6.96 | 7.44 | 7.93 | 6.82 | 7.30 |
| 1932. | 7.93 | 6.96 | 7.44 | 7.93 | 6.82 | 7.30 |
| 1933. | 7.93 | 6.96 | 7.44 | 7.93 | 6.82 | 7.30 |
| 1934. | 7.92 | 6.96 | 7.39 | 7.92 | 6.82 | 7.19 |
| 1935. | 7.92 | 7.02 | 7.32 | 7.92 | 6.47 | 6.91 |
| 1936. | 7.58 | 6.71 | 7.24 | 7.57 | 6.17 | 6.76 |
| 1937. | 7.58 | 6.71 | 7.24 | 7.57 | 6.17 | 6.74 |
| 1938. | 7.58 | 6.71 | 7.22 | 7.57 | 6.17 | 6.71 |
| 1939. | 7.58 | 6.30 | 6.74 | 6.98 | 5.61 | 6.09 |
| 1940. | 7.22 | 6.30 | 6.65 | 6.69 | 5.61 | 5.98 |
| 1941. | 7.16 | 6.30 | 6.60 | 6.56 | 5.61 | 5.93 |
| 1942. | 6.83 | 6.02 | 6.51 | 6.28 | 5.61 | 5.83 |
| 1943. | 6.82 | 6.02 | 6.45 | 6.28 | 5.34 | 5.77 |
| 1944. | 6.82 | 5.75 | 6.37 | 6.28 | 5.08 | 5.69 |
| 1945. | 6.82 | 5.75 | 6.30 | 6.28 | 5.08 | 5.62 |
| 1946. | 6.57 | 5.75 | 6.23 | 5.88 | 5.08 | 5.54 |
| 1947. | 6.57 | 5.75 | 6.22 | 5.88 | 5.08 | 5.53 |
| 1948. | 6.32 | 5.75 | 6.10 | 5.61 | 5.08 | 5.40 |
| 1949. | 6.32 | 5.75 | 6.10 | 5.61 | 5.08 | 5.40 |
| 1950. | 6.32 | 5.75 | 6.09 | 5.60 | 5.08 | 5.39 |
| 1951. | 6.32 | 5.75 | 6.09 | 5.60 | 5.08 | 5.39 |
| 1952. | 6.32 | 5.98 | 6.11 | 5.60 | 5.28 | 5.42 |
| 1953. | 6.32 | 5.98 | 6.11 | 5.60 | 5.28 | 5.42 |
| 1954. | 6.32 | 5.86 | 6.09 | 5.60 | 5.28 | 5.41 |
| 1955. | 6.32 | 5.86 | 6.09 | 5.60 | 5.28 | 5.42 |
| 1956. | 6.32 | 5.86 | 6.09 | 5.60 | 5.28 | 5.41 |
| 1957. | 6.32 | 5.86 | 6.09 | 5.60 | 5.28 | 5.41 |
| 1958. | 6.32 | 5.86 | 6.10 | 5.60 | 5.28 | 5.42 |
| 1959. | 6.32 | 5.90 | 6.10 | 5.60 | 5.28 | 5.42 |
| 1960. | 6.32 | 5.90 | 6.10 | 5.60 | 5.28 | 5.42 |
| 1961. | 6.32 | 5.90 | 6.11 | 5.60 | 5.28 | 5.43 |
| 1962. | 6.32 | 5.90 | 6.11 | 5.60 | 5.28 | 5.43 |
| 1963. | 6.32 | 5.90 | 6.12 | 5.60 | 5.28 | 5.44 |
| 1964. | 6.32 | 5.98 | 6.15 | 5.65 | 5.28 | 5.46 |

[^1]ranged from $\$ 6.76$ to $\$ 7.93$ per thousand of policy proceeds. This range applied to both male and female payees. Between 1930 and 1951 the rates that were guaranteed in newly issued policies were successively reduced. Since 1951 they have been held at a relatively stable level.

The average 1951 guarantee was $\$ 6.09$ per thousand for male payees and $\$ 5.39$ per thousand for female payees. In 1963 the comparable rates were $\$ 6.15$ and $\$ 5.46$, respectively.

It is important to note that, when the earlier policies were issued, it was assumed that the granting of the life-income options would not create any additional liability for the life insurance company. It was originally assumed that the regular life insurance reserve would always measure the company's entire policy liability. Subsequent events have demonstrated that this assumption was unrealistic. It has become clear that the high level of interest returns in the 1920's was only a temporary phenomenon. Experience has also demonstrated that the improving trend in mortality-noted as early as 18991-has been a continuing one. These developments have created the liability for unmatured life-income options.

This problem was first recognized after the publication of the 1936 study of life-income option mortality ${ }^{2}$ and during the preparation of the 1937 Standard Annuity Table. ${ }^{3}$ So far as we could determine, the first reserve was set up on December 31, 1937. The Phoenix Mutual's annual statement of that year shows that $\$ 34,000$ was set aside for its retire-ment-income options.

On October 1, 1938, the supervisory authorities expressed their official concern with the trends in mortality and interest. On that date, the Insurance Department of New York State sent a letter to each company operating in that state asking several questions regarding life-income option reserves.

The New York Department asked: "Has your company set aside . . . any extra reserves for annuities or supplementary contracts written on standards which do not adequately measure present longevity or interest assumptions? If not, does your company have such steps in mind?"4

[^2]Since 1938 life-income options have presented actuaries with three problems:

1. What rates should be guaranteed in new contracts?
2. What should be the reserve basis for matured life-income options?
3. What is the magnitude of the reserve for unmatured options?

It is not our purpose to discuss the first two subjects in this paper. However, it may be pertinent to note in passing that, between 1938 and 1950, actuaries devoted considerable attention to these subjects, and during this period life insurance companies set aside substantial amounts to strengthen matured life-income option reserves.

In 1947 Mr. Edward W. Marshall suggested that each company should make a model-office study to determine whether it should set up a "reserve against future losses under ultraliberal settlement options in old policies." He explained: "The time will come when these policies will largely have been terminated by death or maturity, whereas the settlement options arising from them may continue in force in substantial volume for a generation or so thereafter and be a continual source of loss if present rates of interest continue and annuitant longevity increases, as seems likely." ${ }^{5}$ At that time Mr. J. Gordon Beatty agreed and emphasized that "we should make each series of policies pay its own way as far as we can." ${ }^{6}$

The reports of life insurance company examinations made by the examiners of the National Association of Insurance Commissioners and published by the several insurance departments reveal that many companies have recognized this additional actuarial liability for a long time. This is readily confirmed by the reserve tabulation found in Table 2.

These Association reports indicate that some companies determine their unmatured option reserve periodically by a model-office computation and that others compute it annually. However, so far as the authors could determine, the details of these computations-and the life-income option election rates used-have never been published.

## NML RESERVE

In 1948 and again in 1954, we at the Northwestern Mutual (NML) studied our own unmatured option situation. We had planned to repeat our laborious 1954 punched-card model-office study in 1964. However, in 1963 we found that, if we could write an appropriate program, our electronic data-processing system would be able to determine our reserve in less than an hour, using our group valuation summaries. With this as

[^3]TABLE 2
History of Reserve for Unmatured Options
(United States Companies Showing Reserve in 1954 or Before)
(To Nearest Thousand Dollars-000 Omitted)

