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A DIRECT COMPREHENSIVE APPROACH TO THE CALCULATION OF GROSS NONPARTICIPATING PREMIUMS

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I. INTRODUCTION

Gross premium calculation methods may be arbitrarily separated into: (*a*) Type 1 methods—those which initially have commissions and other "per cent of premium" expenses in the denominator (Cammack [1] and Jenkins [2]) and (*b*) Type 2 methods—those which initially have commissions and other "per cent of premium" expenses in the numerator (Hoskins [3], Rosser [5], and Anderson [6]).

The traditional Type 1 methods directly calculate the gross premium that will yield a specified margin for profit and contingencies over the gross premium calculation period. The main disadvantage of this approach has been the lengthy asset share accumulation required to determine the incidence of profit.

Type 2 methods require the use of a trial or preliminary gross premium in the initial calculations. Rosser's and Anderson's methods do not require the use of an asset share calculation to determine the incidence of profit.

The purpose of this paper is to present:

1. A Type 1 method of gross premium calculation which, in addition to combining the advantages of all previously published Type 1 and Type 2 methods, has the following advantages:

a) The cost of reinsurance is incorporated into the gross premium calculation and may be shown as a level annual cost over the gross premium calculation period.

b) The effect on profit margins of a change in the retention limit may be quickly calculated without redoing the original calculation.

c) The premiums needed to "break even" (e.g., amortize the initial investment in surplus) at the end of *each* policy year are calculated as a by-product.

d) The profit or loss generated in each policy year or over any number of policy years may be quickly calculated for *any* gross premium without redoing the original calculation or first using a trial gross premium.

e) The gross premium that will produce any specified profit over *any* number of policy years may be directly solved for without redoing the original calculation or first using a trial gross premium.

f) The premium that will produce a specified yield on surplus invested in new business [6] may be *directly* solved for by a flexible technique not first involving the use of the renewal net premium or any other preliminary gross premium.

2. A mathematical definition of a financially meaningful "average" policy size which recognizes the cost of reinsurance.

3. Mathematical definitions of common measures of profit, and the mathematical relationships between them.

4. Formulas expressing the effect of changes in various assumptions used in the gross premium calculation as a level annual cost over the gross premium calculation period.

The method of gross premium calculation presented in this paper should eliminate the need for trial or preliminary gross nonparticipating premiums.

This paper will emphasize the basic actuarial theory, as opposed to the art or philosophy, of rate-making.

II. DEFINITION OF SYMBOLS

1. $q^d_{[X]+t-1}$ = Mortality rate in policy year t.
2. $q^w_{[X]+t-1}$ = Withdrawal rate in policy year t (including conversions).
3. $q^{(T)}_{[X]+t-1} \doteq q^d_{[X]+t-1} + q^w_{[X]+t-1}$.
4. ${}_tp^{(T)}_{[X]}$ = Probability of policy issued at age X staying in force at least t years.
5. i_t = Net interest rate earned on invested assets in policy year t.
6. $(1+i)^t = (1+i_1)(1+i_2)\ldots(1+i_t)$.
7. ${}_tE^{(T)}_{[X]} = v^t \cdot {}_tp^{(T)}_{[X]}$.
8. c_t = Total commission rate in policy year t.
9. g_t = Per cent of gross premium paid out in other "per cent of premium" expenses in policy year t.
10. z_t = The proportion of policy year t commissions that are not vested.
11. θ_t = Probability of nonvested commissions being paid in policy year t if premium for that year is collected.
12. Π_X = The level annual gross premium per \$1,000 of insurance (excluding the policy fee).
13. ${}_tf_X = {}_{t-1}E^{(T)}_{[X]}\{1 - g_t - c_t[1 - Z_t(1-\theta_t)]\}$.
14. ${}_tF_X = \sum_{s=1}^{t} {}_sf_X$.
15. j_t = Rate of profit required in policy year t on surplus invested in new business.
16. $(1+j)^t = (1+j_1)(1+j_2)\ldots(1+j_t)$.
17. N = Gross premium calculation period.
18. $f(X,G)$ = Per cent of the policies issued at age X which have a face amount of G (thousands).

19. A_t = Average policy size (in thousands) in policy year t.

20. $_tA$ = Level average policy size (in thousands) over the first t policy years.

21. B = Policy fee.

22. A = Traditional definition of average policy size (in thousands):

$$A = \sum_G f(X, G) \cdot G .$$

23. e_t = Expense per $1,000 of insurance in policy year t.

24. e_t' = Per policy expense in policy year t, including per policy expenses of conversions and payments of cash surrender values, maturities, and death benefits.

25. $_tCV_X$ = Cash value per $1,000 of insurance at end of policy year t.

26. $_tV_X$ = Terminal reserve per $1,000 of insurance at end of policy year t.

27. K = The retention limit (in thousands).

28. $k = \dfrac{_NA - K}{_NA} > 0 .$

29. r_{X+t-1} = Yearly Renewable Term (YRT) reinsurance premium (exclusive of any annual fee) per $1,000 of reinsurance in policy year t.

30. B_t^r = Policy year t annual fee per policy reinsured under select and ultimate YRT reinsurance.

31. g_t^r = Per cent of policy year t YRT premiums (including any annual fee) reinsurer refunds to insurer for payment of premium taxes.

32. e_t^r = Total expense charge in the reinsurer's refund formula covering policy year t.

33. P_X = Valuation renewal net premium.

34. $\ddot{a}'_{X+t-1}$ = The present value at the beginning of policy year t of a $1 life annuity due for the remainder of the premium payment period on the valuation basis ($\ddot{a}'_X$ is defined as equal to zero).

35. $q^c_{[X]+t-1}$ = Probability of conversion in policy year t.

36. $_tCM_X$ = Present value at end of policy year t of future excess mortality per $1 of insurance converted at the end of policy year t.

37. r = Per cent of reinsurer's "profit" that is returned to ceding company under reinsurer's experience rating formula.

38. m_t = Expected proportion of insurer's experience in policy

year t that the reinsurer pools with the experience of other ceding companies for experience refund purposes.

39. $(pr)_t$ = Expected ratio of total experience refund to total earned premium for policy year t under reinsurer's YRT risk pool.

40. $$^AR^1_{X+t-1}=\left(\frac{1{,}000-{}_tV_X}{1{,}000}\right)\left[\left(r_{X+t-1}-1{,}000\cdot v^{1/2}q^d_{[X]+t-1}\right)\right.$$
$$-v\cdot r\cdot(1-m_t)(r_{X+t-1}-1{,}000\cdot v^{1/2}q^d_{[X]+t-1}-e^r_t)$$
$$\left.-v\cdot m_t\cdot(pr)_t\cdot r_{X+t-1}-v\cdot g^r_t\cdot r_{X+t-1}\right].$$

41. $^BR^1_{X+t-1} = B^r_t(1 - v\cdot g^r_t)$.

42. S_t = Survival benefit per $1,000 of insurance to those surviving policy year t.

43. $$h_X=\left[e_t+\frac{e'_t}{A_t}+1{,}000\cdot v^{1/2}q^d_{[X]+t-1}+v\cdot q^w_{[X]+t-1}\cdot{}_tCV_X\right.$$
$$+\frac{A_t-K^*}{A_t}\left({}^AR^1_{X+t-1}+\frac{{}^BR^1_{X+t-1}}{A_t-K}\right)+v\cdot q^c_{[X]+t-1}$$
$$\left.\cdot 1{,}000\cdot{}_tCM_X+v\cdot p^{(T)}_{[X]+t-1}\cdot S_t\right]{}_{t-1}E^{(T)}_{[X]}.$$

44. $${}_tH_X=\sum_{s=1}^{t}{}_sh_X.$$

III. CALCULATION OF PRESENT VALUE FACTORS

The first step in the calculation of the gross premium is to compute the required values of ${}_tE^{(T)}_{[X]}$, ${}_tf_X$, and ${}_tF_X$. In order to calculate these factors it is first necessary to decide upon (1) assumptions as to mortality, lapses, and interest earned on assets, and (2) the commissions and other "per cent of premium" expenses to be paid.

${}_tE^{(T)}_{[X]}$, ${}_tf_X$, and ${}_tF_X$ were defined and employed in Rosser's and Anderson's papers. However, these papers employed Type 2 methods of gross premium calculation, which prevented development of the full potential of these factors.

IV. COST OF REINSURANCE

The second step in the gross premium calculation is to compute the cost of reinsurance. This cost has a definite effect on gross premiums and their profit margins. In this paper reinsurance shall, in all cases, mean Yearly Renewable Term (YRT) reinsurance.

Traditionally, YRT reinsurance has been offered on an aggregate premium basis with the reinsurance premium per $1,000 of reinsurance

* If $K \geq A_t$, $\quad \frac{A_t-K}{A_t}\left({}^AR^1_{X+t-1}+\frac{{}^BR^1_{X+t-1}}{A_t-K}\right)=0$.

for any policy year varying only by attained age, except for the first-year premium rate being substantially (usually about 50 per cent) smaller than the renewal rate for the same attained age.

Recently, reinsurers have also begun offering YRT reinsurance on a select and ultimate premium basis, with an annual fee (which may vary by policy year) per policy reinsured being charged in addition to the YRT premium per $1,000 of reinsurance. There are, at present, several different select and ultimate YRT premium structures being offered. These differ with regard to the select period and/or the size of the annual fee.

If YRT reinsurance is experience-rated, the ceding company's experience refund for any year will depend upon (*a*) its own experience, (*b*) the credibility given to this experience by the reinsurer (this is, of course, a function of the amount of insurance the insurer cedes to the reinsurer), (*c*) the reinsurer's over-all experience on "pooled" YRT reinsurance assumed, and (*d*) the reinsurer's experience rating formula and the basis of its application. As the various reinsurance companies use a number of different methods to calculate the noncreditable (or pooled) portion of an insurer's experience refund, the assumption is made that the noncreditable portion of an insurer's experience refund for a policy year will be a per cent of the noncreditable portion of the YRT premium (excluding any annual fee) paid for that policy year. It has also been assumed that the premium tax and experience refunds for a policy year are, on the average, paid at the end of that policy year.

Before incorporating the cost of reinsurance into the gross premium calculation, it is first necessary to define this cost for gross premium calculation purposes.

Ormsby [7] dealt with the "out-of-pocket" cost of reinsurance, which may be defined prospectively as (*a*) the ceding company's reinsurance handling costs plus the reinsurer's expense and profit charge on the amounts of insurance issued in excess of the retention limit, less (*b*) the margin for profit in the additional gross premiums received on such insurance. This "out-of-pocket" cost of reinsurance is especially suited to decisions concerning the adoption or expansion of a reinsurance program.

Wooddy [8] gives several additional definitions of the cost of reinsurance. The definition that appears to be most suitable for inclusion in the gross premium calculation considers the cost of reinsurance as (*a*) outgo (e.g., reinsurance premiums), plus (*b*) the ceding company's administrative cost of handling the reinsurance, less (*c*) inflow (e.g., claims paid by reinsurer, premium tax and experience refunds).

The ceding company's administrative cost of handling reinsurance is not directly proportional to the amount reinsured or the number of policies reinsured. As this expense may be considered an integral part of a company's issue and record-keeping expense and is, at least partially, offset by the many valuable services provided by reinsurers, the "cost" of reinsurance is now defined *for gross premium calculation purposes* as (*a*) outgo, less (*c*) inflow; with any amounts not actually paid to, or received from, the reinsurer being excluded from this definition. For YRT reinsurance, the cost of reinsurance for a specific policy year will be defined as (*a*) the reinsurance premium (including any annual fee) paid at the beginning of the policy year, less (*b*) the value, at the beginning of the policy year, of (*i*) that policy year's death claims paid by the reinsurer plus (*ii*) the premium tax and experience refunds for that policy year. The "cost" of reinsurance for policy year t may now be expressed as follows:

a) Cost per \$1,000 of insurance reinsured at beginning of policy year t:

$$^{A}R^{1}_{X+t-1}+\frac{^{B}R^{1}_{X+t-1}}{A_t-K}\,.$$

b) Cost per \$1,000 of insurance reinsured at issue:

$$_{t-1}p^{(T)}_{[X]}\left[{}^{A}R^{1}_{X+t-1}+\frac{^{B}R^{1}_{X+t-1}}{A_t-K}\right].$$

c) Cost per \$1,000 of insurance in force at beginning of policy year t:

$$\frac{1}{A_t}[\,(A_t-K)\cdot{}^{A}R^{1}_{X+t-1}+{}^{B}R^{1}_{X+t-1}\,].$$

d) Cost per \$1,000 of insurance issued:

$$_{t-1}p^{(T)}_{[X]}\,\frac{1}{A_t}[\,(A_t-K)\cdot{}^{A}R^{1}_{X+t-1}+{}^{B}R^{1}_{X+t-1}\,].$$

The cost of reinsurance as a level annual cost over the gross premium calculation period may be expressed:

a) In terms of each \$1,000 of insurance reinsured at issue:

$$_{N}R^{1}_{X}=\frac{\sum_{t=1}^{N}{}^{A}R^{1}_{X+t-1}\cdot{}_{t-1}E^{(T)}_{[X]}}{_{N}F_{X}}+\frac{1}{_{N}A-K}\cdot\frac{\sum_{t=1}^{N}{}^{B}R^{1}_{X+t-1}\cdot{}_{t-1}E^{(T)}_{[X]}}{_{N}F_{X}} \qquad (1)$$

$$={}^{A}_{N}R^{1}_{X}+\frac{1}{_{N}A-K}\cdot{}^{B}_{N}R^{1}_{X}\,.$$

b) In terms of each \$1,000 of insurance issued:

$$_{N}R^{2}_{X}=k\cdot{}_{N}R^{1}_{X}=\frac{1}{_{N}A}[\,(_{N}A-K)\,{}^{A}_{N}R^{1}_{X}+{}^{B}_{N}R^{1}_{X}\,]. \qquad (2)$$

As the gross premium calculation is being prepared from the ceding company's point of view, uniform underwriting and, hence, mortality is assumed for amounts of insurance above and below the retention limit. Because the subject of inserting contingency margins into the gross premium calculation has, in the opinion of the author, been adequately covered in other papers (e.g., Anderson [6]), it has been assumed that actual future mortality will not fluctuate from that assumed in the gross premium calculation. This assumption does, however, inject an element of conservatism into the cost of reinsurance as mortality fluctuations can cause occasional negative experience refunds which, because of a limit (often 3 years) on the carryover of losses, may not be fully offset against future positive experience refunds.

