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Stochastic Modeling is on the Rise

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Stochastic modeling is on the rise in the life insurance industry due to a coalescence of regulations on the horizon and an increasing demand for stochastic analysis in many internal modeling exercises. While regulatory developments across the globe certainly have played a part in this increased interest, there are plenty of other reasons why stochastic modeling proficiency is growing among both actuarial modelers and those who interpret stochastic results.

This topic continues to garner attention as the industry increasingly relies upon stochastic models to value its business, design its products, and manage its portfolios. It appears that stochastic models gradually are becoming the industry norm for internal metrics since deterministic models often cannot adequately quantify the risk profile of the industry's increasingly complex business.

STOCHASTIC MODELING PROLIFERATION

As with many other industry trends, regulatory considerations will play a pivotal role in the increasing interest in stochastic modeling. Regulatory bodies in both the European Union (EU) and the United States continue to propose new stochastic modeling requirements, joining efforts from other nations worldwide.

Figure 1
Uses of Internal Stochastic Models

Tail risk analysis
Hedging strategies
Product pricing and design
Business mix optimization
Risk-adjusted merger and acquisition (M&A) pricing
Evaluation of reinsurance programs
Risk structure optimization
Calculation of diversification effects
Corporate strategy development
Risk-adjusted performance measurements and targets
Management compensation strategy
Satisfying parent company requirements

The benefits of incorporating stochastic modeling enterprisewide expand well beyond simply preparing for possible regulatory changes. Though we mention a dozen in this list, we easily could have included many more.

The EU is internally aligning its capital requirements under Solvency II, and the National Association of Insurance Commissioners (NAIC) has introduced VM-20 to address life insurance statutory reserve requirements. Each approach permits the use of internal stochastic models. VM-20 calls for stochastic modeling of economic risks, but does not require stochastic modeling of mortality risk (however, a company may elect to do so). Each of these regulations, when fully implemented, will significantly expand the use and importance of stochastic models.

Leaving aside these looming regulatory changes, however, companies are discovering stochastic models' value to an organization's cash flow projections and risk management activities. Insurers are expanding their use of internal stochastic models as available tools and computing power make this modeling more feasible. Companies are implementing stochastic models not only to determine economic capital, but also to use in product development areas. In reinsurance units, nonproportional reinsurance programs such as stop-loss and catastrophic coverages may necessitate stochastic modeling for both pricing and valuation.

NEED FOR CONTINUED RESEARCH

Given these and other reasons for the ongoing proliferation of stochastic models, the life insurance industry still has room to expand its stochastic modeling knowledge and techniques. While the stochastic modeling of market and credit risks is fairly well established, stochastic modeling of mortality is not as fully developed. In fact, most published research regarding stochastic mortality modeling either has been across general population segments where there are no underwriting selection effects, or has been conducted on longevity risks covering pensioners or annuitants.

Both of these approaches pose challenges. Research on general populations, pensioners and annuitants does not carry over well to the stochastic modeling requirements of fully underwritten life insurance. These insured populations have distinctly different mortality characteristics that require partitioning by product, underwriting class, distribution channel, policy issue year and policy duration. Similar to deterministic modeling, such partitioning should consider the level of credibility within the partitioned segments when determining stochastic distribution metrics such as means and variances. Adjoining segments may need to be combined when segmented credibility is low.

Another consideration that affects fully underwritten portfolios is policyholder lapsation. For example, lapse rates are typically very high at the end of level period for term life insurance products. These rates are difficult to model because they depend upon a number of factors, most of which are highly dependent upon post-level period premium increases and the insured's health status. This is typically not a concern when stochastically modeling general population segments or annuitants.

A good introductory resource addressing stochastic mortality for underwritten life portfolios is a document produced by Ernst & Young LLP titled, "Stochastic Analysis of Long-Term Multiple-Decrement Contracts," published by the Society of Actuaries in 2008. This report evaluates stochastic modeling of life insurance nonmarket risks (i.e., mortality and lapse). It lays out the primary issues and describes potential modeling solutions. However, it also recognizes the need for an increased understanding of the sensitivities associated with stochastic mortality model design. Suggested areas of research are selection of stochastic variable probability distributions, stochastic variable correlations, and other relatively uncharted terrain for fully underwritten life insurance.

The benefits of stochastic modeling cannot be overstated. We have touched on only a few of these benefits, but certainly could have extended the discussion into various areas of pricing, valuation and stakeholder interest.