| Year- <br> End | Phoenix <br> Mutual* | New York Life | Prudential | Connecticut Mutual | Mutual <br> Benefit | Metropolitan | Bankers Life (Iowa) | Travelers |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1946 | 423 | 40,000 | 8,251 | 5,000 | 2,922 |  |  |  |
| 1947 | 508 | 40,000 | 8,044 | 5,000 | 3,458 | 8,934 | 8,497 |  |
| 1948 | 593 | 40,000 | 9,618 | 5,000 | 7,497 | 13,902 | 10,105 | 3,242 |
| 1949 | 678 | 40,000 | 32,292 | 8,000 | 10,469 | 17,662 | 11,928 | 7,242 |
| 1950 | 763 | 40,000 | 32,783 | 12,000 | 12,814 | 20,876 | 14,011 | 12,742 |
| 1951. | 848 | 40,000 | 53,256 | 15,000 | 12,853 | 23,726 | 14,496 | 9,742 |
| 1952. | 933 | 40,000 | 53,022 | 15,234 | 12,882 | 30,866 | 14,889 | 14,742 |
| 1953. | 1,018 | 40,000 | 52,729 | 15,413 | 21,404 | 33,539 | 14,968 | 19,742 |
| 1954 | 1,103 | 40,000 | 52,210 | 15,472 | 21,404 | 35,709 | 15,282 | 27,834 |
| 1955 | 4,036 | 40,000 | 51,676 | 15,917 | 21,404 | 38,027 | 15,289 | 31,169 |
| 1956 | 4,686 | 40,000 | 51,027 | 16,040 | 21,255 | 40,513 | 15,190 | 32,174 |
| 1957 | 4,686 | 40,000 | 50,112 | 16,349 | 18,130 | 23,299 | 15,129 | 31,052 |
| 1958. | 4,686 | 62,000 | 49,145 | 16,696 | 17,753 | 23,711 | 11,297 | 29,930 |
| 1959. | 4,686 | 93,262 | 48,451 | 17,203 | 17,355 | 24,694 | 11,015 | 24,716 |
| 1960 | 6,325 | 82,141 | 47,575 | 17,300 | 17,022 | 25,757 | 10,716 | 20,340 |
| 1961 | 8,529 | 79,687 | 46,246 | 23,281 | 16,680 | 26,173 | 10,422 | 18,485 |
| 1962 | 7,999 | 77,350 | 45,313 | 24,484 | 15,482 | 26,278 | 10,030 | 15,885 |
| 1963 | 7,424 | 75,090 | 44,078 | 25,942 | 14,394 | 26,238 | 10,119 | 13,445 |

* Reserve first appeared on December 31, 1937.

TABLE 2-Continued

| YearEnd | Home Life | Northwestern Mutual | Mutual Life | Equitable of New York | Provident Mutual | New England Mutual | State Mutual (Mass.) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1946 |  |  |  |  |  |  |  |
| 1947. |  |  |  |  |  |  |  |
| 1948 | 600 |  |  |  |  |  |  |
| 1949 | 1,600 |  |  |  |  |  |  |
| 1950 | 2,100 | 3,000 | 8,000 | 3,021 |  |  |  |
| 1951 | 2,750 | 5,000 | 15,075 | 4,305 |  |  |  |
| 1952 | 3,350 | 7,000 | 17,110 | 5,436 | 2,000 |  |  |
| 1953 | 4,000 | 10,000 | 18,410 | 6,424 | 2,000 |  |  |
| 1954. | 4,000 | 12,000 | 19,100 | 7,257 | 3,000 | 14,599 | 350 |
| 1955 | 4,000 | 14,000 | 16,913 | 7,948 | 3,500 | 15,795 | 550 |
| 1956 | 4,000 | 18,000 | 16,525 | 8,507 | 4,000 | 16,210 | 750 |
| 1957 | 3,800 | 22,000 | 16,410 | 8,944 | 4,000 | 15,324 | 1,000 |
| 1958. | 3,565 | 22,550 | 15,400 | 9,246 | 7,092 | 20,259 | 1,250 |
| 1959 | 3,353 | 23,114 | 14,153 | 21,972 | 6,692 | 19,531 | 1,250 |
| 1960. | 3,250 | 23,692 | 13,090 | 21,035 | 5,532 | 18,718 | 1,500 |
| 1961 | 3,185 | 24,284 | 11,868 | 20,107 | 5,273 | 17,896 | 3,500 |
| 1962 | 3,160 | 24,891 | 9,387 | 19,200 | 5,057 | 17,129 | 3,400 |
| 1963 | 3,129 | 25,513 | 7,411 | 18,305 | 4,807 | 16,119 | 2,651 |

an incentive, the necessary program was written, and all the elements needed for the computation, such as life-income election rates, were reexamined. Our objective was to define a computation procedure that was as theoretically precise as would be possible with the data at our disposal. Most of this paper is an exposition of our procedure for determining the reserve for unmatured options.

The formulas will be defined first on a single-premium basis. The extension to the annual premium case will be dealt with in a subsequent section.

## Development of Single-Premium Reserve

Throughout this development it is assumed that each year-end the insurance in force is available on a group valuation basis. That is, each "plan-year of issue-age at issue-sex" group is separate.

Definitions
$a=$ calendar year of issue
$b=$ calendar year of valuation
$r=b-a$
$x=$ age at issue
$c=$ calendar year of maturity for endowment-type plans and the year of expiry for term plans
$q_{\nu}^{(d)}=$ probability of the insured's death between ages $y$ and $y+1$
$q_{\nu}^{(w)}=$ probability of the insured's surrender between ages $y$ and $y+1$
$p_{\nu}^{(T)}=1-q_{\nu}^{(d)}-q_{v}^{(w)}$
$f_{\nu}^{(d p)}=$ proportion of death proceeds settled under a life-income option where the insured was under plan $p$ and died in the calendar year in which age $y$ is attained
$f_{y}^{(w p)}, f_{y}^{(m p)}=$ comparable election rates for surrenders and maturities
${ }_{t} B_{x}^{(w)}=$ cash value at the end of the $t$ th policy year per $\$ 1.00$ of face amount
${ }_{t} B_{x}^{(d)}=$ average death benefit per $\$ 1.00$ of face in the calendar year in which the $t$ th policy year is completed-for retirement plans where reserve may exceed face, or in the case of annuities where death benefit may be less than face
$B_{x}^{(m)}=$ maturity value per $\$ 1.00$ of face-for retirement plans
$a_{d(y)}=$ the net single premium-guaranteed in the contract-to provide a life income of $\$ 1.00$ per year under the typical life-income option to the typical payee associated with an insured attaining age $y$ in the calendar year of death
$a_{w(y)}, a_{m(y)}=$ the comparable net single premiums where the proceeds arise from surrenders and maturities
$a_{d(y)}^{(z)}=$ the net single premium as above, except settlement is assumed to be in calendar year $z$ and the value is based on a mortality table which is representative of life-income settlement option payees and recognizes expected trends in future mortality-for example, the Annuity Table for 1949 (with Projection) ${ }^{7}$ or the 1960 Modification thereof ${ }^{8}$
$a_{w(y)}^{(s)}, a_{m(y)}^{(s)}=$ the comparable net single premiums for surrender and maturity proceeds
If provision is to be made for the expense of handling life-income option payments, the last three single premiums should be placed on a gross premium basis.
${ }_{a}^{p} I_{x}^{b}=$ the face amount of insurance in force on December 31 of calendar year $b$ out of business written on plan $p$ at issue age $x$ in year $a$
${ }_{a}^{p} R_{x}^{b}=$ that part of the fully funded reserve for unmatured options related to ${ }_{a}^{p} I_{x}^{b}$
Note that in this development we have not burdened the notation with sex differentiation. However, some of the elements above vary by sex and require separate calculation for male and female insured lives.

## Formulas

1. Life Plans

$$
\begin{aligned}
&{ }_{a}^{p} R_{x}^{b}={ }_{a}^{p} I_{x}^{b}\left\{\sum_{t=1}^{\infty} v^{t-1 / 2} \cdot t-1 p_{x+r+1 / 2}^{(T)}\left[q_{x+r+t-1 / 2}^{(d)} f_{x+r+t}^{(d p)}\left(\frac{a_{d}^{(b+t)}}{a_{d}(x+r+t+t)}-1\right)\right.\right. \\
&\left.\left.+q_{x+r+t-1 / 2 \cdot r+t}^{(w)} B_{x}^{(w)} f_{x+r+t}^{(w p)}\left(\frac{a_{w}^{(b+t)}}{a_{v p}(x+r+t)}-1\right)\right]\right\} .
\end{aligned}
$$

