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THE GENERALIZED FAMILY OF AGGREGATE ACTUARIAL COST METHODS FOR PENSION FUNDING

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INTRODUCTION

THE two papers written by Trowbridge [1, 2] and the discussions pertaining thereto fairly bristle with ideas for further exploration. In this paper a formula for a generalized family of aggregate actuarial cost methods will be discussed, and certain relationships with the ideas presented by Trowbridge and others will be shown.

TERMINOLOGY

One of the difficulties in describing actuarial cost methods and relating the results to those obtained by others lies in the area of nomenclature. The "new pension terminology" does not quite contemplate the generalized results which are described in this paper, yet it is desirable to use such terminology whenever possible. It is also desirable to co-ordinate the results here with those obtained by others, and the use of "old" terminology is sometimes necessary.

In the "new pension terminology" the generalized family is a "projected benefit cost method with supplemental liability." However, the supplemental liability may be zero, or it may be a function not clearly related to the supplemental liability of the particular actuarial cost method which is involved.

Further, under the generalized concept, a particular "accrued benefit cost method" is included within the generalized formula as a "projected benefit cost method," and identical costs and funds result in the ultimate state.

With these terminology problems in mind, the reader will appreciate the author's problem in some of the areas in which the "old" and the "new" terminologies overlap.

NOTATION

For convenience and because Trowbridge's papers have become the two main chapters in the bible of pension funding, the notation used by him will be used in this paper [2].

Let C_t represent the tth annual *contribution* to the pension plan, payable annually in advance.

Let F_t represent the *fund* built up after t years (before contribution or benefits then due).

Let B_t represent benefits for the tth year, assumed to be payable annually in advance.

Let V_t represent the present *value* of benefits, for both active and retired lives, at the beginning of the tth year, including the B_t payments then due. Benefits for future entrants are not included within V_t.

GENERALIZED FAMILY

In the discussion by Nesbitt of the second Trowbridge paper [2], he states:

It may be noted that another family of modified aggregate funding methods may be obtained by splitting off an amount L_0 of initial accrued liability on which only interest will be paid, and with contribution (for the illustrative plan) determined by

$$C_t = (V_t - F_{t-1} - L_0)/y + dL_0 ,$$

where

$$y = \sum_a^{r-1} l_x \ddot{a}_{x:\overline{r-x}|} \Big/ \sum_a^{r-1} l_x$$

as in the aggregate method. By various choices of L_0, one obtains funding equivalent ultimately to that under given standard methods.

This paper will concern itself with a similar, but more generalized, formula for aggregate actuarial cost methods.

The contribution formula for this generalized family is

$$C_t = \alpha_t(V_t - F_{t-1} - L_t) + \beta_t L_t ,$$

where

L_t is the unfunded amount at the end of year t of a portion of the initial present value of benefits to be specially funded by payments of $\beta_t L_t$.

α_t is a factor which measures the portion of costs to be paid at the beginning of year t, other than the special payment $\beta_t L_t$.

β_t is a factor which measures the payment to be made at the beginning of year t with respect to L_t.

It may be quickly noted that such actuarial cost methods as aggregate (in the traditional use of the word), attained age normal, frozen initial liability–entry age normal, Nesbitt's modified aggregate family, and Trowbridge's unfunded present value family are all special cases of this generalized family by proper choices of α, β, and L.

It is obvious that C_t and the limit of C_t as t becomes very large, and F_t and the limit of F_t as t becomes very large, are functions of α, β, and L. The available choices are without limit. Although this paper will show some of these general results, its main objective is to show the results obtained by conventional choices of the parameters α, β, and L.

Although α, β, and L can be set somewhat arbitrarily, this paper will explore a more traditional pattern, where α is a function of the lives covered for pension benefits, β is a payment toward or interest on an initial supplemental liability, and L is a function of the initial supplemental liability or liabilities. This approach may have some advantages of acceptance by those working in the pension field as compared with an empirical approach to α, β, and L.