V. THE "AVERAGE" POLICY SIZE

A. For the Gross Premium Calculation Period

Whenever gross premiums are calculated, there is invariably a statement made regarding an assumed "average" policy size. This is usually defined as the total insurance issued divided by the number of policies issued (see definition 22). Unfortunately, this definition does not take into account the financial effect of the cost of reinsurance and will normally produce an average policy size which will overstate the profit per $1,000 of insurance issued. This overstatement will, of course, be insignificant for a large company with a very high retention limit and very little reinsurance ceded.

If a company would issue a plan of insurance only for policy size ${}_NA$ and receive over the gross premium calculation period exactly the same profit per $1,000 of insurance issued that would be received if the plan were offered at the usual (from minimum policy size to limit of issue) range of policy sizes, ${}_NA$ is the "average" policy size that should be used in calculating gross premiums.

To define ${}_NA$ more explicitly, the present value at issue of per policy expenses (e'_t) and reinsurance costs to be incurred over the gross premium calculation period per $1,000 of insurance issued at age X under a policy of size ${}_NA$ should equal the present value at issue of the per policy expenses and cost of reinsurance to be incurred over the gross premium calculation period per $1,000 of insurance issued at age X if the policy is offered at the usual range of policy sizes. If ${}_NA$ is greater than the retention limit, K, this definition is satisfied by equation (3). Solving this equation for ${}_NA$ leads to equation (4). If ${}_NA$ is smaller than or equal to K, this definition will be satisfied by equation (5), which, when solved, leads to equation (6).

${}_N A > K$:

$$\frac{\sum_G f(X,G) \sum_{t=1}^{N} e'_t \cdot {}_{t-1}E^{(T)}_{[X]} - B \cdot {}_N F_X + \sum_G f(X,G)(G-K)^* {}_N F_X \left({}^A_N R^1_X + \frac{{}^B_N R^1_X}{(G-K)^*} \right)}{\sum_G f(X,G) \cdot G}$$

$$= \frac{\sum_{t=1}^{N} e'_t \cdot {}_{t-1}E^{(T)}_{[X]} - B \cdot {}_N F_X + {}_N F_X [({}_N A - K) {}^A_N R^1_X + {}^B_N R^1_X]}{{}_N A} . \quad (3)$$

$${}_N A = \frac{\sum_G f(X,G) G \left[\sum_{t=1}^{N} e'_t \cdot {}_{t-1}E^{(T)}_{[X]} - B \cdot {}_N F_X + {}_N F_X ({}^B_N R^1_X - K \cdot {}^A_N R^1_X) \right]}{\sum_G f(X,G) \sum_{t=1}^{N} e'_t \cdot {}_{t-1}E^{(T)}_{[X]} - B \cdot {}_N F_X + {}^A_N R^1_X \cdot {}_N F_X \left[\sum_G f(X,G)(G-K)^* - \sum_G f(X,G) \cdot G \right] + \sum_G f(X,G)(G-K)^* {}_N F_X \frac{{}^B_N R^1_X}{(G-K)^*}} . \quad (4)$$

${}_N A \leq K$:

$$\frac{\sum_G f(X,G) \sum_{t=1}^{N} e'_t \cdot {}_{t-1}E^{(T)}_{[X]} - B \cdot {}_N F_X + \sum_G f(X,G)(G-K)^* {}_N F_X \left[{}^A_N R^1_X + \frac{{}^B_N R^1_X}{(G-K)^*} \right]}{\sum_G f(X,G) \cdot G} = \frac{\sum_{t=1}^{N} e'_t \cdot {}_{t-1}E^{(T)}_{[X]} - B \cdot {}_N F_X}{{}_N A} . \quad (5)$$

$${}_N A = \frac{\sum_G f(X,G) \cdot G \left[\sum_{t=1}^{N} e'_t \cdot {}_{t-1}E^{(T)}_{[X]} - B \cdot {}_N F_X \right]}{\sum_G f(X,G) \sum_{t=1}^{N} e'_t \cdot {}_{t-1}E^{(T)}_{[X]} - B \cdot {}_N F_X + \sum_G f(X,G)(G-K)^* {}_N F_X \left[{}^A_N R^1_X + \frac{{}^B_N R^1_X}{(G-K)^*} \right]} . \quad (6)$$

* When $G \leq K, (G-K) = \left(\frac{G-K}{G-K} \right) = 0$.

In the above equations, the portion of the policy fee not paid out in commissions or other per cent of premium expenses is treated as a negative per policy expense. The values of G used in these equations will, of course, represent appropriate policy size groupings.

There will usually be *two* average policy sizes that will satisfy the definition of the average policy size. If this occurs, the two values of ${}_NA$ will be on opposite sides of the retention limit and will both yield an identical profit over the N-year gross premium calculation period. Their only difference will be in the *incidence* of profit earned over the gross premium calculation period.

B. For Each Policy Year

The level average policy size previously defined does not produce the exact *incidence* of profit per $1,000 of insurance issued that would be produced if the policy were issued at the usual range of policy sizes. In order to do this an average policy size must be calculated for each policy year. The average policy size for policy year t is calculated as shown in formulas (7) and (8) on page 244.

It will not be unusual to have equations (7) and (8) both produce correct values of A_t. A value of A_t produced by equation (7) is correct if it is positive and is not greater than K. Conversely, a value of A_t produced by equation (8) is correct if it is greater than K.

If ${}_NA$ were substituted for $A_t (t = 1, 2, \ldots N)$ in ${}_NH_X$, there would be no change in the value at issue of the total amount of profit to be earned over the gross premium calculation period.

VI. THE BREAK-EVEN PREMIUMS

Once the "average" policy size has been determined, the break-even premiums may be calculated. The t-year break-even premium is the premium that will result in exactly zero profit at the end of policy year t. The t-year break-even premium may be defined in two ways, depending upon what *concept* of profit is referred to. If actuarial profit is the profit referred to, the cash value will represent the insurer's policy liability and equation (9) will be used to define the t-year break-even premium. On the other hand, if annual statement (book) profit is the profit referred to, the

$A_t \leq K$:

$$A_t = \frac{{}_{t-1}E_{[X]}^{(T)} \cdot e'_t}{\frac{1}{{}_tA}\left\{\sum_{s=1}^{t} {}_{s-1}E_{[X]}^{(T)} \cdot e'_s + ({}_tA - K)^* {}_tF_X\left[{}_t^AR_X^1 + \frac{{}_t^BR_X^1}{({}_tA - K)^*}\right]\right\} - \frac{1}{{}_{t-1}A}\left\{\sum_{s=1}^{t-1} {}_{s-1}E_{[X]}^{(T)} e'_s + ({}_{t-1}A - K)^* {}_{t-1}F_X\left[{}_{t-1}^{\ A}R_X^1 + \frac{{}_{t-1}^{\ B}R_X^1}{({}_{t-1}A - K)^*}\right]\right\}} \cdot \quad (7)$$

$A_t > K$:

$A_t =$

$$\frac{[e'_t + {}^BR_{X+t-1}^1 - K \cdot {}^AR_{X+t-1}^1]\, {}_{t-1}E_{[X]}^{(T)}}{\frac{1}{{}_tA}\left\{\sum_{s=1}^{t} {}_{s-1}E_{[X]}^{(T)} \cdot e'_s + ({}_tA - K)^* {}_tF_X\left[{}_t^AR_X^1 + \frac{{}_t^BR_X^1}{({}_tA - K)^*}\right]\right\} - \frac{1}{{}_{t-1}A}\left\{\sum_{s=1}^{t-1} {}_{s-1}E_{[X]}^{(T)} \cdot e'_s + ({}_{t-1}A - K)^* {}_{t-1}F_X\left[{}_{t-1}^{\ A}R_X^1 + \frac{{}_{t-1}^{\ B}R_X^1}{({}_{t-1}A - K)^*}\right]\right\} - {}_{t-1}E_{[X]}^{(T)} \cdot {}^AR_{X+t-1}^1} \cdot$$

(8)

reserve will represent the insurer's policy liability and equation (10) will be used to define the t-year break-even premium.

$$ {}_tP_X^{BE} = \frac{{}_tH_X + {}_tE_{[X]}^{(T)} \cdot {}_tCV_X}{{}_tF_X} . \tag{9} $$

$$ {}_tP_X^{BE} = \frac{{}_tH_X + {}_tE_{[X]}^{(T)} \cdot {}_tV_X}{{}_tF_X} . \tag{10} $$

If there is a $\$B$ policy fee the *ratebook* break-even premium per \$1,000, ${}_tP_X^{BE/M}$, is defined as follows:

$$ {}_tP_X^{BE/M} = {}_tP_X^{BE} - B/{}_tA . \tag{11} $$

VII. CALCULATION OF THE VARIOUS MEASURES OF PROFIT

In an actuarial report dealing with gross premium rates, there is a high probability of finding a phrase such as "This will produce at age 35 a twenty-year profit of \$5 per \$1,000." This statement, unfortunately, is anything but clear. It could conceivably mean any of the following:

a) A \$5 profit accumulated at the end of the twentieth policy year for each \$1,000 of insurance still in force at the end of the twentieth policy year, or

b) A \$5 profit accumulated at the end of the twentieth policy year for each \$1,000 of insurance issued, or

c) A profit accumulated at the end of the twentieth policy year which has a present value at issue of \$5 per \$1,000 of insurance issued, or

d) A profit accumulated at the end of the twentieth policy year with a present value at issue of \$5 per \$1,000 of insurance still in force at the end of twenty years, or

e) A \$5 profit accumulated at the end of the twentieth policy year for each \$1,000 of insurance still in force at the beginning of the twentieth policy year.

The profit criteria referred to usually will depend on the gross premium calculation technique used by the actuary. There seems to be a general tendency to avoid precisely defining the "profit" referred to. As one of the primary functions of the actuary is to communicate his findings to management, more time should be devoted to this subject.

Once ${}_NP_X^{BE/M}$ has been calculated, the N-year profit margin per \$1,000 of insurance issued can be immediately calculated for any gross premium. This is defined as (*a*) the ratebook gross premium per \$1,000, Π_X, less (*b*) the N-year ratebook break-even premium per \$1,000, ${}_NP_X^{BE/M}$, and is a sixth measure of profitability.

The two main advantages of the N-year profit margin are that (*a*) it is particularly meaningful to management, and (*b*) any of the other five measures of profitability mentioned above may be quickly and easily determined for any proposed profit margin. This is shown as follows:

1. The profit accumulated at the end of the Nth policy year for each \$1,000 of insurance in force at the end of the Nth policy year is equal to

$$(\Pi_X - {}_NP_X^{BE/M})\frac{{}_NF_X}{{}_NE_{[X]}^{(T)}}. \tag{12A}$$

2. The profit accumulated at the end of the Nth policy year for each \$1,000 of insurance issued is equal to

$$(\Pi_X - {}_NP_X^{BE/M})_NF_X(1+i)^N. \tag{12B}$$

3. The value at issue of the profit earned over the first N policy years for each \$1,000 issued is equal to

$$(\Pi_X - {}_NP_X^{BE/M})_NF_X\,. \tag{12C}$$

4. The value at issue of the profit accumulated at the end of the Nth policy year for each \$1,000 in force at the end of the Nth policy year is equal to

$$\frac{(\Pi_X - {}_NP_X^{BE/M})\,{}_NF_X}{{}_NE_{[X]}^{(T)}(1+i)^N}. \tag{12D}$$

5. The profit accumulated at the end of the Nth policy year for each \$1,000 of insurance in force at the beginning of the Nth policy year is equal to

$$\frac{(\Pi_X - {}_NP_X^{BE/M})\,{}_NF_X}{v\cdot{}_{N-1}E_{[X]}^{(T)}}. \tag{12E}$$

Conversely, once ${}_NP_X^{BE/M}$ has been calculated, the gross premium that will yield a specified profit in terms of one of the five measures of profitability described above can be quickly solved for. For instance, referring to equation (12A) above, the gross premium, Π_X, that will accumulate a \$50 profit at the end of the Nth policy year for each \$1,000 of insurance still in force at the end of the Nth policy year is equal to

$${}_NP_X^{BE/M} + 50\,\frac{{}_NE_{[X]}^{(T)}}{{}_NF_X}. \tag{13}$$

It can easily be seen how similar equations would be developed from equations (12B) through (12E). This leads to the next step in the gross premium calculation process: determining the gross premium.

VIII. DETERMINATION OF THE GROSS PREMIUM

The final gross premium will depend primarily upon a company's desired competitive position and its profit objectives. Common profit objectives include the following: (*a*) a specified profit over a specified number of policy years, as shown in Section VII, and (*b*) amortization of the initial investment in new business by the end of a specified number of policy years.