## 2. Endowment and Retirement Plans

$$
\begin{aligned}
& { }_{a}^{p} R_{x}^{b}={ }_{a}^{p} I_{x}^{b}\left\{\sum _ { t = 1 } ^ { c - b - 1 } v ^ { t - 1 / 2 } \cdot { } _ { t - 1 } p _ { x + r + 1 / 2 } ^ { ( T ) } \left[q_{x+r+t-1 / 2}^{(d)} \cdot{ }_{r+t} B_{x}^{(d)} f_{x+r+t}^{(d p)}\right.\right. \text { : } \\
& \times\left(\frac{a_{d(x+r+t)}^{(b+t)}}{a_{d(x+r+t)}^{(x)}}-1\right)+q_{\left.x+r+t-1 / 2 \cdot{ }_{r+t}^{(w)} B_{x}^{(w)} f_{x+r+t}^{(w p)}\left(\frac{a_{w(x+r+t)}^{(b+t)}}{a_{w(x+r+t)}^{(b)}}-1\right)\right]} \\
& +v^{c-b-1 / 2} \cdot{ }_{c-b-1} p_{x+r+1 / 2}^{(r)}\left[\frac{1}{2} \cdot q_{x+c-a-1 / 2}^{(d)} \cdot{ }_{c-a} B_{x}^{(d)} f_{x+c-a}^{(d p)}\left(\frac{a_{d(x+c-a)}^{(c)}}{a_{d(x+c-a)}}-1\right)\right. \\
& \left.\left.+\left(1-\frac{1}{2} \cdot q_{x+c-a-1 / 2}^{(d)}\right) \cdot B_{x}^{(m)} f_{x+e-a}^{(m p)}\left(\frac{a_{m}^{(0)}(x+c-a)}{a_{m}(x+c-a)}-1\right)\right]\right\} .
\end{aligned}
$$

Note: ${ }_{\star} B_{x}^{(d)}=B_{x}^{(m)}=1$ for regular endowments.
${ }^{7}$ Wilmer A. Jenkins and Edward A. Lew, "A New Basis for Annuities," TSA, I, 369.
${ }^{8}$ Charles M. Sternhell and Charles H. Page, "The 1960 Modification of the $a-1949$ Table with Projections," TSA, XIII, 127.
3. Term Plans

$$
\begin{aligned}
{ }_{a}^{p} R_{x}^{b}={ }_{a}^{p} I_{x}^{b} & \left\{\sum_{t=1}^{c-b-1} v^{t-1 / 2} \cdot{ }_{t-1} p_{x+r+1 / 2}^{(T)} q_{x+r+t-1 / 2}^{(d)} f_{x+r+t}^{(d p)}\left(\frac{a_{d(x+r+t)}^{(b+t)}}{a_{d}(x+r+t)}-1\right)\right. \\
& \left.+\frac{1}{2} v^{c-b-1 / 2} \cdot{ }_{c-b-1} p_{x+r+1 / 2}^{(T)} q_{x+c-a-1 / 2}^{(d)} f_{x+c-a}^{(d p)}\left(\frac{a_{d}^{(c)}(x+c-a)}{a_{d}(x+c-a)}-1\right)\right\} .
\end{aligned}
$$

## 4. All Business

The total reserve on a single-premium basis as of December 31 in calendar year $b$ is thus:

$$
\sum_{\text {all } p} \sum_{\text {all } a} \sum_{\operatorname{all} x}{ }_{a}^{p} R_{x}^{b} .
$$

## Elements Used in the Reserve Calculation

1. Interest rate.-The unmatured option reserve is a long-term life insurance liability. Life-income payments will be made under the ultraliberal options well into the twenty-first century. This has to be recognized in the valuation interest rate.

In computing the NML reserve, we will use 3 per cent both in the period before policy settlement and during the life-income period.
2. Mortality rates.-The computation of the unmatured option reserve involves the use of two mortality tables. One is needed for insured lives; another for beneficiaries.

For insured lives it would be appropriate to use either the policy-reserve mortality table or a current experience mortality table. The policyreserve table is, of course, a generally recognized table, since it is found in the state statutes that define the legal minimum cash values and policy reserves. It does tend to produce a somewhat higher value for the liability.

For beneficiaries it would be desirable to use a mortality table that is representative of annuitant mortality and that recognizes expected mortality trends.

In computing the NML reserve, we will use the policy-reserve table (the American Experience Table of Mortality) for insured lives and the 1960 Modification of the $a$-1949 Table (with Projection) for beneficiaries.
3. Persistency of life insurance.-Standard persistency tables-such as the Linton A and Linton B Tables-were prepared primarily to describe the characteristics of lapses in the early policy years.

Since the ultraliberal life-income options were guaranteed in policies issued before 1950, and since the rates of surrender change significantly near the ages of retirement, these tables are not appropriate to compute the reserve. Rather, a specifically designed experience table of persistency should be used.

In computing the NML reserve, we will use rates based on our own experience which vary by attained age only. These are illustrated in Table 3.
4. Life-income election rates.-Since it was feasible to program a refined set of life-income election rates into an electronic computation, we searched actuarial literature for such rates but found none. As a consequence, we undertook to prepare some from our own experience.

To this end we examined all of our 1959 settlements by amounts of proceeds. Some 45,000 policies were studied with proceeds of $\$ 160,000,000$, of which $\$ 22,000,000$ was settled under the life-income options. We chose to study the 1959 settlements because we wanted to include as life-income settlements all those amounts which, although initially settled under an option that did not involve life contingencies, were later transferred to a

TABLE 3
SURRENDER Rates per 1,000

| Attained Age | Rate | Attained Age | Rate |
| :--- | :---: | :---: | :---: |
| $35 \ldots \ldots \ldots$ | 15 | $65 \ldots \ldots \ldots$ | 23 |
| $40 \ldots \ldots \ldots$ | 14 | $70 \ldots \ldots \ldots$ | 29 |
| $45 \ldots \ldots \ldots \ldots$ | 13 | $75 \ldots \ldots \ldots$ | 28 |
| $50 \ldots \ldots \ldots \ldots$ | 11 | $80 \ldots \ldots \ldots$ | 16 |
| $60 \ldots \ldots \ldots$ | 17 |  | 0 |

life-income option. Such transfers made before December 31, 1963, were included in the study.

The 1959 policy settlements were each coded so that election rates could have been derived for each of the following variables: (a) mode of termination; (b) attained age and sex of insured; (c) plan; (d) year of issue; (e) regular business; and (f) pension trust business.

While we believe that the life-income election rates are functions of each of these variables, our data were not adequate to produce graduated rates in all the possible subdivisions. This was particularly true where the insureds were female.

Initially we studied the experience of all business terminated in 1959 in order to determine the underlying nature of the election rate curves as accurately as possible. Our final election rates were derived from the American Experience business only (issues 1946 and prior) because this is the business upon which our reserve for unmatured options is based. The final American Experience rates are shown in Tables 4, 5, and 6.

These rates, we feel, are satisfactory at this time. We suspect, though, that in the long run they will tend to be too low and that the ultraliberal
guarantees will gradually get greater utilization. At this time our development of election rates has been confined to only one calendar year, and we have no concrete evidence of a secular trend in these rates. If such a trend can be delineated, it would be a simple matter to incorporate it in our program.

## a) Death proceeds

Female experience was too limited to construct a separate table. The male rates, however, were applied to the exposure for females to obtain a ratio of actual to expected for all ages combined. This ratio was 7 per cent for life, endowment, and term plans and 12 per cent for retirement endowment plans.

TABLE 4
Life-Income Election Rates
Male Insured-for Death Proceeds

| Attained Age | Life, Endowment, and Term Plans | Retirement <br> Endowment <br> Plans* |
| :---: | :---: | :---: |
| Under 27. | 0\% | 0\% |
| 28-32. | 2 | 1 |
| 33-37. | 4 | 1 |
| 38-42. | 4 | 1 |
| 43-47. | 4 | 2 |
| 48-52. | 5 | 4 |
| 53-57. | 7 | 7 |
| 58-62. | 11 | 10 |
| 63-67. | 12 | 31 |
| 68-72. | 10 | 42 |
| 73-77. | 6 | . .... |
| 78-82. | 6 |  |
| 83-87. | 5 |  |
| 88-92. | 4 |  |
| 93 and over. | 4 | . . . . . |

[^4]
## b) Maturity proceeds

As with death proceeds, an over-all ratio of actual to expected was obtained for female regular endowment plans using the male rates. This ratio was 125 per cent.

## c) Surrender proceeds

For female insureds, the over-all ratios of actual to expected were 295 per cent for life and endowment plans, 95 per cent for personal retirement plans, and 77 per cent for pension trust retirement plans. (Male rates were used to compute expected elections.)

