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Interactions Between Dynamic Lapses and Interest Rates in Stochastic Modeling

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The Joint Risk Management Section of the Society of Actuaries recently published the Policyholder Behavior in the Tail Variable Annuity Guaranteed Benefits Survey/C3 phase II 2009 Results. According to the survey, the goal "was to gain insight into companies' assumptions of variable annuity policyholder behavior in the tail of the C3 Phase II calculation."

The survey observed that "an overwhelming majority of insurers use dynamic lapses for living benefits. The percentage of insurers using dynamic lapses has risen from 83 percent in 2005 to 90 percent in 2009, with a peak of 95 percent in 2008." Since most companies are leveraging the expertise gained through their C3 phase II efforts in the VACARVM implementation, it is safe to assume that the same observation can be made in the VACARVM stochastic model as well.

Dynamic lapses for variable annuities reflect the phenomenon that policyholders tend not to surrender their policies when the guarantees embedded in the contracts are "in-the-money." A policy is said to be in-the-money when the guaranteed value exceeds the account value. It is "out-of-the-money" when the account value is sufficient to cover the value of the guarantees.

Reducing lapse rates when the policies are in-the-money tends to increase liabilities. Hence, it is generally reasonable to model dynamic lapses in a stochastic model to avoid understating liabilities. Consequently, it is not surprising to see that most of the companies have incorporated dynamic lapses in their stochastic models for statutory reserve and capital calculations. But does it mean that the work is done? Let's take a closer look.

The modeling is often achieved by using a dynamic lapse formula which acts to increase or decrease the base lapse rates when policies are out-of- or in-the-money. A formula that reduces the lapse rate when in-the-money and increases it when out-of-the-money is said to be two-sided. One that only decreases the lapse rate when in-the-money but does not increase it when out-of-the-money is said to be one-sided.

The extent to which the base lapse rate is increased or decreased obviously depends on the parameters chosen. It also depends on the definition of the guaranteed value which determines the level of in-the-moneyness, the factor that ultimately drives the lapse rate. Take the following formula for example:

$$\text{lapse rate} = \text{base lapse rate} \times e^{2\,[\text{MIN}(\text{account value} / \text{guaranteed value},\, 1) - 1]} \qquad (1)$$

This is a one-sided dynamic lapse formula. When guaranteed value exceeds account value in formula (1), the base lapse rate will be multiplied by a factor less than one, serving to reduce the base lapse rate. In fact, when the guaranteed value is twice the account value, or 200 percent in-the-money, the base lapse rate will be reduced to just 37 percent of its original value.

But what is the guaranteed value? Take a life-time guaranteed minimum withdrawal benefit (GMWB) rider on a variable annuity contract for example: Is it the Guaranteed Withdrawal Balance (GWB) defined in the contract? Or is it the present value of the stream of future guaranteed payments? What interest rates should be used to discount the stream of payments? Should a constant rate be used throughout the model? Or should the forward rates at the point of calculation be used?

In a stochastic model, the forward rates are specific to the time step and the scenario under consideration. Naturally, how the interest rates are modeled also has significant implications on the dynamic lapse function, since the guaranteed value that determines the in-the-moneyness is influenced by the stochastically modeled discount rates. As if it is not already complicated enough, the account value, another factor in the in-the-moneyness calculation, is influenced primarily by the equity markets whose performances are often correlated with the interest rates.

All these factors: dynamic lapses, stochastic interest rates, and correlations with the equity markets, are all inter-related in a stochastic model. Needless to say, considering all of their interactions can make the model extremely complex—not to mention resource consuming. It exemplifies perfectly the balancing act between model complexity and accuracy. But can we simply ignore the impact of interest rate modeling on dynamic lapses?

In practice, we have a tendency to simply either define the guaranteed value at a constant discount rate or to assume interest rates to be independent of the equity markets. However, one can easily imagine a scenario where extremely low equity returns and low interest rates drive the account value low but the guaranteed value high, yielding high in-the-moneyness and very low lapse rates. The liabilities in this scenario can be very high because all these generate high claims due to the guarantees. Therefore, one should carefully ensure that a simple definition of the guaranteed value does not underestimate the liabilities in the tail. This is particularly important when the measure of liability is a percentile or conditional tail expectation (CTE) of the distribution such as in the case of VACARVM and C3 Phase II.