Last but not least, ratings agencies have been increasingly supportive of the improved risk management metrics derived from stochastic modeling, making it even more vital that companies continue to develop their internal stochastic models to keep pace with what is rapidly becoming an industrywide best practice.

MONTE CARLO SIMULATION

Having set the stage as to the "whys" of stochastic modeling let's discuss some of the "hows," presenting a practical example of designing a stochastic model of death benefits on fully underwritten life insurance.

Stochastic models typically incorporate Monte Carlo simulation to reflect complex stochastic variable interactions in which alternative analytic approaches would be either unworkable or untenable at best. For the illustrative projection discussed in this article, we developed a Monte Carlo simulation model to stochastically project 30 years of annual claims on a large, fully underwritten, term life insurance portfolio. The implemented modeling process can be described in the following four high-level steps:

1. Input variable analysis and specification
2. Random sampling of input stochastic variables
3. Computation of death benefit projections
4. Aggregation and analysis of results

INPUT VARIABLE SPECIFICATION

We define input variables as either stochastic or deterministic. Deterministic variables are assigned a predetermined fixed value or may be the result of a fixed nonrandom formula. Stochastic input variables are assigned statistical distributions and may correlate with other stochastic variables.

In our model, we defined three stochastic input variables: base mortality rate, mortality improvement rate and catastrophic mortality rate. We also defined one deterministic variable: policy lapse rate.

DETERMINISTIC POLICY LAPSE RATE VARIABLE

We could have modeled policy lapse rates stochastically based upon some real-world model of policyholder behavior. However, determining appropriate statistical distributions and correlations for our particular project proved to be difficult: The policyholder's decision to lapse term insurance is typically not driven by external fluctuating forces such as interest rates or stock market indices, but by other less tractable criteria. We chose instead to use predetermined best estimate lapse rates in the Monte Carlo simulation to lapse individual policies randomly.[1]

STOCHASTIC BASE MORTALITY VARIABLE

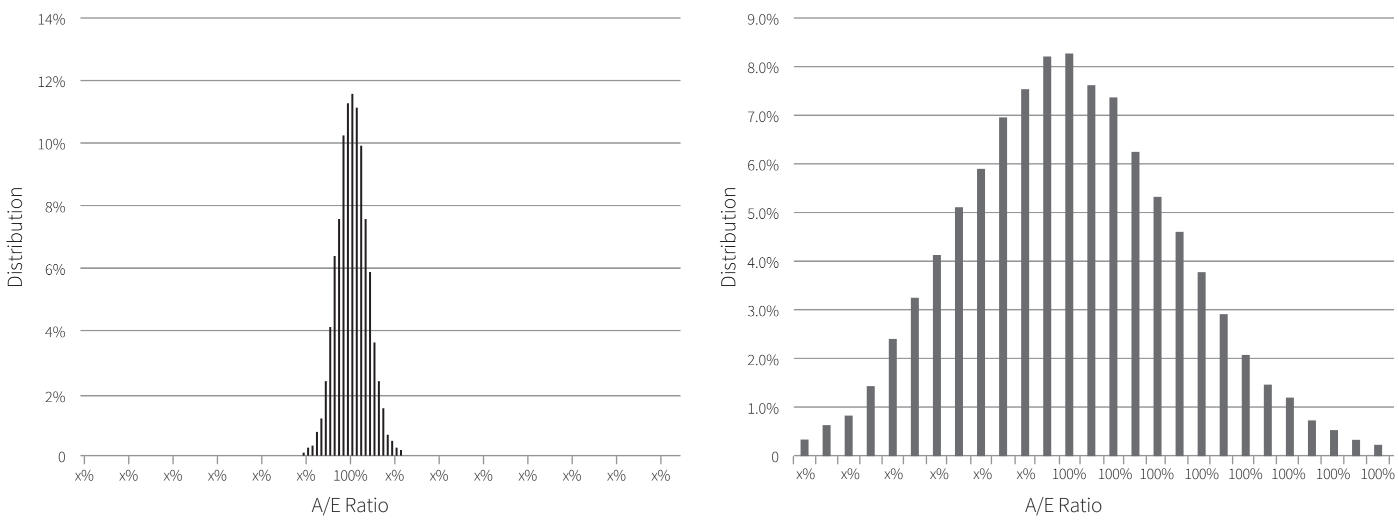
This stochastic variable reflects the uncertainty in determining an underlying best estimate mortality assumption for our portfolio. For this exercise, we referenced a recent mortality experience study for the portfolio. We can think of a mortality study as one random sample from the portfolio's "true" mortality. Just as with any random sample, uncertainty exists as to whether the sample is a good representation of the population (Figure 2). The left chart in Figure 2 shows the range of actual-to-expected (A/E) ratios that an experience study might produce for a portfolio where we expect only 25 claims. The right chart shows the range for a portfolio with 1,250 expected claims. The uncertainty about a particular study's credible representation of the population is a function of the expected claim count, and decreases as the count increases.