Even when the company's desired competitive position has been the dominating force in determining the final gross premium, management may have questions regarding (*a*) amount of "profit" and (*b*) time required to recover the company's investment in new business (to "break-even"). The break-even premiums and present value factors may be used to answer such questions with little or no additional work.

If, on the other hand, the final gross premium depends primarily upon the company's profit objectives, the gross premium that will satisfy these objectives has already been calculated (t-year break-even premiums) or may be readily calculated with little additional work (as shown in Section VII).

Anderson's paper [6] introduced the revolutionary concept of solving for the gross premium that would produce a predetermined yield (which may vary by policy year) on surplus invested in new business. Anderson's method involved the use of the renewal net premium as a preliminary gross premium.

The same gross premium may be solved for directly by a less lengthy technique not requiring the use of a preliminary gross premium. The gross premium, P'_X, as defined in Anderson's paper, may be solved for as follows:

$P'_X \geq P_X$ (no deficiency reserves):

$$P'_X = \frac{\sum_{t=1}^{N}\left({}_tP_X^{BE/M}\cdot {}_tF_X - {}_{t-1}P_X^{BE/M}\cdot {}_{t-1}F_X\right)\left(\frac{1+i}{1+j}\right)^{t-1}}{\sum_{t=1}^{N} {}_tf_X\left(\frac{1+i}{1+j}\right)^{t-1}}, \quad (14A)$$

with

$${}_tP_X^{BE} = \frac{{}_tH_X + {}_tE_{[X]}^{(T)}\cdot {}_tV_x}{{}_tF_X}.$$

$P'_X < P_X$ (deficiency reserves):

$$P'_X = \frac{\sum_{t=1}^{N} [{}_tP_X^{BE/M} \cdot ({}_tF_X + {}_tE_{[X]}^{(T)} \ddot{a}'_{X+t}) - {}_{t-1}P_X^{BE/M} \cdot ({}_{t-1}F_X + {}_{t-1}E_{[X]}^{(T)} \ddot{a}'_{X+t-1})] \left(\frac{1+i}{1+j}\right)^{t-1}}{\sum_{t=1}^{N} ({}_tf_X + {}_tE_{[X]}^{(T)} \cdot \ddot{a}'_{X+t} - {}_{t-1}E_{[X]}^{(T)} \ddot{a}'_{X+t-1}) \left(\frac{1+i}{1+j}\right)^{t-1}} \quad (14$$

with

$${}_tP_X^{BE} = \frac{{}_tH_X + {}_tE_{[X]}^{(T)} ({}_tV_X + P_X \ddot{a}'_{X+t})}{{}_tF_X + {}_tE_{[X]}^{(T)} \ddot{a}'_{X+t}},$$

where

$${}_0F_X = {}_0P_X^{BE/M} = 0 .$$

Mr. Anderson also introduces the following two concepts: (*a*) treating indirect or other expense (e_t in this paper) as a multiple of the present value of commissions; (*b*) expressing the present value of the profit required to compensate for the use of surplus invested in the agency organization as a multiple of the present value of commissions.

If "*a*" is defined as the multiple of the present value of commissions equal to the present value of indirect or other expenses and if "*b*" is the multiple of the present value of commissions equal to the present value of the profit required to compensate for the use of surplus invested in the agency organization, then ${}_Nf_X$ and ${}_NF_X$ would be modified as follows:

$${}_NF_X = \sum_{t=1}^{N} {}_tf_X = \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} \{1 - g_t - c_t(1+a+b)[1 - z_t(1-\theta_t)]\} . \quad (15)$$

P'_X could then be solved for as shown in equations (14A) and (14B). Equation (15) shows that *a* and *b* may vary by policy year.

IX. OBTAINING THE INCIDENCE OF PROFIT

After the gross premium has been established along with the amount of (expected) profit over the gross premium calculation period, the next thing management will want to know is the incidence of profit. This may be easily accomplished once (*a*) the break-even premiums for the first *N* years and (*b*) the gross premium, Π_X, have been determined. The following equations illustrate how this may be done:

1. Value at issue of the *t*th policy year's profit per $1,000 issued, ${}_tW_X$:

$${}_tW_X = (\Pi_X - {}_tP_X^{BE/M}) {}_tf_X + ({}_{t-1}P_X^{BE/M} - {}_tP_X^{BE/M}) {}_{t-1}F_X . \quad (16A)$$

2. Value at beginning of policy year t of that policy year's profit per \$1,000 issued, ${}_tX_X$:

$$ {}_tX_X = {}_tW_X(1+i)^{t-1} . \tag{16B} $$

3. Value at end of policy year t of that policy year's profit per \$1,000 in force at the beginning of policy year t, ${}_tZ_X$:

$$ {}_tZ_X = \frac{{}_tW_X(1+i)}{{}_{t-1}E_{[X]}^{(T)}} . \tag{16C} $$

4. Value at the beginning of policy year t of that policy year's profit per \$1,000 in force at the beginning of policy year t, ${}_tY_X$:

$$ {}_tY_X = \frac{{}_tW_X}{{}_{t-1}E_{[X]}^{(T)}} . \tag{16D} $$

5. Value at end of policy year t of that policy year's profit per \$1,000 issued, ${}_tU_X$:

$$ {}_tU_X = {}_tW_X(1+i)^t . \tag{16E} $$

X. THE VALUE OF BUSINESS IN FORCE

Management may desire forecasts of the future values of business in force. While the actuary may not predict exactly what the actual market value will be m years in the future, he can give an estimate of this market value based on the best information at hand.

The projected market value per \$1,000 of insurance in force at the end of m years, ${}_m(MV)_X^i$, may be expressed as follows:

$$ {}_m(MV)_X^i = \frac{\sum_{t=m+1} {}_tW_X}{{}_mE_{[X]}^{(T)}} . \tag{17} $$

This definition of market value discounts future book profits by the interest rate earned on assets. If the future book profits were discounted by the rate of profit required on surplus invested in new business, equation (17) would be replaced by equation (18):

$$ {}_m(MV)_X^j = \frac{\sum_{t=m+1} {}_tW_X \left(\frac{1+i}{1+j}\right)^{t-1}}{{}_mE_{[X]}^{(T)} \left(\frac{1+i}{1+j}\right)^m} . \tag{18} $$

This is the projected market value as defined in Mr. Anderson's paper [6].

XI. VARIATION OF ASSUMPTIONS

A. Commissions and Other Per Cent of Premium Expenses

These expenses are included in the denominator of the N-year break-even premium. A change of Δ_t in c_t will result in a new break-even premium, ${}_NP_X'^{BE}$, which is defined in equation (19). If the change of Δ_t is in g_t, the new break-even premium would be defined by equation (20):

$$ {}_NP_X'^{BE} = {}_NP_X^{BE} \frac{{}_NF_X}{{}_NF_X - \sum_{t=1}^{N} \Delta_t [1 - z_t(1-\theta_t)]\,{}_{t-1}E_{[X]}^{(T)}} . \qquad (19) $$

$$ {}_NP_X'^{BE} = {}_NP_X^{BE} \frac{{}_NF_X}{{}_NF_X - \sum_{t=1}^{N} \Delta_t \cdot {}_{t-1}E_{[X]}^{(T)}} . \qquad (20) $$

Equations (19) and (20) become approximations if there is a policy fee, as a change in ${}_NF_X$ will then result in a change in the average policy size.

B. Other Expenses

Other expenses are primarily of two types: (*a*) "per policy" expenses and (*b*) expenses per $1,000 of insurance.

A change, Δ_t, in the policy year t "per policy" expenses will cause the following change, $\Delta_N P_X^{BE}$, in the N-year break-even premium:

$$ \Delta_N P_X^{BE} = \frac{\sum_{t=1}^{N} \Delta_t \cdot {}_{t-1}E_{[X]}^{(T)}}{A \cdot {}_NF_X} . \qquad (21) $$

If the change, Δ_t, were in the tth year expenses per $1,000 of insurance, the term "A" would be eliminated from the right-hand side of equation (21).

C. Average Policy Size

If the average policy size changes from ${}_NA$ to ${}_NA'$, the change, $\Delta_N P_X^{BE}$, in the N-year break-even premium, ${}_NP_X^{BE}$, will be as follows:

Case 1: ${}_NA \le K$ *and* ${}_NA' \le K$:

$$ \Delta_N P_X^{BE} = \frac{\frac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} e_t'}{{}_NF_X} . \qquad (22) $$

Case 2: ${}_NA \le K < {}_NA'$:

$$\Delta_N P_X^{BE} = \frac{\dfrac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} \cdot e_t'}{{}_NF_X} + \frac{{}_NA' - K}{{}_NA'} {}_N^AR^1_X + \frac{{}_N^BR^1_X}{{}_NA'} . \quad (23)$$

Case 3: ${}_NA' \le K < {}_NA$:

$$\Delta_N P_X^{BE} = \frac{\dfrac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} \cdot e_t'}{{}_NF_X} - \frac{{}_NA - K}{{}_NA} {}_N^AR^1_X - \frac{{}_N^BR^1_X}{{}_NA} . \quad (24)$$

Case 4: ${}_NA > K$ *and* ${}_NA' > K$:

$$\Delta_N P_X^{BE} = \frac{\dfrac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} \cdot e_t'}{{}_NF_X} + \frac{K({}_NA' - {}_NA)}{{}_NA \cdot {}_NA'} {}_N^AR^1_X + \frac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} {}_N^BR^1_X . \quad (25)$$

A simultaneous change, Δ_t, in per policy expenses, will cause

$$\frac{\dfrac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} e_t'}{{}_NF_X}$$

to be replaced in equations (22) through (25) by

$$\frac{{}_NA \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} \Delta_t + ({}_NA - {}_NA') \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} e_t'}{{}_NA \cdot {}_NA' {}_NF_X} .$$

D. Retention Limit

A change in the retention limit from K to K' will result in a change in the average size policy. Therefore, the effect on the N-year break-even premium may be found by first calculating the new average policy size and then applying the appropriate equation shown below:

Case 1: ${}_NA \le K$ *and* ${}_NA' \le K'$:

$$\Delta_N P_X^{BE} = \frac{\dfrac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} \cdot e_t'}{{}_NF_X} . \quad (26)$$

Case 2: ${}_NA \leq K$ *and* ${}_NA' > K'$:

$$\Delta_N P_X^{BE} = \frac{\dfrac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} e'_t}{{}_NF_X} + \frac{{}_NA' - K'}{{}_NA'} \cdot {}_N^AR^1_X + \frac{{}_N^BR^1_X}{{}_NA'}. \tag{27}$$

Case 3: ${}_NA > K$ *and* ${}_NA' \leq K'$:

$$\Delta_N P_X^{BE} = \frac{\dfrac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} e'_t}{{}_NF_X} - \frac{{}_NA - K}{{}_NA} \cdot {}_N^AR^1_X - \frac{{}_N^BR^1_X}{{}_NA}. \tag{28}$$

Case 4: ${}_NA > K$ *and* ${}_NA' > K'$:

$$\Delta_N P_X^{BE} = \frac{\dfrac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} e'_t}{{}_NF_X} + \frac{{}_NA' \cdot K - {}_NA \cdot K'}{{}_NA \cdot {}_NA'} \cdot {}_N^AR^1_X + \frac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \cdot {}_N^BR^1_X. \tag{29}$$

A simultaneous change, Δ_t, in per policy expenses will again cause

$$\frac{\dfrac{{}_NA - {}_NA'}{{}_NA \cdot {}_NA'} \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} \cdot e'_t}{{}_NF_X}$$

to be replaced by

$$\frac{{}_NA \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} \cdot \Delta_t + ({}_NA - {}_NA') \sum_{t=1}^{N} {}_{t-1}E_{[X]}^{(T)} \cdot e'_t}{{}_NA \cdot {}_NA' \cdot {}_NF_X}.$$

E. Combining Interest Rates

If three N-year break-even premiums are calculated using interest rates of $3\frac{1}{2}$ per cent, $3\frac{3}{4}$ per cent, and 4 per cent (with all other assumptions the same), new N-year break-even premiums, based on different interest assumptions, may be calculated without redoing the original calculation. Thus the N-year break-even premium that is based upon 4 per cent interest the first a policy years, $3\frac{3}{4}$ per cent the next $c(a + c = b)$ policy years, and $3\frac{1}{2}$ per cent thereafter would be calculated as follows:

$$
{}_{N}P_{X}^{BE} = \frac{{}^{.04}_{a}H_X + \frac{{}^{.04}_{a}E^{(T)}_{[X]}}{{}^{.0375}_{a}E^{(T)}_{[X]}}\left[{}^{.0375}_{b}H_X - {}^{.0375}_{a}H_X + \frac{{}^{.0375}_{b}E^{(T)}_{[X]}}{{}^{.035}_{b}E^{(T)}_{[X]}}\left({}^{.035}_{N}H_X - {}^{.035}_{b}H_X + {}^{.035}_{N}E^{(T)}_{[X]} \cdot {}_{N}V_X\right)\right]}{{}^{.04}_{a}F_X + \frac{{}^{.04}_{a}E^{(T)}_{[X]}}{{}^{.0375}_{a}E^{(T)}_{[X]}}\left[{}^{.0375}_{b}F_X - {}^{.0375}_{a}F_X + \frac{{}^{.0375}_{b}E^{(T)}_{[X]}}{{}^{.035}_{b}E^{(T)}_{[X]}}\left({}^{.035}_{N}F_X - {}^{.035}_{b}F_X\right)\right]} . \qquad (30)
$$

In equation (30), a and b may assume any values that satisfy the inequality $\overline{a + c} < \overline{N + 1}$.

