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GROSS PREMIUMS FOR TERM INSURANCE WITH VARYING BENEFITS AND PREMIUMS

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ABSTRACT

In this paper a model of term insurance experience is constructed on the basis of mortality classes theory and utility theory. The model generates a complete set of mortality, withdrawal, and conversion rates on the term insurance and of mortality rates on converted policies for any plan/age combination. The resulting decrement rates, which are believed to be realistic, can then be used in a Jenkins-type formula to calculate gross premiums.

After the basic operation of the model is explained, commencing with the section "Selection of Risks" several details which are necessary to the operation of the model are presented and discussed.

In the section "The Term Insurance Model Revisited" the results of several executions of the model are presented and compared to known data and expectations. Also, gross premiums are calculated which illustrate cost differentials between different plans of term insurance.

INTRODUCTION

ACTUARIES are often requested to make premium calculations for new and novel plans of term insurance. Oftentimes such premium calculations are based upon rates of mortality, rates of lapse, and rates of conversion which have been experienced on other, more familiar types of term insurance plans. This is usually done with full knowledge that the rates of decrement being used will not necessarily be exactly the same as those expected on the new plan; in view of the fact that experience does not exist on the new plan, however, the best available data have to be used.

Suppose that as an actuary you have been asked to calculate a premium for a new term insurance plan. The death benefits under this new term insurance plan are to be the following: \$1,000, years 1–5; \$667, years 6–10; \$333, years 11–15. This plan is to have level premiums throughout the period of coverage. What rates of lapse would you base your calculations on? What rates of conversion would you use, and what conversion single premiums would you use if the conversion period were to be five, ten, or fifteen years? Many problems can arise in choosing assumptions for such a coverage. For example, if the conversion period is to be a full fifteen years, it may well be expected that some of the impaired lives will convert their coverage fairly early, say, at the end of five or ten years, thus preserving their coverage amount and leaving the original body of policyholders with a slightly less aggregate mortality than otherwise. Should this be recognized, and, if so, what should the conversion cost be? Also, if the plan is to be nonconvertible, at the end of five years we may expect some of the healthier lives, upon seeing their coverage dropping to two-thirds of the former amount while they at the same time must pay the same premium, to be inclined to discontinue their coverage. If enough healthy lives do this, we may expect the mortality to turn somewhat worse among the persisting body of term insurance policyholders.

For another problem, consider the following plan of insurance which has been proposed. The coverage is to be level for fifteen years, but the premium at the end of five years is to rise by 50 per cent, remain level to the tenth premium, then go to a multiple of three of the initial premium for the remaining five years. This would be, in effect, the inverse of the step-down coverage just mentioned. In both cases, the effective premium rate to the policyholder is the same for each corresponding policy year. Would there be any reason to expect different experience on this plan in comparison with the step-down term plan? Naturally, there would be quite a bit of difference in the rate of exercise of the term conversion option.

The two coverages mentioned may be considered as just two of an infinite number of choices which could be designed. Concern on the author's part over the problems of adequate and equitable pricing for various plans of term insurance has led to a consideration of the dynamics of the decisions that the average policyholder will make—that is, his decisions on lapsation and conversion as well as his choice of a particular term plan in the first place. Before going into the model created to represent the dynamics of the actions of the body of term insurance policyholders, it is appropriate to examine the types of experience which have been accumulated under two plans of term insurance having either varying benefits or varying premiums.

EXPERIENCE ON TWO POPULAR PLANS OF TERM INSURANCE

A. Some characteristics of experience on level renewable convertible term insurance (usually convertible to 65 and renewable to 70) are:

1. Standard or better-than-standard mortality on converted policies bought before the end of the conversion period.
2. Substandard mortality (150–200 per cent) on converted policies bought at the last possible moment, that is, the end of the conversion period.
3. Higher lapse rates in the year just before a premium increase reflecting the probability of nonrenewal.
4. Higher mortality, especially at the later durations and especially in comparison with the mortality on decreasing term plans.

B. Some of the characteristics of experience on decreasing convertible term plans are:

1. Substandard mortality (about 165 per cent) on converted policies bought before and at the end of the conversion period.
2. Higher lapse rates in the second and subsequent policy years relative to the first-year lapse rate than are found with level renewable or nonrenewable term insurance. That is, the lapse rates are decidedly more level by duration.
3. Lower mortality, especially at the later durations, in view of the fact that some or all of the impaired lives will have converted to preserve their coverage.

