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A NEW APPROACH TO THE CALCULATION OF ACTIVE LIFE DISABILITY RESERVES

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ABSTRACT

This paper was written to provide a new flexible approach to the calculation of active life disability net premiums and reserves that is easily adaptable to modern computerized systems.

A whole new set of special disability commutation functions is presented in this paper. Through the utilization of these functions, which are more powerful and flexible than the traditional disability functions, the actuary can routinely and effectively calculate net premiums and reserves for policies with nonlevel disability benefits.

I. INTRODUCTION

BECAUSE of the relatively small size of disability reserves and the practical difficulties of applying published disability functions to calculate actuarially correct net premiums and reserves, most companies use crude approximations or even the "nearest" published disability reserves for (1) renewable term policies, (2) policies with nonlevel gross premiums, and even (3) level premium policies for which disability reserves are not published.

A result of all this is that most, if not all, companies keep a separate active life disability reserve for each rider covered by this benefit in addition to the active life disability reserve for the base policy.

This paper presents a new approach to the calculation of individual active life disability reserves that is easily adaptable to modern computerized systems and that, through the utilization of special new functions, provides the power routinely and efficiently to perform the following:

1. Calculate disability net premiums and reserves for policies with a nonlevel disability benefit (e.g., premium waiver reserves for a policy with nonlevel gross premiums).

2. Keep a *single* active life premium waiver disability reserve for a policy and all of its riders.
3. Automatically handle the premature termination of a rider or the addition of a new rider after a policy is issued (where the rider's premium will be waived in the event of disability).

While the approach shown in this paper is applicable to active life disability income reserves, to save space and to avoid redundancy, all formulas will be for premium waiver benefits. The minor modifications required to use these formulas for disability income benefits should be obvious.

II. DEFINITIONS

f_t = Attained age at which tth different base policy or rider gross premium is first payable.

x = Issue age.

y = Age disability coverage terminates.

u = Age to which premiums are waived.

z = Age at which disability premiums terminate.

LL = Number of years in disabled life select period (i.e., 15 years for the 1952 Disability Study).

$L = LL - 1$.

$k = f_t - L$.

m = Waiting period in months.

n = Duration since disablement, which takes on values in twelfths of a year from $n = m/12$ to $n = 24/12$, and integral values thereafter.

w = Last age in life mortality table.

r'_x = Rate of disability at age x.

$$C^r_x = \bar{D}_x \cdot r'_x .$$

$$M^r_x = \sum_{t=x}^{w} C^r_t .$$

$$^w\bar{C}^r_x = v^{m/12} \cdot C^r_x \cdot \bar{a}^i_{[x+1/2]+m/12} .$$

$$^w\bar{M}^r_x = \sum_{t=x}^{w} {}^w\bar{C}^r_t .$$

$$D^i_{[x+1/2]+n} = v^{x+1/2+n} \cdot l^i_{[x+1/2]+n} .$$

$$^u_y\bar{M}^r_x = \sum_{t=x}^{y-1} {}^u\bar{C}^r_t .$$

$$^{u}\bar{C}_{x}^{r} = v^{1/2}\cdot C_{x}^{r}\cdot \bar{a}^{i}_{[x+1/2]+m/12:\overline{u-x-1/2-m/12}|}\,.$$

$$_{v}M_{x}^{r} = \sum_{t=x}^{v-1} C_{t}^{r}\,.$$

$$\bar{N}^{i}_{[x+1/2]+n} = \sum_{s=n}^{\infty}\tfrac{1}{2}\cdot(D^{i}_{[x+1/2]+s} + D^{i}_{[x+1/2]+s+1}) \qquad \text{for} \quad n \geq 2\,;$$

$$= \sum_{s=12n}^{23}\tfrac{1}{24}\cdot(D^{i}_{[x+1/2]+s/12} + D^{i}_{[x+1/2]+(s+1)/12}) + \bar{N}^{i}_{[x+1/2]+2}$$

$$\text{for} \quad m/12 \leq n < 2\,.$$

$$^{w}\bar{M}_{x}^{i} = {}^{w}\bar{M}_{x}^{r} + \frac{m}{12}\cdot v^{m/12}\cdot M_{x}^{r}\,.$$

$$^{w}A_{x}^{vr} = \sum_{t=x}^{w}\frac{v^{m/12}\cdot C_{t}^{r}}{D^{i}_{[t+1/2]+m/12}}\,.$$

$$_{u-k}\bar{A}^{nr}_{[u]} = \sum_{t=k}^{u-1}\frac{v^{m/12}\cdot C_{t}^{r}\cdot \bar{N}^{i}_{[t+1/2]+m/12+u-t-(6+m)/12}}{D^{i}_{[t+1/2]+m/12}}\,;$$

NOTE: $_{0}\bar{A}^{nr}_{[u]} = 0$, where $0 \leq (u-k) \leq 14$.

$$\bar{N}^{i}_{[x+1/2]+t-1/2} \doteq \tfrac{1}{8}\cdot D^{i}_{[x+1/2]+t-1} + \tfrac{3}{8}\cdot D^{i}_{[x+1/2]+t} + \bar{N}^{i}_{[x+1/2]+t}\,.$$

III. CALCULATION OF DISABILITY NET PREMIUMS

The availability of the $^{w}\bar{M}^{i}$, $^{w}A^{vr}$, and $_{u-k}\bar{A}^{nr}_{[u]}$ functions allows the straightforward, routine calculation of actuarially correct disability (including payor) net premiums.* Changes in the amounts of total premium waived (or disability income paid) can be handled routinely. As a consequence, the disability net premium provides for the disability gross premium being waived under, say, a whole life plan, only until the company's terminal age for this rider (i.e., 60 or 65).

The formula by which the traditional method calculates the disability net premium per $100 of total gross premium for a policy under which the total annual gross premium (including the premium waiver gross) does not change (e.g., twenty-year endowment policy issued at age 35) is as follows:

* Except for the usual assumption that the disability net premiums will be payable if the insured lives, whether or not he is disabled.

$$P^{w}_{35} = 100\,({}^{55}_{55}\bar{M}^{r}_{35} + \tfrac{1}{2}\cdot v^{1/2}\cdot {}_{55}M^{r}_{35}) \div (N_{35} - N_{55})$$

$$= 100\left(\sum_{x=35}^{54} v^{1/2}\cdot C^{r}_{x}\cdot \bar{a}^{i}_{[x+1/2]+1/2:\overline{55-x-1}|} + \tfrac{1}{2}\cdot v^{1/2}\cdot \sum_{x=35}^{54} C^{r}_{x}\right) \div (N_{35} - N_{55})$$

$$= 100\left\{\sum_{x=35}^{40} v^{1/2}\cdot \frac{C^{r}_{x}}{D^{i}_{[x+1/2]+1/2}}\cdot(\bar{N}^{i}_{[x+1/2]+1/2} - \bar{N}^{i}_{55}) + \sum_{x=41}^{54} v^{1/2}\cdot \frac{C^{r}_{x}}{D^{i}_{[x+1/2]+1/2}}\cdot(\bar{N}^{i}_{[x+1/2]+1/2} - \bar{N}^{i}_{[x+1/2]+55-x-1/2}) + \tfrac{1}{2}\cdot v^{1/2}\cdot \sum_{x=35}^{54} C^{r}_{x}\right\} \div (N_{35} - N_{55})\,. \tag{1}$$

This formula assumes that, following disability, a level temporary disability annuity is payable until the end of the premium-payment period and that the waiting period, m, is equal to six months.