We can model this uncertainty stochastically. With mortality as a binomial process, the experience study's overall mortality is our mean assumption and $1 / \sqrt{(\#claims)}$ is an approximation to its standard deviation. Then, for a given stochastic iteration, we used the normal approximation to the binomial to randomly select a base mortality assumption for that iteration).[2]

STOCHASTIC MORTALITY IMPROVEMENT RATE VARIABLE

In our model, mortality improves as we project our portfolio into the future. However, just as with base mortality, uncertainty surrounds the rate at which this improvement will occur. We calculated long-term mean improvement rates, along with corresponding standard deviations, based upon an analysis of U.S. population mortality. We reviewed historical trends over the past 20–30 years to select appropriate periods for the analysis (3). A significant and seemingly permanent change in the pattern of mortality occurred around 1982, so we used data from only 1982 to 2007 in our analysis. For this period, we determined that trended mortality had an annualized mean improvement rate of 0.8 percent with a standard deviation of approximately 0.4 percent.

Figure 2
Distribution of Experience Study A/E Ratios



The uncertainty surrounding a particular study's credibility in representing the population is a function of the expected claim count. The lower the count (left), the greater the uncertainty.

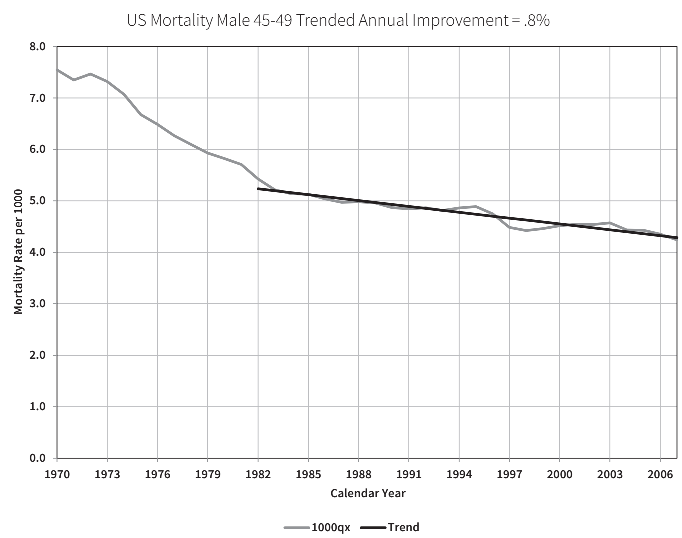
Mortality improvement rates vary significantly by attained age, so we created a vector of improvement assumptions by age group. Recognizing that mortality improvement is correlated among age groups, we also determined a correlation matrix reflecting historical correlations in improvement rates.

Using the U.S. population data, we determined that a normal distribution best represented the fluctuation of improvement rates around the long-term mean. Given the mean and standard deviation parameters, we stochastically generated 10,000 mortality improvement rate scenarios by attained-age group across the projection horizon. We then randomly selected a single scenario from these 10,000 scenarios for application in a single stochastic projection iteration of the portfolio.

STOCHASTIC CATASTROPHIC MORTALITY VARIABLE

Unlike the property/casualty sector, we are concerned only about catastrophes that result in significant loss of life. Natural disasters were less impactful than pandemics and other disasters, which have the potential for loss of life in far greater numbers. Our model includes a stochastic variable representing additional lives lost in a given calendar year from three types of disasters: pandemics, earthquakes and terrorist attacks. From third-party data sources, we developed frequency and severity distributions for each of these three types of disasters and randomly sampled these distributions for each projection year (Figure 4).

