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VELOCITY OF **DIVERSIFICATION**

by Doug Robbins

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Given the technician in me, it's always pleasant to discover a new area in which basic actuarial mathematics can be put to work on the job. Just in the last year, I applied a Calculus concept (first time ever!) to a product development project at my company. A few months later, I then stumbled upon what I think could be—at least for some—a useful application of basic probability and statistics.

For many years at the Valuation Actuary Symposium, I have led and co-taught a session called "Avoiding Statistical Pitfalls in Actuarial Work." Discussions in this session have generally covered economic scenario analysis and use (and misuse) of linear regression. However, one of the areas on which I did some new thinking in 2013 happens to be strongly related to product development.

A New Line of Thought

This new line of thought began essentially as an intuition and is based on nothing more than a personal impression picked up over the course of my career. The gut-level impression is that there are two potential pitfalls regarding the way many actuaries talk and think about mortality:

- Rather than speaking about a set of probabilities (estimated statistically) of death by age and future duration, I often hear actuaries speak of "my mortality assumption." This assumption is seen as a fixed array of values, which when plugged in, helps a model spit out a profit figure.
- Actuaries who might quail at a "CTE99" set of market-driven losses for a given product (and the immense volatility therein) often blithely assume they know very well just what a 65-year-old's mortality rate will be 25 years from now. In fact, the factors that will impact this future mortality rate could be just as volatile as those impacting the economy.

I will discuss the first pitfall more extensively in this article. It is an easy thought pattern to get into, and gets even easier as more time is spent on refining and perfecting the "mortality assumption." This includes extensive time that is also likely spent on considering slope of the mortality curve, and turning the curve into a time series by including mortality improvement. After spending all that time and effort, what more is there to consider?

My response to that would be to consider a product design technique that truly takes the overall mortality assumption into account from a company's risk perspective.

As noted previously, a product development actuary concerned about the markets might run 10,000 scenarios or more and analyze possible results. However, valuation and risk folks will surely remind him or her that only one

Table 1: Profit Streams for Life-only Annuity Due

Life Only PolYr	q	q(x+t)	Premium	Commission	Expense	Profit Margin: Payments	8.0% Profit	Profit Margin By Year Of Death
1	10%	10%	$100,000	$3,750	$100	$24,000	$72,150	72.2%
2	20%	18%	$0	$0	$90	$21,600	-$21,690	48.1%
3	30%	22%	$0	$0	$72	$17,280	-$17,352	24.0%
4	40%	20%	$0	$0	$50	$12,096	-$12,146	-0.2%
5	50%	15%	$0	$0	$30	$7,258	-$7,288	-24.3%
6	60%	9%	$0	$0	$15	$3,629	-$3,644	-48.4%
7	70%	4%	$0	$0	$6	$1,452	-$1,458	-72.5%
8	80%	1%	$0	$0	$2	$435	-$437	-96.6%
9	90%	0.3%	$0	$0	$0	$87	-$87	-120.7%
10	100%	0.04%	$0	$0	$0	$9	-$9	-144.8%

Table 2: Profit Streams for 5yr Certain and Life Option

Life w/ 5 PolYr	q	q(x+t)	Premium	Commission	Expense	Profit Margin: Payments	8.1% Profit	Profit Margin By Year Of Death
1	10%	10%	$100,000	$3,750	$100	$16,750	$79,400	12.0%
2	20%	18%	$0	$0	$100	$16,750	-$16,850	12.0%
3	30%	22%	$0	$0	$100	$16,750	-$16,850	12.0%
4	40%	20%	$0	$0	$100	$16,750	-$16,850	12.0%
5	50%	15%	$0	$0	$100	$16,750	-$16,850	12.0%
6	60%	9%	$0	$0	$15	$2,533	-$2,548	-4.9%
7	70%	4%	$0	$0	$6	$1,013	-$1,019	-21.7%
8	80%	1%	$0	$0	$2	$304	-$306	-38.6%
9	90%	0.3%	$0	$0	$0	$61	-$61	-55.4%
10	100%	0.04%	$0	$0	$0	$6	-$6	-72.3%

future economic scenario will actually occur. The same concept applies with mortality: the holder will die at one-and-only one future point, one's finely honed mortality assumptions notwithstanding. Of course, we count on diversification to solve this conundrum. (And that's surely more reasonable to do with regard to mortality than the economy, since you can have multiple lives, but not multiple economies, at any point in time!) But how well will that work for you? It depends on product design, and that's what this article is really about.

My Line of Thought Illustrated

I am going to work through just one example of what I mean. In Table 1 (page 6), I will assume an annual annuity due (so that lapse/withdrawal issues don't confuse the issue), a 0 percent interest rate for simplicity, along with a matching 0 percent discount rate on profits. (That combined assumption eliminates the need to worry about reserves as well.) The annuity due is issued at age 105, and my mortality table assumes certainty of death in year 10. My other assumptions can be inferred from the output that I show in Table 1. Under those assumptions, a life-only annuity of 24 percent of premium per year gives me roughly an 8 percent profit margin.

On the other hand, the five-year certain and life option shown below also gives me about an 8 percent profit margin. Due to the impact of the certain period on the otherwise high mortality at this issue age, I can only pay out 16.75 percent to obtain that result. This is shown in Table 2.

On this life and with $100k of premium, I expect to make roughly $8k of profit either way. Does that make me indifferent to which is sold?

I would argue that (all other things equal) I should absolutely not be indifferent—I should prefer the second case. To see why, you would need to look at the profit streams for the individual life at the various points where that life might expire. For this, I use the same product examples—along with an additional illustrative column of values.