SOME GENERAL RESULTS

Proofs and relationships throughout this paper will apply to the limiting case, where it is assumed that

$$\alpha = \lim_{t\to\infty} \alpha_t . \qquad B = \lim_{t\to\infty} B_t .$$

$$\beta = \lim_{t\to\infty} \beta_t . \qquad L = \lim_{t\to\infty} L_t .$$

$$V = \lim_{t\to\infty} V_t .$$

Thus, the results obtained apply to the initially mature population or to an initially immature population after it matures. In the limiting case, it can be shown (see Appendix I) that the ultimate contribution is

$$C_\infty = \lim_{t\to\infty} C_t = \alpha (V - F_\infty - L) + \beta L = \frac{\alpha(\beta - dV) + dL(\alpha - \beta)}{\alpha - d}$$

and the ultimate fund is

$$F_\infty = \lim_{t\to\infty} F_t = \frac{\alpha V - L(\alpha - \beta) - B}{\alpha - d} ,$$

where $d = i/(1+i)$. The equation of maturity is satisfied by $C_\infty + dF_\infty = B$.

The ultimate contribution, C_∞, is equal to ${}^{a}C_\infty + C$, where

$$^{a}C_\infty = \alpha\,(V - {}^{a}F_\infty) = \frac{\alpha\,(B - dV)}{\alpha - d}$$

is the ultimate contribution for actuarial cost method α, after the initial supplemental liability has been funded, and

$$C = \alpha({}^{a}F_\infty - F_\infty - L) + \beta L\ .$$

The ultimate fund F_∞ is equal to ${}^{a}F_\infty - F$, where

$$^{a}F_\infty = \frac{\alpha V - B}{\alpha - d}$$

is the ultimate fund for actuarial cost method α, after the initial supplemental liability has been funded, and

$$F = \frac{L(\alpha - \beta)}{\alpha - d}\ .$$

CONVENTIONAL CHOICES OF THE PARAMETERS α, β, AND L

As previously indicated, a large number of the common actuarial cost methods result from the appropriate choices and combinations of the parameters α, β, and L. We shall consider some of these.

L = *0*

The yearly contribution formula then becomes $C_t = \alpha_t(V_t - F_{t-1})$, and the ultimate contribution is $C_\infty = \alpha(V - F_\infty)$. This is the contribution formula for Trowbridge's unfunded present value family discussed in his second paper [2] with $\alpha = k + d$. With respect to Nesbitt's discussion of this paper, $\alpha = c$. In his discussion, Nesbitt shows that $c = 1/\ddot{a}_{\overline{z+1}|}$, where $z + 1$ is an average amortization period. It is worth emphasizing, as Nesbitt pointed out in his discussion, that $\alpha = c = k + d$ completely determines the ultimate contribution level C_∞. If

$$\alpha = \sum_{a}^{r-1} l_x \Big/ \sum_{a}^{r-1} l_x \ddot{a}_{x:\overline{r-x}|}\,,$$

C_∞ becomes the ultimate contribution for entry age normal funding, ${}^{EAN}C_\infty$. If the ultimate contribution for unit credit funding, ${}^{uc}C_\infty$, is desired,

$$\alpha = \sum_{a}^{r-1} (1+i)^x \Big/ \sum_{a}^{r-1} (1+i)^x (r - x)\,.$$

The latter development is contained in Appendix II. Further discussion of this point will be made later in this paper.

L *Is a Constant* > 0 *and* $\beta = \mathrm{d}$

In this case interest in advance at rate d is paid with respect to L. Thus L does not change, and the yearly contribution formula is $C_t = \alpha_t(V_t - F_{t-1} - L) + dL$. The ultimate contribution is $C_\infty = \alpha(V - F_\infty - L) + dL$.

If

$$\alpha = \sum_a^{r-1} l_x \Big/ \sum_a^{r-1} l_x \ddot{a}_{x:\overline{r-x}|},$$

Nesbitt's modified aggregate family results. If

$$\alpha = \sum_a^{r-1} (1+i)^x \Big/ \sum_a^{r-1} (1+i)^x (r-x),$$

another modified aggregate family results.