TABLE 5
Life-Income Election Rates
For Maturity Proceeds

| Attained Age | Endownent <br> Plans <br> Male <br> Insured | Retirement Endownent Plans |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Regular Business |  | Pension Trust Business |  |
|  |  | Male | Female | Male | Female |
| Under 52. | 0\% |  |  |  |  |
| 53-57. | 2 | $21 \%$ | 34\% | 0\% |  |
| 58-62. | 7 | 26 | 37 | 37 | 79\% |
| 63-67. | 11 | 36 | 46 | 82 | 83 |
| 68-72. | 10 | 42 | 44 | 87 |  |
| 73-77. . . . . | 8 |  |  |  |  |
| 78-82. | 5 |  |  |  |  |
| 83 and over | 0 |  |  |  |  |

TABLE 6
Life-Income Election Rates
Male Insured-for Surrender Proceeds

| $\underset{\substack{\text { Atringed }}}{\substack{\text { Atrind }}}$ | Life andEndowimentPlans | Retirement Endownent Pians |  |
| :---: | :---: | :---: | :---: |
|  |  | Regular <br> Business | Pension Trust Business |
| Under 27. | 0\% | 0\% | 0\% |
| 28-32. | 0 | 2 |  |
| 33-37. | 0 | 2 | 0 |
| 38-42. | 0 | 3 | 0 |
| 43-47. | 1 | 3 | 0 |
| 48-52. | 1 | 3 | 0 |
| 53-57. | 2 | 6 | 11 |
| 58-62. | 6 | 8 | 38 |
| 63-67. | 10 | 15 | 51 |
| 68-72. | 9 | 33 | 49 |
| 73-77. | 7 |  |  |
| 78-82. | 4 |  |  |
| 83-87. | 1 |  |  |
| 88 and over | 0 |  |  |

5. Typical beneficiary.-The typical beneficiary is defined by the twin characteristics of sex and age. For maturities and surrenders, the definition is quite simple. We found that, in the case of 97.5 per cent of the maturity and surrender proceeds settled under the life-income options, the sex of the payee was the same as the sex of the insured. We also found that the age of the payee was quite close to the age of the insured. For all the data combined, the weighted age differential was only one-half year, and this small difference was typical of the entire age range.

With regard to death proceeds, our basic hypothesis was that the payee was a female. This was found to be true for 97.7 per cent of the total lifeincome proceeds. The age difference was determined for quinquennial age groups of the insured. The results were used to derive an expression for

TABLE 7
Age Relationship for Death Proceeds

| $\begin{aligned} & \text { Insured's } \\ & \text { Age } \end{aligned}$ | Payee's Age |
| :---: | :---: |
| 35. | 38 |
| 45 | 48 |
| 55 | 55 |
| 65 | 61 |
| 75 | 67 |
| 85 | 72 |
| 95 | 78 |

the payee's age as a function of the insured's age. Thus, for an insured age $y$, the payee's age $h(y)$ was found to be:

$$
\begin{array}{lr}
h(y)=y+3 & \text { for } y \leq 48 ; \\
h(y)=0.58 y+23 & y>48 .
\end{array}
$$

It was concluded, then, that the typical payee could be defined as follows: (a) for maturities and surrenders, the payee is the same person as the insured; (b) for deaths, the payee is a female and her age is related to the age of the insured as illustrated in Table 7.
6. Typical life-income option.--In our study 95 per cent of the lifeincome options were single-life settlements. The average certain period was 11.4 years, and the average payment frequency was 11.8 per year. In view of this, the NML reserve is being determined, using net single premiums based on monthly single-life options with a ten-year-certain period. We have not chosen, at this time, to make provision in our liability for the expense of handling the life-income payments. To this extent our reserve is understated.

## ANNUAL PREMIUM FORMULA

Mr. Charles Dubuar states that the reserve for unmatured options has been calculated on the " (a) single premium basis, (b) the annual premium basis with the annual premium calculated to commence as of the date of issue of the policies involved, or (c) the annual premium basis with the annual premium calculated to commence as of the year in which the reserve strengthening is commenced." ${ }^{9}$

If an annual premium is to be involved, there must be a source from which the income is derived. In the case of nonparticipating business, this may be anticipated future profit margins, while for participating business it may be a charge assessed against the dividends payable to those

TABLE 8
Liability Line Used To Report the Reserve for Unmatured Options by Fifteen Large Companies, 1954-63

| Liability Line | Exhibit | 1954 | 1958 | 1963 |
| :---: | :---: | :---: | :---: | :---: |
| 1. | Exhibit 8-A Exhibit 8-G Subtotal | 3 | 4 | 6 |
| 1.......... |  | 4 | 7 | 9 |
|  |  | 7 | 11 | 15 |
| $\begin{aligned} & 11 \\ & 25 \\ & 27 \end{aligned}$ |  | 1 |  |  |
|  |  | 5 | 2 |  |
|  |  | 2 | 2 |  |
|  | Grand Total | 15 | 15 | 15 |

policies creating the losses. The point is that the premium is not rigidly defined, and, as a practical matter, the level must be set by management after giving due consideration to the margins that are available.

In any event, if ${ }_{a}^{p} P_{x}$ is the level annual premium applicable to ${ }_{a}^{p} I_{x}^{b}$, then the reserve is obtained by deducting the value of the following expression from the single-premium reserve

$$
\sum_{\text {all } p} \sum_{\text {all } a} \sum_{\text {all } x}{ }_{a}^{p} P_{x}{ }_{a}^{p} I_{x}^{b} \sum_{t=1}^{k} v^{t-1 / 2} \cdot{ }_{t-1 / 2} p_{x+r+1 / 2}^{(r)},
$$

where $k$ is the number of years in the future for which the premium is assumed to be credited.

## CONVENTION BLANK

Two questions must be answered in reporting the reserve for unmatured options in the Convention Blank.

[^5]1. In which liability line should it appear on page 3?
2. In which Analysis of Operations column should it appear on pages 5 and 6?

## Liability Line

The liability lines used by fifteen companies during the period from 1946 to 1963 are shown in Table 9 and are summarized in Table 8.

It is readily apparent that, while in the past there was a certain variety in the reporting of the reserve for unmatured options on the liability page, by 1963 uniformity had been achieved. In that year-for the first timeall fifteen companies studied used line 1.

Since the reserve for unmatured options involves life contingencies and a rate of interest, there is no doubt that it belongs in line 1 on the liability page.

TABLE 9
Annual Statement Reporting of the Reserve for Unmatured Options (United States Companies Showing the Reserve in 1954 or Before)

| Year- <br> End | Phoenix <br> Mutual | New <br> York <br> Life | Prudential | Connecticut Mutual | Mutual <br> Benefit | Metropolitan | Bankers Life (Iowa) | Travelers |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Location of Liability |  |  |  |  |  |  |  |
| 1946. | V. 41 | V.4A | V. 4 | V. 38 | V. 4 |  |  |  |
| 1947. | V. 41 | V. 38 | V. 4 | V. 38 | V. 4 | V. 4 | V. 4 |  |
| 1948. | V. 41 | V. 38 | V. 4 | V. 38 | V. 4 | V. 4 | V. 1 | V. 4 |
| 1949 | V. 41 | V. 38 | V. 4 | V. 38 | V. 4 | V. 4 | V. 1 | V. 4 |
| 1950 | V. 41 | V. 38 | V. 4 | V. 38 | V. 4 | V. 4 | V.1,2 | V. 4 |
| 1951. | L 27.1 | L 25.3 | E 8-G | L 25.1 | E 8-A | E 8-G | E 8-A | E 8-G |
| 1952 | L 27.1 | L 25.3 | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1953 | L 27.1 | L 25.3 | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1954 | L 27.1 | L 25.3 | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1955 | L 27.1 | L 25.3 | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1956 | L 27.1 | L 25.3 | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1957. | L 27.1 | L 25.3 | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1958. | L 27.1 | E 8-G | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1959. | L 27.1 | E 8-A | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1960. | E 8-G | E 8-A | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1961 | E 8-G | E 8-A | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1962. | E 8-G | E 8-A | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
| 1963... | E 8-G | E 8-A | E 8-G | E 8-G | E 8-A | E 8-G | E 8-A | E 8-G |
|  | Analysis of Operations |  |  |  |  |  |  |  |
| 1963... | Col. 3 | Col. 3 | Col. 3 | Col. 7 | Col. 3 | Col. 3 | Col. 3 | Col. 3 |