F. Cash Value Replacing Reserve as Policy Liability

Many of the older established companies which are not faced with depletion of surplus problems use the cash value instead of the reserve to represent the insurer's policy liability in the gross premium calculation. This will have the following effect on ${}_{t}W_X$:

$$
\Delta_t W_X = ({}_{t}CV_X - {}_{t}V_X)_{t}E^{(T)}_{[X]} - ({}_{t-1}CV_X - {}_{t-1}V_X)_{t-1}E^{(T)}_{[X]} . \qquad (31)
$$

Similar formulas may be derived from Section IX to calculate the corresponding changes in ${}_{t}X_X$, ${}_{t}Z_X$, ${}_{t}U_X$, and ${}_{t}Y_X$.

XII. DEFICIENCY RESERVES

Deficiency reserves need not have any effect upon the calculation of gross premiums. From an actuarial viewpoint deficiency reserves are simply surplus artificially earmarked by law.

If annual statement (or book) profit is the profit used in the gross premium calculation, deficiency reserves *will not* affect the *total* profit earned over the *life* of the policy, *may* affect the *total* profit earned over the *gross premium calculation period* (depending upon whether they have disappeared by the end of this period), and *will* affect the *incidence* of profit.

Deficiency reserves should be provided for in the gross premium calculation when the incidence of annual statement profit is of particular interest to management, particularly when a predetermined profit objective influencing the selection of the gross premium is based on a desired *incidence* of annual statement profit (e.g., "amortization" of investment in surplus by the end of the *t*th policy year).

The incidence of annual statement profit is most likely to be of concern to the management of small and/or new stock life insurers, particularly under situations such as the following:

a) The insurer has a small surplus and will be in danger of becoming legally insolvent from depletion of surplus caused by new business production if production goals are based on earnings projections and gross premiums which have not taken the effect of deficiency reserves into account.

b) The insurer's stock is wholly owned by a parent corporation which is interested in making a (legal) profit on, and receiving dividends from, this stock as soon as possible.

c) The insurer plans to raise additional capital in a few years by means of a public stock offering and is concerned with the operating results which will be shown on the prospectus.

If deficiency reserves are to be provided for, the gross premium calculation process will, of necessity, become more complicated and lengthy. The method of gross premium calculation presented in this paper can be adopted to handle this complication. This is illustrated by equations (32) and (33), which redefine the formulas for the t-year break-even premium and the value at issue of the tth policy year's profit per $1,000 issued, to provide for deficiency reserves.

$$ {}_tP_X^{BE} = \frac{{}_tH_X + {}_tE_{[X]}^{(T)}({}_tV_X + P_X\ddot{a}'_{X+t})}{{}_tF_x + {}_tE_{[X]}^{(T)}\ddot{a}'_{X+t}} . \qquad (32) $$

$$ \begin{aligned} {}_tW_X = (\Pi_X - {}_tP_X^{BE/M})\,{}_tf_X + ({}_{t-1}P_X^{BE/M} - {}_tP_X^{BE/M})\,{}_{t-1}F_X \\ + \Pi_X({}_tE_{[X]}^{(T)}\ddot{a}'_{X+t} - {}_{t-1}E_{[X]}^{(T)}\ddot{a}'_{X+t-1}) \\ + ({}_{t-1}P_X^{BE/M}\cdot {}_{t-1}E_{[X]}^{(T)}\ddot{a}'_{X+t-1} - {}_tP_X^{BE/M}\cdot {}_tE_{[X]}^{(T)}\cdot \ddot{a}'_{X+t}) . \qquad (33) \end{aligned} $$

XIII. SUMMARY

The primary goal of this paper was to present the basic principles, and some of the more useful applications, of a direct comprehensive approach to the calculation of gross premiums. Level death benefits and level premiums were assumed in order to stress basic theory, which, in turn, can be applied to develop formula modifications for plans with special characteristics.

The sections of this paper pertaining to the cost of reinsurance and the average policy size are equally applicable to participating insurance. On the other hand, determination of participating gross premiums and dividends would necessitate the involvement of substantial additional basic theory and techniques beyond the scope of this paper.

I would like to thank Mr. Barnet N. Berin, Dr. Paul Markham Kahn, and the Committee on Papers for their invaluable suggestions on the presentation of this paper.

XIV. ILLUSTRATIVE EXAMPLES

Example 1

A—Assumptions

Plan	30-Year Endowment
Issue age (X)	35
Gross premium calculation period (N)	30 years
Policy fee (B)	Zero
Policy size band	$10,000 through $75,000
Retention limit (K)	$20,000
Premium payment mode	Annual
Reinsurance	Nonrefund aggregate YRT

Commissions (c_t):

Policy Year	Commission (100% vested)
1	70%
2	10
3–10	5
11–30	2

Other "per cent of premium" expenses (g_t) . . . 3%

Other expenses:

Type of Expense	Policy Year(s) 1	2–30
Per policy (excluding cost of paying death claims, cash surrender values, and maturities)	$75.00	$ 7.50
Per $1,000	4.50	.50
Per cash surrender or maturity	30.00	30.00
Per death claim	50.00	50.00

Net rate earned on invested assets (i_t)	3½% (all years)
Other assumptions	See Table 1

B—Profit Objectives

1. 20 per cent to 25 per cent yield on surplus invested in new business.
2. Amortization of surplus invested in new business within seven policy years.

C—Calculations

The expected distribution of new business by policy size is shown in Table 2. As the most profitable policy size is the retention limit, there are two values of ${}_{30}A$: $29,970 (equation [4]) and $16,850 (equation [6]).

Table 3 shows the following information: (1) the present value factors;

TABLE 1

Schedule of Assumptions

t	$_tV_{35}$	$_tCV_{35}$	$q^d_{[35]+t-1}$	$q^w_{[35]+t-1}$	r_{35+t-1}
1	....	....	.00085	.250	$ 1.62
2	$ 23.35	$ 13.00	.00107	.150	3.39
3	47.31	37.00	.00130	.100	3.60
4	71.86	62.00	.00154	.088	3.84
5	97.01	88.00	.00179	.080	4.09
6	122.76	114.00	.00205	.072	4.37
7	149.10	141.00	.00233	.064	4.69
8	176.06	168.00	.00264	.058	5.03
9	203.67	197.00	.00298	.054	5.39
10	231.92	225.00	.00337	.050	5.79
11	260.86	255.00	.00382	.048	6.24
12	290.47	285.00	.00436	.046	6.74
13	320.80	316.00	.00499	.044	7.28
14	351.84	348.00	.00573	.042	7.86
15	383.64	380.00	.00661	.040	8.50
16	416.22	414.00	.00768	.040	9.16
17	449.62	448.00	.00855	.040	9.87
18	483.90	483.00	.00944	.040	10.62
19	519.11	519.00	.01035	.040	11.42
20	555.32	556.00	.01131	.040	12.29
21	592.61	593.00	.01232	.040	13.22
22	631.07	632.00	.01339	.040	14.24
23	670.82	671.00	.01454	.040	15.32
24	711.97	712.00	.01591	.040	16.48
25	754.70	755.00	.01740	.040	17.67
26	799.18	800.00	.01902	.040	18.98
27	845.65	846.00	.02077	.040	20.60
28	894.38	895.00	.02265	.040	22.30
29	945.70	946.00	.02464	.040	24.25
30	1,000.00	1,000.00	.02670	.040	....

TABLE 2

Distribution of New Business by Policy Size

$1,000G	f(35, G)
$10,000	.05
12,000	.06
14,000	.08
16,000	.13
18,000	.15
20,000	.32
25,000	.10
30,000	.05
40,000	.03
50,000	.02
75,000	.01
	1.00

$$\$1{,}000 \sum_G G \cdot f(35, G) = \$20{,}470$$

$$\$1{,}000 \sum_{G=25} (G - 20) \cdot f(35, G) = \$2{,}750$$

$$\$1{,}000 \cdot {}_{30}A = \$29{,}970 \text{ — Equation (4)}$$

$$\$1{,}000 \cdot {}_{30}A = \$16{,}850 \text{ — Equation (6)}$$

TABLE 3

CALCULATION OF BREAK-EVEN PREMIUMS

t	1,000 A_t		$_{t-1}E^{(T)}_{[35]}$	$_th_{35}$	$_tH_{35}$	$_tE^{(T)}_{[35]}\cdot {_tV_{35}}$	$_tH_{35}$ $+{_tE^{(T)}_{[35]}}\cdot {_tV_{35}}$	$_tf_{35}$	$_tF_{35}$	$_tP^{BE}_{35}$
	$A_t<K$	$A_t>K$								
1	19,948	....	1.00000	9.4611	9.4611	....	9.4611	.27000	.27000	$35.0411
2	13,406	24,205	.72381	3.1300	12.5911	13.8627	26.4538	.62971	.89971	29.4026
3	12,953	24,065	.59369	3.6580	16.2491	24.3888	40.6379	.54619	1.44590	28.1056
4	12,912	24,054	.51551	4.1604	20.4095	32.5871	52.9966	.47427	1.92017	27.6000
5	12,935	24,060	.45348	4.4570	24.8665	39.0281	63.8946	.41720	2.33737	27.3361
6	12,915	24,054	.40231	4.5049	29.3714	44.1838	73.5552	.37013	2.70750	27.1672
7	12,867	24,042	.35992	4.4081	33.7795	48.4098	82.1893	.33133	3.03883	27.0464
8	12,872	24,044	.32468	4.2972	38.0767	51.8814	89.9581	.29871	3.33754	26.9534
9	12,933	24,060	.29468	4.2497	42.3264	54.6834	97.0098	.27111	3.60865	26.8826
10	13,023	24,085	.26849	4.1306	46.4570	56.9526	103.4096	.24701	3.85566	26.8202
11	13,172	24,128	.24557	4.1192	50.5762	58.6858	109.2620	.23329	4.08895	26.7213
12	13,410	24,206	.22497	4.0784	54.6546	59.9588	114.6134	.21372	4.30267	26.6377
13	13,748	24,329	.20642	4.0247	58.6793	60.8461	119.5254	.19610	4.49877	26.5685
14	14,272	24,568	.18967	3.9658	62.6451	61.3961	124.0412	.18019	4.67896	26.5104
15	14,888	24,965	.17450	3.8900	66.5351	61.6663	128.2014	.16578	4.84474	26.4620
16	15,950	26,292	.16074	3.9576	70.4927	61.5589	132.0516	.15270	4.99744	26.4238
17	16,482	27,819	.14790	3.9601	74.4528	61.1303	135.5831	.14051	5.13795	26.3886
18	16,968	31,080	.13596	3.9415	78.3943	60.4246	138.8189	.12916	5.26711	26.3558
19	17,385	41,460	.12487	3.9042	82.2985	59.4796	141.7781	.11863	5.38574	26.3247
20	17,745	....	.11458	3.8536	86.1521	58.3197	144.4718	.10885	5.49459	26.2935
21	18,077	....	.10502	3.7858	89.9379	56.9854	146.9233	.09977	5.59436	26.2628
22	18,351	....	.09616	3.7121	93.6500	55.5026	149.1526	.09135	5.68571	26.2329
23	18,648	....	.08795	3.6268	97.2768	53.8937	151.1705	.08355	5.76926	26.2028
24	19,141	....	.08034	3.5477	100.8245	52.1732	152.9977	.07632	5.84558	26.1732
25	19,694	....	.07328	3.4644	104.2889	50.3687	154.6576	.06962	5.91520	26.1458
26	....	20,167	.06674	3.3775	107.6664	48.4942	156.1606	.06340	5.97860	26.1199
27	....	20,329	.06068	3.2831	110.9495	46.5615	157.5110	.05765	6.03625	26.0942
28	....	20,452	.05506	3.1850	114.1345	44.6027	158.7372	.05231	6.08856	26.0714
29	....	20,462	.04987	3.0809	117.2154	42.6227	159.8381	.04738	6.13594	26.0495
30	....	20,470	.04507	3.0311	120.2465	40.6400	160.8865	.04282	6.17876	26.0386
			.04064							

(2) the average size policy, A_t, for each policy year; and (3) the calculation of the break-even premiums.

The gross premiums that will yield 20 per cent to 25 per cent on surplus invested in new business are calculated as shown in Table 4.

TABLE 4

GROSS PREMIUMS THAT WILL YIELD j PER CENT ON SURPLUS INVESTED IN NEW BUSINESS

j	${}^jP'_{35}$
20%	$27.08
21	27.12
22	27.16
23	27.21
24	27.25
25	27.30

$$ {}^jP'_{35} = \frac{\sum_{t=1}^{30} \left({}_tP^{BE}_{35} \cdot {}_tF_{35} - {}_{t-1}P^{BE}_{35} \cdot {}_{t-1}F_{35} \right) \left(\frac{1.035}{1+j} \right)^{t-1}}{\sum_{t=1}^{30} {}_tf_{35} \left(\frac{1.035}{1+j} \right)^{t-1}} . $$

After a careful review a final gross premium per $1,000 of $27.17 is chosen. This should yield slightly over 22 per cent on surplus invested in new business. The insurer may expect to amortize surplus invested in new business at the end of the sixth policy year. Table 5 analyzes the expected financial results as follows:

Column (1)........ t-year profit margin.

Column (5)........ Value at issue of tth policy year's profit per $1,000 of insurance issued.

Column (6)........ Value at issue of profit earned over first t policy years per $1,000 of insurance issued.

Column (7)........ Value at beginning of policy year t of that policy year's profit per $1,000 of insurance in force at the beginning of policy year t.

Column (8)........ Value at end of policy year t of that policy year's profit per $1,000 of insurance in force at the beginning of policy year t.