These characteristics have been verified by my company's experience and the "Report on Mortality under Term Conversions and Guaranteed Insurability Options" (*TSA, 1968 Reports*). The mortality ratios cited for converted policies were computed, using as a basis for expected deaths, deaths expected according to duration since issue of the original term insurance policy.

Characteristics A, 4, and B, 3, are verifiable in the aggregate from the ratios in the accompanying tabulation. These are ratios of deaths on male standard medical business to expected deaths on the 1955–60 male select table in my company. Other term is mainly decreasing term but includes about 18 per cent of nonrenewable level term.

	By Number	By Amounts	Number of Deaths
Other term	94.2%	92.8%	242
Renewable term	112.2	142.9	167

A model which reflects known characteristics and the dynamics of decisions made by term insurance policyholders can be constructed. The model employed here uses two theories: (1) a "theory of mortality classes," as presented by Louis Levinson in Volume XI of the *Transactions*, and the (2) utility theory.

The model also employs the following assumptions:

1. At the time of receipt of notice of premium due (annual mode assumed) a re-examination of the benefits of the policy occurs. This re-examination takes into account the relationship of the benefits, premiums, and expected mortality.
2. Each policyholder has a clear and accurate estimate of his chances for survival for another year at each year end. Changes in prospects of longevity occur from time to time, but the prediction of the following year's chance of survival is good.

In the next two sections the two theories will be discussed.

BASIC THEORY OF MORTALITY CLASSES

This theory assumes that each of the persons in the living population at any given age x is classifiable by probability of death, which varies from close to zero to nearly unity. If x is a young age, most of the lives will not have deteriorated much and hence will be subject to a very low rate of mortality. A few, however, will be as near to death as the average man many years their senior. At the older ages the opposite will be true, with considerable deterioration affecting most lives but with a few as spry as twenty-year-olds. In the real population at a given age, then, the various lives are distributed in accordance with some frequency function. In order to facilitate a practical application of this theory, each of the lives at each age is assumed to be classifiable into one of fourteen different mortality strata. Each mortality stratum defines a unique probability of death. The fourteen mortality strata define a point distribution of which the points are chosen, arbitrarily, to cover the extremes of the range of mortality rates. If the probability of death for lives in stratum s is ${}_sq$ and the proportion of lives which are in stratum s (according to the frequency function) is ${}_s\rho_x$, then the rate of mortality in the population at age x will be

$$q_x = \sum_1^{14} {}_s\rho_x \cdot {}_sq \,. \tag{1}$$

The dynamics of the theory of mortality classes is embodied in the rates of deterioration. Although deterioration of the human organism probably occurs continuously, for practical purposes it is assumed to occur once each year. The rates of deterioration express the chance of remaining in the present stratum for one more year or of moving to any of the higher strata. No lives move to lower strata, since there is no inverse process to deterioration. Thus for stratum n there will be $(15 - n)$ probabilities (rates of deterioration), one representing the probability of remaining in stratum n and $(14 - n)$ representing the probability of jumping from stratum n to stratum $(n + 1)$, $(n + 2)$, . . . or (14). At a given age there will be 105 [$=(15 \cdot 14)/2$] rates of deterioration.

Let ${}_{s+t}^{\ \ s}a_x$ be the rate of deterioration which represents the probability that a life age x in stratum s at the beginning of the year will be in stratum $s + t$, $(t \geq 0)$, at age $x + 1$ at the end of the year, if it survives the year. Thus the distribution of the lives in the mortality strata at age $x + 1$ can be derived from the distribution at age x and the rates of deterioration.

$$ {}_u\rho_{x+1} = \frac{\sum_{s=1}^{s=u} {}_s\rho_x(1 - {}_sq)\,{}_u^s a_x}{\sum_{s=1}^{s=14} {}_s\rho_x(1 - {}_sq)} \qquad (u = 1, 2, \ldots 14)\,. \quad (2) $$

The rates of deterioration derived from and applicable to the general population are also applicable to any subgroup of the population, including, for example, insured lives. The only difference between the population and insured lives at any attained age is in their respective distribution among the fourteen mortality strata. They are both assumed to be subject to the same rates of deterioration.

In order to derive the strata distributions and the rates of deterioration, the following assumptions were used: (1) the distribution of lives into the fourteen strata was assumed to be binomial and (2) the $(15 - s)$ rates of deterioration for each stratum s were also assumed to follow a binomial distribution.