TABLE 9-Continued

| $\begin{aligned} & \text { Year- } \\ & \text { End } \end{aligned}$ | Home Life | Northwestern Mutual Life | $\begin{aligned} & \text { Mutual } \\ & \text { Life } \end{aligned}$ |  | Provident Mutual | New England Mutual | State Mutual (Mass.) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Location of Liability |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| 1947. |  |  |  |  |  |  |  |
| 1948. | V.38A |  |  |  |  |  |  |
| 1949. | V. 38 |  |  |  |  |  |  |
| 1950. | V .38 | V. 37 | V. 38 | V. 4 |  |  |  |
| 1951. | L 25.3 | L 25.1 | L 25.4 | E 8-A |  |  |  |
| 1952. | L 25.3 | ${ }^{\mathrm{L}} 25.1$ | L 25.4 | E 8-A | L 25.1 |  |  |
| 1953. | L 25.2 L 25.2 | L 25.2 L 25.2 | L 25.4 L 25.4 | ${ }_{\text {E }}^{\text {E 8-A }}$ | L 25.2 L 25.2 | L 11.3 | L 27.1 |
| 1955. | ${ }_{L} \mathrm{~L} 25.2$ | L 25.2 | $\mathrm{E}_{\mathrm{E}}^{2} 8 \mathrm{~A}$ - | E 8-A | ${ }^{L} 25.2$ | E 8-G | ${ }^{L} 27.1$ |
| 1956. | L 25.2 | L 25.2 | E 8-A | E 8-A | L 25.2 | E 8-G | L 27.1 |
| 1957. | L 25.2 | L 25.2 | E 8-A | E 8-A | L 25.2 | E 8-G | L 27.1 |
| 1958. | L 25.2 | L 25.2 | E8-A | E 8-A | E 8-G | E8-G | L 27.1 |
| 1959. | E 8-G | E 8-G | E8-A | E 8-A | E 8-A | E8-G | L 27.1 |
| 1960. | E 8-G | E 8-G | E 8-A | E 8-A | E 8-A | E 8-G | L 27.1 |
| 1961. | E 8-G | E 8-G | E 8-A | E 8-A | E 8-A | E 8-G | L 27.1 |
| 1962. | E 8-G | E 8-G | E 8-A | E 8-A | E 8-A | E 8-G | L 27.1 |
| 1963... | E 8-G | E 8-G | E 8-A | E 8-A | E 8-A | E 8-G | E 8-G |
|  | Analysis of Operations |  |  |  |  |  |  |
| 1963. | Col. 3 | Col. 7 | Col. 3 | Col. 3 | Col. 3 | Col. 3 | Col. 3 |

Explanation of Notation.-1946-50: In these years, the liability section was designated by the Roman numeral V. For example, in the above table, V. 4 means line 4 of the liability section. 1951-63: Where the reserve is carried in liability line 1 , the above table shows either E 8 -A or E 8 -G, depending on whether the reserve is shown in Block $A$ or Block $G$ of Exhibit 8. In all other cases, the liability line is given following the letter L.

## Analysis of Operations

As indicated in Table 9, two general approaches are currently followed in reporting the reserve for unmatured options in the analysis of operations.

Thirteen of the fifteen companies studied use Method A under which the reserve is included in column 3 ("Life Insurance"). The remaining two companies use Method B under which the reserve is included in column 7 ("Supplementary Contracts").

In the schematic below, we have outlined a means of handling the reserve under each of the two methods. We realize that they are not unique, but feel they are sufficient to illustrate some of the more basic considera-
tions involved. To simplify the comparison, it will be helpful to start the discussion by setting down two premises.

1. It is assumed that the actual experience will precisely follow all the assumptions inherent in the calculation of the reserve for unmatured options.
2. It is assumed that the premium (if any) for this extra reserve is a predetermined net premium applied in the customary sense and not a form of reserve strengthening; thus reserve strengthening will not affect the net gain from operations.

Methods A and B may be compared symbolically as follows:
ANALYSIS OF OPERATIONS BY LINE OF BUSINESS

| Line | Title | Method A |  | Method B |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Column 3 | Column 7 | Column 3 | Column 7 |
| $\begin{aligned} & 2 . \\ & 4 . \end{aligned}$ | Considerations, etc. Net investment income | I | $-L$ |  | $-L$ $I$ |
| $\begin{aligned} & 17 . \\ & 19 . \end{aligned}$ | Increase in reserves, etc. Transfers on account of unmatured options | $P+I-L$ |  | $+P$ | $\underset{-P}{P+I-L}$ |
| 28 | Net gain, etc. | $-P+L$ | $-L$ | $-P$ | 0 |

$P$ represents the annual premium for the reserve for unmatured options; $I$, the required interest; and $L$, the reserves released. Under our premise, $L$ is equal to the excess of the tabular premium over the actual considerations received for life-income options. In fact, we plan to use that as the definition of $L$ in actual practice. The annual increase in reserves is $P+I-L$.

Under Method A, the reserve for unmatured options is included in the "Life Insurance" column. Each year this branch of the "Ordinary Life" line is charged with the annual premium and credited with the reserves released. The "Supplementary Contracts" branch is charged with the excess of the tabular premiums over the actual considerations. Thus, as settlements are made and losses occur, these losses are reflected under "Supplementary Contracts," and the reserve released acts, in effect, as a repayment to "Life Insurance" for current and prior contributions. The authors feel that this creates somewhat of a distortion in the "Life Insurance" gain, particularly if the reserve was built up by way of Exhibit 8-A, using unassigned surplus. In addition, "Supplementary Contracts"
continue to reflect a loss, even though such losses may have already been provided for. For these reasons we prefer Method B.

Under Method B, the "Life Insurance" branch is charged with the year's premium. Of course, this premium comes either from policyholder's dividends or profit margins. The premium is an income item for the "Supplementary Contract" branch of the "Ordinary Life" line, and, since the reserves released offset the deficiency in the considerations, there is no gain or loss in column 7.

In actual practice, experience will not always follow the assumptions. In the Analysis of Increase in Reserves, under Method A, Tabular Cost $(C)$ becomes the balancing item and will reflect the adequacy of the reserve; under Method B, Tabular less Actual Reserve released ( $T-A$ ) performs that function. As implied, in the schematic above, the Increase in Reserves is thus $P+I-C-L$ under Method A and $P+I+T-$ $A-L$ under Method B.

CONCLUSION
The primary objective of this paper has been to describe one method by which the reserve for unmatured options may be calculated. The authors know that this is not the only method of determining this reserve liability. However, we believe that it has the merit of setting out explicitly all the elements that affect its size.

This description demonstrates that the reserve for unmatured options is a complex life insurance reserve. It also demonstrates that its computation involves, either explicitly or implicitly, the use of (a) a rate of interest; (b) a mortality table of insured lives; (c) a mortality table of lifeincome beneficiaries; (d) a table of persistency rates; (e) a table of lifeincome election rates; and $(f)$ a definition of the typical beneficiary and the typical life-income option.

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Association Examination Reports

| Company | Year of Examination |
| :---: | :---: |
| Bankers Life (Iowa). | 1959 |
| Connecticut Mutual. | 1961 |
| Equitable (Iowa) | 1958 |
| Equitable Life (New York) | 1959 |
| Home Life (New York). | 1960 |
| John Hancock. | 1961 |
| Lincoln National Life | 1959, 1962 |
| Metropolitan Life. | 1960 |
| Mutual Benefit. | 1958 |
| Mutual (New York) | 1959 |
| New England Life. | 1962 |
| New York Life | 1958 |
| Northwestern Mutual Life. | 1957 |
| Provident Mutual. | 1958, 1961 |
| Prudential. | 1957, 1960 |
| State Mutual (Mass.) | 1957 |
| Travelers Life. | 1956 |

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## DISCUSSION OF PRECEDING PAPER

CHARLES M. STERNHELL:

Messrs. Lewis and Noback have presented a very interesting and comprehensive review of the Northwestern Mutual's method of determining its reserve for unmatured life-income options. Since the method differs in some respects from that used by the New York Life, a few comments on our method seem to be in order.

From Table 8 of the paper it can be seen that some companies, including my own, include their reserve for unmatured options with life insurance reserves in Part A of Exhibit 8, while others, including the authors' company, show it as a separate item under miscellaneous reserves in Part G of Exhibit 8. Perhaps this difference in treatment reflects a difference in philosophy regarding the nature of this reserve.

In our own case we include this reserve along with the basic policy reserve in Part A of Exhibit 8 because we believe it to be an integral part of the liability under the policies to which it relates.