Another way of expressing this is to replace each temporary disability annuity by a lifetime disability annuity *less* a deferred lifetime disability annuity. Formula (1) is rearranged in this manner as follows:

$$P^{w}_{35} = 100\left\{\sum_{x=35}^{54}\left(v^{1/2}\cdot \frac{C^{r}_{x}}{D^{i}_{[x+1/2]+1/2}}\cdot \bar{N}^{i}_{[x+1/2]+1/2} + \tfrac{1}{2}\cdot v^{1/2}\cdot C^{r}_{x}\right) - \bar{N}^{i}_{55}\cdot \sum_{x=35}^{40} v^{1/2}\cdot \frac{C^{r}_{x}}{D^{i}_{[x+1/2]+1/2}} - \sum_{x=41}^{54} v^{1/2}\cdot \frac{C^{r}_{x}}{D^{i}_{[x+1/2]+1/2}}\cdot \bar{N}^{i}_{[x+1/2]+55-x-1/2}\right\} \div (N_{35} - N_{55})\,. \tag{1a}$$

The first term of equation (1a), which assumes a lifetime premium waiver benefit for disability occurring between the ages of 35 and 55, can be split into (*a*) a lifetime premium waiver benefit for disability occurring after age 35, less (*b*) a lifetime premium waiver benefit for disability occurring after age 55.

This would result in equation (1b):

$$P^{w}_{35} = 100\left\{\sum_{x=35}^{w}\left(v^{1/2}\cdot \frac{C^{r}_{x}}{D^{i}_{[x+1/2]+1/2}}\cdot \bar{N}^{i}_{[x+1/2]+1/2} + \tfrac{1}{2}\cdot v^{1/2}\cdot C^{r}_{x}\right) - \sum_{x=55}^{w}\left(v^{1/2}\cdot C^{r}_{x}\cdot \frac{\bar{N}^{i}_{[x+1/2]+1/2}}{D^{i}_{[x+1/2]+1/2}} + \tfrac{1}{2}\cdot v^{1/2}\cdot C^{r}_{x}\right) - \bar{N}^{i}_{55}\cdot \sum_{x=35}^{40} v^{1/2}\cdot \frac{C^{r}_{x}}{D^{i}_{[x+1/2]+1/2}} - \sum_{x=41}^{54} v^{1/2}\cdot \frac{C^{r}_{x}}{D^{i}_{[x+1/2]+1/2}}\cdot \bar{N}^{i}_{[x+1/2]+55-x-1/2}\right\} \div (N_{35} - N_{55})\,. \tag{1b}$$

Going one step further, the next-to-last term in equation (1b) can be broken up into

$$-\bar{N}^i_{55}\cdot\left(\sum_{x=35}^{w} v^{1/2}\cdot\frac{C^r_x}{D^i_{[x+1/2]+1/2}} - \sum_{x=41}^{w} v^{1/2}\cdot\frac{C^r_x}{D^i_{[x+1/2]+1/2}}\right).$$

Substituting this into equation (1b) completes the transformation of equation (1) as follows:

$$\begin{aligned} P^w_{35} = 100 \Bigg\{ & \sum_{x=35}^{w}\left(v^{1/2}\cdot\frac{C^r_x}{D^i_{[x+1/2]+1/2}}\cdot\bar{N}^i_{[x+1/2]+1/2} + \tfrac{1}{2}\cdot v^{1/2}\cdot C^r_x\right) \\ & - \sum_{x=55}^{w}\left(v^{1/2}\cdot\frac{C^r_x}{D^i_{[x+1/2]+1/2}}\cdot\bar{N}^i_{[x+1/2]+1/2} + \tfrac{1}{2}\cdot v^{1/2}\cdot C^r_x\right) \\ & - \left(\bar{N}^i_{55}\cdot\sum_{x=35}^{w} v^{1/2}\cdot\frac{C^r_x}{D^i_{[x+1/2]+1/2}} - \bar{N}^i_{55}\cdot\sum_{x=41}^{w} v^{1/2}\cdot\frac{C^r_x}{D^i_{[x+1/2]+1/2}}\right) \\ & - \sum_{x=41}^{54} v^{1/2}\cdot\frac{C^r_x}{D^i_{[x+1/2]+1/2}}\cdot\bar{N}^i_{[x+1/2]+55-x-1/2}\Bigg\} \div (N_{35} - N_{55}) . \end{aligned} \tag{1c}$$

If we use the special functions defined earlier, this now can be rewritten as equation (1d):

$$P^w_{35} = 100$$

$$\times \frac{\{({}^w\bar{M}^i_{35} - {}^w\bar{M}^i_{55}) - \bar{N}^i_{55}\cdot({}^wA^{vr}_{35} - {}^wA^{vr}_{41}) - ({}_{14}\bar{A}^{nr}_{[55]} - {}_0\bar{A}^{nr}_{[55]})\}}{N_{35} - N_{55}} . \tag{1d}$$

NOTE: ${}_0\bar{A}^{nr}_{[55]} = 0$.

Before showing the net premium formulas for policies with varying gross premiums, the following terms must be defined:

GP_t = The tth different gross annual premium. Where the policy does not provide for lifetime waiver of premiums, the last annual gross premium is *defined* as *zero*.

g = The number of different annual gross premiums under a covered policy or rider.

f_t = The attained age at which the tth different total gross premium is first payable, where $f_1 = x$ and $f_g = u$.

The net annual premium is comprised of g different pieces, one for each different annual gross premium and the period during which it is payable.

