TRANSACTIONS OF SOCIETY OF ACTUARIES
1967 VOL. 19 PT. 1 NO. 55

THE EFFECT OF INTEREST ON PENSION CONTRIBUTIONS

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I. INTRODUCTION

In discussions of pension plan costs the actuary often needs to describe the effect on contributions of a variation in the interest rate. The rule of thumb that an increase of $\frac{1}{4}$ of 1 per cent in the interest rate will result in a 6 per cent reduction in contributions is believed to be widely used by actuaries and others. That this "rule" is not applicable to all situations is obvious considering that under pay-as-you-go funding interest has no effect on contributions.

The importance of interest is referred to in a general way in many texts and papers on the subject of pensions. McGill has stated: "In a typical plan . . . a variation of one fourth of 1 per cent in the interest assumption can be expected to produce a differential of 6 or 7 per cent in the overall valuation of liabilities."[1] Crabbe and Poyser have suggested that, on a money-purchase plan, an increase in the rate of interest from 3 to $3\frac{1}{4}$ per cent would result in a $7\frac{1}{4}$ per cent reduction in the contributions required to purchase the same benefit.[2] Hamilton and Bronson have demonstrated the effect of variations in several assumptions, including the interest rate, on a level premium to retirement age for entry ages 25 and 45.[3] The *Study Notes* contain a brief reference to the impact of the interest rate on pension fund costs and include the following illustrations:[4]

Age	Function	$3\frac{3}{4}$% Factor as Percentage of $3\frac{1}{2}$% Factor	4% Factor as Percentage of $3\frac{1}{2}$% Factor
30......	Individual Level Cost—payable to age 65	92.9%	86.3%
50......	Deferred Annuity—commencing at age 65	94.7	89.6
70......	Life Annuity	98.4	96.9

Source.—*The Pension Actuary's Handbook* (Turnover Scale T-1; see Appendix A).

[1] Dan Mays McGill, *Fundamentals of Private Pensions* (Homewood: Irwin & Sons, 1964), p. 208.

[2] R. J. W. Crabbe and C. A. Poyser, *Pensions and Widows' and Orphans' Funds* (Cambridge: Cambridge University Press, 1953) p. 40.

[3] James A. Hamilton and Dorrance C. Bronson, *Pensions* (New York: McGraw-Hill Book Co., Inc., 1958) p. 270.

[4] "Valuation Assumptions and Budgeting Methods," *Society of Actuaries Study Notes*, Part 9E, 2-2-66, p. 16.

II. PURPOSE

This paper will attempt (1) to establish an index of interest variation for the more commonly used actuarial cost methods assuming certain population models, (2) to indicate how the index is affected by altering the models, and (3) to check on the acceptability of the rule of thumb.

III. MODELS AND TERMINOLOGY, NOTATION, AND GENERAL-CONCEPT TERMINOLOGY AND MODELS

Illustrations appearing in Tables 1 and 2 are based on mature and immature population models used by Trowbridge.[5] Contributions and funds at interest rates of $3\frac{1}{2}$ and $4\frac{1}{2}$ per cent for both models are shown in Appendix B. Trowbridge has shown these values assuming $2\frac{1}{2}$ per cent.

The actuarial cost terminology follows the recommendations of the Committee on Pension and Profit-sharing Terminology. Trowbridge's funding class terminology is included to facilitate reference to his paper.

NOTATION

The following notation has been used to describe the calculations appearing in Tables 1, 2, 3, and 4.

${}^kI_r$ = Index of effect on contributions of $\frac{1}{4}$ per cent increase in interest for actuarial cost method k over first r plan years.

${}^kC_t^j$ = Pension contribution in year t for actuarial cost method k with interest at rate j.

B_t = Annual retirement benefits paid at beginning of year t.

${}^kF_t^j$ = Pension fund at beginning of year t (before contributions or benefits then due) for actuarial cost method k and interest at rate j.

$d^j = \dfrac{j}{1+j}$.

∞ = Subscript denoting ultimate level of benefits, contributions, funds, and index of effect.

GENERAL CONCEPT

The ratio of total contributions based on interest at $4\frac{1}{2}$ per cent to total contributions based on $3\frac{1}{2}$ per cent is used to determine the index of effect. The general concept can be expressed symbolically as

$$ {}^kI_r = 0.25 \times \left(1 - \frac{\sum_1^r {}^kC_t^{0.045}}{\sum_1^r {}^kC_t^{0.035}} \right). \qquad (1) $$

[5] C. L. Trowbridge, "Fundamentals of Pension Funding," *TSA*, IV, 17.

Most discussions involving the difference in contributions produced by an increase in interest are concerned with the long range. At the limit point, total plan contributions are dominated by the ultimate level contribution ${}^{k}C^{i}_{\infty}$, so that it is sufficient to use

$$ {}^{k}I_{\infty} = 0.25 \times \left(1 - \frac{{}^{k}C^{0.045}_{\infty}}{{}^{k}C^{0.035}_{\infty}}\right) \qquad (2) $$

to measure the long-range effect.

The pension fund level created by the excess of contributions over benefits is fundamental to the impact of interest earnings. This can be demonstrated algebraically by making the following substitution in formula 2:[6]

$$ {}^{k}C^{i}_{\infty} \equiv B_{\infty} - d^{i}\,{}^{k}F^{i}_{\infty} \,. $$

The ultimate index of effect becomes

$$ {}^{k}I_{\infty} = 0.25 \times \left(\frac{d^{0.045}\,{}^{k}F^{0.045}_{\infty} - d^{0.035}\,{}^{k}F^{0.035}_{\infty}}{B_{\infty} - d^{0.035}\,{}^{k}F^{0.035}_{\infty}}\right). \qquad (3) $$

This expression is a function of the fund developed by the plan which, in turn, is a function of the actuarial cost method used for determining contributions.

IV. ILLUSTRATIONS

The results of applying formulas (1) and (2) to the contributions illustrated in Appendix B are shown in Tables 1 and 2. The annual contributions sections of Tables 1 and 2 show the effect of an increase of $\frac{1}{4}$ per cent in interest on each year's contribution; that is,

$$ 0.25 \times \left(1 - \frac{{}^{k}C^{0.045}_{t}}{{}^{k}C^{0.035}_{t}}\right). $$

The accumulated contributions sections show the cumulative effect, ${}^{k}I_{r}$.