Column (9)........ Value at end of policy year t of that year's profit per $1,000 of insurance issued.

Column (10)........ Profit accumulated at end of policy year t per $1,000 of insurance issued.

Column (11)........ Projected market value of insurance in force at end of

TABLE 5

FINANCIAL ANALYSIS

t	$\Pi_{35} - {}_tP_{35}^{BE}$ (1)	${}_tf_{35}$ (2)	${}_{t-1}P_{35}^{BE} - {}_tP_{35}^{BE}$ (3)	${}_{t-1}F_{35}$ (4)	${}_tW_{35}$ (5)	$(1) \cdot {}_tF_{35} = \sum_t {}_tW_{35}$ (6)	${}_tY_{35}$ (7)	${}_tZ_{35}$ (8)	${}_tU_{35}$ (9)	$(6) \cdot (1.035)^t$ (10)	${}_{t-1}(MV)_{35}^i$ (11)	${}_{t-1}(MV)_{35}^j$ (12)
1.........	$-7.8711	.27000	....	....	$-2.1252	$-2.1252	$-2.1252	$-2.1996	$-2.1996	$- 2.1996	$ 6.99	$ .01
2.........	-2.2326	.62971	$5.6385	.27000	+ .1165	-2.0087	+ .1610	+ .1666	+ .1248	- 2.1518	12.59	3.48
3.........	- .9356	.54619	1.2970	.89971	.6559	-1.3528	1.1048	1.1435	.7272	- 1.4999	15.16	4.77
4.........	- .4300	.47427	.5056	1.44590	.5271	- .8257	1.0225	1.0583	.6049	- .9475	16.18	4.97
5.........	- .1661	.41720	.2639	1.92017	.4374	- .3883	.9645	.9983	.5195	- .4612	17.24	5.29
6.........	+ .0028	.37013	.1689	2.33737	.3958	+ .0075	.9838	1.0182	.4865	+ .0092	18.34	5.75
7.........	.1236	.33133	.1208	2.70750	.3680	.3755	1.0224	1.0582	.4682	.4777	19.40	6.27
8.........	.2166	.29871	.0930	3.03883	.3473	.7228	1.0697	1.1071	.4573	.9518	20.37	6.86
9.........	.2874	.27111	.0708	3.33754	.3142	1.0370	1.0662	1.1035	.4282	1.4133	21.27	7.52
10.........	.3498	.24701	.0624	3.60865	.3116	1.3486	1.1606	1.2072	.4395	1.9023	22.17	8.36
11.........	.4487	.23329	.0989	3.85566	.4860	1.8346	1.9791	2.0484	.7095	2.6785	22.97	9.28
12.........	.5323	.21372	.0836	4.08895	.4556	2.2902	2.0252	2.0961	.6884	3.4607	22.92	9.38
13.........	.6015	.19610	.0692	4.30267	.4157	2.7059	2.0139	2.0844	.6501	4.2319	22.77	9.46
14.........	.6596	.18019	.0581	4.49877	.3802	3.0861	2.0045	2.0747	.6154	4.9954	22.59	9.54
15.........	.7080	.16578	.0484	4.67896	.3438	3.4299	1.9702	2.0392	.5760	5.7463	22.37	9.65
16.........	.7462	.15270	.0382	4.84474	.2990	3.7289	1.8601	1.9252	.5185	6.4659	22.15	9.88
17.........	.7814	.14051	.0352	4.99744	.2857	4.0146	1.9317	1.9993	.5127	7.2049	22.05	10.28
18.........	.8142	.12916	.0328	5.13795	.2737	4.2883	2.0131	2.0836	.5084	7.9655	21.89	10.65
19.........	.8453	.11863	.0311	5.26711	.2641	4.5524	2.1150	2.1890	.5077	8.7520	21.64	11.03
20.........	.8765	.10885	.0312	5.38574	.2634	4.8158	2.2988	2.3793	.5241	9.5824	21.28	11.60
21.........	.9072	.09977	.0307	5.49459	.2592	5.0750	2.4681	2.5545	.5338	10.4516	20.71	11.90
22.........	.9371	.09135	.0299	5.59436	.2529	5.3279	2.6300	2.7221	.5391	11.3565	19.92	12.23
23.........	.9672	.08355	.0301	5.68571	.2519	5.5798	2.8641	2.9643	.5557	12.3097	18.90	11.96
24.........	.9968	.07632	.0296	5.76926	.2468	5.8266	3.0719	3.1794	.5635	13.3041	17.56	12.17
25.........	1.0242	.06962	.0274	5.84558	.2315	6.0581	3.1591	3.2697	.6374	14.3167	15.88	11.64
26.........	1.0501	.06340	.0259	5.91520	.2198	6.2779	3.2934	3.4087	.9997	15.3555	13.97	10.73
27.........	1.0758	.05765	.0257	5.97860	.2157	6.4936	3.5547	3.6791	.5461	16.4390	11.74	10.25
28.........	1.0986	.05231	.0228	6.03625	.1951	6.6887	3.5434	3.6674	.5112	17.5255	9.02	8.67
29.........	1.1205	.04738	.0219	6.08856	.1864	6.8751	3.7377	3.8685	.5055	18.6444	6.05	5.80
30.........	1.1314	.04282	.0109	6.13594	.1153	6.9904	2.5582	2.6477	.3236	19.6206	2.56	2.50
				6.17876								

policy year $\overline{t-1}$($_0MV^i_{35}$ = value at issue), with future book profits discounted at 3½ per cent (i_t).

Column (12)........ Projected market value of insurance in force at end of policy year $\overline{t-1}$, with future book profits discounted at 22 per cent(j_t).*

In Table 6 the 30-year break-even premiums for the different policy sizes (see Table 2), which are calculated using the appropriate equations in subsection XI, C, are used to calculate the average 30-year break-even premium and profit margin per $1,000 of insurance issued. These are shown to be equal to the 30-year break-even premium (see Table 3) and the 30-year profit margin (see Column [1], Table 5) that are produced by the 30-year level average policy size ($16,850 or $29,970) shown in Table 2.

TABLE 6

MODEL OFFICE CALCULATION OF AVERAGE 30-YEAR PROFIT MARGIN PER $1,000 OF INSURANCE ISSUED

G (1)	$_{30}P^{BE}_{35}$ (2)	$G\cdot f(35, G)$ (3)	(2)×(3) (4)
10.........	27.0017	.50	13.5009
12.........	26.6069	.72	19.1570
14.........	26.3250	1.12	29.4840
16.........	26.1132	2.08	54.3155
18.........	25.9488	2.70	70.0618
20.........	25.8171	6.40	165.2294
25.........	25.9501	2.50	64.8753
30.........	26.0390	1.50	39.0585
40.........	26.1497	1.20	31.3796
50.........	26.2163	1.00	26.2163
75.........	26.3051	.75	19.7288
	....	20.47	533.0071

Average $_{30}P^{BE}_{35}$ = 533.0071 ÷ 20.47 = $26.0385
Average 30-year profit margin per $1,000 of insurance issued
= $27.17 − $26.0385 = $1.1315

Example 2

A—Assumptions

Policy fee........... $10.

Reinsurance........ Nonrefund select and ultimate YRT, with an annual fee of $2 the first policy year and $4 all renewal years. The YRT reinsurance rates per $1,000 of reinsurance are shown in Column (1) of Table 7.

All others.......... Same as Illustrative Example Number 1.

* j_t is really slightly higher than 22 per cent.

B—Profit Objective

A profit earned over the first 20 policy years with a value at issue of $5 per $1,000 of insurance issued.

C—Calculations

Table 7 shows the calculation of the ratebook break-even premiums

TABLE 7

CALCULATION OF RATEBOOK BREAK-EVEN PREMIUMS
($10 POLICY FEE)

t	(Select and Ultimate) r_{35+t-1} (1)	$1{,}000A_t$ (2)	${}_tH_{35}$ (3)	${}_tH_{35}+{}_tE^{(T)}_{[35]}\cdot{}_tV_{35}$ (4)	${}_tF_{35}$ (5)	${}_tP^{BE}_{35}$ (6)	$1{,}000\,{}_tA$ (7)	${}_tP^{BE/M}_{35}$ (8)
1....	$ 1.13	20,719	9.4143	9.4143	.27000	$34.8678	20,719	$34.3852
2....	1.68	16,310	12.4298	26.2925	.89971	29.2233	19,733	28.7165
3....	2.23	14,568	16.0346	40.4234	1.44590	27.9573	19,295	27.4390
4....	2.46	14,126	20.1602	52.7473	1.92017	27.4701	18,941	26.9421
5....	2.70	13,806	24.5952	63.6233	2.33737	27.2200	18,649	26.6838
6....	3.00	13,385	29.0895	73.2733	2.70750	27.0631	18,394	26.5194
7....	3.35	12,927	33.4964	81.9062	3.03883	26.9532	18,164	26.4027
8....	3.75	12,466	37.8012	89.6826	3.33754	26.8709	17,952	26.3139
9....	4.21	12,023	42.0668	96.7502	3.60865	26.8106	17,753	26.2473
10....	4.76	11,510	46.2223	103.1749	3.85566	26.7593	17,562	26.1899
11....	5.19	11,446	50.3671	109.0529	4.08895	26.6701	17,395	26.0952
12....	5.66	11,522	54.4706	114.4294	4.30267	26.5950	17,253	26.0154
13....	6.18	11,746	58.5186	119.3647	4.49877	26.5327	17,136	25.9491
14....	6.73	12,122	62.5057	123.9018	4.67896	26.4806	17,043	25.8938
15....	7.33	12,716	66.4138	128.0801	4.84474	26.4369	16,974	25.8478
16....	7.98	13,729	70.3863	131.9452	4.99744	26.4026	16,930	25.8119
17....	8.67	14,253	74.3592	135.4895	5.13795	26.3703	16,898	25.7785
18....	9.39	14,641	78.3125	138.7371	5.26711	26.3403	16,874	25.7477
19....	10.12	15,076	82.2269	141.7065	5.38574	26.3114	16,857	25.7182
20....	10.92	15,424	86.0895	144.4092	5.49459	26.2821	16,845	25.6885
21....	11.81	15,550	89.8842	146.8696	5.59436	26.2532	16,835	25.6592
22....	13.13	15,048	93.6072	149.1098	5.68571	26.2254	16,822	25.6309
23....	14.41	14,870	97.2454	151.1391	5.76926	26.1973	16,809	25.6024
24....	16.01	14,444	100.8062	152.9794	5.84558	26.1701	16,794	25.5746
25....	16.67	15,673	104.2798	154.6485	5.91520	26.1443	16,788	25.5486
26....	18.15	15,585	107.6664	156.1606	5.97860	26.1199	16,782	25.5240
27....	19.69	15,873	110.9575	157.5190	6.03625	26.0955	16,778	25.4995
28....	21.30	15,777	114.1504	158.7531	6.08856	26.0740	16,774	25.4778
29....	22.98	15,475	117.2392	159.8619	6.13594	26.0534	16,769	25.4571
30....	....	22,023	120.2644	160.9044	6.17876	26.0415	16,816	25.4468

along with the values of the calculated average policy sizes used.

A ratebook gross premium of $26.60 is solved for as follows:

$$\Pi_{35} = {}_{20}P^{BE/M}_{35} + \$5/{}_{20}F_{35} = 25.6885 + .9100 \doteq \$26.60\,.$$

Table 8, which consists of an abridged profit analysis, demonstrates that \$26.60 is the ratebook gross premium which will yield the desired 20-year profit.

Table 9 shows a model office calculation of the average 10-, 20-, and 30-year ratebook break-even premiums. These are the same ratebook premiums calculated in Table 7.