Each piece of the net annual disability premium will be calculated by use of one of seven general formulas. The formula applicable to a specified gross premium will depend upon the attained age at which the premium is first payable. The seven general formulas and the conditions for their use are as follows:

CASE I. $f_t < y, f_t \leq (x+L), f_{t+1} \leq y$:

$$P(t) = \text{GP}_t \cdot ({}^w\bar{M}^i_{f_t} - {}^w\bar{M}^i_{f_{t+1}}) + (\text{GP}_t - \text{GP}_{t-1}) \cdot {}_{f_t - x}\bar{A}^{nr}_{[f_t]} \,. \tag{2}$$

CASE II. $f_t < y, f_t \leq (x+L), f_{t+1} > y$:

$$P(t) = \text{GP}_t \cdot ({}^w\bar{M}^i_{f_t} - {}^w\bar{M}^i_y) + (\text{GP}_t - \text{GP}_{t-1}) \cdot {}_{f_t - x}\bar{A}^{nr}_{[f_t]} \,. \tag{3}$$

CASE III. $f_t < y, f_t > (x+L), f_{t+1} \leq y$:

$$P(t) = \text{GP}_t \cdot ({}^w\bar{M}^i_{f_t} - {}^w\bar{M}^i_{f_{t+1}}) + (\text{GP}_t - \text{GP}_{t-1}) \times \{\bar{N}^i_{f_t} \cdot ({}^wA^{vr}_x - {}^wA^{vr}_{f_t - L}) + {}_L\bar{A}^{nr}_{[f_t]}\} \,. \tag{4}$$

CASE IV. $f_t < y, f_t > (x+L), f_{t+1} > y$:

$$P(t) = \text{GP}_t \cdot ({}^w\bar{M}^i_{f_t} - {}^w\bar{M}^i_y) + (\text{GP}_t - \text{GP}_{t-1}) \times \{\bar{N}^i_{f_t} \cdot ({}^wA^{vr}_x - {}^wA^{vr}_{f_t - L}) + {}_L\bar{A}^{nr}_{[f_t]}\} \,. \tag{5}$$

CASE V. $f_t > (y+L)$:

$$P(t) = (\text{GP}_t - \text{GP}_{t-1}) \cdot \bar{N}^i_{f_t} \cdot ({}^wA^{vr}_x - {}^wA^{vr}_y) \,. \tag{6}$$

CASE VI. $(y+L) \geq f_t \geq y, f_t \leq (x+L)$:

$$P(t) = (\text{GP}_t - \text{GP}_{t-1}) \cdot ({}_{\overline{f_t - x}|}\bar{A}^{nr}_{[f_t]} - {}_{\overline{f_t - y}|}\bar{A}^{nr}_{[f_t]}) \,. \tag{7}$$

CASE VII. $(y+L) \geq f_t \geq y, f_t > (x+L)$:

$$P(t) = (\text{GP}_t - \text{GP}_{t-1}) \cdot \{\bar{N}^i_{f_t} \cdot ({}^wA^{vr}_x - {}^wA^{vr}_k) + ({}_{\overline{f_t - k}|}\bar{A}^{nr}_{[f_t]} - {}_{\overline{f_t - y}|}\bar{A}^{nr}_{[f_t]})\} \,. \tag{8}$$

The logic underlying formulas (2)–(8) is as follows:

a) The $\text{GP}_t \cdot ({}^w\bar{M}^i_{f_t} - {}^w\bar{M}^i_{f_{t+1}})$ terms represent lifetime disability annuities of GP_t payable to insureds who are disabled during the premium paying period of GP_t.

b) The $(\text{GP}_t - \text{GP}_{t-1}) \cdot \{N^i_{f_t} \cdot ({}^wA^{vr} - {}^wA^{vr})$ and/or $\bar{A}^{nr}_{[f_t]}\}$ terms represent the adjustment of the lifetime disability annuities payable to those who became (and still are) disabled prior to the period during which GP_t is payable. The annual annuity adjustment is from GP_{t-1} to GP_t.

The general expression for a policy's or rider's disability net premium is concisely stated by formula (9):

$$P_x^w = \frac{\sum_{t=1}^{g} P(t)}{N_x - N_s} . \tag{9}$$

This formula assumes that the base policy or the rider has a level disability premium. If, for some reason, a nonlevel disability premium is desired, the denominator of formula (9) may be modified accordingly.

IV. CALCULATION OF TERMINAL RESERVES

The general equation for calculating terminal reserves is shown by formula (10):

$$_tV_x^w = \frac{1}{D_{x+t}} \cdot \left[\sum_{s=c}^{g'} {_t\mathrm{PV}_x(s)} - \sum_{e=1}^{Q} {^eP_x^w} \cdot (N_{x+t} - N_{z_e}) \right] . \tag{10}$$

In equations (11)–(17) $_t\mathrm{PV}_x(s)$ is defined, while GP_c and $^eP_x^w$ are defined below:

GP_c = Total covered gross premium payable at the beginning of policy year $t + 1$.
$^eP_x^w$ = Disability net premium payable for policy or rider e.
Z_e = Attained age at which $^eP_x^w$ terminates.
Q = Number of covered riders plus 1 (for the base policy) at the time the reserve is calculated.
g' = Number of different *total* annual gross premiums under a covered policy and its riders.

In the following equations, $\mathrm{GP}_{c-1} = 0$, $\mathrm{GP}_{g'} = 0$ if $u < w$, and f_s is the attained age at which GP_s is first payable.

CASE I. $f_s < y, f_s \leq (x + t + L), f_{s+1} \leq y$:

$$_t\mathrm{PV}_x(s) = \mathrm{GP}_s \cdot ({^w\bar{M}^i_{f_s}} - {^w\bar{M}^i_{f_{s+1}}}) + (\mathrm{GP}_s - \mathrm{GP}_{s-1}) \cdot {_{\overline{f_s - x - t}|}\bar{A}^{nr}_{[f_s]}} . \tag{11}$$

CASE II. $f_s < y, f_s \leq (x + t + L), f_{s+1} > y$:

$$_t\mathrm{PV}_x(s) = \mathrm{GP}_s \cdot ({^w\bar{M}^i_{f_s}} - {^w\bar{M}^i_{y}}) + (\mathrm{GP}_s - \mathrm{GP}_{s-1}) \cdot {_{\overline{f_s - x - t}|}\bar{A}^{nr}_{[f_s]}} . \tag{12}$$

CASE III. $f_s < y, f_s > (x + t + L), f_{s+1} \leq y$:

$$_t\mathrm{PV}_x(s) = \mathrm{GP}_s \cdot ({^w\bar{M}^i_{f_s}} - {^w\bar{M}^i_{f_{s+1}}}) + (\mathrm{GP}_s - \mathrm{GP}_{s-1}) \times \{\bar{N}^i_{f_s} \cdot ({^wA^{vr}_{x+t}} - {^wA^{vr}_{f_s - L}}) + {_L\bar{A}^{nr}_{[f_s]}}\} . \tag{13}$$

CASE IV. $f_s < y, f_s > (x + t + L), f_{s+1} > y$:

$$
{}_t\mathrm{PV}_x(s) = \mathrm{GP}_s\cdot({}^w\bar{M}^i_{f_s} - {}^w\bar{M}^i_y) + (\mathrm{GP}_s - \mathrm{GP}_{s-1}) \times \{\bar{N}^i_{f_s}\cdot({}^wA^{vr}_{x+t} - {}^wA^{vr}_{f_s-L}) + {}_L\bar{A}^{nr}_{[f_s]}\}\,. \tag{14}
$$