Tables 1 and 2 indicate that ${}^{k}I_{r}$ increases as the level of funds generated by the actuarial cost method increases; that is, higher fund, larger index; lower fund, smaller index. The relationship between fund levels and index of effect also can be seen by comparing ${}^{k}I_{r}$ for the same actuarial cost methods in the mature and immature models. Although contributions for the mature group are higher than those for the immature population, the lower initial benefit payout to retirees in the immature group results in higher funds relative to contributions in early years.

[6] *Ibid.*, p. 18.

TABLE 1

INDEX OF EFFECT ON CONTRIBUTIONS
MATURE POPULATION

Year	Class I	Class II	Class III	Class IV	
	Pay-as-You-Go	Terminal Funding	Accrued Benefit—20-Yr.	Individual Level Cost—20-Yr.	Aggregate Level Cost
	Annual Contributions				
1	0.00%	1.37%	1.84%	2.16%	1.99%
2	.00	1.75	1.84	2.16	2.03
3	.00	1.75	1.84	2.16	2.08
4	.00	1.75	1.84	2.16	2.14
5	.00	1.75	1.84	2.16	2.19
10	.00	1.75	1.84	2.16	2.54
15	.00	1.75	1.84	2.16	3.00
20	.00	1.75	1.84	2.16	3.56
21	.00	1.75	4.70	7.13	3.68
25	.00	1.75	4.70	7.13	4.18
30	.00	1.75	4.70	7.13	4.81
35	.00	1.75	4.70	7.13	5.38
40	.00	1.75	4.70	7.13	5.86
50	.00	1.75	4.70	7.13	6.52
Limit	0.00	1.75	4.70	7.13	7.13
	Accumulated Contributions				
1	0.00%	1.37%	1.84%	2.16%	1.99%
2	.00	1.40	1.84	2.16	2.01
3	.00	1.43	1.84	2.16	2.03
4	.00	1.45	1.84	2.16	2.05
5	.00	1.47	1.84	2.16	2.08
10	.00	1.54	1.84	2.16	2.20
15	.00	1.58	1.84	2.16	2.34
20	.00	1.61	1.84	2.16	2.48
21	.00	1.62	1.88	2.21	2.51
25	.00	1.63	2.02	2.38	2.62
30	.00	1.65	2.18	2.58	2.77
35	.00	1.66	2.32	2.76	2.91
40	.00	1.67	2.45	2.92	3.05
50	.00	1.68	2.67	3.22	3.32
Limit	0.00	1.75	4.70	7.13	7.13

TABLE 2

INDEX OF EFFECT ON CONTRIBUTIONS
IMMATURE POPULATION

Year	Class I: Individual Level Cost—Interest Only	Class II: Terminal Funding	Class III: Accrued Benefit—20-Yr.	Class IV: Individual Level Cost—20-Yr.	Class IV: Aggregate Level Cost
	Annual Contributions				
1	3.02%	....	3.85%	4.19%	3.86%
2	3.02	1.75%	3.84	4.19	3.93
3	3.02	1.75	3.83	4.19	4.00
4	3.02	1.75	3.82	4.19	4.07
5	3.02	1.75	3.81	4.19	4.14
10	3.02	1.75	3.78	4.19	4.53
15	3.02	1.75	3.77	4.19	4.95
20	3.02	1.75	3.74	4.19	5.39
21	3.02	1.75	4.73	7.13	5.48
25	3.02	1.75	4.64	7.13	5.80
30	3.02	1.75	4.56	7.13	6.16
35	3.02	1.75	4.67	7.13	6.45
40	3.02	1.75	4.73	7.13	6.66
50	3.02	1.75	4.73	7.13	6.92
Limit	3.02	1.75	4.70	7.13	7.13
	Accumulated Contributions				
1	3.02%	....	3.85%	4.19%	3.86%
2	3.02	1.75%	3.85	4.19	3.90
3	3.02	1.75	3.84	4.19	3.93
4	3.02	1.75	3.84	4.19	3.96
5	3.02	1.75	3.83	4.19	3.99
10	3.02	1.75	3.81	4.19	4.15
15	3.02	1.75	3.80	4.19	4.31
20	3.02	1.75	3.79	4.19	4.46
21	3.02	1.75	3.81	4.24	4.49
25	3.02	1.75	3.90	4.42	4.61
30	3.02	1.75	3.98	4.62	4.75
35	3.02	1.75	4.04	4.79	4.89
40	3.02	1.75	4.10	4.94	5.01
50	3.02	1.75	4.19	5.19	5.23
Limit	3.02	1.75	4.70	7.13	7.13

In the particular models chosen, ${}^{k}I_r$ approaches the limiting point very slowly. After fifty years the cumulative effect for Funding Class IV, immature, has reached only 73 per cent of its ultimate level. The increasing nature of the index as r increases is due to the increasing importance of the ultimate level contribution. In early years contributions include larger amounts for funding accrued benefits of active and retired employees than they do in later years. The value of accrued benefits is weighted toward the ages which are at or close to retirement age. As illustrated in the *Study Notes*,[7] interest has a lesser impact at the ages close to retirement, due to the shorter interest-earning period before benefit payments begin to reduce the invested funds.

TABLE 3

EFFECT OF CHANGING POPULATION MODEL

	Retirement Age	Entry Age	Preretirement Terminations	Postretirement Mortality	$_{ILC}I$
1.....	65	30	1951 *Ga*	1951 *Ga*	6.16%
2.....	65	25	1951 *Ga*	1951 *Ga*	6.79
3.....	65	35	1951 *Ga*	1951 *Ga*	5.52
4.....	60	30	1951 *Ga*	1951 *Ga*	5.75
5.....	70	30	1951 *Ga*	1951 *Ga*	6.61
6.....	65	30	1951 *Ga*	1951 *Ga*—projected to 1970(C)	6.25
7.....	65	30	1951 *Ga*	1951 *Ga*—generation age 65 in 2005(C)	6.40
8.....	65	30	T-3*	1951 *Ga*	6.63
9.....	65	30	T-11*	1951 *Ga*	7.83

* Turnover from *The Actuary's Pension Handbook* (see Appendix A).