The population was assumed to exhibit mortality represented by the 1959–61 United States white males mortality rates interpolated one-half age to approximate mortality by age nearest birthday. From these rates of mortality (with excess accidental deaths of ages 17–31 removed), the above two assumptions, and formulas (1) and (2), 14 rho's and 105 rates of deterioration were derived for each attained age from 10 to 109.

For details on the methods of derivation, see Appendix B, pages 435 ff., of "Changes in American Mortality 1901—1949–1951," by Louis Levinson, in Volume II of the *1960 Transactions* of the XVI International Congress of Actuaries, or pages 55 ff. of "A Theory of Mortality Classes," by Louis Levinson, in Volume XI of the *1959 Transactions*.

Table 1 shows the mortality strata used. Table 2 shows the rates of deterioration at age 37. Table 4 (see p. 31) shows the distribution of lives among the mortality strata in the population at age 37 in column 2.

UTILITY THEORY

Utility theory provides a theoretical basis for predicting the choice a person will make when faced with two alternatives: (1) a certain payoff

(A) now or (2) a larger payoff (B) with a chance of p or no payoff with a chance of $(1 - p)$.

The mathematical value of these two alternatives is A for alternative 1 and pB for alternative 2. If the values of alternatives 1 and 2 were the same, most people would take the sum certain under alternative 1 instead of running the risk of getting nothing under alternative 2. In fact, most people will take a lesser value certain rather than take a chance on a much higher sum in spite of a higher mathematical expectation. Such persons are known as risk-averters. Their counterparts, the gamblers, will choose

TABLE 1

MORTALITY STRATA

Stratum s	Rate of Mortality $_sq$	Stratum s	Rate of Mortality $_sq$
1	0.000122	8	0.015616
2	.000244	9	.031232
3	.000488	10	.062464
4	.000976	11	.124928
5	.001952	12	.249856
6	.003904	13	.499712
7	0.007808	14	0.999424

TABLE 2

RATES OF DETERIORATION ($_{s+t}^{s}a_{37}$) AT AGE 37

($_{s+t}^{s}a_{37}$ = Probability of Transfer from Stratum s to Stratum $s + t$ at the End of the Year of Age 37)

s	t*			
	0	1	2	3
1	0.8572	0.1329	0.0095	0.0004
2	0.8669	.1245	.0082	.0003
3	0.8767	.1161	.0070	.0003
4	0.8863	.1076	.0059	.0002
5	0.8958	.0992	.0049	.0001
6	0.9050	.0909	.0040	.0001
7	0.9136	.0830	.0032	.0001
8	0.9216	.0758	.0026	0
9	0.9285	.0694	.0021	0
10	0.9340	.0643	.0017	0
11	0.9381	.0606	.0013	0
12	0.9403	.0588	.0009	0
13	0.9421	.0579	0	0
14	1.0000	0	0	0

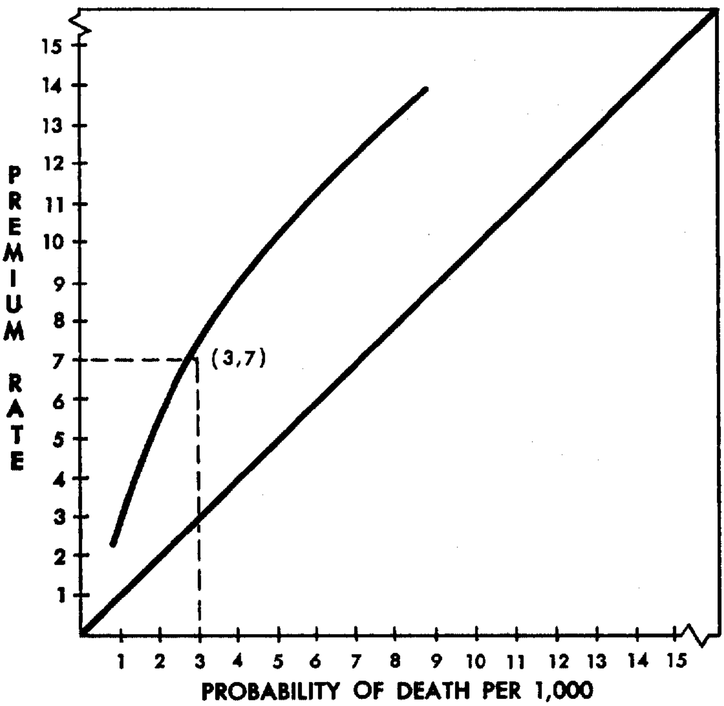
* Rates are negligible for values of $t \geq 4$.

alternative 2 even though it may have a smaller value than alternative 1. Most people are risk-averters or conservative by nature.