CASE V. $f_s > (y + L)$:

$$
{}_t\mathrm{PV}_x(s) = (\mathrm{GP}_s - \mathrm{GP}_{s-1})\cdot\bar{N}^i_{f_s}\cdot({}^wA^{vr}_{x+t} - {}^wA^{vr}_y)\,. \tag{15}
$$

CASE VI. $(y + L) \geq f_s \geq y, f_s \leq (x + t + L)$:

$$
{}_t\mathrm{PV}_x(s) = (\mathrm{GP}_s - \mathrm{GP}_{s-1})\cdot({}_{\overline{f_s-x-t}|}\bar{A}^{nr}_{[f_s]} - {}_{\overline{f_s-y}|}\bar{A}^{nr}_{[f_s]})\,. \tag{16}
$$

CASE VII. $(y + L) \geq f_s \geq y, f_s > (x + t + L)$:

$$
{}_t\mathrm{PV}_x(s) = (\mathrm{GP}_s - \mathrm{GP}_{s-1})\cdot\{\bar{N}^i_{f_s}\cdot({}^wA^{vr}_{x+t} - {}^wA^{vr}_k) + ({}_{\overline{f_s-k}|}\bar{A}^{nr}_{[f_s]} - {}_{\overline{f_s-y}|}\bar{A}^{nr}_{[f_s]})\}\,. \tag{17}
$$

V. MEAN RESERVES

Where one reserve is kept for a policy and its covered riders, the mean reserve would be defined as the greater of (*a*) or (*b*):

a) 50 per cent of the two disability terminal reserves plus 50 per cent of the *sum* of the disability net premiums of the policy and each of its riders; for example,

$$
{}_t\mathrm{MV}^i_x = \tfrac{1}{2}\Big({}_{t-1}V^i_x + {}_tV^i_x + \sum_e {}^eP^w_x\Big)\,.
$$

b) 50 per cent of the sum of the disability net premiums of the policy and its riders; for example,

$$
{}_t\mathrm{MV}^i_x = \tfrac{1}{2}\cdot\sum_e {}^eP^w_x\,.
$$

This definition, in the author's opinion, would show adequate conservatism even if disability reserves were not such an insignificant per cent of total life reserves.

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APPENDIX A

ABRIDGED TABLES OF SPECIAL DISABILITY FUNCTIONS

Tables 1 and 2 contain the special functions needed for the calculation of net premiums and reserves for policies issued at ages 30 and above.

TABLE 1

ABRIDGED ULTIMATE DISABILITY FUNCTIONS
1952 DISABILITY STUDY—PERIOD 2
COMBINED WITH 1958 C.S.O. AT 3 PER CENT INTEREST
(Benefit 5–6-Month Waiting Period)

x	${}^{w}\bar{M}_x^i$	A_x^{ir}	$\bar{N}_x^i$
30	1,060,774.81	1.606323081	...
31	1,045,785.25	1.604330914	...
32	1,030,739.65	1.602104448	...
33	1,015,629.48	1.599616630	...
34	1,000,441.64	1.596834568	...
35	985,151.14	1.593719310	...
36	969,507.64	1.590173559	...
37	953,507.59	1.586141000	...
38	937,025.29	1.581523406	...
39	919,939.95	1.576200442	...
40	902,130.38	1.570026176	...
41	883,487.74	1.562834627	...
42	864,024.22	1.554485508	...
43	843,538.58	1.544730638	...
44	822,053.69	1.533387037	...
45	799,469.39	1.520166716	1,142,596
46	775,595.96	1.504670997	1,042,929
47	750,355.18	1.486492453	951,674
48	723,244.03	1.464803530	868,057
49	693,987.53	1.438757723	791,383
50	661,902.08	1.406924046	721,021
51	626,239.82	1.367447301	656,415
52	586,090.03	1.317829805	597,052
53	540,512.47	1.254951819	542,473
54	488,504.21	1.174766055	492,273
55	428,836.77	1.071919048	446,074
56	360,426.11	0.940023157	403,560
57	281,312.00	0.769308649	364,445
58	194,944.10	0.560662850	328,468
59	101,175.40	0.306887289	295,412
60	...	...	265,077
61	...	...	237,281
62	...	...	211,855
63	...	...	188,640
64	...	...	167,485
65	...	...	148,243
66	...	...	130,779
67	...	...	114,963
68	...	...	100,671
69	...	...	87,787
70	...	...	76,202
71	...	...	65,813
72	...	...	56,523
73	...	...	48,241
74	...	...	40,887
75	...	...	34,388
76	...	...	28,680
77	...	...	23,706
78	...	...	19,405
79	...	...	15,719
80	...	...	12,591

TABLE 2

ABRIDGED SELECT DISABILITY FUNCTIONS
1952 DISABILITY STUDY—PERIOD 2 COMBINED WITH 1958 C.S.O. AT 3 PER CENT INTEREST

(Benefit 5–6-Month Waiting Period)