TABLE 8

ABRIDGED PROFIT ANALYSIS

t	$\Pi_{35} - {}_tP_{35}^{BE/M}$ (1)	${}_tf_{35}$ (2)	${}_{t-1}P_{35}^{BE/M} - {}_tP_{35}^{BE/M}$ (3)	${}_{t-1}F_{35}$ (4)	$(1)(2)+(3)(4) = {}_tW_{35}$ (5)	$(1)\cdot{}_tF_{35} = \sum_{t=1}(5)_t$ (6)
1....	\$−7.7852	.27000	....	....	\$−2.1020	\$−2.1020
2....	−2.1165	.62971	\$5.6687	.27000	+ .1978	−1.9042
3....	− .8390	.54619	1.2775	.89971	+ .6911	−1.2131
4....	− .3421	.47427	.4969	1.44590	+ .5562	− .6569
5....	− .0838	.41720	.2583	1.92017	+ .4610	− .1959
6....	+ .0806	.37013	.1644	2.33737	+ .4141	+ .2182
7....	+ .1973	.33133	.1167	2.70750	+ .3814	+ .5996
8....	+ .2861	.29871	.0888	3.03883	+ .3553	+ .9549
9....	+ .3527	.27111	.0666	3.33754	+ .3179	+1.2728
10....	+ .4101	.24701	.0574	3.60865	+ .3084	+1.5812
11....	+ .5048	.23329	.0947	3.85566	+ .4829	+2.0641
12....	+ .5846	.21372	.0798	4.08895	+ .4512	+2.5153
13....	+ .6509	.19610	.0663	4.30267	+ .4129	+2.9282
14....	+ .7062	.18019	.0553	4.49877	+ .3760	+3.3042
15....	+ .7522	.16578	.0460	4.67896	+ .3399	+3.6441
16....	+ .7881	.15270	.0359	4.84474	+ .2943	+3.9384
17....	+ .8215	.14051	.0334	4.99744	+ .2823	+4.2207
18....	+ .8523	.12916	.0308	5.13795	+ .2683	+4.4890
19....	+ .8818	.11863	.0295	5.26711	+ .2600	+4.7490
20....	+ .9115	.10885	.0297	5.38574	+ .2592	+5.0082
21....	+ .9408	.09977	.0293	5.49459	+ .2549	+5.2631
22....	+ .9691	.09135	.0283	5.59436	+ .2469	+5.5100
23....	+ .9976	.08355	.0285	5.68571	+ .2454	+5.7554
24....	+1.0254	.07632	.0278	5.76926	+ .2386	+5.9940
25....	+1.0514	.06962	.0260	5.84558	+ .2252	+6.2192
26....	+1.0760	.06340	.0246	5.91520	+ .2137	+6.4329
27....	+1.1005	.05765	.0245	5.97860	+ .2099	+6.6428
28....	+1.1222	.05231	.0217	6.03625	+ .1897	+6.8325
29....	+1.1429	.04738	.0207	6.08856	+ .1802	+7.0127
30....	+1.1532	.04282	.0103	6.13594	+ .1126	+7.1253
				6.17876		

TABLE 9

MODEL OFFICE CALCULATION OF AVERAGE 10-, 20-, AND 30-YEAR RATEBOOK BREAK-EVEN PREMIUMS PER $1,000

G (1)	${}_{10}P_{35}^{BE}$* (2)	${}_{10}P_{35}^{BE/M}$ (3)	${}_{20}P_{35}^{BE}$* (4)	${}_{20}P_{35}^{BE/M}$ (5)	${}_{30}P_{35}^{BE}$* (6)	${}_{30}P_{35}^{BE/M}$ (7)	$G\cdot f(35,G)$ (8)
10..	$28.1262	$27.1262	$27.3038	$26.3038	$27.0018	$26.0018	.50
12..	27.5971	26.7638	26.8848	26.0515	26.6070	25.7737	.72
14..	27.2192	26.5049	26.5854	25.8711	26.3249	25.6106	1.12
16..	26.9358	26.3108	26.3609	25.7359	26.1134	25.4884	2.08
18..	26.7153	26.1597	26.1863	25.6307	25.9488	25.3932	2.70
20..	26.5389	26.0389	26.0466	25.5466	25.8172	25.3172	6.40
25..	26.5972	26.1972	26.1432	25.7432	25.9061	25.5061	2.50
30..	26.4826	26.1493	26.0580	25.7247	25.8179	25.4846	1.50
40..	26.3393	26.0893	25.9515	25.7015	25.7076	25.4576	1.20
50..	26.2533	26.0533	25.8876	25.6876	25.6414	25.4414	1.00
75..	26.1387	26.0054	25.8024	25.6691	25.5531	25.4198	.75
							20.47

$$\text{Average } {}_{10}P_{35}^{BE/M} = \frac{\Sigma(3)(8)}{\Sigma(8)} = \frac{536.1066}{20.47} = 26.1899$$

$$\text{Average } {}_{20}P_{35}^{BE/M} = \frac{\Sigma(5)(8)}{\Sigma(8)} = \frac{525.8427}{20.47} = 25.6885$$

$$\text{Average } {}_{30}P_{35}^{BE/M} = \frac{\Sigma(7)(8)}{\Sigma(8)} = \frac{520.8969}{20.47} = 25.4468$$

* Calculated using formulas from Subsection XI, *C*.

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DISCUSSION OF PRECEDING PAPER

JAMES E. HOSKINS:

Mr. Stein has made a valuable addition to the literature of nonparticipating premium calculation. His treatment of the subject is both ingenious and thorough. His method of obtaining the incidence of profit arising from a given gross premium is especially interesting.

He also introduces the concept of an average policy size which takes into account the cost of reinsurance. This procedure emphasizes the point that any excess of net reinsurance premiums over the assumed mortality cost of the amount reinsured adds to the expense of policies whose amounts exceed the retention limit. It may even cause the total expense per \$1,000 to increase with size after the retention limit is passed, as is suggested by Mr. Stein's Table 6. The procedure, however, increases the work of premium calculation. The method presented in the paper requires the calculation of this nonconventional average size for the successive periods from issue to the end of each policy year and then for each individual year. In each case there are two possible formulas, and, if calculation by one of these formulas does not produce a meaningful result, then the average size must be recalculated by the other formula. It seems to me that these steps can be avoided in the calculation of his fundamental function ${}_th_x$ which represents the net outgo per \$1,000 of insurance in the year t. I suggest multiplying the terms representing the reinsurance cost by the proportion of issues assumed to be reinsured, by amount and by number, respectively, and then dividing by the conventional average size those expenses which are "per policy." Under the assumption which Mr. Stein uses—that mortality and persistency are independent of policy size—these proportions reinsured are constant for all durations.

Specifically, in Mr. Stein's definition 43, e'_t/A would be substituted for e'_t/A_t and

$$\frac{\sum_{G=K} f(x,G)(G-K)}{\sum_{G=1} f(x,G)\cdot G}\cdot {}^{A}R^{1}_{x+t-1}+\left(\frac{\sum_{G=K} f(x,G)}{\sum_{G=1} f(x,G)}\cdot {}^{B}R^{1}_{x+t-1}\right)\div A$$

$$=\frac{\sum_{G=K} f(x,G)(G-K)\cdot {}^{A}R^{1}_{x+t-1}+\sum_{G=K} f(x,G)\cdot {}^{B}R^{1}_{x+t-1}}{A}$$

for

$$\frac{A_t - K^*}{A_t}\left({}^{A}R^1_{x+t-1} + \frac{{}^{B}R^1_{x+t-1}}{A_t - K}\right)$$

in the definition of ${}_th_x$.

This alternative procedure merely introduces into the total outgo of the issues of a given age and plan an expense arising from only a part of those issues, just as is done with, say, the cost of paying cash surrender values. The alternative appears to produce the same result as Mr. Stein's procedure and to retain all the advantages of his method.

The procedure in event of a change in average size would be analogous to that given by Mr. Stein, except that a single formula would replace formulas (22)–(25). The formula for the change in break-even premium on account of a change in retention limit would be analogous to Mr. Stein's formula (21) for a change in "per policy" expenses.

CECIL J. NESBITT:

Mr. Stein has pushed forward the analysis and formulation of gross nonparticipating premiums in several ways. For one thing, he has given a careful analysis of reinsurance costs and brought them into his premium formulas. Also, he has given some indication of the depth of analysis that is possible with modern computer means and would hardly be feasible otherwise. There are other interesting features, such as his formulas (14) for the gross premium by Anderson's principle requiring a predetermined yield on surplus invested in new business; also, his formulas for incidence of profit and for the variation of assumptions.

The author's formula (16A) for the value at issue of the tth policy year's profit per \$1,000 issued, namely,

$${}_tW_x = (\Pi_x - {}_tP_x^{BE/M})\,{}_tf_x + ({}_{t-1}P_x^{BE/M} - {}_tP_x^{BE/M})\,{}_{t-1}F_x\,,$$

may be written in the alternative forms

$${}_tW_x = (\Pi_x\,{}_tf_x - ({}_tP_x^{BE/M}\,{}_tF_x - {}_{t-1}P_x^{BE/M}\,{}_{t-1}F_x)\ ; \qquad (a)$$

$${}_tW_x = (\Pi_x - {}_tP_x^{BE/M})_t\,F_x - (\Pi_x - {}_{t-1}P_x^{BE/M})_{t-1}\,F_x\ ; \qquad (b)$$

$${}_tW_x = (\Pi_x - {}_{t-1}P_x^{BE/M})\,{}_tf_x + ({}_{t-1}\,P_x^{BE/M} - {}_tP_x^{BE/M})\,{}_tF_x\ ; \qquad (c)$$

each of which has an interpretation. Thus the author's formula exhibits ${}_tW_x$ as the discounted value of the premium margin in the tth year plus the value over $t-1$ years of the saving obtained by spreading costs over t years rather than $t-1$. Formula (a) expresses ${}_tW_x$ as the present value of the tth year gross premium less the present value of operating the insurance from the end of the $(t-1)$th to the end of the tth year. Formula (b) gives ${}_tW_x$ as the discounted value of premium margins on a t-year basis over the corresponding value on a $(t-1)$-year basis. Finally, formula (c) is somewhat similar to the author's but uses the premium margin based on the $(t-1)$-year break-even premium so that the second term includes saving over t years.

In order to take into account the financial effect of the cost of reinsurance, and to get the exact incidence of profit, the author introduces complex level average policy sizes ${}_tA$ and A_t. The use of these average policy sizes is to lead to the same average profit results that would be produced if the policy were issued with the usual distribution of policy sizes. If the policy fee is zero and if there is no reinsurance, then

$${}_tA = A_t = \sum_G G\cdot f(x, G),$$

that is, ${}_tA$ and A_t are constant and equal to the average size at issue; however, under other circumstances ${}_tA$ and A_t may vary with t.

The question arises whether the author's objectives can be obtained without the use of such complex averages as ${}_tA$ and A_t. Making free use of the computer, one might make independent calculations of such quantities as ${}^G_tP^{BE}_x$, ${}^G_tP^{BE/M}_x$, ${}^G_NP'_x$, ${}^G_tW_x$, where the left superscript indicates that these are for a policy of size G. After computations are completed for the various values of G, then corresponding weighted averaged quantities would be obtained by such formulas as

$${}_tP^{BE/M}_x = \frac{\sum_G G\cdot f(x, G)\,{}^G_tP^{BE/M}_x}{\sum_G G\cdot f(x, G)} \qquad (d)$$

and

$${}_tW_x = \frac{\sum_G G\cdot f(x, G)\,{}^G_tW_x}{\sum_G G\cdot f(x, G)}. \qquad (e)$$

To my mind, this procedure with its multiple formulas and tabulations would be simpler, more flexible, and more illuminating than the composite formulas and tabulations proposed by the author, utilizing the average policy sizes ${}_tA$ and A_t. Also, the results might be a better realization of the actual operation of the policy than would be yielded by the author's method. It is quite possible that the procedure that I have indicated would yield much the same results as would be obtained by the formulas of the paper, but I would be surprised if the results were *exactly* the same. Perhaps Mr. Stein would be willing to explore in relation to one of his illustrative examples the procedure that I have suggested and compare it with what he has shown in his paper.

As the factors $f(x, G)$ come in only at the end in the procedure I have suggested, it would be very easy to use various assumptions as to the distribution of policy sizes. On the other hand, this procedure probably entails more extensive computation and certainly more tabulation than the author's method.

The foregoing has bearing on Tables 6 and 9 of the paper. In these tables, the author shows break-even premiums for various assumed policy sizes. However, such break-even premiums are not calculated *independently* but are obtained by corrections to the previously calculated break-even premium based on the author's definitions of average policy size. Thus, for size G, the break-even premium is given as

$$_{N}P_x^{BE} + \Delta_N^G P_x^{BE} - B/G\,, \qquad (f)$$

where $\Delta_N^G P_x^{BE}$ is the change in the break-even premium when the average size changes from $_NA$ to G.

It may be shown that the weighted average

$$\frac{\sum_G G\cdot f(x, G)\left(\Delta_N^G P_x^{BE} - \frac{B}{G}\right)}{\sum_G G\cdot f(x, G)}$$

is $-B/_NA$ by reason of the author's definition of $_NA$, and hence the weighted average of the formula (f) is simply $_NP_x^{BE} - B/_NA = {_NP_x^{BE/M}}$. In other words, by definition of the break-even premiums for size G appearing in Tables 6 and 9, it was certain that they would average out to the values previously obtained. What is uncertain is how closely premiums calculated by formula (f) would come to break-even premiums $_N^GP_x^{BE/M}$ calculated independently for the various policy sizes G.

Mr. Stein's paper was of special interest to us in the actuarial mathematics seminar at Michigan, not only because he was a former student but because of the formulations he has achieved. His paper should prove to be a valuable contribution to the literature on gross premiums.

HARWOOD ROSSER:

One reaction to Mr. Stein's excellent paper is gratification that, unlike the Education and Examination Committee, he included me in a list of contributors to this general subject. My 1951 paper is the only one in his list that touches on participating business also. This may be a factor in the Committee's attitude—a paper that discusses two syllabus subjects could be confusing. Mr. Stein is not guilty on this score, and his paper may well appear on some future required-reading list. Thus I advise him to order some extra reprints.

If it does so appear, I hope that it will be accompanied by the suggestion that not every company will wish to explore all his theoretical refinements, such as those for reinsurance cost and average policy size. One could wish that he had illustrated the numerical effect of some of the variations in the latter. Perhaps he will do us this additional favor in his reply.

He makes quite a point of his belief that a trial gross premium is mandatory in the approach used by Jim Anderson and by me but unnecessary in his. Actually, it could be dispensed with in mine, at least, and a "direct" approach used.

If the calculation in my Table 1 is made with a zero gross premium and with corresponding adjustments to the expenses, the end product, with sign reversed, will represent the discounted value of all outlays, including setting up the cash value, except those expenses that are a percentage of the ratebook premium. This will be equivalent to the numerator of Mr. Stein's formula (9), without some of the refinements mentioned above. His ${}_tF_x$ would seem to be identical with the final figure in my Table 8, labeled "Effect on Profit of $1 Change in Gross Premium." Division by this produces a break-even premium, as in his formula (9).

While my illustration is keyed to twenty years, the t-year break-even premiums for all previous years could readily be obtained also, if desired. If column 11 of my Table 1, modified as above, and the final column of my Table 8 are both summed downward, division of the first by the second, line by line, and with a sign change, would give a column corresponding to the final one in Mr. Stein's Table 3, except for his use therein of his formula (10) rather than his formula (9).