If

$$\alpha = \sum_a^{r-1} l_x \Big/ \sum_a^{r-1} l_x \ddot{a}_{x:\overline{r-x}|},$$

$C_\infty = {}^{EAN}C_\infty + C$ for the results shown in Appendix III, where ${}^{EAN}C_\infty$ is the ultimate normal cost under the entry age normal actuarial cost method. If

$$\alpha = \sum_a^{r-1} (1+i)^x \Big/ \sum_a^{r-1} (1+i)^x (r-x),$$

$C_\infty = {}^{UC}C_\infty + C$ for the results shown in Appendix III, where ${}^{UC}C_\infty$ is the ultimate normal cost under the unit credit actuarial cost method.

If L is set equal to $({}^{\alpha}F_\infty - {}^{\gamma}F_\infty)$, where ${}^{\gamma}F_\infty$ is the ultimate fund under some particular actuarial cost method γ, $F_\infty = {}^{\gamma}F_\infty$, and $C_\infty = {}^{\gamma}C_\infty$ (see Appendix III). This is the required L, given α, if ultimate results under the particular actuarial cost method γ are desired.

$\beta > \mathrm{d}$

Under this case, payments greater than interest are made, and L eventually becomes 0. This case has been discussed under $L = 0$.

Other Combinations of α, β, *and* L

Other practical and impractical combinations of α, β, and L are possible and can be analyzed for the ultimate state using the results shown in the appendices. For example, another example involves L held in the calculation as a constant > 0 and $\beta = 0$.

The yearly contribution formula then becomes $C_t = \alpha_t(V_t - F_{t-1} -$

L), and the ultimate contribution is $C_\infty = \alpha(V - F_\infty - L)$. Using Appendix III, it is demonstrated that the ultimate contribution becomes $C_\infty = {}^{a}C_\infty + \alpha({}^{a}F_\infty - F_\infty - L)$, and the ultimate fund becomes $F_\infty = {}^{a}F_\infty - \alpha L/(\alpha - d)$. If L is set equal to $[(\alpha - d)/\alpha]({}^{a}F_\infty - {}^{\gamma}F_\infty)$, where ${}^{\gamma}F_\infty$ is the ultimate fund under some particular actuarial cost method γ, $F_\infty = {}^{\gamma}F_\infty$, and $C_\infty = {}^{\gamma}C_\infty$ (see Appendix IV). This is the required L, given α, if ultimate results under the particular actuarial cost method γ are desired.

THE PARAMETER α

Note that

$$\alpha = \sum_{a}^{r-1} l_x \Big/ \sum_{a}^{r-1} l_x \ddot{a}_{x:\overline{r-x}|}$$

is the reciprocal of the average temporary annuity, weighted by l_x. When

$$\alpha = \sum_{a}^{r-1} (1+i)^x \Big/ \sum_{a}^{r-1} (1+i)^x (r-x)\,,$$

we obtain the reciprocal of the average years to retirement, weighted by $(1 + i)^x$. There is an α corresponding to every actuarial cost method.

PRACTICAL ACTUARIAL COST METHODS

Some Familiar Old Friends

It is worth mentioning three of the actuarial cost methods in current use which are special cases of the generalized formula. They are:

Actuarial Cost Method	Choice of Parameters
Entry age normal–frozen initial liability.........	$\alpha = \sum_{a}^{r-1} l_x \Big/ \sum_{a}^{r-1} l_x \ddot{a}_{x:\overline{r-x}|}$. $\beta = d$, or $\beta = 1/\ddot{a}_{\overline{n-t}|} > d$. L_0 = Initial accrued liability. If $\beta = d$, $L = L_0$. If $\beta = 1/\ddot{a}_{\overline{n-t}|}$, $L = 0$ for $t > n$.
Attained age normal.....	$\alpha = \sum_{a}^{r-1} l_x \Big/ \sum_{a}^{r-1} l_x \ddot{a}_{x:\overline{r-x}|}$. $\beta = d$, or $\beta = 1/\ddot{a}_{\overline{n-t}|} > d$. L_0 = Initial past-service liability. If $\beta = d$, $L = L_0$. If $\beta = 1/\ddot{a}_{\overline{n-t}|}$, $L = 0$ for $t > n$.

Aggregate.............. $\alpha = \sum_{a}^{r-1} l_x \Big/ \sum_{a}^{r-1} l_x \ddot{a}_{x:\overline{r-x}|}$.

$\beta = 0$.

$L = 0$.