For this article, the author studied the interactions between dynamic lapses and interest rate modeling through a stochastic model built for lifetime GMWB riders. The dynamic lapse function is as described in formula (1). The guaranteed value in the formula is defined to be the present value of future payment stream. Three alternatives of the discount rate are considered: constant, stochastic interest rates independent of equity returns, and stochastic interest rates with correlations to equity returns.

The GMWB Rider

For illustration purposes only, the author modeled a life time GMWB rider which charges 80 basis points and guarantees the following withdrawal rates for life:

Age of First Withdrawal	Guaranteed Withdrawal Rate
50	4.0%
60	5.0%
70	6.0%
80	7.0%

If no withdrawals are taken for 10 years, the rider guarantees 180 percent of the initial premium as the withdrawal base at the end of the 10th year. It also has an annual ratchet feature which steps up the withdrawal base if the account value is higher than the base on anniversaries.

The Cash Flow Model

Only cash flows due to the rider, specifically rider charges and claim payments, are modeled explicitly.

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The profit of the rider is defined to be the present value of the rider charges less the present value of the claims. The rider profit is often expressed either as a ratio or as basis points (bps) of the present value of the withdrawal base. For example, if the withdrawal base is $100,000 every year for the next 10 years, a profit of 10 bps means $100 each year for the next 10 years.

The cash flows are projected over 1,000 equity and interest rate scenarios. The average profit over the 1,000 scenarios is used as the measure of the value of the business. Since profit has an inverse relationship with the value of the liability due to the rider, it can be used as an indirect measure of the liability.

“The profit of the rider is defined to be the present value of the rider charges less the present value of the claims.”

Other cash flows such as M&E fees, revenue sharing, expenses, and commissions are not part of the cash flows that go into the profit calculation, although they serve to reduce the account value in the projection.

Other modeling approaches are certainly possible. For example, we can model the base contract and the rider together and consider all cash flows. We could use a different measure of liability such as a percentile or CTE of the distribution of the rider profit, or we could measure liability through accumulated deficiency or surplus as defined in AG 43 and RBC C3 Phase II. However, for understanding the interactions of dynamic lapses and interest rate modeling, the above simplified approach serves the purpose.

Equity and Interest Rate Scenarios

Six equity indices are modeled stochastically through a lognormal process. Means, volatilities and correlations between the six indices are based on historical data. The six indices are S&P500, Russell2000, NASDAQ COMPOSITE, EAFE, BOND, and Money Market. The funds of the policy with the GMWB rider are assumed to have a 60/40 allocation between stocks and bonds.

The short rate is modeled using the Cox, Ingersoll, and Ross model, which is a one-factor equilibrium model that reverts to a long-term mean.

The formula for changes in the short-term rate is as follows:

$$dr = a(b - r_{t-1})dt + \sigma\sqrt{r_{t-1}}\,dz$$

$$r_t = dr + r_{t-1}$$

The parameters are chosen as below:

r_0	1.24%
a	15.0%
b	5.0% or 3.0%
σ	7.5%

The parameter $\mathbf{r_0}$ is the initial short rate; **a** is referred to as the strength to mean reversion; **b** is the long-term mean target; and σ is the short rate volatility. *dz* is the sampling error of the standard normal distribution.