V. EFFECT IN OTHER MODELS

The combined mortality, turnover, salary-scale, entry-age, and retirement-age characteristics peculiar to a given group of employees will affect the relative level of funds required to provide retirement benefits and, though to a somewhat lesser extent than the actuarial cost method, will influence ${}^{k}I_r$.

Table 3 illustrates the effect of changing the population model on a Class IV (Individual Level Cost—Twenty-Year) actuarial cost method. Lines 1, 2, and 3 indicate that a lower entry age results in a greater ultimate index. Lines 1, 4, and 5 show that delay in retirement date increases the interest impact. In general, longer periods of advance funding show a higher index of interest variation.

Lines 1, 6, and 7 involve changing the longevity of retired employees.

[7] *Study Notes*, Part 9E, 2-2-66.

Lower postretirement mortality rates result in the maintenance of higher retired life funds for a longer period of time leading to greater interest earnings.

The effect of turnover is illustrated in lines 1, 8, and 9. T-3 is low turnover and T-11 high turnover (see Appendix A). The ultimate contribution required for a group with a high-turnover characteristic is lower than that for a no-turnover group, since a smaller number of survivors reach retirement date. However, in these examples, ultimate funds generated by the high-turnover group are larger, relative to the ultimate contribution, than those for the low-turnover group due to a greater impact of benefit of survivorship on the accrual of contributions. The result is a larger index of interest variation in the higher-turnover model.

VI. CHOICE OF INTEREST INTERVAL

It has been assumed in this paper that the index of variation is independent of the interest interval over which it is measured. Table 4 compares $^kI_\infty$ calculated over two different interest intervals. Column 1 is the limit index shown in Tables 1 and 2, and column 2 is based on one-fourth of the interval from $2\frac{1}{2}$ to $3\frac{1}{2}$ per cent. Column 3 indicates that, although the index is not independent of interest rate, the choice of interval is not a critical factor in the range from $2\frac{1}{2}$ to $4\frac{1}{2}$ per cent. Choice of the $3\frac{1}{2}$ to $4\frac{1}{2}$ per cent interval is perhaps more appropriate today considering current interest levels.

TABLE 4

Actuarial Cost Class	Actuarial Cost Method	$3\frac{1}{2}$–$4\frac{1}{2}$ Per Cent (1)	$2\frac{1}{2}$–$3\frac{1}{2}$ Per Cent (2)	(1)–(2) (3)
I.......	Individual Level Cost—Interest Only	3.02%	3.19%	−0.17%
II......	Terminal Funding	1.75	1.86	− .12
III.....	Accrued Benefit	4.70	5.01	− .31
IV......	Individual Level Cost—20-Year	7.13	7.15	−0.02

VII. CONCLUSION

Based on the population models and actuarial cost methods examined in this paper, the 6 per cent rule of thumb is appropriate *only* for Class IV and *only* in the very long range. Even for Class IV the index may be more than or less than 6 per cent, depending on the actual population characteristics. For general use, a 3-5-7 rule of thumb, properly hedged as applying to a particular model, does a better job of approximating the index for actuarial cost methods typically encountered, that is, 3 per cent for Class I—Individual Level Cost—Interest Only; 5 per cent for Class

III—Accrued Benefit; and 7 per cent for Class IV—Individual Level Cost—Twenty-Year, or Aggregate Level Cost. The 3-5-7 rule applies only to long-range total contributions. It does not apply to present value of liabilities, for which the index is between 3 and 4 per cent, or to funds, for which the appropriate indices are about one-half of those for contributions, or to other parameters.

For those interested in the short range, Tables 1 and 2 provide a rough and ready reference. For example, for the thirty-five-year period during which the initial active employee group will completely turn over, a 3-4-5 rule of thumb is indicated. The tables in Appendix B may be used to calculate short-range contributions for Individual Level Cost using other than twenty-year amortization of the initial supplemental liability.

ACKNOWLEDGMENT

The author is indebted to C. L. Trowbridge for his helpful suggestions.

APPENDIX A

TURNOVER SCALES

Selected rates from turnover scales used in the illustrations appearing in Sections I and V are given in Table A1.

TABLE A1

Age	Annual Rate of Termination* (From All Causes)		
	T-1	T-3	T-11
27	0.045486	0.051196	0.244036
32	.032715	.048287	.224176
37	.019922	.043963	.197717
42	.009700	.038589	.165138
47	.005397	.031699	.127836
52	.007938	.017020	.085788
57	.012298	.013470	.044781
62	.018353	.018353	.022184
67	0.030112	0.030112	0.030112

* Thomas F. Crocker, Jr., Harry M. Sarason, and Byron W. Straight, *The Actuary's Pension Handbook* (Los Angeles: Pension Publications, 1955). Quoted by special permission of Mr. Thomas F. Crocker, Jr.

APPENDIX B

TABLES OF CONTRIBUTIONS AND FUNDS

The tables on the following pages show the contributions and funds developed by Mr. Trowbridge's[8] mature and immature populations for selected actuarial cost methods, assuming interest at 3½ and 4½ per cent.