When profit test calculations were done by clerks with desk computers, however, there were collateral advantages to using a trial premium fairly close to the final one. For instance, it gave some indication as to the duration at which a profit first emerged. Mr. Stein's Table 5 does this also, but at the expense of considerably more calculation. This was important fifteen years ago. The lightness with which the author tosses formulas around is one indication of how far we have come since then.

Finally, I welcome another avowed member to the minority group that believes, especially in this electronic age, that asset shares can be bypassed and that we can proceed directly to the question of the profitability of the premium.

One is tempted to speculate that yet another paper on gross premiums —this one—will be a strong contender for a Triennial Prize.

FRANK P. DIPAOLO:

Mr. Stein has made an outstanding contribution to the science of gross premium calculation. It may be that some of us will not share the author's opinion on a number of basic concepts, but, whatever disagreement there may be, we must recognize the usefulness of the techniques set forth in this excellent paper.

The author seems to be overly preoccupied with profit objectives and profit margins to the point of meticulously measuring the latter according to five different definitions. Yet he does not mention two factors that have a significant bearing on the emergence of profit—mortality fluctuations and income tax. Mr. Anderson, in his paper on gross premiums, suggested a contingency margin of 25 deaths per 1,000 per century. While the idea of including an allowance for mortality swings in the mortality rate itself is commendable, it cannot be said that a margin of 0.00025 deaths will suit the contingency need of every company. The magnitude of the margin will depend on the size and claim distribution of the net direct portfolio as well as the size of the risk reserve. And, of course, if there is profit, there will be one of the proverbial certainties of life—taxes.

This punctilious concern with profit leads the author to a paradoxical treatment of reinsurance costs. Mr. Stein defines the cost of reinsurance as reinsurance premiums plus ceding company's administrative costs of handling the reinsurance less claims and other refunds paid by the reinsurer. This cost, with due regard to the probability of $1,000 being reinsured out, is priced into the gross premium so that every $1,000 of insurance issued, regardless of size, will generate exactly the same profit over the gross premium calculation period. But what happens if the company is requested to issue amounts greatly in excess of the largest amount now in the portfolio? It seems to me that the company must decline such amounts unless it wishes to disregard its previously set profit objectives. This is tantamount to imposing an issue limit which may well be either too low or not necessary at all.

Apart from the various services provided by the reinsurer, a ceding company will seek reinsurance mainly to protect its own surplus against mortality fluctuations. Through reinsurance the ceding company acquires, for a price, the right to use the reinsurer's surplus in the event of adverse mortality and thus avoid ruin. If the price is convenient, why shouldn't the ceding company acquire unlimited right to the reinsurer's surplus? For example, if we use Ormsby's "out of pocket" definition, and the same YRT rates and other costs assumed by Mr. Stein in his two examples, the two rate book premiums of $27.17 and $26.60 produce negative reinsurance costs that have a present value at issue of $5.81 and $5.31 per $1,000 before income tax. With such a profit on reinsured business, why shouldn't the ceding company free itself of issue limit restrictions and let its agency force loose after small as well as big policyholders?

The adoption of the "out of pocket" concept would involve a recalculation of the "break-even premium." The expense rates per $1,000 would slightly increase since the administrative expenses would now be allocated entirely to the retained business. However, this increase would be partially offset by the elimination of the reinsurance cost from the gross premium calculation.

Mr. Stein and, to a certain extent, Mr. Anderson distinguish between (*a*) the science of gross premium calculation and (*b*) the art or philosophy of rate-making. I would rather think of both as a science—better, I would rather not dichotomize. I find it difficult to visualize the insurance rate-maker holding hands with Thalia, the muse of idyllic poetry; rather, I see him as a cool-minded scientist who uses basic linear programming techniques to minimize competition and maximize profit.

Let $f(\Pi_r)$ be the new business we may expect from a gross premium structure, the average of which (on a given distribution by plan, size, and age) is r deciles from the median average structure (based on the same distribution) used by competitors in a geographically well-defined market. If $\Delta\Pi_r$ is the average gross premium reduction necessary to move the structure up s deciles, which may result in a new business increase of $\Delta f(\Pi_r)$, then the total profit will be greater than or equal to that generated by $f(\Pi_r)$, if

$$\Delta f(\Pi_r)\sum_{t=1}^{n} {}_tW-[f(\Pi_r)+\Delta f(\Pi_r)]\Delta\Pi_{rn}F\geq 0\,,$$

where ${}_nF$ and ${}_tW$ are as defined in Sections II and IX but for the entire gross premium structure rather than for one specific plan and age. It seems to me that the above inequality leads clearly to the conclusion that rate-making is indeed a science and cannot be dichotomized from the science of gross premium calculation.

I realize that a more competitive premium structure alone is not sufficient to guarantee a larger share of the market. Other factors, such as agency expansion, improved field training techniques, and more effective advertising, also play a determining role. However, in the case of a small stock company like the one envisaged by Mr. Stein, a competitive premium structure is of paramount importance.

FRANKLIN B. DANA:

I am sure that all of us who are concerned with the calculation of nonparticipating gross premiums will want to re-examine our procedures in the light of Mr. Stein's excellent paper. Although the paper states that basic actuarial theory has been emphasized as opposed to the art or philosophy of rate-making, those of us concerned with rate-making will be interested in whatever practical applications the paper may have.

An area in our present procedures which we will want to re-examine is the extent to which we should refine our estimates of the financial effect of the various elements entering into the calculation. How fully should we subdivide? As an example, the paper goes into great detail in calculating the cost of reinsurance and the average policy size. While we are in complete agreement that the treatment given in the paper is theoretically desirable, it has been our experience that few except perhaps the very largest companies would have sufficient data in these areas to justify working them into the gross premium calculation as elaborately as Mr. Stein suggests. While theoretically the analysis of the various items should be complete and precise, practically we wonder whether the end result is actually improved by trying to estimate and project reliable bases for all the 44 items mentioned in the "Definition of Symbols."

Also, there should no longer be a problem of solving for the equality in the classic Type 1 or Type 2 formulas. A small computer (1401 or H-200) can develop a gross premium in a trial asset share calculation, for a number of sets of conditions, in less than a minute per issue age for a given plan—you might want a 5 per cent profit after 20 years but at least a 1 per cent profit in each year after the third. The "solve-for" premium will satisfy only a single set of conditions (e.g., the 5 per cent profit after 20 years).

The "computer asset share" approach is also directly applicable to participating gross premiums. However, since the dividend is a function of the gross premium, a derivation by the "solve-for" technique would be extremely complex.

THOMAS P. BLEAKNEY:

The modern student must be both pleased by the precision of the technique and overwhelmed by the detail that presently goes into the calculation of nonparticipating gross premiums, particularly when he compares, for example, this paper with some of the earlier papers on nonparticipating gross premiums. Mr. Stein's devotion to the problem is apparent, and his paper presents the student with many ideas worthy of pursuit.

The author's opening remarks set the pace for the paper: a Type 1, or explicit, method of developing gross premiums is presented in contrast to Type 2, or implicit, methods which have been presented in previous papers by Messrs. Hoskins, Rosser, and Anderson. The advantages Mr. Stein lists in favor of his explicit formulas are impressive. But, as one who is generally partisan by nature, I would like to cast my vote for the implicit method, acknowledging at the same time that the difference between the two methods is of small importance once the concept of present value at issue is adopted.

The relationship between the two methods is expressed in probably its simplest form in formula (12C):

$$\sum_t {}_tW_{35} = (\Pi_X - {}_NP_X^{BE/M})\,{}_NF_X\,.$$

If a profit margin approach has been used in the original calculations, using a test premium Π_X, the break-even premium can be derived from easy manipulation of formula (12C). This relationship is illustrated in a comparison of the last column of Table 3 with column 6 of Table 5, the former displaying the break-even premiums and the latter the values at issue of profit earned over the first t policy years per $1,000 of insurance issued. The Table 5 column gives the actuary measures of both the initial surplus investment and the ultimate profitability of the policy, items not readily available from the column of break-even premiums. If he also has available a table of ${}_tF_x$, the next to the last column of Table 3, he can readily determine the effect on profits of any change from Π_X, the tested gross premium.

The ability to measure the effect of a change in premium from a given one, whether that be an arbitrarily assumed test premium or one computed by means of Mr. Stein's technique, seems quite important to me. If such were not available, pity the poor actuary who has determined the premium, particularly if his calculations are by hand, only to find the premium is not within the range of competition. Even more important in some instances, however, can be an analysis of the elements making up the calculation, similar to the analysis found in Table 3. This analysis can break the final result down into components of premium, expense, mortality, reinsurance, and so forth. But here again, an analysis of a profit margin calculation seems easier to interpret and pass along to management than an analysis of a break-even premium calculation. Moreover, any changes in the output resulting from changes in assumptions (such as changes in the commission schedule) can be inferred directly from a profit margin analysis. Such a change should be simpler to understand, and simpler to explain to management, than a change in a portion of a formula designed to yield a calculated break-even premium.

Table A is an illustration of the application of the profit margin technique to the author's sample data. All columns except that entitled "Premiums" are taken directly from the paper, but the rearrangement emphasizes the additive nature of the profit margin. In practice, a further breakdown of the ${}_tH_{35}$ column would be desirable for the reasons mentioned earlier.

All of this is primarily a quibble, of course, since the actuary has the same data available using either technique, the choice being merely a matter of preference. Either technique could give recognition to the author's interesting treatment of reinsurance, particularly of the relationship of the average policy size to the retention limit. An alternative and, to my mind, preferable procedure would be the calculation of a series of studies at each age, one for each of several pivotal policy sizes. This technique not only would point out the possible low spots in profitability, thus aiding in minimum size decisions, for example, but also would seem essential when the company is using a policy fee or band system for its premium structure. Since the basic calculations are carried out by computer anyway, the problem of getting several runs at each age is generally minimal.

The use of computers for actuarial calculations, although it has reached a substantial level, undoubtedly has a great distance to go. One possible usage is brought to mind by Mr. Stein's paper. His formulas might be used to replace the conventional loading formulas for calculation of gross premiums on an age-by-age basis, once the basic assumptions and profit goals had been resolved by use of either an explicit or implicit method. Such a technique would have the built-in advantage of following the desired premiums exactly at all test points rather than requiring the compromises of fit so often faced in the conventional design of a simple loading formula.

Even if this last suggestion might be considered a practical application, and this is a moot point, my reaction is that the actuary faced with the problem of setting a premium structure will find this paper wanting and will turn to Mr. Anderson's paper for practical guidance. Nevertheless, Mr. Stein's work should be well received by the student wishing a com-

TABLE A

PROFIT MARGIN ANALYSIS

Year	Premiums	Expenses, Benefits, Reinsurance	Reserves	Profit Margin	Effect of $1 Premium Change
	$_tF_{35}$	$_tH_{35}$	$_tE^{(T)}_{[35]}\cdot {}_tV_{35}$	$\sum_t {}_tW_{35}$	$_tF_{35}$
	(1)	(2)	(3)	(4) = (1) − (2) − (3)	(5)
1	7.3359	9.4611	....	$−2.1252	0.27000
2	24.4451	12.5911	13.8627	−2.0087	0.89971
3	39.2851	16.2491	24.3888	−1.3528	1.44590
4	52.1709	20.4095	32.5871	−0.8257	1.92017
5	63.5063	24.8665	39.0281	−0.3883	2.33737
6	73.5627	29.3714	44.1838	0.0075	2.70750
7	82.5648	33.7795	48.4098	0.3755	3.03883
8	90.6809	38.0767	51.8814	0.7228	3.33754
9	98.0468	42.3264	54.6834	1.0370	3.60865
10	104.7582	46.4570	56.9526	1.3486	3.85566
11	111.0966	50.5762	58.6858	1.8346	4.08895
12	116.9036	54.6546	59.9588	2.2902	4.30267
13	122.2313	58.6793	60.8461	2.7059	4.49877
14	127.1273	62.6451	61.3961	3.0861	4.67896
15	131.6313	66.5351	61.6663	3.4299	4.84474
16	135.7805	70.4927	61.5589	3.7289	4.99744
17	139.5977	74.4528	61.1303	4.0146	5.13795
18	143.1072	78.3943	60.4246	4.2883	5.26711
19	146.3305	82.2985	59.4796	4.5524	5.38574
20	149.2876	86.1521	58.3197	4.8158	5.49459
21	151.9983	89.9379	56.9854	5.0750	5.59436
22	154.4805	93.6500	55.5026	5.3279	5.68571
23	156.7503	97.2768	53.8937	5.5798	5.76926
24	158.8243	100.8245	52.1732	5.8266	5.84558
25	160.7157	104.2889	50.3687	6.0581	5.91520
26	162.4385	107.6664	48.4942	6.2779	5.97860
27	164.0046	110.9495	46.5615	6.4936	6.03625
28	165.4259	114.1345	44.6027	6.6887	6.08856
29	166.7132	117.2154	42.6227	6.8751	6.13594
30	167.8769	120.2465	40.6400	6.9904	6.17876

prehensive study of the elements of nonparticipating gross premium calculation.

(AUTHOR'S REVIEW OF DISCUSSION)

MEL STEIN:

I am gratified by the amount of discussion my paper has received. A number of the discussions questioned the desirability of the laborious average policy size calculations. Mr. Hoskins, however, deftly duplicated the much shorter, practical alternative to these calculations that I believe should be used when calculating gross premiums. Table 10 shows the calculation of the first ten years' break-even premiums of Example 2 using the alternative formulas shown by Mr. Hoskins.