$_1\bar{A}^{nr}_{[x]}$	$_2\bar{A}^{nr}_{[x]}$	$_3\bar{A}^{nr}_{[x]}$	$_4\bar{A}^{nr}_{[x]}$	$_5\bar{A}^{nr}_{[x]}$	x	$_6\bar{A}^{nr}_{[x]}$	$_7\bar{A}^{nr}_{[x]}$	$_8\bar{A}^{nr}_{[x]}$	$_9\bar{A}^{nr}_{[x]}$	$_{10}\bar{A}^{nr}_{[x]}$	$_{11}\bar{A}^{nr}_{[x]}$	$_{12}\bar{A}^{nr}_{[x]}$	$_{13}\bar{A}^{nr}_{[x]}$	$_{14}\bar{A}^{nr}_{[x]}$
12,868.4	....	....	....	....	31	....	....	....	....	....	....	....	....	....
12,937.1	22,633.8	....	....	....	32	....	....	....	....	....	....	....	....	....
13,013.1	22,807.6	30,730.4	....	....	33	....	....	....	....	....	....	....	....	....
13,106.6	23,007.0	31,042.4	37,808.8	....	34	....	....	....	....	....	....	....	....	....
13,221.9	23,246.0	31,401.0	38,282.8	44,224.6	35	....	....	....	....	....	....	....	....	....
13,560.6	23,730.7	32,021.2	39,024.2	45,077.8	36	50,381.1	....	....	....	....	....	....	....	....
13,905.8	24,398.0	32,843.6	39,981.5	46,151.6	37	51,560.0	56,339.5	....	....	....	....	....	....	....
14,362.3	25,186.1	33,934.7	41,224.9	47,523.4	38	53,040.7	57,917.4	62,250.0	....	....	....	....	....	....
14,928.7	26,176.1	35,236.8	42,807.2	49,249.2	39	54,885.6	59,862.5	64,284.2	68,225.2	....	....	....	....	....
15,609.1	27,372.4	36,823.5	44,682.5	51,381.2	40	57,149.9	62,235.8	66,749.0	70,771.3	74,363.0	....	....	....	....
16,389.5	28,767.2	38,688.6	46,904.1	53,866.6	41	59,868.8	65,075.2	69,687.3	73,792.8	77,458.7	80,735.6	....	....	....
17,168.1	30,246.4	40,724.2	49,366.5	56,652.8	42	62,894.5	68,312.3	73,033.3	77,228.2	80,969.6	84,314.1	87,305.2	....	....
18,124.2	31,908.3	43,018.2	52,163.6	59,836.2	43	66,370.8	72,004.9	76,916.8	81,209.7	85,031.9	88,444.9	91,497.5	94,228.3	....
19,065.0	33,701.4	45,449.0	55,164.2	63,291.0	44	70,174.3	76,072.5	81,179.1	85,644.2	89,554.4	93,040.3	96,155.0	98,941.7	101,435.0
20,101.1	35,583.0	48,093.8	58,383.5	67,023.8	45	74,316.6	80,529.0	85,873.2	90,513.3	94,578.7	98,143.6	101,324.0	104,166.9	106,710.9
21,314.3	37,727.3	50,996.2	61,969.7	71,126.7	46	78,881.6	85,462.6	91,089.6	95,943.4	100,166.1	103,870.9	107,122.1	110,024.1	112,618.8
22,601.3	40,096.3	54,198.5	65,850.5	75,620.1	47	83,838.8	90,835.3	96,793.9	101,902.0	106,316.6	110,162.6	113,539.9	116,505.3	119,153.0
24,346.8	42,991.7	58,057.9	70,453.8	80,829.6	48	89,596.6	97,008.7	103,340.5	108,746.6	113,389.8	117,408.2	120,912.3	123,991.2	126,695.6
26,349.6	46,531.0	62,620.7	75,874.7	86,913.6	49	96,221.1	104,123.2	110,826.8	116,567.9	121,478.9	125,702.6	129,361.2	132,553.7	135,360.0
28,983.0	50,925.5	68,373.4	82,536.7	94,338.7	50	104,235.9	112,618.8	119,759.8	125,833.4	131,044.9	135,509.0	139,351.8	142,682.7	145,590.5
32,307.1	56,550.6	75,553.0	90,919.0	103,527.9	51	114,102.5	123,008.5	130,576.3	137,039.7	142,548.1	147,281.5	151,339.7	154,835.4	157,866.8
36,472.8	63,611.7	84,638.0	101,378.5	115,053.1	52	126,342.1	135,848.0	143,878.7	150,720.2	156,575.7	161,573.9	165,873.0	169,561.2	172,739.6
41,515.3	72,274.7	95,843.3	114,369.8	129,260.8	53	141,493.6	151,630.3	160,190.8	167,440.9	173,630.6	178,937.3	183,471.9	187,374.9	190,724.7
47,502.2	82,639.8	109,380.3	130,146.9	146,616.8	54	159,925.8	170,897.3	180,013.4	187,730.2	194,279.7	199,881.0	204,689.0	208,800.7	212,341.3
54,625.3	94,965.4	125,534.7	149,091.0	167,538.8	55	182,244.0	194,166.6	204,019.7	212,224.6	219,184.2	225,101.4	230,168.4	234,521.6	238,246.2

TABLE 2—*Continued*

$_{1}\bar{A}^{nr}_{[x]}$	$_{2}\bar{A}^{nr}_{[x]}$	$_{3}\bar{A}^{nr}_{[x]}$	$_{4}\bar{A}^{nr}_{[x]}$	$_{5}\bar{A}^{nr}_{[x]}$	x	$_{6}\bar{A}^{nr}_{[x]}$	$_{7}\bar{A}^{nr}_{[x]}$	$_{8}\bar{A}^{nr}_{[x]}$	$_{9}\bar{A}^{nr}_{[x]}$	$_{10}\bar{A}^{nr}_{[x]}$	$_{11}\bar{A}^{nr}_{[x]}$	$_{12}\bar{A}^{nr}_{[x]}$	$_{13}\bar{A}^{nr}_{[x]}$	$_{14}\bar{A}^{nr}_{[x]}$
62,785.0	109,316.0	144,429.3	171,345.5	192,252.7	56	208,705.4	221,862.1	232,554.5	241,408.7	248,795.8	255,072.2	260,415.5	264,995.5	268,932.5
72,771.6	126,399.2	166,910.1	197,805.8	221,667.9	57	240,290.5	254,990.6	266,772.7	276,366.1	284,323.8	290,973.3	296,630.1	301,450.7	305,585.2
79,615.8	141,926.2	188,612.5	224,223.7	251,577.9	58	272,801.4	289,415.5	302,559.6	313,114.0	321,721.3	328,870.7	334,851.7	339,944.6	344,287.5
86,604.6	154,920.5	209,147.6	250,137.6	281,618.7	59	305,908.9	324,811.8	339,642.9	351,397.7	360,850.5	368,568.8	374,986.2	380,359.5	384,937.8
93,624.3	168,073.1	227,487.5	275,028.1	311,199.6	60	339,103.9	360,699.2	377,542.5	390,782.0	401,290.6	409,750.7	416,664.3	422,416.7	427,236.0
0	80,602.7	145,288.0	197,283.4	239,152.0	61	271,149.0	295,909.2	315,112.4	330,117.5	341,929.7	351,315.6	358,877.3	365,060.3	370,207.3
0	0	69,940.6	126,430.5	172,118.1	62	209,072.5	237,400.8	259,371.0	276,442.3	289,799.9	300,326.8	308,696.8	315,443.3	320,961.7
0	0	0	60,931.3	110,442.6	63	150,668.3	183,307.9	208,385.1	227,868.8	243,029.2	254,903.9	264,268.4	271,717.1	277,722.7
0	0	0	0	53,250.8	64	96,725.0	132,160.0	160,978.4	183,158.6	200,416.0	213,857.6	224,392.5	232,703.7	239,315.8
0	0	0	0	0	65	46,617.4	84,803.7	115,999.4	141,414.6	161,002.8	176,259.2	188,149.2	197,471.2	204,826.7
0	0	0	0	0	66	0	40,816.9	74,328.2	101,751.0	124,123.2	141,383.8	154,836.1	165,323.3	173,546.5
0	0	0	0	0	67	0	0	35,694.1	65,046.8	89,098.3	108,741.7	123,907.4	135,731.2	144,949.6
0	0	0	0	0	68	0	0	0	31,139.2	56,778.9	77,811.7	95,003.3	108,281.6	118,635.4
0	0	0	0	0	69	0	0	0	0	27,076.4	49,397.2	67,724.2	82,712.2	94,290.9
0	0	0	0	0	70	0	0	0	0	0	23,452.3	42,806.7	58,709.1	71,718.2
0	0	0	0	0	71	0	0	0	0	0	0	20,223.9	36,929.7	50,661.7
0	0	0	0	0	72	0	0	0	0	0	0	0	17,353.6	31,698.0
0	0	0	0	0	73	0	0	0	0	0	0	0	0	14,805.8
0	0	0	0	0	74	0	0	0	0	0	0	0	0	0

These functions, which are calculated by use of the formulas shown in section II, are used in the calculations required for the illustrative examples of Appendix B.