[8] C. L. Trowbridge, *op. cit.*

TABLE B1

CONTRIBUTIONS AND FUNDS
MATURE POPULATION
INTEREST AT 3½ PER CENT

	CLASS I	CLASS II	CLASS III	CLASS IV	
	Pay-as-You-Go	Terminal Funding	Accrued Benefit—20-Yr.	Individual Level Cost—20-Yr.	Aggregate Level Cost
	Initial Supplemental Liability				
	None	$473,812	$1,069,156	$1,290,793	$1,290,793
	Individual Level Cost				
	$63,000	$ 46,977	$ 26,845	$ 19,350	$ 19,350
Beg. of Year	Contributions				
1.....	$63,000	$520,789	$ 99,528	$ 107,101	$ 166,635
2.....	63,000	46,977	99,528	107,101	154,396
3.....	63,000	46,977	99,528	107,101	143,173
4.....	63,000	46,977	99,528	107,101	132,884
5.....	63,000	46,977	99,528	107,101	123,449
10.....	63,000	46,977	99,528	107,101	86,812
15.....	63,000	46,977	99,528	107,101	63,069
20.....	63,000	46,977	99,528	107,101	47,683
21.....	63,000	46,977	26,845	19,350	45,328
25.....	63,000	46,977	26,845	19,350	37,712
30.....	63,000	46,977	26,845	19,350	31,250
35.....	63,000	46,977	26,845	19,350	27,062
40.....	63,000	46,977	26,845	19,350	24,348
50.....	63,000	46,977	26,845	19,350	21,450
Limit...	63,000	46,977	26,845	19,350	19,350
End of Year	Funds				
1.....	None	$473,812	$ 37,807	$ 45,644	$ 107,262
2.....	None	473,812	76,937	92,885	205,611
3.....	None	473,812	117,436	141,780	295,787
4.....	None	473,812	159,353	192,385	378,469
5.....	None	473,812	202,737	244,763	454,280
10.....	None	473,812	443,523	535,465	748,677
15.....	None	473,812	729,503	880,729	939,461
20.....	None	473,812	1,069,156	1,290,793	1,063,099
21.....	None	473,812	1,069,156	1,290,793	1,082,017
25.....	None	473,812	1,069,156	1,290,793	1,143,222
30.....	None	473,812	1,069,156	1,290,793	1,195,146
35.....	None	473,812	1,069,156	1,290,793	1,228,796
40.....	None	473,812	1,069,156	1,290,793	1,250,602
50.....	None	473,812	1,069,156	1,290,793	1,273,892
Limit...	None	473,812	1,069,156	1,290,793	1,290,793

TABLE B2

Contributions and Funds
Mature Population
Interest at 4½ Per Cent

	Class I	Class II	Class III	Class IV	
	Pay-as-You-Go	Terminal Funding	Accrued Benefit—20-Yr.	Individual Level Cost—20-Yr.	Aggregate Level Cost
	Initial Supplemental Liability				
	None	$448,529	$ 956,897	$1,141,857	$1,141,857
	Individual Level Cost				
	$63,000	$ 43,685	$ 21,794	$ 13,829	$ 13,829
Beg. of Year	Contributions				
1.....	$63,000	$492,214	$ 92,189	$ 97,831	$ 153,384
2.....	63,000	43,685	92,189	97,831	141,840
3.....	63,000	43,685	92,189	97,831	131,251
4.....	63,000	43,685	92,189	97,831	121,538
5.....	63,000	43,685	92,189	97,831	112,629
10.....	63,000	43,685	92,189	97,831	77,990
15.....	63,000	43,685	92,189	97,831	55,495
20.....	63,000	43,685	92,189	97,831	40,888
21.....	63,000	43,685	21,794	13,829	38,649
25.....	63,000	43,685	21,794	13,829	31,401
30.....	63,000	43,685	21,794	13,829	25,241
35.....	63,000	43,685	21,794	13,829	21,240
40.....	63,000	43,685	21,794	13,829	18,642
50.....	63,000	43,685	21,794	13,829	15,860
Limit...	63,000	43,685	21,794	13,829	13,829
End of Year	Funds				
1.....	None	$448,529	$ 30,502	$ 36,398	$ 94,452
2.....	None	448,529	62,377	74,434	181,090
3.....	None	448,529	95,686	114,181	260,562
4.....	None	448,529	130,494	155,717	333,460
5.....	None	448,529	166,869	199,123	400,328
10.....	None	448,529	374,817	447,266	660,299
15.....	None	448,529	633,959	756,497	829,124
20.....	None	448,529	956,897	1,141,857	938,759
21.....	None	448,529	956,897	1,141,857	955,557
25.....	None	448,529	956,897	1,141,857	1,009,955
30.....	None	448,529	956,897	1,141,857	1,056,190
35.....	None	448,529	956,897	1,141,857	1,086,215
40.....	None	448,529	956,897	1,141,857	1,105,713
50.....	None	448,529	956,897	1,141,857	1,126,597
Limit...	None	448,529	956,897	1,141,857	1,141,857

TABLE B3

Contributions and Funds
Immature Population
Interest at 3½ Per Cent

Beg. of Year	Class I	Class II	Class III	Class IV	
	Individual Level Cost—Interest Only	Terminal Funding	Accrued Benefit—20-Yr.	Individual Level Cost—20-Yr.	Aggregate Level Cost
	Initial Supplemental Liability				
	$ 543,034	None	$ 353,509	$ 543,034	$ 543,034
	Ultimate Supplemental Liability				
	$1,290,793	$473,812	$1,069,156	$1,290,793	$1,290,793
	Initial Individual Level Cost				
	$ 19,350	None	$ 20,461	$ 19,350	$ 19,350
	Ultimate Individual Level Cost				
	$ 19,350	$ 46,977	$ 26,845	$ 19,350	$ 19,350
	Contributions				
1.	$ 37,713	. . . .	$ 44,493	$ 56,266	$ 80,052
2.	37,713	$ 9,395	45,354	56,266	75,000
3.	37,713	14,093	46,106	56,266	70,424
4.	37,713	17,083	46,794	56,266	66,283
5.	37,713	21,353	47,380	56,266	62,514
10.	37,713	36,136	49,206	56,266	47,937
15.	37,713	39,148	50,192	56,266	38,261
20.	37,713	40,850	51,046	56,266	31,773
21.	37,713	41,105	27,180	19,350	30,761
25.	37,713	41,944	27,842	19,350	27,458
30.	37,713	51,675	28,131	19,350	24,605
35.	37,713	58,722	26,813	19,350	22,712
40.	37,713	46,621	26,418	19,350	21,515
50.	37,713	45,565	26,676	19,350	20,263
Limit. . .	37,713	46,977	26,845	19,350	19,350