Table 11 shows the independent calculation of the first ten years' break-even premiums for a \$10,000 policy. The ten-year break-even and ratebook break-even premiums are seen to be *exactly* the same as those calculated in Table 9 by means of Dr. Nesbitt's formula (*f*). What I believe Dr. Nesbitt is pointing out is that where there is a policy fee the average policy size formulas produce artificial break-even premiums in order to arrive at the correct *ratebook* break-even premiums. This can be seen by comparing columns 6 and 8 of Table 7 with columns 10 and 11 of Table 10. If ${}^A_t/P^{BE}_x$ represents the artificial break-even premiums shown in column 6 of Table 7 and ${}_tP^{BE}_x$ the correct break-even premiums shown in column 10 of Table 10, the following equation will illustrate the relationship between ${}^A_tP^{BE}_x$, ${}_tP^{BE}_x$, and ${}_tP^{BE/M}_x$:

$${}^A_tP^{BE}_x - B/{}_tA = {}_tP^{BE/M}_x = {}_tP^{BE}_x - B/A \, .$$

It should be noted that, when there is a policy fee, only the *ratebook* break-even premium is of importance.

I would like to thank Mr. Rosser for his flattering comments and the illuminating demonstration of how his gross premium calculation method may be modified to produce break-even premiums and avoid trial gross premiums. I would also like to acknowledge the extreme value of the present value concept Mr. Rosser introduced to the gross premium calculation in his fine paper.

Mr. DiPaolo is correct in his observation that I left out the treatment of federal income taxes and mortality fluctuations. The treatment of federal income taxes and contingency margins is covered in Mr. Anderson's paper and in the Part 9-I study notes. As I had no major innovation to add, I did my best to avoid duplicating previously published material.

Mr. Anderson did not propose that *every* company use a contingency margin of *exactly* 0.00025. This figure was but an example, perhaps even the figure that Mr. Anderson felt was appropriate for his own company.

The purpose of measuring profit according to six (not five) definitions was to show the basic mathematical relationship between them and how to convert any one into the other five. I was hardly advocating that *all six* be used to measure the profitability of any one set of gross premiums.

Mr. DiPaolo seems to have misunderstood my "paradoxical" treatment of reinsurance costs. He evidently gave the same significance to the \$75,000 figure in Table 2 that he did to the 0.00025 in Mr. Anderson's

TABLE 10

CALCULATION OF BREAK-EVEN PREMIUMS BY ALTERNATIVE METHOD NOT INVOLVING AVERAGE POLICY SIZE CALCULATIONS

e_t (1)	$e'_t/20.47$ (2)	$0.13434 \cdot {}^1R^A_{35+t-1}$ (3)	$0.010259 \cdot {}^1R^B_{35+t-1}$ (4)	t	$1{,}000 \cdot v^{1/2} \cdot q^d_{[35]+t-1}$ (5)	$v \cdot q^W_{[35]+t-1} \cdot {}_tCV_{35}$ (6)	${}_th_{35}$ (7)	${}_tH_{35}$ (8)	${}_tH_{35} + {}_tV_{35} \cdot {}_tE^{(T)}_{[35]}$ (9)	${}_tP^{BE}_{35}$ (10)	${}_tP^{BE/M}_{35}$ (11)
$4.50	$4.0204	$0.0396	$0.0205	1	$0.8355	....	$9.4160	$ 9.4160	$ 9.4160	$34.8741	$34.3856
0.50	0.5819	.0824	.0410	2	1.0518	$ 1.8841	2.9974	12.4134	26.2761	29.2051	28.7166
0.50	0.5117	.1218	.0410	3	1.2778	3.5749	3.5783	15.9917	40.3805	27.9276	27.4391
0.50	0.4954	.1180	.0410	4	1.5137	5.2712	4.0928	20.0845	52.6716	27.4307	26.9422
0.50	0.4848	.1140	.0410	5	1.7595	6.8015	4.3991	24.4836	63.5117	27.1723	26.6838
0.50	0.4743	.1160	.0410	6	2.0150	7.9310	4.4565	28.9401	73.1239	27.0079	26.5194
0.50	0.4637	.1210	.0410	7	2.2903	8.7194	4.3678	33.3079	81.7177	26.8912	26.4027
0.50	0.4562	.1277	.0410	8	2.5950	9.4147	4.2645	37.5724	89.4538	26.8023	26.3138
0.50	0.4515	.1370	.0410	9	2.9292	10.2775	4.2246	41.7970	96.4804	26.7359	26.2474
0.50	0.4469	0.1493	0.0410	10	3.3126	10.8698	4.1132	45.9102	102.8628	26.6784	26.1899

$$\frac{\sum_{G=25} f(35, G)}{A} = \frac{0.21}{20.47} = 0.010259 . \qquad \frac{\sum_{G=25} f(35, G) \cdot G}{A} = \frac{2.75}{20.47} = 0.13434 .$$

TABLE 11

INDEPENDENT CALCULATION OF BREAK-EVEN PREMIUMS FOR $10,000 POLICY

e_t (1)	$e'_t/10$ (2)	$1{,}000 \cdot v^{1/2} \cdot q^d_{[35]+t-1}$ (3)	$v \cdot q^W_{[35]+t-1} \cdot {}_tCV_{35}$ (4)	t	${}_th_{35}$ (5)	${}_tH_{35}$ (6)	${}_tH_{35} + {}_tV_{35} \cdot {}_tE_{[35]}$ (7)	${}_tP^{BE}_{35}$ (8)	${}_tP^{BE/M}_{35}$ (9)
$4.50	$8.2297	$0.8355	....	1	$13.5652	$13.5652	$ 13.5652	$50.2415	$49.2415
0.50	1.1911	1.0518	$ 1.8841	2	3.3491	16.9143	30.7770	34.2076	33.2076
0.50	1.0475	1.2778	3.5749	3	3.7997	20.7140	45.1028	31.1936	30.1936
0.50	1.0142	1.5137	5.2712	4	4.2783	24.9923	57.5794	29.9866	28.9866
0.50	0.9925	1.7595	6.8015	5	4.5591	29.5514	68.5795	29.3405	28.3405
0.50	0.9708	2.0150	7.9310	6	4.5931	34.1445	78.3283	28.9301	27.9301
0.50	0.9493	2.2903	8.7194	7	4.4842	38.6287	87.0385	28.6421	27.6421
0.50	0.9338	2.5950	9.4147	8	4.3648	42.9935	94.8749	28.4266	27.4266
0.50	0.9242	2.9292	10.2775	9	4.3114	47.3049	101.9883	28.2622	27.2622
0.50	0.9148	3.3126	10.8698	10	4.1877	51.4926	108.4452	28.1262	27.1262

paper. That $75,000 is the largest policy size used in the distribution of business by policy size is purely chance. No significance should be attached to this figure or any other used in the two illustrative examples. The *only* purpose of these examples was to give a numerical demonstration of the formulas presented in the paper.

Nowhere in my paper did I suggest that a company decline to issue policies for amounts greater than the largest policy size used in the gross premium calculations (e.g., Table 2). Mr. DiPaolo says, "It seems to me that the company must decline such amounts unless it wishes to disregard its previously set profit objectives." This is like saying that the company should stop using electricity and use candles when its electricity bill through November exceeds the amount budgeted for the calendar year. The maximum amount of insurance a company will issue is determined by its limit of issue (the determination of which is outside the scope of my paper and is covered in actuarial literature).

I have always thought that gross premium calculations were based on many assumptions, approximations, and, when necessary, educated guesses. It has also been my understanding that projected profits are *never exactly duplicated* by actual operating results, no matter *what* a company may do.

Mr. DiPaolo then says:

> This cost (of reinsurance), with due regard to the probability of $1,000 being insured out, is priced into the gross premium calculation so that every $1,000 of insurance issued, regardless of size, will generate exactly the same profit over the gross premium calculation period.

This is not quite correct. The treatment of reinsurance costs and the average policy size concept include the total reinsurance cost in the gross premium calculation and avoid the overstatement of profit that results from using the traditional definition of the average policy size. This results in determining a *ratebook* gross premium which will, all assumptions being realized, yield a specified *average* profit (however defined) per $1,000 of insurance issued.

Unfortunately, Mr. DiPaolo's figures of −$5.81 and —$5.31 for the negative "out of pocket" reinsurance costs for each $1,000 issued in excess of $75,000 should instead be −$4.25 and −$7.56. These figures may be calculated and checked as follows:

Illustrative Example 1 (from Table 6)

$$\frac{[30(27.17-{}^{30}_{30}P^{BE/M}_{35})-75(27.17-{}^{75}_{30}P^{BE/M}_{35})]}{75-30}\,{}_{30}F_{35}$$

$$=\frac{[30(1.1310)-75(0.8649)]6.17876}{45}=-4.25\,.$$

$$\frac{[20(27.17 - {}^{20}_{30}P^{BE/M}_{35}) - 25(27.17 - {}^{25}_{30}P^{BE/M}_{35})]_{30}F_{35}}{25 - 20}$$

$$= \frac{[20(1.3529) - 25(1.2199)]6.17876}{5} = -4.25 .$$

Illustrative Example 2 (from Table 9)

$$\frac{[30(26.60 - {}^{30}_{30}P^{BE/M}_{35}) - 50(26.60 - {}^{50}_{30}P^{BE/M}_{35})]_{30}F_{35}}{50 - 30}$$

$$= \frac{[30(1.1154) - 50(1.1586)]6.17876}{20} = -7.56 .$$

$$\frac{[50(26.60 - {}^{50}_{30}P^{BE/M}_{35}) - 75(26.60 - {}^{75}_{30}P^{BE/M}_{35})]_{30}F_{35}}{75 - 50}$$

$$= \frac{[50(1.1586) - 75(1.1802)]6.17876}{25} = -7.56 .$$

Mr. DiPaolo's thoughts that gross premium calculations and rate-making are both pure science without an element of "art" are most interesting. I hope he will still feel that way if he ever becomes an actuary for a small insurance company (particularly if it issues accident and health, as well as life, insurance).

His thoughts on the application of linear programming to gross premium calculations and his illustrative equation are quite intriguing. While the equation is algebraically correct and seems simple enough to apply, Mr. DiPaolo has neglected to show how $\Delta\Pi_r$ and $\Delta f(\Pi_r)$ can be accurately determined. While a large company may be able to develop reasonably accurate values for these parameters, I have grave doubts whether this is also true of a small company (even if it had the personnel, IBM equipment, time, and money). Certain volatile variables (e.g., type of agency organization, rate of agency expansion, quality and training of agency personnel, turnover of agents, amount of brokerage business, etc.) would seem to make $\Delta\Pi_r$ and $\Delta f(\Pi_r)$ among the most uncertain variables of all.

Mr. Dana quite reasonably questioned whether it is practical to include the cost of reinsurance and average policy size calculations in the gross premium calculation. Mr. Hoskins showed the very practical alternative to the lengthy theoretical average policy size calculations. In all companies except a very new one there should be adequate data on which to forecast a distribution of business by policy size. An experienced actuary can, even in the case of the new company, call upon past experience to make a reasonable (as reasonable as, say, expense assumptions) forecast. To handle reinsurance costs as generally suggested in my paper, it is only necessary to determine the per cent of insurance and policies reinsured (from the distribution of business by policy size) and to estimate the experience refund. The reinsurer can usually give the actuary a fairly good picture of probable future experience refunds. With a small computer like the 1401, this can be handled with little real additional effort. In any case, I feel that the cost of reinsurance is too important to partially or completely ignore.

To quote Mr. Dana, "The 'solve-for' premium will satisfy only a single set of conditions (e.g., the 5 per cent profit after 20 years)." Definitions 15 and 16 explicitly show that the required rate of profit on the new business investment in surplus may vary by policy year. Furthermore, equation (15) shows that a and b may also vary by policy year, even when deficiency reserves are involved.

While participating premiums and dividends were deliberately excluded from the scope of the paper, participating gross premiums and dividends that will meet predetermined objectives may be *simultaneously solved for* on a small computer (e.g., 1401) with a relatively simple program not involving linear programming.

I agree with Mr. Bleakney's feeling regarding the practical guidance offered by Mr. Anderson's paper to the actuary faced with the problem of setting a premium structure. I attempted, as mentioned previously, to avoid duplicating previous literature. My paper is intended to supplement, rather than replace, Mr. Anderson's fine paper. It is intended for the reader who is already thoroughly familiar with existing literature on gross premiums.

While Mr. Bleakney's Table A would seem to be understandable to a nonactuary (more so than my Table 5, which is not meant for such an individual), I still feel that it has too many numbers and that it is unnecessary and undesirable for a nonactuary to become involved in the manipulations of column 5. If the nonactuary is to help "choose" a final gross premium, I believe that the following will be most understandable and helpful to him:

1. Rates of his company's chief competitors.
2. A schedule of break-even premiums (e.g., When will we get our money back?).
3. A schedule of "Anderson's Premiums" as shown in Table 4 (e.g., What yield will our investment in surplus return?).

It cannot be too strongly emphasized that there is no *one correct* premium. The "best" premium will depend upon a company's objectives and their relative weights (e.g., to maximum profits, rate of growth of in force, assets, rate of growth of "market" value of company, dividends to stockholders, etc.). If these objectives are to be realized, management should, with the assistance of the actuary, balance production against surplus, offset profit against competitiveness, and, after a careful weighing of its different objectives, "select" a gross premium which it feels will best expedite these objectives.

I strongly recommend that my paper be read with the purpose of learning basic actuarial theory. This may then be applied to modify the formulas in the paper to maximize practical application. This was brilliantly demonstrated by Mr. Hoskins. Other modifications may be made for nonlevel premiums, nonlevel death benefits, coinsurance (by the reinsurer as well as the ceding company), participating premiums, and dividends.