APPENDIX B

ILLUSTRATIVE EXAMPLES

Example 1: Policy with Five Riders Issued at Age 30

CALCULATION OF NET PREMIUMS

a) $100 Waived to Age 40: Cases I, VI

$$P^i_{30} = \frac{100\,({}^w\bar{M}^i_{30} - {}^w\bar{M}^i_{40}) - 100 \cdot {}_{10}\bar{A}^{nr}_{[40]}}{N_{30} - N_{40}} = 0.2480\,.$$

b) $100 Waived to Age 45: Cases I, VII

$$P^i_{30} = \frac{100\,({}^w\bar{M}^i_{30} - {}^w\bar{M}^i_{45}) - 100\,\{\bar{N}^i_{45} \cdot ({}^wA^{vr}_{30} - {}^wA^{vr}_{31}) + {}_{14}\bar{A}^{nr}_{[45]}\}}{N_{30} - N_{45}}$$
$$= 0.3224\,.$$

c) $100 Waived to Age 50: Cases I, VII

$$P^i_{30} = \frac{100\,({}^w\bar{M}^i_{30} - {}^w\bar{M}^i_{50}) - 100\,\{\bar{N}^i_{50} \cdot ({}^wA^{vr}_{30} - {}^wA^{vr}_{36}) + {}_{14}\bar{A}^{nr}_{[50]}\}}{N_{30} - N_{50}}$$
$$= 0.4137\,.$$

d) $100 Waived to Age 55: Cases I, VII

$$P^i_{30} = \frac{100\,({}^w\bar{M}^i_{30} - {}^w\bar{M}^i_{55}) - 100\,\{\bar{N}^i_{55} \cdot ({}^wA^{vr}_{30} - {}^wA^{vr}_{41}) + {}_{14}\bar{A}^{nr}_{[55]}\}}{N_{30} - N_{55}}$$
$$= 0.5532\,.$$

e) $100 Waived to Age 65: Cases II, VII

$$P^i_{30} = \frac{100\,({}^w\bar{M}^i_{30} - {}^w\bar{M}^i_{60}) - 100\,\{\bar{N}^i_{65} \cdot ({}^wA^{vr}_{30} - {}^wA^{vr}_{51}) + ({}_{14}\bar{A}^{nr}_{[65]} - {}_{5}\bar{A}^{nr}_{[65]})\}}{N_{30} - N_{60}}$$
$$= 1.0913\,.$$

f) $100 Waived to Age 80: Cases II, V

$$P^i_{30} = \frac{100\,({}^w\bar{M}^i_{30} - {}^w\bar{M}^i_{60}) - 100 \cdot \bar{N}^i_{80} \cdot ({}^wA^{vr}_{30} - {}^wA^{vr}_{60})}{N_{30} - N_{60}} = 1.3839\,.$$

CALCULATION OF TERMINAL RESERVES ON AN INDIVIDUAL BASIS

Terminal Reserves at End of Policy Year 4

a) $100 Waived to Age 40: Cases I, VI

$${}_4V^i_{30} = \frac{100\,({}^w\bar{M}^i_{34} - {}^w\bar{M}^i_{40}) - 100 \cdot {}_6\bar{A}^{nr}_{[40]}}{D_{34}} - 0.2480 \cdot \ddot{a}_{34:\overline{6}|} = -0.1785\,.$$

b) \$100 Waived to Age 45: Cases I, VI

$$ {}_{4}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{34} - {}^{w}\bar{M}^{i}_{45}) - 100\cdot{}_{11}\bar{A}^{nr}_{[45]}}{D_{34}} - 0.3224\cdot\ddot{a}_{34:\overline{11}|} = -0.0415\,. $$

c) \$100 Waived to Age 50: Cases I, VII

$$ {}_{4}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{34} - {}^{w}\bar{M}^{i}_{50}) - 100\,\{\bar{N}^{i}_{50}\cdot({}^{w}A^{vr}_{34} - {}^{w}A^{vr}_{36}) + {}_{14}\bar{A}^{nr}_{[50]}\}}{D_{34}} - 0.4137\cdot\ddot{a}_{34:\overline{16}|} = 0.2384\,. $$

d) \$100 Waived to Age 55: Cases I, VII

$$ {}_{4}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{34} - {}^{w}\bar{M}^{i}_{55}) - 100\,\{\bar{N}^{i}_{55}\cdot({}^{w}A^{vr}_{34} - {}^{w}A^{vr}_{41}) + {}_{14}\bar{A}^{nr}_{[55]}\}}{D_{34}} - 0.5532\cdot\ddot{a}_{34:\overline{21}|} = 0.7653\,. $$

e) \$100 Waived to Age 65: Cases II, VII

$$ {}_{4}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{34} - {}^{w}\bar{M}^{i}_{60}) - 100\,\{\bar{N}^{i}_{65}\cdot({}^{w}A^{vr}_{34} - {}^{w}A^{vr}_{51}) + ({}_{14}\bar{A}^{nr}_{[65]} - {}_{5}\bar{A}^{nr}_{[65]})\}}{D_{34}} - 1.0913\cdot\ddot{a}_{34:\overline{26}|} = 3.0217\,. $$

f) \$100 Waived to Age 80: Cases II, V

$$ {}_{4}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{34} - {}^{w}\bar{M}^{i}_{60}) - 100\cdot\bar{N}^{i}_{80}\cdot({}^{w}A^{vr}_{34} - {}^{w}A^{vr}_{60})}{D_{34}} - 1.3839\cdot\ddot{a}_{34:\overline{26}|} = 4.2458\,. $$