TABLE B3—*Continued*

	Class I	Class II	Class III	Class IV	
	Individual Level Cost—Interest Only	Terminal Funding	Accrued Benefit—20-Yr.	Individual Level Cost—20-Yr.	Aggregate Level Cost
End of Year	Funds				
1.....	$ 39,033	....	$ 46,050	$ 58,236	$ 82,854
2.....	78,564	$ 8,855	93,734	117,640	162,509
3.....	118,173	21,578	142,561	177,820	238,913
4.....	157,675	36,346	192,314	238,612	312,210
5.....	196,715	54,206	242,571	299,686	382,327
10.....	376,116	166,389	490,871	601,386	684,007
15.....	518,060	270,898	721,447	888,581	907,239
20.....	622,546	342,871	936,168	1,165,580	1,065,526
21.....	639,530	353,576	953,227	1,182,564	1,090,819
25.....	697,265	386,261	1,013,061	1,240,299	1,175,565
30.....	751,149	428,682	1,073,136	1,294,184	1,252,502
35.....	770,834	497,389	1,096,106	1,313,869	1,287,001
40.....	763,674	505,569	1,087,475	1,306,709	1,289,309
50.....	743,878	476,464	1,064,885	1,286,914	1,279,624
Limit...	747,760	473,812	1,069,156	1,290,793	1,290,793

TABLE B4

CONTRIBUTIONS AND FUNDS
IMMATURE POPULATION
INTEREST AT 4½ PER CENT

	CLASS I	CLASS II	CLASS III	CLASS IV	
	Individual Level Cost—Interest Only	Terminal Funding	Accrued Benefit—20-Yr.	Individual Level Cost—20-Yr.	Aggregate Level Cost
	Initial Supplemental Liability				
	$ 448,756	None	$292,552	$ 448,756	$ 448,756
	Ultimate Supplemental Liability				
	$1,141,857	$448,529	$956,897	$1,141,857	$1,141,857
	Initial Individual Level Cost				
	$ 13,829	None	$ 16,114	$ 13,829	$ 13,829
	Ultimate Individual Level Cost				
	$ 13,829	$ 43,685	$ 21,794	$ 13,829	$ 13,829
Beg. of Year	Contributions				
1......	$ 33,154	....	$ 37,636	$ 46,842	$ 67,683
2......	33,154	$ 8,737	38,385	46,842	63,212
3......	33,154	13,106	39,041	46,842	59,162
4......	33,154	15,886	39,644	46,842	55,496
5......	33,154	19,857	40,157	46,842	52,159
10......	33,154	33,604	41,757	46,842	39,252
15......	33,154	36,404	42,626	46,842	30,678
20......	33,154	37,987	43,403	46,842	24,922
21......	33,154	38,225	22,036	13,829	24,023
25......	33,154	39,005	22,670	13,829	21,085
30......	33,154	48,054	22,996	13,829	18,543
35......	33,154	54,606	21,802	13,829	16,851
40......	33,154	43,354	21,419	13,829	15,780
50......	33,154	42,372	21,633	13,829	14,655
Limit....	33,154	43,685	21,794	13,829	13,829

TABLE B4—*Continued*

	Class I	Class II	Class III	Class IV	
	Individual Level Cost—Interest Only	Terminal Funding	Accrued Benefit—20-Yr.	Individual Level Cost—20-Yr.	Aggregate Level Cost
End of Year	Funds				
1......	$ 34,645	....	$ 39,329	$ 48,950	$ 70,729
2......	69,972	$ 8,252	80,334	99,225	139,090
3......	105,572	20,125	122,553	150,446	204,979
4......	141,265	33,928	165,793	202,463	268,493
5......	176,702	50,639	209,652	254,958	329,515
10......	341,164	155,994	428,229	516,942	594,735
15......	472,856	254,850	633,512	770,163	793,560
20......	570,893	323,436	827,852	1,019,649	936,160
21......	586,967	333,673	843,871	1,035,723	959,129
25......	642,139	364,970	900,521	1,090,895	1,036,751
30......	694,999	405,226	958,868	1,143,755	1,108,828
35......	715,273	469,974	982,395	1,164,030	1,141,480
40......	708,927	478,308	974,929	1,157,684	1,143,057
50......	689,682	451,286	953,244	1,138,439	1,132,300
Limit....	693,101	448,529	956,897	1,141,857	1,141,857

DISCUSSION OF PRECEDING PAPER

BARNET N. BERIN:

We come much closer to the practical everyday problem of describing "the effect on contributions of a variation in the interest rate" by considering (*a*) the effect on actual accrued liabilities and on actual current-service costs and (*b*) the long-range effect as measured in the year-to-year change in expected interest earnings.

The first is capable of an exact answer, based on the latest available data, or at least a good estimate. We could respond to (*b*) by indicating how actual interest earnings and expected interest earnings act to determine yearly interest gains or losses under either the immediate or spread method of recognizing gains or losses and by showing how this result will be affected by a change in the expected interest rate.

The author has raised an interesting question. He responds by resorting to *ultimate* employee distributions which have little application to any one company's experience. I personally feel that the stationary population concept is misleading in pension calculations.

RICHARD DASKAIS:

The 3 per cent rule of thumb (for Class I, Individual Level Cost—Interest Only) is for use in practical situations. Caution should be exercised to make sure that it is properly applicable to any practical situation for which it is used.

The index increases as the ratio of the individual level cost to the unfunded supplemental liability increases. The ratio and the index will increase if there is an increase in the number of employees covered by the plan or, in a plan whose benefits are pay-related, if there is an increase in aggregate pay of the covered employees. In the latter case, if the pay increases were greater than any assumed in the valuation assumptions, the method of adjusting costs for the actuarial losses would affect the results. It should be noted that, even if no employee receives a pay increase but new entrants' pay is greater than the pay of retiring and terminating employees, there is an increase in the aggregate pay of covered employees. The effect of pay increases will be particularly great in a plan integrated with social security by means of a stepped-up benefit rate, where the benefit percentages and breaking point have not been changed with increases in social security benefits and the social security maximum wage base.

Employee contributions produce leverage in the effect on employer costs of contributory plans. Whatever rule of thumb is applicable must be applied to the total contributions used to provide retirement benefits (i.e., not to the portion of employee contributions which will be used to provide preretirement lump-sum death and termination benefits) in order to approximate the total effect of a change in interest, and this effect must then be related to the employer cost.

Based on the limited sample of the plans with which I have worked, a large proportion of salaried employees' plans has experienced pay and coverage increases, and it is likely that the same will be true of new plans.

DONALD S. GRUBBS, JR.:

Mr. Adams has provided very helpful information to assist in estimating the effect of interest-rate changes without extensive calculations.