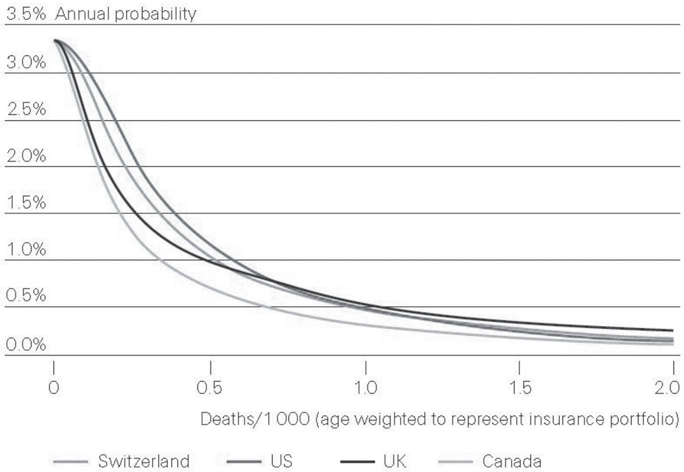
Figure 3
U.S. Population Mortality Improvement, Male 45-49



Mortality among males age 45-49 improved noticeably 1970-1982, but since has flattened out.

For each projection year, we randomly sampled the additional catastrophic mortality rate that was then added to the base mortality of each individual life. Having identified our approach and variables, we can now apply the stochastic process and analyze results.

Figure 4
Frequency and Severity of Pandemics



The graph shows the annual probability that an influenza pandemic will cause mortality greater than the X-axis values shown. (*Influenza pandemics: Time for a reality check?* Swiss Re, March 2007.)

DEATH BENEFIT COMPUTATION VIA BERNOULLI PROCESSING

In our model, we use a Bernoulli process to randomly decrement (via death or lapse) each life in the portfolio. To achieve this, we first stochastically generate base mortality rates, mortality improvement factors and catastrophic mortality rates. These stochastically generated rates we then combine into a composite set of projection year mortality rates for each insured life. Along with the deterministic lapse rates, these stochastically generated composite mortality rates are applied using a Bernoulli process.

A simplified explanation of our Bernoulli policy decrement process begins by sampling a uniformly distributed random variable for each insured life. These sampled random variables are then compared against the previously determined composite mortality rates for a given projection year as described in the following "if-then" process:

- If the random variable is less than the composite mortality rate, then a death results in the given projection year.
- Otherwise, the life survives and we generate a second uniformly distributed random variable. If this sampled value is less than the deterministic lapse rate, then a policy lapse occurs in the given projection year.
- Otherwise, the policy remains in force and continues into the next projection year where we repeat the process.

ANALYSIS OF MODEL RESULTS

After the model generates an adequate number of iterations—typically 10,000 or more depending on the portfolio size and modeling objective—we then validate and analyze model results. One of the first steps is to validate the output against other modeling sources and conduct a high-level evaluation of results given the model's input assumptions. After the model has passed these initial validations, we then conduct sensitivity analyses to further validate the model and to better understand how changes in input assumptions affect model results. Once the model is satisfactorily validated, we can then evaluate various value at risk (VaRs), and conditional tail expectation (CTE) measures.

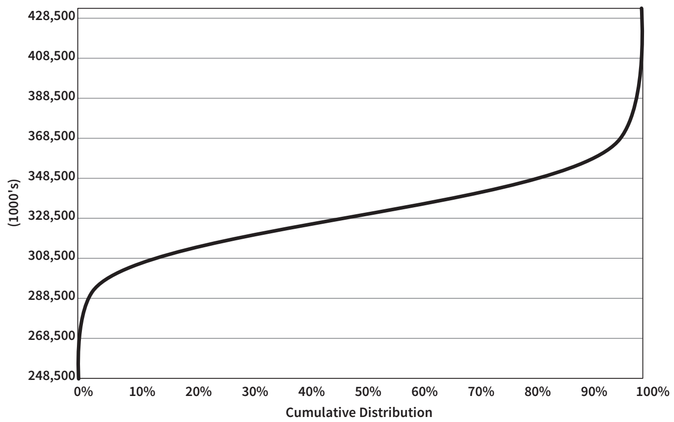
Model validation and sensitivity analysis. Some stochastic model validation criteria can be obtained from corresponding deterministic modeling results. For example, the average stochastic results of a given stochastic variable can often be validated against the corresponding best estimate result of a deterministic model. If the stochastic mean differs from the deterministic best estimate, this may raise a red flag. If the model is newly constructed, we should activate stochastic variables individually to assess their impact as they flow through the model. When considering the effect of catastrophes in our stochastic model, the resulting overall mean mortality rate should increase by the summed products of the respective frequencies and severities of each catastrophe variable. Numerous additional validation exercises can also be a part of the modeling process, including validation of interim calculations.

Once we have determined that the model is producing results in line with the input assumptions and other validation criteria, further sensitivity testing of model parameters can add value to the current project and enhance understanding of the model for future uses.