Terminal Reserves at End of Policy Year 12

b) \$100 Waived to Age 45: Cases I, VI

$$ {}_{12}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{42} - {}^{w}\bar{M}^{i}_{45}) - 100\cdot{}_{3}\bar{A}^{nr}_{[45]}}{D_{42}} - 0.3224\cdot\ddot{a}_{42:\overline{3}|} = -0.3144\,. $$

c) \$100 Waived to Age 50: Cases I, VI

$$ {}_{12}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{42} - {}^{w}\bar{M}^{i}_{50}) - 100\cdot{}_{8}\bar{A}^{nr}_{[50]}}{D_{42}} - 0.4137\cdot\ddot{a}_{42:\overline{8}|} = 0.1651\,. $$

d) \$100 Waived to Age 55: Cases I, VI

$$ {}_{12}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{42} - {}^{w}\bar{M}^{i}_{55}) - 100\cdot{}_{13}\bar{A}^{nr}_{[55]}}{D_{42}} - 0.5532\cdot\ddot{a}_{42:\overline{13}|} = 1.7030\,. $$

e) \$100 Waived to Age 65: Cases II, VII

$$ {}_{12}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{42} - {}^{w}\bar{M}^{i}_{60}) - 100\,\{\bar{N}^{i}_{65}\cdot({}^{w}A^{vr}_{42} - {}^{w}A^{vr}_{51}) + {}_{14}\bar{A}^{nr}_{[65]}\}}{D_{42}} - 1.0913\cdot\ddot{a}_{42:\overline{18}|} = 9.1514\,. $$

f) $100 Waived to Age 80: Cases II, V

$$ {}_{12}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{42} - {}^{w}\bar{M}^{i}_{60}) - 100\cdot N^{i}_{80}\cdot({}^{w}A^{vr}_{42} - {}^{w}A^{vr}_{60})}{D_{42}} - 1.3839\cdot\ddot{a}_{42:\overline{18|}} $$

$$ = 13.2527\,. $$

Terminal Reserves at End of Policy Year 21

d) $100 Waived to Age 55: Cases I, VI

$$ {}_{21}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{51} - {}^{w}\bar{M}^{i}_{55}) - 100\cdot{}_{4}\bar{A}^{nr}_{[55]}}{D_{51}} - 0.5532\cdot\ddot{a}_{51:\overline{4|}} = 0.4223\,. $$

e) $100 Waived to Age 65: Cases II, VI

$$ {}_{21}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{51} - {}^{w}\bar{M}^{i}_{60}) - 100\,({}_{14}\bar{A}^{nr}_{[65]} - {}_{5}\bar{A}^{nr}_{[65]})}{D_{51}} - 1.0913\cdot\ddot{a}_{51:\overline{9|}} $$

$$ = 13.5135\,. $$

f) $100 Waived to Age 80: Cases II, V

$$ {}_{21}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{51} - {}^{w}\bar{M}^{i}_{60}) - 100\,({}^{w}A^{vr}_{51} - {}^{w}A^{vr}_{60})}{D_{51}} - 1.3839\cdot\ddot{a}_{51:\overline{9|}} $$

$$ = 21.0144\,. $$

CALCULATION OF TERMINAL RESERVES ON AN AGGREGATE BASIS

Terminal Reserve at End of Policy Year 4: Cases I, I, I, III, III, VII, V

$$ {}_{4}V^{i}_{30} = \frac{600\,({}^{w}\bar{M}^{i}_{34} - {}^{w}\bar{M}^{i}_{40})}{D_{34}} + \frac{500\,({}^{w}\bar{M}^{i}_{40} - {}^{w}\bar{M}^{i}_{45}) - 100\cdot{}_{6}\bar{A}^{nr}_{[40]}}{D_{34}} $$

$$ + \frac{400\,({}^{w}\bar{M}^{i}_{45} - {}^{w}\bar{M}^{i}_{50}) - 100\cdot{}_{11}\bar{A}^{nr}_{[45]}}{D_{34}} $$

$$ + \frac{300\,({}^{w}\bar{M}^{i}_{50} - {}^{w}\bar{M}^{i}_{55}) - 100\,\{\bar{N}^{i}_{50}\cdot({}^{w}A^{vr}_{34} - {}^{w}A^{vr}_{36}) + {}_{14}\bar{A}^{nr}_{[50]}\}}{D_{34}} $$

$$ + \frac{200\,({}^{w}\bar{M}^{i}_{55} - {}^{w}\bar{M}^{i}_{60}) - 100\,\{\bar{N}^{i}_{55}\cdot({}^{w}A^{vr}_{34} - {}^{w}A^{vr}_{41}) + {}_{14}\bar{A}^{nr}_{[55]}\}}{D_{34}} $$

$$ - \frac{100\,\{\bar{N}^{i}_{65}\cdot({}^{w}A^{vr}_{34} - {}^{w}A^{vr}_{51}) + ({}_{14}\bar{A}^{nr}_{[65]} - {}_{5}\bar{A}^{nr}_{[65]})\}}{D_{34}} $$

$$ - \frac{100\cdot\bar{N}^{i}_{80}\cdot({}^{w}A^{vr}_{34} - {}^{w}A^{vr}_{60})}{D_{34}} - 0.2480\cdot\ddot{a}_{34:\overline{6|}} - 0.3224\cdot\ddot{a}_{34:\overline{11|}} $$

$$ - 0.4137\cdot\ddot{a}_{34:\overline{16|}} - 0.5532\cdot\ddot{a}_{34:\overline{21|}} - 1.0913\cdot\ddot{a}_{34:\overline{26|}} $$

$$ - 1.3839\cdot\ddot{a}_{34:\overline{26|}} = 8.0513\,. $$

The sum of the reserves calculated on an individual basis is 8.0512; this is identical to 3 places:

$$\begin{array}{r} -0.1785 \\ -0.0415 \\ +0.2384 \\ +0.7653 \\ +3.0217 \\ +4.2458 \\ \hline 8.0512 \end{array}$$

Terminal Reserve at End of Policy Year 12: Cases I, I, I, I, VII, V

$${}_{12}V^i_{30} = \frac{500\,({}^w\bar{M}^i_{42} - {}^w\bar{M}^i_{45})}{D_{42}} + \frac{400\,({}^w\bar{M}^i_{45} - {}^w\bar{M}^i_{50}) - 100\cdot{}_3\bar{A}^{nr}_{[45]}}{D_{42}}$$

$$+ \frac{300\,({}^w\bar{M}^i_{50} - {}^w\bar{M}^i_{55}) - 100\cdot{}_8\bar{A}^{nr}_{[50]}}{D_{42}} + \frac{200\,({}^w\bar{M}^i_{55} - {}^w\bar{M}^i_{60}) - 100\cdot{}_{13}\bar{A}^{nr}_{[55]}}{D_{42}}$$

$$- \frac{100\,\{\bar{N}^i_{65}\cdot({}^wA^{vr}_{42} - {}^wA^{vr}_{51}) + {}_{14}\bar{A}^{nr}_{[65]}\}}{D_{42}} - \frac{100\cdot\bar{N}^i_{80}\cdot({}^wA^{vr}_{42} - {}^wA^{vr}_{60})}{D_{42}}$$

$$- 0.3224\cdot\ddot{a}_{42:\overline{3}|} - 0.4137\cdot\ddot{a}_{42:\overline{8}|} - 0.5532\cdot\ddot{a}_{42:\overline{13}|}$$

$$- (1.0913 + 1.3839)\cdot\ddot{a}_{42:\overline{18}|} = 23.9579\,.$$

The sum of the reserves calculated on an individual basis is:

$$\begin{array}{r} -\ 0.3144 \\ +\ 0.1651 \\ +\ 1.7030 \\ +\ 9.1514 \\ +13.2527 \\ \hline 23.9578 \end{array}$$

(which is again identical to 3 decimal places).