As before, the profit margin is 8 percent. But now, I have calculated this 8 percent a different way—as a mortality-weighted average of the 10 possible profit margin figures based on year of death. These numbers vary a lot and represent staggering losses in the worst cases. (This phenomenon is well understood in the life insurance industry and is a likely driver of the presence of a deep life reinsurance market.) Let us also look at the same "revised" analysis for the life annuity with five-year certain period in Table 4.

In this case, the first thing that you—as the product development actuary—might notice is the loss of the chance of making a huge early profit, if the owner dies "young." But the risk and valuation actuaries looking over your shoulder might be more attracted to the much reduced the tail loss picture. Which is the more important of those two

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Table 3: Updated Profit Streams for Annuity Due

Life Only PolYr	q	q(x+t)	Premium	Commission	Expense	Profit Margin: Payments	8.0% Profit	Profit Margin By Year Of Death
1	10%	10%	$100,000	$3,750	$100	$24,000	$72,150	72.2%
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9	90%	0.3%	$0	$0	$0	$87	-$87	-120.7%
10	100%	0.04%	$0	$0	$0	$9	-$9	-144.8%

considerations? The answer is actually a combination of both considerations, along with the probability distribution implied by the array of mortality rates.

Here's the real issue that I'm trying to shed new light on: If you looked at case 1 (life only), you would see that in this case you have almost exactly a ***50 percent*** chance of either a profit or a loss. In case 2 (life w/ 5 certain), your chance of ***turning a profit*** on a single case has risen to about ***85 percent***! Keep in mind that all I've done is adjust the payout patterns of my annuity in a way that many annuitants tend to prefer anyway. And yet I've ***dramatically*** altered the stochastic nature of the transaction.

What are the implications of that? Isn't it true that by selling a lot of cases, I diversify and achieve profitability for my overall portfolio with little risk? Yes, that is true (-ish), but how ***quickly*** does that diversification occur and reduce my risk?

- In the life-only product version, the shape of my overall mortality variable would require about 70-75 similar cases sold, to be 95 percent confident that I will not lose money overall. And even then (assuming I do achieve a gain) the amount of that gain could be very large or very small.

- In the life with five-year certain product, one only needs to sell seven or so cases to achieve the same probability of being profitable overall! (Needless to say, if you were to sell the 70-75 cases as before, you'd also have ***much*** higher confidence on the overall profit level.)

I thus refer to the speed at which confidence in profitability is attained as the "velocity of diversification." I'm not sure yet of the best theoretical form for the statistic in question—I do believe it is a valid concept for actuaries to consider. One form the statistic might take would be {100 * (1 / # similar policies needed to be 95 percent confident of overall profitability)}, for example. So the velocity of diversification of the life-only policy above would be in the ballpark of 1.4; for life w/ five, it would be 10 times that, coming in at about 14.

Conclusion and Application

Looking at the extreme age and mortality probabilities used above, a reader might be tempted to dismiss this concept as less important in more run-of-the-mill cases. I would encourage everyone to experiment with some of the different product options for any product line you deal in (term life insurance with return-of-premium options versus standard term life insurance comes to mind.) You may be surprised how often this effect is still quite material!

For a company that expects to sell a very large number of homogenous cases in a product line type, this effect might

Table 4: Updated Profit Streams for Five-Year Certain and Life Annuity

Life w/ 5 PolYr	q	q(x+t)	Premium	Commission	Expense	Profit Margin: Payments	8.1% Profit	Profit Margin By Year Of Death
1	10%	10%	$100,000	$3,750	$100	$16,750	$79,400	12.0%
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10	100%	0.04%	$0	$0	$0	$6	-$6	-72.3%

be immaterial. Instances in which I would suggest that diversification velocity be considered carefully would include:

- New blocks for where you are concerned that you might only sell a few cases that might still be large enough to be of concern to management
- Lines of business for which premiums are received infrequently (e.g., structured settlements)
- Blocks of business for which you have reasons to believe your case sizes will be very heterogeneous (imagine selling a hundred $150,000 cases, but having a star agent in the local country club who might sell a few cases with premium of $10,000,000 or so each).

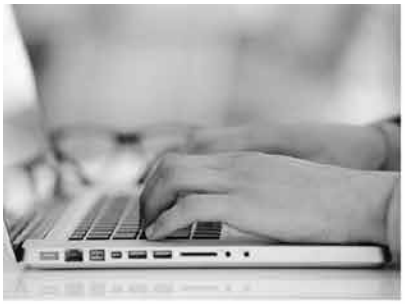
In each of those cases, I believe it makes sense to use product design to nudge sales toward product forms that diversify quickly, all else equal. In annuity product examples earlier referenced, policyholders could very well approve of the reduction in their estate's risk by taking the life with certain period option. This would lead to a "win-win" situation for all parties.

Final Thoughts

What happens in the examples if my mortality experience unfolds such that the mortality improves by five percent (from 30 percent to 25 percent) in year three, 10 percent in year four, and 15 percent for years five and later? The answer is that my expected profit margin is now slightly negative for the life only annuity. However, it is only down to just over four percent for the life annuity with period certain—not nearly as bad.

With any time series (and a mortality variable over many future years clearly is one of those), it is worth considering that you are much more confident of your values in early projection periods than you are in later ones. Anything that can be done in product development to make the accuracy of any assumption—inclusive of but not limited to mortality—less critical in later projection periods is worth considering. Aiming for products with a high velocity of diversification may help in this regard. □

Please note a correction has been made to tables 3 and 4 within this article. The original article that was printed and distributed to members had incorrect tables due to a mistake made in the layout process. We apologize for any inconvenience.



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