One minor problem is his assumption that the effect of an increase of $\frac{1}{4}$ per cent would be 0.25 times the effect of an increase of 1 per cent. The interpolation should be geometric rather than arithmetical. For example, if increasing the interest rate 1 per cent decreases the contributions by 24 per cent, Mr. Adams would conclude that the reduction for each $\frac{1}{4}$ per cent reduction is 6 per cent; when a geometric approach is used, the reduction is 6.63 per cent.

$$1-(0.76)^{1/4}=0.0663 .$$

Mr. Adams shows the effect if both the assumed interest rate and the actual interest rate are changed. If only one of these two is changed, the effect will be significantly different. In his example Mr. Adams shows that, if the assumed interest rate and actual interest rate are $3\frac{1}{2}$ per cent under Class IV, the ultimate fund is $1,290,793 and the ultimate employer contribution will be $19,350, whereas if the assumed and actual interest rates are $4\frac{1}{2}$ per cent, the ultimate fund is $1,141,857 and the ultimate contribution is $13,829. But, if the assumed interest rate is maintained at $3\frac{1}{2}$ per cent while the actual interest rate increases to $4\frac{1}{2}$ per cent, the ultimate fund will be determined by the assumed interest rate and will be $1,290,793. The ultimate rate of contribution will be the normal cost of $19,350 less the interest gain of $12,908, or $6,442. This would be a reduction of 66.71 per cent, or 16.68 per cent for each $\frac{1}{4}$ per cent change in interest rate according to Mr. Adams' method, compared to 7.13 per cent shown in Table 2. This is a measurement of the fact that, if the interest assumption is not increased to agree with investment income received, the cost reduction will be deferred, so that the cost in the earlier years will be higher and the ultimate cost lower than if the assumed interest rate were increased.

ROBERT F. LINK:

Mr. Adams' paper appropriately shows that statements about interest effects on pension costs are often oversimplified and overoptimistic. To illustrate additional dimensions of the problem, here is an imaginary conversation between Mr. Smith, president of Amalgamated Washers, and Mr. Johnson, his actuary.

MR. SMITH: What would happen to my pension costs if I could raise the yield on the fund by 1 per cent?

MR. JOHNSON: Do you mean if you could raise it instantly?

MR. SMITH: Yes.

MR. JOHNSON: You would have an additional amount of 1 per cent of your fund each year to reduce costs. This reduction would grow each year as your fund grows.

MR. SMITH: Do you mean that my contribution would drop by 1 per cent of the fund starting in the first year?

MR. JOHNSON: Not exactly. We are spreading gains, remember?

MR. SMITH: Well, how can I get this 25 per cent reduction that I read about?

MR. JOHNSON: We could reduce costs by changing the interest assumption, but I would be inclined to be cautious and raise it by only $\frac{1}{2}$ or $\frac{3}{4}$ per cent.

MR. SMITH: Then I would get a reduction of 18 or 19 per cent right away?

MR. JOHNSON: Well, taking into account your unfunded liability payments, you might reduce contributions at the start by, say, 12–14 per cent and, of course, considerably more later on if everything works out.

MR. SMITH: Oh.

The statements that we read about interest and pension costs almost always make—and almost never mention—an assumption. The assumption is that actual yield and the assumed yield for funding purposes will be simultaneously changed by the same increment. This, of course, seldom happens. Subject to the limitations that Mr. Adams illustrates, such statements have validity in two frames of reference: first, they reveal the effect of a change in interest assumption if gains are excluded and, second, they may measure reasonably the total effect, over long periods, of a change in actual earnings.

CHARLES L. TROWBRIDGE:

Mr. Adams has done a real service for pension actuaries. Many have used the rule of thumb that he quotes, but no one before him has made the effort to substitute demonstrations for impressions. I am personally gratified that he has done so, because his is a first paper from a relatively new FSA and because he has found it useful to use a mathematical model from my paper of fifteen years ago.

The identity of the original maker of this useful rule may well be lost.

The sources from actuarial literature that Mr. Adams quotes are all relatively recent, and I feel sure that the rule has been around the pension field longer than it has appeared in print. To some extent, at least, one may well have taken from another, or from some pre-World War II source not clearly identified.

My personal version of this rule (and I have been as guilty as any of using it without a really satisfactory demonstration) has been that a difference of $\frac{1}{4}$ per cent in i means about 6 per cent in contribution). We could call it, then, a 1-to-24 rule. To satisfy myself that the rule makes sense, I have used two rationalizations, neither of them satisfactory and both of them related to the number 24.

One of them hinged around the undisputed fact that $(1.0025)^{24} = 1.06$, and $(1 + i + 0.0025)^{24}/(1 + i)^{24}$ for usual values of i is not much different. If one had any reason to think that the average time between pay in and pay out of a dollar in a pension fund was 24 years, the rule would seem reasonable. Although 24 in this context does not seem unreasonable, neither do 18 or 36. The rationalization appears to be the rule itself, stated in a different form. Perhaps Mr. Adams can show us, from his interesting tables, that 24 years more or less is correct.

A second rationalization came from the concept of the equation of equilibrium. Suppose that we had a mature pension fund with contributions (C), funds (F), and benefits (B) stabilized at interest rate i such that $C + iF = B$. Assume also that $F/C = N$. If F and B are undisturbed by a change in interest rate Δi, then ΔC is $-\Delta iNC$. Since the rule tells us that if $\Delta i = 0.0025$, $\Delta C = -0.06C$, then N must be 24. This *appears* to justify the rule *if* the ultimate fund is typically 24 times the ultimate contribution. Unfortunately, the typical N appears to be substantially higher, and this approach seems to disprove the rule of thumb.

Mr. Adams has now shown how to remove the contradiction, by recognizing that F is not independent of i. His tables tell us that a larger i produces a smaller F as well as a smaller C and that the effect of $\frac{1}{4}$ per cent interest on F is about half its effect on C. Thus, as i increases by 0.0025, F must decrease by about 3 per cent, if, as the rule tells us, C decreases by 6 per cent. $F/C = N$ must therefore increase by about 3 per cent. Putting this all together at $i = 0.035$,

$$C_{0.035} = \frac{B}{1+0.035N},$$

$$C_{0.0375} = \frac{B}{1+0.0375N(1.03)},$$

$$\frac{C_{0.0375}}{C_{0.035}} = \frac{1+0.035N}{1+0.0375N(1.03)} = 1 - 0.06,$$

and we force out an N of approximately 46, which lies between the N's he illustrates for Class III (about 40) and Class IV (about 66). Mr. Adams clearly tells us that the 1-to-24 rule lies between Class III and Class IV—so, in a way, we have arrived at the same place.