Terminal Reserve at End of Policy Year 21: Cases I, II, VI, V

$${}_{21}V^i_{30} = \frac{300\,({}^w\bar{M}^i_{51} - {}^w\bar{M}^i_{55})}{D_{51}} + \frac{200\,({}^w\bar{M}^i_{55} - {}^w\bar{M}^i_{60}) - 100\cdot{}_4\bar{A}^{nr}_{[55]}}{D_{51}}$$

$$- \frac{100\,({}_{14}\bar{A}^{nr}_{[65]} - {}_5\bar{A}^{nr}_{[65]})}{D_{51}} - \frac{100\cdot\bar{N}^i_{80}\cdot({}^wA^{vr}_{51} - {}^wA^{vr}_{60})}{D_{51}} - 0.5532\cdot\ddot{a}_{51:\overline{4}|}$$

$$- (1.0913 + 1.3839)\cdot\ddot{a}_{51:\overline{9}|} = 34.9503\,.$$

The sum of the reserves calculated on an individual basis is:

$$\begin{array}{r} 0.4223 \\ 13.5135 \\ \underline{21.0144} \\ 34.9502 \end{array}$$

(which is again identical to 3 decimal places).

EXAMPLE 2: POLICY, WITH PREMIUM DOUBLING AT END OF TEN YEARS, ISSUED AT AGE 30; RIDER ADDED AT AGE 35

CALCULATION OF NET PREMIUMS

a) $100 Waived to Age 40; $200 Waived Thereafter for Life: Cases I, II

$$P^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{30} - {}^{w}\bar{M}^{i}_{40}) + 200\,({}^{w}\bar{M}^{i}_{40} - {}^{w}\bar{M}^{i}_{60}) + 100\cdot{}_{10}\bar{A}^{nr}_{[40]}}{N_{30} - N_{60}}$$

$$= 2.7095\,.$$

b) $50 Waived to Age 65: Cases II, VII

$$P^{i}_{35} = \frac{50\,({}^{w}\bar{M}^{i}_{35} - {}^{w}\bar{M}^{i}_{60}) - 50\,\{\bar{N}^{i}_{65}\cdot({}^{w}A^{vr}_{35} - {}^{w}A^{vr}_{51}) + ({}_{14}\bar{A}^{nr}_{[65]} - {}_{5}\bar{A}^{nr}_{[65]})\}}{N_{35} - N_{60}}$$

$$= 0.6569\,.$$

CALCULATION OF TERMINAL RESERVES

a) Terminal Reserve at Age 34: Cases I, II

$${}_{4}V^{i}_{30} = \frac{100\,({}^{w}\bar{M}^{i}_{34} - {}^{w}\bar{M}^{i}_{40}) + 200\,({}^{w}\bar{M}^{i}_{40} - {}^{w}\bar{M}^{i}_{60}) + 100\cdot{}_{6}\bar{A}^{nr}_{[40]}}{D_{34}} - 2.7095\cdot\ddot{a}_{34:\overline{26}|} = 9.4892\,.$$

b) Terminal Reserve (for Policy and Rider) at Age 37: Cases I, II, VII

$${}_{7}V^{i}_{30} = \frac{150\,({}^{w}\bar{M}^{i}_{37} - {}^{w}\bar{M}^{i}_{40})}{D_{37}} + \frac{250\,({}^{w}\bar{M}^{i}_{40} - {}^{w}\bar{M}^{i}_{60}) + 100\cdot{}_{3}\bar{A}^{nr}_{[40]}}{D_{37}}$$

$$- \frac{50\,\{\bar{N}^{i}_{65}\cdot({}^{w}A^{vr}_{37} - {}^{w}A^{vr}_{61}) + ({}_{14}\bar{A}^{nr}_{[65]} - {}_{5}\bar{A}^{nr}_{[65]})\}}{D_{37}} - 3.3664\cdot\ddot{a}_{37:\overline{28}|}$$

$$= 17.8556\,.$$

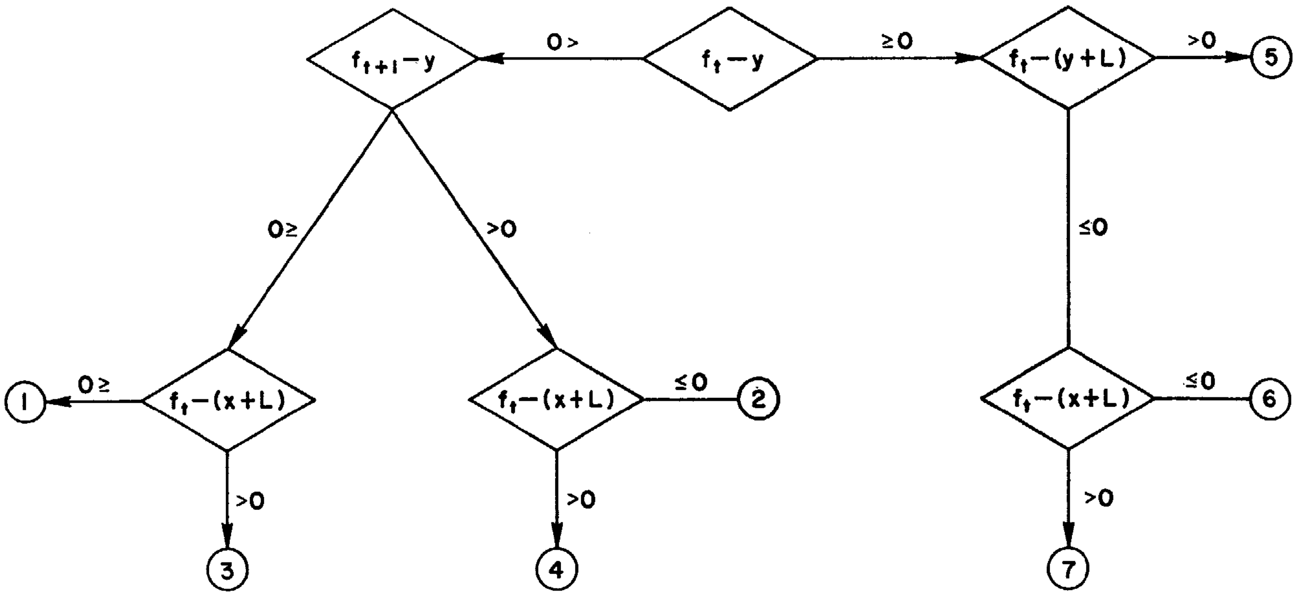
APPENDIX C

DISABILITY RESERVE SYSTEM DECISION CHART

Chart I demonstrates the tests gone through in determining the formula applicable for any $P(t)$ during the calculation of a net premium. A decision chart for the calculation of terminal reserves would be identical, except for $\overline{x+t}$ replacing x.

CHART I

DISABILITY RESERVE DECISION



①, Case I; ②, Case II; and so forth.