The short rate can be independent, or it can be correlated with equity returns, in which case the correlation is defined as:

Correlation	S&P500	Russell2000	Nasdaq Comp	Bond	EAFE	Money Market
Short Rate	(0.12)	(0.10)	(0.08)	(0.14)	(0.55)	-

Discount Rate to Determine Guaranteed Value

The dynamic lapses formula used is as described in formula (1). At any given time-step of the projection, the guaranteed value in the formula is the present value of the future payment stream that a policyholder could receive if he or she starts the lifetime withdrawal immediately. The discount rate can be a constant independent of the short rates stochastically generated in the model; or it can be a function of the short rates. Theoretically, the guaranteed value is a measure of how much the guarantee is worth to the policyholder if withdrawal is taken immediately. The discount rate to determine this value should be comparable to the rate at which the policyholder can annuitize the contract. This rate is often derived from a long-term treasury rate which is on the other end of the term structure of interest rate. For our purpose, the author approximated the annuitization rate by adding 100 bps to the short rate, and defined this rate to be the discount rate at which the guaranteed value is calculated.

> "... the guaranteed value in the formula is the present value of the future payment stream that a policyholder could receive if he or she starts the lifetime withdrawal immediately."

Discussion of Results

The model is run with only the following variations:

As for the discount rate to determine the guaranteed value in formula (1),
- 4 percent constant discount rate, or
- Short rate + 100 bps

As for the correlation between the short rate in the above bullet point and the equity returns,
- independent short rates
- correlated short rates with equity returns

As for the long-term reversion target of the short rate model,
- 3 percent long-term target
- 5 percent long-term target

The table on pg. 12 shows the average rider profit over the 1,000 scenarios.

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One immediate observation from the table below is that no matter what the long-term view of interest rates is (whether it's 3 percent or 5 percent); the linkage of the dynamic lapse formula and the interest rates has a material impact on the value of the liability. For example, if the short-term interest rate reverts to 3 percent long term and it varies independently with the equity returns, the profit is only 14 bps compared to the 23 bps if dynamic lapses are not linked to interest rates.

On closer inspection, when dynamic lapses and interest rates are linked, a higher long-term interest rate generally reduces liability and helps profitability. The higher discount rates reduce the guaranteed value of the contract, making them less in-the-money, hence lapse rates remain close to the base level.

The author has assumed a slightly inverse correlation between interest rates and equity returns. In other words, when equity returns are low, interest rates tend to be high, and vice versa. This inverse correlation helps to reduce liability since when equity returns are low, the high discount rates serve to reduce the guaranteed value of the contract. The dynamic lapse formula generates a higher lapse rate than it would have if the interest rates and the equity returns were not correlated.

Perhaps the biggest surprise is the 14 bps profit when the short-term interest rate reverts to the 3 percent level and the equity returns move independently of the interest rates. It is less than half of the 29 bps profit if the interest rates and equity returns are simply correlated, everything else being equal. This result could be even more dramatic if a percentile or a CTE measure was used. The explanation lies in the fact that when the short rate and the equity returns are not correlated, there are some scenarios with very low interest rates and low equity returns. The low equity returns result in reduced account values. The low discount rate exacerbates the situation by increasing the guaranteed value of the contract in the dynamic lapse formula, causing the contract to be more in-the-money. The resulting lapse rates from applying the formula are the lowest, which tends to increase liability. This is a classic case of increased tail risk due to the interactions of two or more variables.

Final Words

How do we address the question in the first section: can we be satisfied after building in the dynamic lapse formula in a stochastic model? The answer is that we need to carefully study the interactions between dynamic lapses and interest rates, making sure tail risks are not overlooked. Even when a simplified approach is preferable, such as using a constant discount rate to determine the guaranteed value, we need to ensure that it is consistent with the various interest rate assumptions such as long-term mean and correlations to equity returns.

As illustrated in the previous sections, not fully understanding the interaction can result in material differences in the calculation of liabilities. For pricing applications, this could mean not fully understanding profitability. For valuation models, this could lead to understating or overstating VACARVM reserves, FAS 133 reserves, RBC, or Economic Capital. For hedging applications, this could result in under or over hedging the liability. □

Guaranteed Value Discount Rate	3% Long Term Mean Reversion Target	5% Long Term Mean Reversion Target
4% Constant Discount Rate	0.23%	0.23%
Stochastic Short Rate Independent of Equity Returns	0.14%	0.25%
Stochastic Short Rate with Correlation to Equity Returns	0.29%	0.31%