As we see it, the existence of more favorable settlement option lifeincome guarantees in prior policy series than those considered appropriate at the present time has the effect of increasing the value of the insurance for which we are liable (i.e., the effective amount insured each year is in excess of the face amount). The increase in the effective amount insured each year, of course, reflects the excess of the present value of the guaranteed life income on a current settlement option mortality basis over the actual face amount of the policy after proper adjustment for the probability of election of a life income settlement.

We see no reason for not valuing the increase in the effective amount insured on mortality and interest bases consistent with those used for valuing the actual face amount of the policy. Similarly, we do not see any need to use withdrawal rates in calculating the reserve for the increase in the effective amount insured, as withdrawal rates are generally not used in calculating the reserve for the actual face amount of the policy. Our basic concept is that the reserve for unmatured life-income options is an integral part of the in solido (or whole) reserve for the life insurance policy after taking account of the required increase in the effective amount insured.

We do not make a cell-by-cell determination of the reserve, as the Northwestern does, but make use of average reserve factors in year of
issue blocks, using projected average attained ages of the insured and the payee at settlement and election rates for each block of issues.

Our reserve for unmatured options is not limited to policies with old 3 per cent interest guarantees for life-income options but also covers policies with 2 per cent and $2 \frac{1}{2}$ per cent interest guarantees. We have found that, for practical reasons, it is not desirable to have different declared rates of interest on settlement option proceeds arising from different policy series. We generally pay excess interest on the basis of the excess of the uniform declared interest rate for all policy series over the guaranteed life-income interest rate in each policy series. Consequently, we do not attempt to recover settlement option mortality losses from excess interest earnings but require each element to be self-sustaining. This means that we would use the guaranteed life-income interest rate in the policy or that used for reserve purposes, with an appropriate current life-income mortality basis to compute the present value of the more favorable settlement option life-income guaranteed in the prior policy series.

Messrs. Lewis and Noback carefully point out in the paper that, "if an annual premium is to be involved, there must be a source from which the income is derived." Our reserve is held on a single-premium basis, since we do not use a negative settlement option dividend element in our dividend formulas.

## EDWIN B. LANCASTER:

Messrs. Lewis and Noback are to be complimented for focusing our attention on the facts and actuarial considerations on the important matter of reserves for unmatured settlement options.

The purpose of this discussion is to describe Metropolitan's approach to this matter.

In 1947 Metropolitan, with the specific approval of the New York Insurance Department, began to accumulate an unmatured option reserve. It is determined by accumulating from the policy anniversary in 1947 a net annual premium for each policy in force on which the settlement option guarantees are overly liberal as measured by current standards. The net annual premium is calculated to provide an additional amount per $\$ 1,000$ of insurance at death or maturity based on what we believe to be realistic annuitant mortality and interest rates. Settlement option election rates and life insurance mortality rates are based on recent Metropolitan experience. Our dividend formula for those blocks of policies containing the overly liberal settlement options takes into consideration the extra cost of those options. The treatment in the dividend formula and the net annual premium reflected in the unmatured option reserve are
consistent. Thus, as I see it, our practice has the effect of making specific provision for a future contingency and releasing reserves following the same actuarial principles (although the mathematics may be a bit more complex) as the basic level-premium life insurance reserve.

Since Metropolitan began holding a reserve for unmatured options in 1947, it has been the practice to examine periodically the basis used in determining the reserve-taking account of then current conditions. Because of this, the basis of calculating the reserve was changed in 1952 and again in 1957, and we expect to change it again for the 1964 Annual Statement. The change in basis in 1952 and 1957 was approved by the New York Insurance Department, and we will shortly submit to the Department the basis proposed for the 1964 Statement, the effect of which will be to reduce our reserve for unmatured options due principally to the use of a higher interest rate.

The intended effect of our convention blank handling of the unmatured option reserve, the associated transfer of excess consideration (difference between the consideration on current realistic mortality and interest assumptions and the amount of policy proceeds), and the dividend practice is to reflect any added cost resulting from the overly liberal settlement option guarantees in the ordinary life insurance column (col. 3) of the Gain and Loss Exhibit and to place the settlement option column (col. 7) on a basis which shows a relatively small net gain (or loss) after dividends and federal income taxes.

## FRANK W. KLINZMAN:

Mr. Lewis and Mr. Noback are to be congratulated on their paper. I felt it was a very well-written and complete paper covering everything from the history of this reserve to how it is handled in the Annual Statement. In the hopes that it might cast further light upon the subject, I have given the following description of my company's method of computing this reserve.

The Connecticut Mutual used a model office approach in computing this reserve which is as follows:

1. We obtained a model office of paid-for arranged by plan and age groups at issue on a per $\$ 100,000$ paid-for basis.
2. By assuming 90 per cent of the 1946-49 Basic Table for mortality and Linton's A for withdrawals, we projected the model office to find the amount in-force at the end of each policy year and the amount for the deads and matureds for each policy year. For term plans, we used two times Linton's A for withdrawals to take into account a higher withdrawal rate because of term conversions.
3. We adjusted the face amount of the retirement income endowment plans that are included with the amounts shown as maturing each year in our projected model office. They were adjusted to reflect the cash value at maturity, since this is the amount that will be applied under the settlement options. We did not make any corresponding adjustment for death benefits where the death benefit was greater than the face amount.
4. We broke our year-end in-force into year of issue groups which had the same guaranteed settlement option rates. For example, our 1935-38 year-of-issue group had the same guaranteed settlement option rates.
5. By using our projected model office for $\$ 100,000$ paid-for in a given year, we found separately for each of the year-of-issue groups the amount in-force at the end of the current year and the amount of death benefit and matured endowment proceeds for each calendar year in the future.
6. For every fifth calendar year in the future, we calculated from our model office projections for each of the year-of-issue groups, the average age at death for the deads, and the average age at maturity for the matureds. For matureds, we assumed the payee was the insured. For deads, we assumed that, if the beneficiary was a male, he was 5 years older than the insured and, if the beneficiary was a female, she was 5 years younger than the insured. This gave us the average age of the beneficiaries for every fifth calendar year in the future for each of the year-of-issue groups.
7. By looking at settlement options that had been elected in a previous year, we were able to determine an average certain period and the percentage of male and female beneficiaries for the average ages of the beneficiaries found in 6 above. We found, as did Mr. Lewis and Mr. Noback, that, in practically all cases, the beneficiary arising from a death claim was a female. Therefore, we assumed for the deads the beneficiary was a female 5 years younger than the insured, and for matureds the beneficiary was the same age as the insured but with varying percentages of male and female. We also found that the certain period that was elected tended to vary with the age of the payee. At the younger ages the certain period seemed to be significantly longer than at the older ages. This seemed to reflect the wider differences in income that are received per $\$ 1,000$ of proceeds between short and long duration certain periods for the older aged payees. Therefore, we graded our certain periods for the payees down from about 20 years certain at the very young ages to about 9 or 10 years certain at the higher ages. This gave us the average age of the beneficiaries, a certain period, and the percentage of male and female beneficiaries for every fifth calendar year in the future for each of the year-of-issue groups.
8. Since for a given year of issue group we had for every fifth calendar year in the future the age, sex, and length of certain period of the bene-
ficiary, we could calculate the amount of loss per $\$ 1,000$ of proceeds. This loss would be the difference between the reserve required to be set up to provide the income under the settlement option and the policy proceeds. Now the Connecticut Mutual offers alternate option rates which provide 104 per cent of the income yielded by a current single premium annuity purchase. This led to the problem as to what the income rates would be in the future for those year-of-issue groups that have a guaranteed rate lower than the current nonguaranteed alternate rate. We felt that the general trend of income rates in the future would be down, so we assumed that the alternate option rates that would be used in the future would be somewhere between the guaranteed rates and the current alternate rates. This is what led us to the decision to assume that the future income rates would be a weighted average of the guaranteed rate and the current alternate rate, where the current alternate rate was more favorable. Where the guaranteed rate was more favorable, the guaranteed rate would apply.
9. Having the age and sex of the beneficiary, the length of the certain period, and the amount of income, we can calculate the losses per $\$ 1,000$ of proceeds that are applied under a settlement option. In determining these losses, the settlement option reserve basis that was used was the 1955 American Annuity Table set back two years with 33 per cent interest. For losses on options arising from matureds, we had for every fifth calendar year within each year-of-issue group the beneficiaries split into what percentages were male and female. Here we figured the loss for the male and female separately and weighted each of their losses according to the per cent of male and female beneficiaries there were for that particular calendar year. This gave a weighted loss per $\$ 1,000$ of proceeds for matureds. This approach was not necessary for the deads, since it was assumed all the beneficiaries were female. This gave losses per $\$ 1,000$ or proceeds for every fifth calendar year in the future for each of the year-ofissue groups. The losses per $\$ 1,000$ of proceeds for the intermediate calendar years were found by interpolation.
10. We had kept a record of the percentage electing a life-income option for 1949 and subsequent years. This covered $\$ 25,050,594$ of considerations received and $\$ 315,532,653$ of claims paid for deads and $\$ 33,339,007$ of considerations received and $\$ 134,881,592$ of claims paid for matureds. These results are shown in Table 1. Using this table as a guide, we projected graphically what we thought the percentage electing options might be in the future for each year-of-issue group separately. In doing this, we noticed that within each year-of-issue group the percentage electing options from the deads formed somewhat of a bell-shaped curve, whereas