CECIL J. NESBITT:

As a means for examining the effect of interest on pension contributions, the author has gone back to the now classical pension-funding models presented by Trowbridge in his paper "Fundamentals of Pension Funding" (*TSA*, Vol. IV [1952]). He is thereby able to investigate for a number of standard funding methods both the short-range and the long-range effects of changes in the interest assumption. He does this for both the mature and the immature population models and obtains a comprehensive view of the effect of interest on pension contributions. His investigations lead him to observe that the higher the fund generated by the actuarial cost method, the higher the index of effect on contributions of an increase in interest. As the fund generated is in opposite relation to the ultimate contribution, one might rephrase the observation as "the lower the ultimate contribution, the higher the index," or, again, "the lower the ultimate contribution, the longer the average term of investment of the contribution before it is utilized for pension outgo—hence, the longer the average investment term, the higher the index."

One might argue that the Trowbridge models are oversimplified and unrepresentative. It is part of their usefulness that they are simple enough to be examined easily both arithmetically and algebraically. Moreover, the l_x function of the model may be thought to be based on withdrawal and salary-increase assumptions as well as mortality and therefore may not be unrealistic. In considering these points, I was reminded of a note entitled "The Effect on Pension Fund Contributions of a Change in the Rate of Interest," by M. T. L. Bizley (*JIASS*, X [1950], 47). Bizley let $100c(x)$ be the percentage contribution for entry age x required to support a pension per annum equal to $100k$ per cent of pensionable salary π for each year of service, with pension payable from retirement age r, say. Then

$$c(x) = kD_r\bar{a}_r\pi(r-x) \Big/ \int_x^r D_y s_y dy\,,$$

where s_y is the salary-scale factor at age y. This may be rearranged as

$$c(x) = R\int_r^\infty D'_y dy \Big/ \int_x^r D_y s_y dy\,,$$

where R does not depend on i, and D'_y is based on the mortality for retired lives. On taking logarithms of each side and differentiating, one obtains

$$[dc(x)/di]/c(x) = -v(y_2 - y_1)$$

where y_1 is the weighted average age

$$\int_x^r yD_y s_y dy \bigg/ \int_x^r D_y dy$$

and y_2 is the weighted average age

$$\int_r^\infty D'_y dy \bigg/ \int_r^\infty D'_y dy .$$

From this, one obtains

$$-\Delta c(x)/c(x) \doteq v(y_2 - y_1)\Delta i .$$

Bizley goes on to suggest that quick estimates of the effect of a change in the interest rate may be found by graphing $D_y s_y$ and D'_y and then cut-

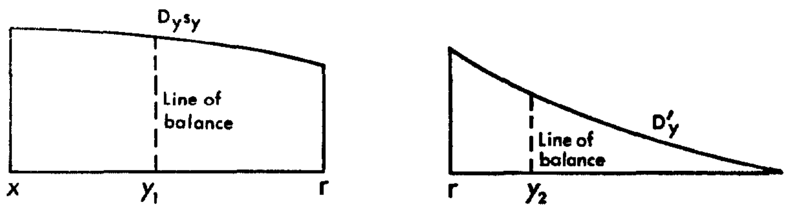

FIG. 1

ting out the areas bounded by the graphs and balancing the cutouts to estimate the centroid abscissas y_1 and y_2 (see Fig. 1). By doing this for the earliest age x first and by successively cutting off from the left portions of the $D_y s_y$ cutout, one can quickly estimate $-\Delta c(x)/c(x)$ for various entry ages x.

An advantage of Bizley's approach is that one can use it to visualize what would be the effect on $-\Delta c(x)/c(x)$ if the withdrawal, salary-scale, or retirement-age assumptions are modified. If higher withdrawal rates are assumed, then the $D_y s_y$'s at the older ages are of less weight, so y_1 shifts to the left and $-\Delta c(x)/c(x)$ increases. If a steeper salary scale is introduced, then the $D_y s_y$'s at the older ages have increased weight, y_1 shifts to the right, and $-\Delta c(x)/c(x)$ decreases. If the retirement age r is increased, then both y_1 and y_2 shift to the right, but one would expect the shift in y_2 to be greater than that for y_1. These observations concur with the author's illustrations in Table 3.

I was interested to try Bizley's approach in regard to some of the families of funding methods that have been developed recently by Trowbridge, Taylor, and Cooper and Hickman. In my discussion of Taylor's paper, "The Generalized Family of Aggregate Cost Methods for Pension Funding" (*TSA*, Vol. XIX), I mentioned a modified aggregate cost method with $a = 1/\ddot{a}_{\overline{n}|}$, that is, with contribution at each valuation date equal to the annual payment to amortize the unfunded liabilities over the next n years. If this method is applied to Trowbridge's model plan for a mature population, one finds that the ultimate contribution rate ${}^aC_\infty$ is given by

$${}^aC_\infty = (1+i)^{n-1}\,{}^IN\,,$$

where ${}^IN = l_a \cdot {}_{r-a}|\bar{a}_a$ is the annual normal cost under initial funding. This may be rewritten as

$${}^aC_\infty = (1+i)^{a+n-1}\bar{N}_r\,.$$

If one now takes logarithmic derivatives, assuming n is independent of i, and puts the result in differential form, one gets

$$-\Delta^aC_\infty/{}^aC_\infty \doteq v\,[\,y_2 - (a+n-1)\,]\Delta i\,,$$

where in this case

$$y_2 = \int_r^\infty yD_y dy \Big/ \int_r^\infty D_y dy\,.$$

If $n = y_2 - a + 1$, then $-\Delta^aC_\infty/{}^aC_\infty \doteq 0$, and the modified aggregate cost funding approximates pay-as-you-go funding (the exact n that brings out pay-as-you-go funding is a function of i and would alter the differentiation slightly). If $n > y_2 - a + 1$, then $\Delta^aC_\infty/{}^aC_\infty \doteq v[n - (y_2 - a + 1)]\Delta i$ is positive, and an increase in the interest rate increases the contribution level. This can happen in the somewhat heretical situation where ${}^aF_\infty$ is negative, that is, represents debt, and ${}^aC_\infty$ is the benefit outgo plus interest on the debt. In other words, contributions would be stabilized at a level higher than pay-as-you-go costs. This indicates the possibility of actuarial cost methods even weaker than those of Class I.