Under Trowbridge's classification [1], these are all Funding Class IV methods.

It is interesting that the generalized formula, with a different choice of α, can lead us to similar methods in other funding classes. Two of these, of Funding Class III, will next be presented.

Some Worthwhile New Acquaintances

Let us consider two actuarial cost methods, categorized in tabular form:

Actuarial Cost Method	Choice of Parameters
Unit credit–frozen initial liability...............	$\alpha = \sum_{a}^{r-1} (1+i)^x \Big/ \sum_{a}^{r-1} (1+i)^x (r-x)$.
	$\beta = d$, or $\beta = 1/\ddot{a}_{\overline{n-t}|} > d$.
	L_0 = Initial past-service liability.
	If $\beta = d$, $L = L_0$.
	If $1/\ddot{a}_{\overline{n-t}|}$, $L = 0$ for $t > n$.
Aggregate (Class III).....	$\alpha = \sum_{a}^{r-1} (1+i)^x \Big/ \sum_{a}^{r-1} (1+i)^x (r-x)$.
	$\beta = 0$.
	$L = 0$.

In Appendix V it is shown that the ultimate contribution and fund are those for Trowbridge's Class III—that is, contributions and funds for unit credit funding. Note that the unit credit–frozen initial liability method is an accrued benefit cost method, which has been expressed as a projected benefit cost method. Its ultimate cost and fund are identical with those for the aggregate (Class III) method, which is a true projected benefit cost method.

The unit credit–frozen initial liability method has been in use in a limited way for several years. It was probably developed originally by the late William M. Rae, but William G. Schneider, Richard P. Peterson, and Charles E. Farr all contributed to its development. It is possible that others have used this method. As far as the author knows, its significance as a Class III method, because of its parameter a, has not been noted previously.

It is also possible that the aggregate (Class III) method has been in use, but its existence has not hitherto been noted in the literature or elsewhere to the author's knowledge, except as a special case of Trowbridge's unfunded present value family. This should serve to show the interrelationships of actuarial cost methods. Trowbridge's unfunded present value family is a special case of the generalized family of aggregate actuarial cost methods, and one of the most important members of the latter family is a special case of the former family!

These two methods—unit credit–frozen initial liability and aggregate (Class III)—have some attractive features. Spreading of gains and losses under Class III (unit credit type) methods is technically feasible. This means that spreading of gains or losses under an accrued benefit cost method is possible if it is expressed as a projected benefit cost method. The ultimate accumulation of funds under each method is less than that which develops under Class IV methods. This may be the answer to some of the points raised by Griffin [3]. The initial costs under aggregate (Class III) are less than those for the "usual" Class IV aggregate method; this may be attractive.

CALCULATION TECHNIQUES FOR AN IMMATURE POPULATION

For practical use, a does not simplify as nicely as might be indicated in the ultimate state.

For Class IV methods, a_t is an average temporary annuity, weighted by normal costs (or by benefits, if a common entry age is assumed). For aggregate Class III or the unit credit–frozen initial liability methods, a_t is an average of years to retirement, weighted by current-service cost. This would be calculated in each year t, with respect to the then existent employee group.

YEARLY CONTRIBUTION AND FUNDS

Trowbridge has shown [2] the contributions and funds for aggregate (Class III) and the regular form of aggregate (Class IV) actuarial cost methods. For the mature situation, a_t would equal $k + d$ for all t, and Trowbridge's Table 1 [2] would produce initial results identical with those which could be obtained using the techniques of this paper. For the immature situation, a_t would not equal $k + d$ for all t, because a_t would depend on the actual employee group in year t. Thus Trowbridge's Table 2 [2] would not show exactly the results which would be obtained using the techniques shown in this paper, except in the ultimate state. However, the differences should be fairly small, and a general comparison of aggregate (Class III) and aggregate (Class IV) can be obtained there.

CONCLUSIONS

The basic goal of this paper has been to develop a generalized statement of aggregate actuarial cost methods and relate to it certain ideas developed by Trowbridge, Nesbitt, and others. A secondary goal has been to describe practical methods which bear the same relationship to unit credit funding that the "usual" forms of frozen initial liability and aggregate bear to entry age normal funding. It is the author's hope that the general knowledge of actuarial cost methods has been advanced in the process.