TABLE 1
Percentage of Proceeds Left under Life-Income Options
(INSURANCE ONLY)

| Year of Issue | Calendar Year of Settlement |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1949 | 1950 | 1951 | 1952 | 1953 | 1954 | 1955 | 1956 | 1957 | 1958 | 1959 | 1960 | 1961 | 1962 | 1963 |
|  | Death Claims |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1912 and prior | 6.14 | 6.40 | 3.02 | 1.08 | 5.26 | 3.59 | 4.86 | 0.68 | 2.24 | 0.18 | 1.84 | 1.75 | 2.96 | 0.94 | 4.19 |
| 1913-27. | 9.35 | 11.05 | 11.84 | 6.02 | 11.25 | 13.58 | 6.53 | 9.03 | 8.67 | 10.04 | 6.66 | 7.32 | 5.69 | 11.73 | 7.56 |
| 1928-34. | 9.50 | 11.43 | 16.04 | 9.13 | 11.39 | 20.78 | 8.80 | 11.63 | 8.53 | 10.56 | 6.68 | 9.03 | 5.95 | 10.01 | 11.16 |
| 1935-38. | 17.28 | 12.50 | 6.83 | 14.98 | 14.77 | 15.80 | 8.94 | 17.15 | 17.15 | 19.30 | 12.90 | 12.82 | 13.11 | 14.18 | 7.52 |
| 1939-40 | 15.64 | 10.32 | 12.02 | 7.55 | 17.48 | 15.24 | 14.54 | 12.76 | 17.39 | 6.29 | 20.81 | 21.87 | 9.08 | 11.46 | 19.67 |
| 1941-42 | 3.17 | 2.14 | 7.43 | 8.83 | 7.46 | 3.30 | 2.92 | 9.10 | 5.72 | 8.62 | 8.75 | 10.79 | 4.81 | 9.64 | 8.71 |
| 1943-49 | 9.01 | 4.56 | 5.79 | 7.81 | 4.43 | 5.92 | 13.45 | 13.02 | 5.33 | 5.20 | 6.45 | 8.14 | 8.59 | 4.55 | 5.86 |
| 1950-57 |  | 14.50 | 3.47 | 2.61 | 1.53 | 6.06 | 7.31 | 4.50 | 2.22 | 2.58 | 3.26 | 4.46 | 5.55 | 3.74 | 2.24 |
| 1958-61 |  |  |  |  |  |  |  |  |  |  |  | 0.29 | 2.28 | 0.84 | 0.75 |
| 1962-63. |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1.29 |
|  | Matured Endowments |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1912 and prior. | 7.29 | 12.10 | 6.31 | 1.66 | 12.72 | 8.18 | 6.64 | 8.82 | 2.74 | 4.40 | 3.74 | 5.79 | 0 | 0 | 8.10 |
| 1913-27. | 10.24 | 8.86 | 9.39 | 9.27 | 12.83 | 9.53 | 9.25 | 10.61 | 13.33 | 9.02 | 8.67 | 6.53 | 5.19 | 4.99 | 8.14 |
| 1928-34. | 17.51 | 21.63 | 31.08 | 25.35 | 23.25 | 28.48 | 27.28 | 42.13 | 34.11 | 24.02 | 20.78 | 23.12 | 20.85 | 30.93 | 30.48 |
| 1935-38 | 30.78 | 38.85 | 48.09 | 37.60 | 29.59 | 48.63 | 25.92 | 39.16 | 32.74 | 29.17 | 37.92 | 38.63 | 43.45 | 42.44 | 27.16 |
| 1939-40 | 21.48 | 24.49 | 36.52 | 53.83 | 40.79 | 19.40 | 11.23 | 31.20 | 26.77 | 40.14 | 13.69 | 13.33 | 34.63 | 31.23 | 37.16 |
| 1941-42. | 21.79 | 54.39 | 10.77 | 37.82 | 57.20 | 59.31 | 95.39 | 37.99 | 24.17 | 52.39 | 47.11 | 40.21 | 20.43 | 15.98 | 23.74 |
| 1943-49 | 102.48* | 78.83 | 160.57* | 93.27 | 52.00 | 40.28 | 35.62 | 39.47 | 34.65 | 35.95 | 36.01 | 39.51 | 29.10 | 30.93 | 23.77 |
| 1950-57 |  |  |  |  |  |  |  |  |  | 78.92 | 60.05 | 29.64 | 48.76 | 55.53 | 48.02 |
| 1958-61 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13.73 |
| 1962-63. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

* Because of the sparsity of data and due to the fact that transfers from claims in previous years from the interest only option are included with considerations received, this percentage is greater than 100 per cent.
the percentage electing options from the matureds was fairly constant. We followed this same kind of pattern in our projection of the percentage electing an option. We also felt that varying the percentages electing options by the various year-of-issue groups would reflect any differences due to some having more favorable guaranteed rates than others.

11. We now have for every calendar year in the future for each year-of-issue group separately the amount of death benefit or matured endowment benefit, the percentage electing an option, and the loss per $\$ 1,000$ of proceeds that are applied under a life-income option. By multiplying these three items together, we have the total loss for each calendar year in the future for each year-of-issue group separately. We then discounted these losses back to the current year-end with interest. Adding up the discounted losses for the future calendar years within a year-of-issue group, we have the present value of all the future losses for the in-force at the current year-end determined by our model office. We then multiply the present value of the future losses determined from our model office by the ratio of our actual in-force for the same year-of-issue group with respect to the model office in-force for this year-of-issue group. This gives us the present value of all future losses for a given year-of-issue group for its actual in-force as of the current year-end. This is done for all the year-ofissue groups involved and for the deads and matureds and all the present values of losses are totaled.
12. We keep a record of the actual losses that are incurred each year under new life-income options. This is the difference between the reserve that is set up to provide the income and the policy proceeds, and these figures are shown separately for deads, matureds, and surrenders. Since 1949, the total losses on surrenders have run about 25 per cent of the total losses on matureds. We, therefore, estimated that the reserve to cover the future losses arising from surrenders was 25 per cent of the reserve to cover the losses arising from matureds.
13. By adding the reserve to cover losses arising from surrenders to the reserve to cover losses arising from deads and matureds, we then had the total reserve we wanted. This type of approach was also used on our annual premium-deferred annuities and was added to the reserve found for insurance.
14. This reserve is usually carried forward from year to year on an operating basis by adjusting for interest, for current losses covered, and for any new future losses which may have developed during the year. Only periodically do we use the model-office approach I have mentioned to compute this reserve. This is done to check the reserve we have computed by the method of carrying forward the previous year's reserve and to bring it back in line if it should happen to vary.

One could approximate the reserve required to cover the losses arising from AI death claims by taking a percentage of the reserve to cover losses arising from regular death claims. This percentage would be the percentage that the AI death claims have been to the regular death claims if they have exhibited a fairly stable relationship over recent years. Of course, this would assume that the assumptions with respect to the losses arising from AI are similar to the assumptions with respect to losses arising from the other deads.

Again I would like to congratulate Mr. Lewis and Mr. Noback on their paper and feel it is one that will be helpful to many of us.

JULIUS VOGEL:
The authors are to be congratulated on their thorough paper on the determination of this important reserve. Their formulas furnish a clear idea of the factors determining the amount of a company's liability with respect to unmatured settlement options. It seems to me, however, that the use of election rates reflecting essentially current experience might be open to some question. The frequency with which a settlement option rather than cash payment of policy proceeds is elected must depend to a considerable extent on the relationship between the interest rates underlying the settlement option guarantee and those available in the market place at the time the policy proceeds become payable. The size of the unmatured settlement option reserve depends directly on the assumed election rates, and it might be appropriate to determine these rates by examining experience at a time when interest yields obtainable on bank accounts, government bonds, etc., were a good deal lower than they have been recently.