As an application of Bizley's method to Cooper and Hickman's paper on "A Family of Accrued Benefit Actuarial Cost Methods" (*TSA*, Vol. XIX), I considered

$$b(x) = m(x)\cdot{}_{r-x}|\,\bar{a}_x = m(x)(1+i)^x\bar{N}_r/l_x\,,$$

where $m(x)$ is their pension-purchase density function and $b(x)$ is the annual rate of contribution at attained age x. One finds, after logarithmic differentiation, that

$$-\Delta b(x)/b(x) \doteq \{\,v(y_2 - x) - [\,dm(x)/di\,]/m(x)\,\}\Delta i\,.$$

If $m(x)$ is independent of i, this becomes $v(y_2 - x)\Delta i$, which measures the effect of an interest-rate increase on the unit single premium at attained age x. If $m(x)$ varies as v^x, that is,

$$m(x) = v_x \Big/ \int_a^r v_y dy ,$$

then the formula becomes

$$v(y_2 - y_1')\Delta i ,$$

where

$$y_1' = \int_a^r y v^y dy \Big/ \int_a^r v^y dy .$$

If $m(x)$ varies as $(1 + i)^x$, that is,

$$m(x) = (1+i)^x \Big/ \int_a^r (1+i)^y dy ,$$

then the formula is

$$v(y_1'' + y_2 - 2x)\Delta i ,$$

where

$$y_1'' = \int_a^r y(1+i)^y dy \Big/ \int_a^r (1+i)^y dy .$$

It should be noted that these formulas measure the effect of interest on the annual rate of contribution at attained age x and not the aggregate effect on the total contribution for the covered group.

The author has provided useful guides for estimating the effect of interest on pension contributions and has thereby contributed to our knowledge of pension funding.

(AUTHOR'S REVIEW OF DISCUSSION)

WARREN R. ADAMS:

I would like to thank Messrs. Berin, Daskais, Grubbs, Link, Trowbridge, and Nesbitt for their comments and contributions.

Messrs. Berin and Grubbs consider the situation in which actual investment return differs from that assumed in determining plan liabilities and contributions. Messrs. Berin and Daskais raise the point that a stationary population concept does not fit any one group's experience. These and other considerations give rise to the need for properly hedging statements made in discussing the operation of a pension plan. These points were intentionally omitted in the paper to avoid complicating what is intended to be a simple conversational tool.

Mr. Grubbs suggests that the interpolation for ${}^kI_r$ should be geometric rather than linear. A geometric index, however, would be impractical since it is difficult to handle mentally in cases where the effect of two or more quarters of 1 per cent is desired. For example, if the geometric index for $\frac{1}{4}$ of 1 per cent is 0.0663, then the effect of $\frac{3}{4}$ of 1 per cent is $1 - (1 - 0.0663)^3$.

Mr. Link's entertaining imaginary conversation effectively illustrates the difficulties faced by a conscientious pension actuary in his attempt to communicate a complicated answer to a vital question. Within the framework of this paper his answer to Mr. Smith's initial question might be, "Based on certain assumptions, in the long run your contributions would be reduced by roughly 20 per cent (or 12 or 28 per cent)." It is doubtful that Mr. Johnson would want to hear more than this. One might argue that the actuary does not properly perform his function when he waters down an answer to the extent that it loses its relevance and the client is left completely befuddled.

In searching for a rationalization of the 6 per cent rule, Mr. Trowbridge considers the fact that $(1.0025)^{24} = 1.06$ and argues that this may be reasonable if the average time between pay in and pay out of a dollar in a pension fund is 24 years. Using ultimate funds for $i = 0.035$ and an ultimate benefit payroll of \$63,000, the average time, n, between pay in and pay out can be determined from the relationship

$$63{,}000\,\ddot{a}_{\overline{n}|0.035} = {}^kF_\infty^{0.035} .$$

The index can then be calculated as

$${}^kI_\infty = 0.25\left[1-\left(\frac{1.035}{1.045}\right)^n\right].$$

These calculations result in $n = 15$, 24, and 34 and indices of 3.36, 5.15, and 6.97 per cent for Funding Classes I, III, IV, respectively. Mr. Trowbridge's 24 years is generally correct, and his rationalization leads to indices which are close to those derived in the paper. Also, Dr. Nesbitt suggests this as a possible approach to the problem by restating the observation in the paper as "the longer the average investment term, the higher the index."

Mr. Trowbridge considers an alternate rationalization of the 6 per cent rule which uses the equation of maturity

$$C + dF = B .$$

If $0.01{}^kI_\infty$ is the reduction in ${}^kC_\infty$ caused by an increase in i of 0.0025 and ${}^kf(0.01{}^kI_\infty)$ is the reduction in ${}^kF_\infty$, we can determine ${}^kI_\infty$ from the relationship

$$ {}^kI_\infty = \frac{{}^kN(d'-d)}{1+d'\,{}^kf({}^kN)} \times 100\% \doteq \frac{0.23({}^kN)}{1+0.038\,{}^kf({}^kN)}\%, $$

where

$$ {}^kN = {}^kF_\infty \mid {}^kC_\infty $$

and

$$ d = i/(1+i), \qquad d' = (i+0.0025)/(1+i+0.0025). $$

Using the tables in the Appendix, this results in ${}^kN = 20$, 40, and 68 and indices of 3.02, 4.70, and 7.13 for Funding Classes I, III, IV, respectively. The proposed 3-5-7 rule implies ${}^kN = 17$, 37, 72, which are reasonably close to the actual values.

I am especially grateful to Dr. Nesbitt for introducing Mr. Bizley's approach and discussing its application to some of the recently developed families of funding methods. This is a valuable addition to the paper.