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APPENDIX I

$$C_t = \alpha_t(V_t - F_{t-1} - L_t) + \beta_t L_t\,.$$

$$F_t = (F_{t-1} + C_t - B_t)(1+i)$$

$$= [F_{t-1} + \alpha_t(V_t - F_{t-1} - L_t) + \beta_t L_t - B_t](1+i)\,.$$

$$F_{t-1}[(1+i)\alpha_t - i] + \Delta F_{t-1} = [\alpha_t V_t - L_t(\alpha_t - \beta_t) - B_t](1+i)\,.$$

Let us assume:

$$\lim_{t\to\infty} \alpha_t = \alpha\,. \qquad \lim_{t\to\infty} V_t = V\,.$$

$$\lim_{t\to\infty} \beta_t = \beta\,. \qquad \lim_{t\to\infty} B_t = B\,.$$

$$\lim_{t\to\infty} L_t = L\,.$$

C_t and F_t have limits as t increases without limit. Then

$$F_\infty = \lim_{t\to\infty} F_t = \lim_{t\to\infty} F_{t-1} = \frac{[\alpha V - L(\alpha-\beta) - B]}{(1+i)\alpha - i}\,(1+i)$$

$$= \frac{\alpha V - L(\alpha-\beta) - B}{\alpha - d}\,,$$

where

$$d = i/(1+i)$$

and

$$C_\infty = \lim_{t\to\infty} C_t = \alpha(V - F_\infty - L) + \beta L$$

$$= \alpha\left\{V - \left[\frac{\alpha V - L(\alpha-\beta) - B}{\alpha - d}\right] - L\right\} + \beta L$$

$$= \frac{\alpha(B - dV) + dL(\alpha-\beta)}{\alpha - d}\,.$$

For the equation of maturity,

$$C_\infty + dF_\infty = \frac{\alpha(B - dV) + dL(\alpha - \beta)}{\alpha - d} + d\left[\frac{\alpha V - L(\alpha - \beta) - B}{\alpha - d}\right]$$

$$= \frac{B(\alpha - d)}{\alpha - d} = B .$$

APPENDIX II

$$C_\infty = \alpha(V - F_\infty) .$$

If

$$C_\infty = {}^{UC}C_\infty = \frac{1}{r-a}\sum_a^{r-1} l_x \frac{N_r}{D_x},$$

and

$$F_\infty = {}^{UC}F_\infty = \frac{1}{r-a}\sum_a^{r-1} l_x (x-a)\frac{N_r}{D_x} + \sum_r^{w} l_x \frac{N_x}{D_x},$$

and

$$V = \sum_a^{r-1} l_x \frac{N_r}{D_x} + \sum_r^{w} l_x \frac{N_x}{D_x},$$

then

$$\alpha = \frac{[1/(r-a)]\sum_a^{r-1} l_x \frac{N_r}{D_x}}{\left[\sum_a^{r-1} l_x \frac{N_r}{D_x} + \sum_r^{w} l_x \frac{N_x}{D_x}\right] - \left[[1/(r-a)]\sum_a^{r-1} l_x (x-a)\frac{N_r}{D_x} + \sum_r^{w} l_x \frac{N_x}{D_x}\right]}$$

$$= \frac{[1/(r-a)]\sum_a^{r-1} l_x \frac{N_r}{D_x}}{[1/(r-a)]\sum_a^{r-1} (r-x) l_x \frac{N_r}{D_x}} = \frac{\sum_a^{r-1} (1+i)^x}{\sum_a^{r-1} (1+i)^x (r-x)} .$$

The converse is easily proved, also. Thus if α is of the form indicated, α is a necessary and sufficient condition for $C_\infty = {}^{UC}C_\infty$ and $F_\infty = {}^{UC}F_\infty$.