Cumulative distribution of results. Once we constructed our model we ran 10,000 simulations. Each simulation produced a net present value (NPV) of death benefits that we collectively ordered from lowest to highest. We graphed these ordered y-axis values with corresponding x-axis values set equal to the ordered rank divided by 10,000, producing values from 0 to 100 percent as shown in Figure 5. We then used the resulting cumulative distribution of NPV of death benefits to evaluate suitable measures for this variable.

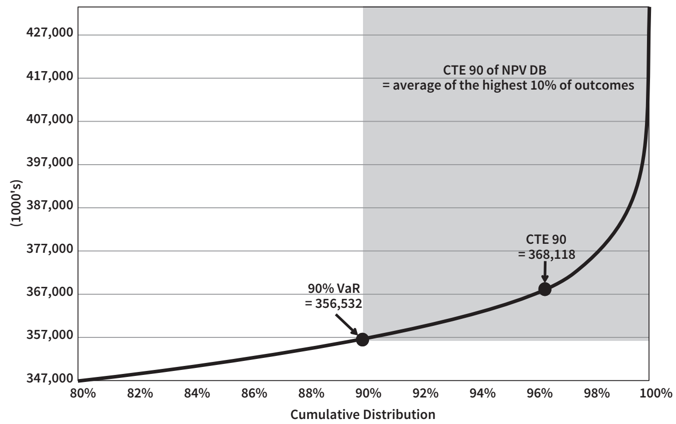
VaR and CTE. Two well-utilized measures obtained from stochastically generated cumulative distributions are the VaR and CTE. Each of these measures has its own strengths and weaknesses, but both can be easily ascertained once a modeler has produced an adequate number of stochastic iterations.

Figure 5
Distribution of NPV of Death Benefits



The 10,000 stochastic simulations yielded a fairly smooth cumulative distribution of net present value of death benefits.

Figure 6
Tail Distribution of NPV of Death Benefits



For clarification of the two measures, the above graph shows the CTE and VaR each evaluated at α = 90.

VaR is specified with a confidence level α (typically α is selected ≥ 95%) and is the point on the cumulative distribution curve at x = α. Generally, α-VaR is defined as the loss amount that will not be exceeded with probability α. For example, maintaining capital at a 99.5% VaR on next year's projected cash flows should sustain all but a 1-in-200-year scenario, or a 0.5 percent risk of insolvency. Note that some call this particular scenario tail a 1-in-200-year *event*; however, the tail may contain the culmination of different compounded *events* in the Monte Carlo process. VaR is typically measured over short time periods—for example, Solvency II incorporates a 99.5% VaR over a one-year period.

While VaR is a useful measure, we often want to know more about potential tail losses. For example, what is the expected size of a tail loss? The answer to this question is CTE α, which measures the expected loss given that the loss falls within the (1- α) quantile tail. For example, CTE 90 is the average of the worst 10 percent of modeled outcomes—which is easily calculated from the cumulative distribution.

Figure 6 shows the tail of our cumulative distribution of NPV of death benefits, along with illustrative CTE 90 and 90% VaR points on the curve. The plotted values in the blue shaded area each equally contribute to the CTE 90 calculation of $368 million in NPV of death benefits. In contrast, the 90% VaR of $357 million is the point in which only 10 percent of simulated NPVs exceed.

CTE measures may be more sensitive to severe low-frequency loss scenarios, whereas VaR measures may stop short of recognizing such rare loss events. However, even though the CTE may include extreme losses, their impact upon the CTE measure may be significantly tempered by the remaining tail. Furthermore, any comparison between CTE and VaR measures, and their sensitivities to rare events, will invariably depend on their respective quantiles (i.e., CTE 90 vs. 99.5% VaR).

CONCLUSION

Whether due to external requirements (i.e., principle-based approach (PBA), Solvency II) or internal needs, the importance of stochastic modeling is growing in the life insurance industry. Models should be built using an approach that will result in the most realistic simulation, incorporating the critical variables that affect the solvency of a block of business.

The basic model we described is only one of many possible designs that actuaries could use to stochastically model mortality and lapsation. While we did not touch on the increasing availability of stochastic modeling software, we covered some fundamental aspects that can be modeled independently of commercial applications. More importantly, our intention was to share a high-level illustration of some basic modeling components and how they can be assembled into a practical solution. ■

ENDNOTES

1 See "Lapse Rates in a Principles-Based World," *The Messenger*, June 2007.

2 "Credibility Analysis for Mortality Experience Studies—Part 1," *The Messenger,* March 2008

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