The Prudential is one of the companies which uses Method A, under which the unmatured settlement option reserve is carried in the "Life Insurance" column of page 5 of the annual statement. Our treatment of this reserve is somewhat different from that described in the paper for Method A and, we believe, avoids some of the difficulties mentioned.

In our company the emerging strengthening for the current year (defined as the excess of the tabular premiums over the actual considerations received for the settlement options and denoted in the paper by $L$ ) is entered as an income item in line 6 of the "Supplementary Contracts" column of the Analysis of Operations by Lines of Business and offset by a corresponding negative entry in line 6 of the "Life Insurance" column. The aggregate increase in the reserve for unmatured settlement options is included in line 17 of the "Life Insurance" column. The effect of this treatment is that the net gain from "Supplementary Contracts" opera-
tions reported in line 28 is not directly affected by either the termination rate among policies for which an unmatured settlement options reserve is held or the settlement option election rate. Put very simply, "Life Insurance" transfers to "Supplementary Contracts" (via line 6 on page 5) a sum sufficient to enable the latter to set up the strengthened initial reserve on each newly issued supplementary contract without any effect on line 28 of the "Supplementary Contracts" column. In subsequent years, line 28 of the "Supplementary Contracts" column reflects the extent to which actual experience differs from the experience contemplated by the "Supplementary Contract" strengthened reserves. Any effect on operations of the year that may be attributed to variations in the rate of termination among policies for which an unmatured settlement option reserve is held, or in the frequency with which proceeds of such policies are actually applied under the settlement options, is reflected in line 28 of the "Life Insurance" column.

## FREDERICK S. TOWNSEND:

Mr. Lewis and Mr. Noback are to be commended for presenting statistical data which have been sought after but which have remained unavailable until now. I hope that several of the companies mentioned in this paper will enhance its value by submitting similar statistical data on their own experience.

But this paper is much more than a compilation of statistical data. The authors' treatment of the subject is thought-provoking, as they have brought the following thoughts to mind.

About seven years ago the guaranteed insurability option began to come into widespread use in the life insurance industry. One of the immediate problems falling upon any company which wished to add the guaranteed insurability option to its portfolio was the determination of an actuarially sound basis for calculating premiums and reserves. I believe the question of a premium scale was easily solved-many companies introduced a premium scale identical to that of the first company to introduce the product. But each company was left to its own devices to determine an adequate reserve liability.

There are at least three unique problems in determining reserves for unmatured life-income options and for guaranteed insurability options which normally do not enter into the reserve calculation for other life insurance products.

1. Attained ages at which options may be elected;
2. Percentage of insureds (or payees) electing options at each option age; and
3. Net single premiums at each option age representing the increase in the
present value of the newly opted life insurance policy or in the present value of the newly elected life-income option. This so-called increase in present value reflects the change in mortality, deterioration, and improvement, respectively, in determining the costs of guaranteed insurability options and ultraliberal life-income options.
The great point of dissimilarity between these two types of options is that reserves for unmatured life-income options have been established either by contributions from surplus funds or by infringing upon future profit margins, whereas reserves for guaranteed insurability options have been created from premium income.

What I am leading up to is this. No one will deny either that programming is a sales tool used quite extensively in today's market or that competitive considerations are often the deciding factor in the selection of a table of life-income settlement option rates.

Therefore, if the cost of ultraliberal life-income options can be calculated on as adequate a basis as (we hope) we have all calculated the cost of providing guaranteed insurability options, I would not be surprised if in the near future one or more companies intentionally promulgates a scale of ultraliberal life-income option rates and at the same time increases its gross premium rates so that the increased premium income will generate the necessary reserve funds for their future unmatured life-income options.

An oversimplified example might be as follows: A company determines that, for a particular block of new business, its maximum election rate of life-income options on death, surrender, and maturity proceeds will not exceed 10 per cent. It is then decided to increase the life-income option rates by 20 per cent, a very liberal adjustment. As a result, the present value of death, surrender, and maturity proceeds would be increased by 10 per cent of 20 per cent, requiring an increase of 2 per cent in the policy gross premium to provide adequate reserve funds for an ultraliberal scale of life-income option rates.

There is only one stumbling block that comes to mind. If companies purposely introduce ultraliberal life-income option rates, this fact may ultimately be recognized by the public, and the rate of election of these options would be likely to increase over the election rate which had been built into the gross premium charge for these options.
(AUTHORS' REVIEW OF DISCUSSION)
CLAIR A. LEWIS AND JOSEPH C. NOBACK:
We are indebted to each member who has discussed our paper and helped to focus attention on some of the important aspects of this Reserve.

Messrs. Sternhell, Lancaster, and Klinzman have described the methods by which the New York Life, Metropolitan, and Connecticut Mutual each determine the size of their respective liabilities. The New York Life's method emphasizes that the effect of granting ultraliberal lifeincome options is equivalent to increasing the amount of protection provided. The Metropolitan's method introduces a net annual premium collected by dividend deduction from a specified date, while Connecticut Mutual uses a model-office approach quite similar to that we used in the past.

Mr. Vogel noted, as we did in the paper, that there is every expectation that the level of election rates will tend to increase in the future. We recognize that-to the extent this happens-our reserve is understated. Mr. Vogel also pointed out that, in its Annual Statement, the Prudential uses a variation of Method A which avoids the difficulties inherent in this procedure. Each year Prudential enters $L$ as income in line 6 of the "Supplementary Contract" column.

After presenting a very interesting contrast between "guaranteed insurability options" and "unmatured life-income options," Mr. Townsend raised the somewhat disturbing thought that he would not be surprised if-for competitive reasons--"in the near future one or more companies intentionally promulgates a scale of ultraliberal life-income option rates and at the same time increases its gross premium rates." If this were done, the net annual premium for a reserve for unmatured options would be built into the premium structure at original issue. We firmly believe that any such action would be a serious mistake.

The reserves that are now held for unmatured life-income options are needed to fund deficiencies that arose some time after issue. Unanticipated changes occurred in mortality and interest experience. A company that would issue policies with ultraliberal rates is ignoring the lessons of the past sixty years.

Since 1959 the Northwestern Mutual has taken the opposite approach. We carefully grade the life-income rate by year of settlement as well as age at settlement. Thus we provide a conservative guarantee. If, on the date of settlement, the mortality and interest situation is favorable, then we shall grant a higher income to the beneficiary than that originally guaranteed.


[^0]:    SOURCE, SETTLEMENT OPTIONS IFGA; FLITCRAFTIMC. (NEW TORK 19641

[^1]:    * Bankers of Iowa; Canada Life; Connecticut Mutual; Equitable of New York; Home Life, New York;

    John Hancock; Lincoln National; Metropolitan Life; Mutual Benefit; Mutual of New York; New England Mutual; New York Life; Northwestern Mutual Life; Phoenix Mutual; Provident Mutual (Penn.); Prudential; State Mutual (Mass.); Sun Life (Canada); Travelers; and Union Central.

    Source: Setllement Options 1964 (New York: Flitcraft Inc., 1964).

[^2]:    ${ }^{1}$ Emory McClintock, "Special Tables for the Estimation of Mortality among Annuitants," TASA, VI, 13.
    ${ }^{2}$ Arthur Hunter, "Note on Mortality Under Settlement Options," TASA, XXXVII, 207.
    ${ }^{8}$ Frank D. Kineke, "A New Annuity Mortality Table," TASA, XXXIX, 8.
    ${ }^{4}$ Charles Dubuar, "Policy Reserves of Life Insurance Companies and Fraternal Societies," Chapter I of Examination of Insurance Companies, ed. Adelbert G. Straub, Jr. (New York: New York State Insurance Department, 1954), III, 1-94.

[^3]:    ${ }^{5}$ Edward W. Marshall, "Settlement Options," RAIA, XXXVI, 147.
    ${ }^{6}$ J. Gordon Beatty, "Settlement Options," RAIA, XXXVI, 149.

[^4]:    * For regular business and pension trust business combined.

[^5]:    ${ }^{9}$ Charles Dubuar, op. cit., p. 91.