APPENDIX III

If $\beta = d$,

$$C_\infty = {}^{\alpha}C_\infty + \alpha({}^{\alpha}F_\infty - F_\infty - L) + dL ;$$

$$F_\infty = {}^{\alpha}F_\infty - \frac{\alpha - d}{\alpha - d} L = {}^{\alpha}F_\infty - L .$$

If it is desired that $C_\infty = {}^\gamma C_\infty$ and $F_\infty = {}^\gamma F_\infty$, where γ is the desired actuarial cost method in the ultimate state, set $L = {}^a F_\infty - {}^\gamma F_\infty$. Then

$$F_\infty = {}^a F_\infty - ({}^a F_\infty - {}^\gamma F_\infty) = {}^\gamma F_\infty$$

and

$$\begin{aligned} C_\infty &= {}^a C_\infty + a({}^a F_\infty - {}^\gamma F_\infty - {}^a F_\infty + {}^\gamma F_\infty) + d({}^a F_\infty - {}^\gamma F_\infty) \\ &= {}^a C_\infty + d({}^a F_\infty - {}^\gamma F_\infty) \\ &= {}^a C_\infty + d\,{}^a F_\infty - d\,{}^\gamma F_\infty \\ &= B - d\,{}^\gamma F_\infty \\ &= {}^\gamma C_\infty\,. \end{aligned}$$

APPENDIX IV

If $\beta = 0$,

$$C_\infty = {}^a C_\infty + a\,({}^a F_\infty - F_\infty - L)\,;$$

$$F_\infty = {}^a F_\infty - \frac{aL}{a-d}\,.$$

If it is desired that $C_\infty = {}^\gamma C_\infty$ and $F_\infty = {}^\gamma F_\infty$, where γ is the desired actuarial cost method in the ultimate state, set

$$L = \frac{a-d}{a}({}^a F_\infty - {}^\gamma F_\infty)\,.$$

Then

$$F_\infty = {}^a F_\infty - ({}^a F_\infty - {}^\gamma F_\infty) = {}^\gamma F_\infty$$

and

$$\begin{aligned} C_\infty &= {}^a C_\infty + a\left[{}^a F_\infty - {}^\gamma F_\infty - \left(\frac{a-d}{a}\right)({}^a F_\infty - {}^\gamma F_\infty)\right] \\ &= {}^a C_\infty + a\,({}^a F_\infty - {}^\gamma F_\infty) - (a-d)\,({}^a F_\infty - {}^\gamma F_\infty) \\ &= {}^a C_\infty + d\,({}^a F_\infty - {}^\gamma F_\infty) \\ &= {}^a C_\infty + d\,{}^a F_\infty - d\,{}^\gamma F_\infty \\ &= B - d\,{}^\gamma F_\infty \\ &= {}^\gamma C_\infty\,. \end{aligned}$$

APPENDIX V

Using the results of Appendix III,

$${}^{A(\mathrm{III})}C_\infty = {}^a C_\infty + a[{}^a F_\infty - {}^{A(\mathrm{III})}F_\infty - L] + dL\,;$$

$${}^{A(\mathrm{III})}F_\infty = {}^a F_\infty - L\,.$$

Now, a is that for the unit credit actuarial cost method and $L = 0$; then

$$^{a}C_{\infty} = {}^{UC}C_{\infty}\ ; \qquad {}^{a}F_{\infty} = {}^{UC}F_{\infty}\ ;$$

then

$$^{A(III)}F_{\infty} = {}^{UC}F_{\infty}\ ; \qquad {}^{A(III)}C_{\infty} = {}^{UC}C_{\infty} + a[{}^{UC}F_{\infty} - {}^{A(III)}F_{\infty}] = {}^{UC}C_{\infty}\ .$$

If L_0 = initial past-service liability and has been funded, so that $L = 0$, the above results apply to the unit credit–frozen initial liability method.

If $L = L_0$ and $\beta = d$,

$$^{UC\text{-}FIL}C_{\infty} = {}^{UC}C_{\infty} + a({}^{UC}F_{\infty} - {}^{UC\text{-}FIL}F_{\infty} - L_0) + dL_0\ ;$$

$$^{UC\text{-}FIL}F_{\infty} = {}^{UC}F_{\infty} - L_0\ ;$$

$$^{UC\text{-}FIL}C_{\infty} = {}^{UC}C_{\infty} + dL_0\ .$$

Here the unit credit–frozen initial liability method is Class III, with an interest payment dL_0 added to the contribution and a fund shortage L_0, if $\beta = d$.

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