In order to adapt this theory to the insurance policyholder, we shall refer to Graph I. This graph assumes an insured of a given age x, \$1,000 of one-year renewable term insurance, and a chance of death which is known to him. This insured must decide whether or not to renew.

GRAPH I

UTILITY OF INSURANCE RENEWAL



We can see for the utility function shown that, if the probability of death is 3 per 1,000, the insured would be willing to spend \$7 or less on insurance to avoid the chance of his beneficiary not getting \$1,000 (or a net payoff to both of \$993, i.e., \$1,000 − \$7). To state this another way, the insured would be willing to pay 233 per cent of the cost in order to avoid the risk to the beneficiary. This ratio, that is, of premium rate to probability of death, would be expected to decrease as the probability of death increases. In the extreme case, of certain death within one year, the most any insured would be willing to pay would be \$1,000, so that, at that point, the ratio would be 100 per cent. At the other extreme, where the probability of death is infinitesimal, the insured would be willing to pay a large sum (relative to the loss expectancy) to avoid the risk. Thus, at this extreme there would be a huge ratio.

Graph I has thus illustrated the maximum premium that the insured would be willing to pay at various chances of death. In reality, however, premium rates are fixed once the risk classification has been determined, so that in our case, where the chance of death is 3 per 1,000, if the renewal premium rate for the insured's risk class were $8, the insured would decline to renew. Thus we can see that, with his particular utility curve, each insured will renew or not according as his maximum premium is more or less than the insurance company premium. For some individuals (utility curves) there will be no point at which the premium rate is economical; for others, renewal will always seem prudent; for still others, it will depend upon the rate of death expected or, stated another way, on the segment or part of the utility curve in question. This difference in utility curves reflects differences in risk-aversion attitudes on the part of the insureds. Risk-aversion attitudes, as far as insurance renewals are concerned, are a reflection of the individual's security needs, marital status, income, dependents' projected and present needs, and his awareness or acceptance of insurance.

An examination of the utility curves of all insureds at age x who are classified standard, with a current chance of death of 3 per 1,000, would reveal that a certain number of the curves fell below the standard renewal premium rate. These people would decline to renew insurance at standard rates. The remainder would renew. If we repeated this examination at each death rate, we would get a schedule showing, for each death rate, the percentage renewing. Since the ratio, mentioned earlier, of premium to probability of death is solely dependent upon the death rate at a given premium, we could produce a schedule showing the percentage renewing at each ratio. This schedule is called a "preference function" in this paper. Since all utility functions applicable to a given age will increase with increase in the probability of death, at the smaller probabilities of death (higher ratios) a smaller percentage will be willing to pay the fixed standard premium rate than the percentage at the higher probabilities of death (lower ratios).

This preference function will be seen to be analogous to the demand curves of economics by the following argument: The complement of the preference function can be graphed with the ratio along the y axis and the percentage renewing along the x axis. The demand curves of the theory of economics show the price along the y axis and the quantity bought or consumed along the x axis. These curves usually slope downward, showing a tendency to consume less with higher per unit prices. This will also be the case with the complement of the preference function, which will show, at the lower ratios (analogous to lower price), a large percentage renewing and, at the higher ratios, a small percentage renewing. This analogy is not coincidental but reflects the insured's desire to, within limits, maximize the utility of his fixed income (in the terminology of economics).

THE TERM INSURANCE MODEL

Now, at long last, after reviewing the theory of mortality classes and utility theory, we are finally ready to describe the term insurance model. Critical to the working of this term insurance model is the one-year outlook. At the time of the receipt of the premium notice, the policyholder looks ahead one year, knowing his expected mortality rate, the benefits under the policy during the ensuing year, the premium now due less the expected increase in cash value, the benefits on a converted policy during the ensuing year, and the premium under a converted policy. This one-year outlook according to our preference functions will determine the policyholder's desire to continue, lapse, or convert his insurance. It is certainly debatable whether all policyholders are so short-sighted as to look only one year ahead. However, for term insurance, it is a reasonable basis. The one-year outlook will be used exclusively in place of a better assumption, subject to some modifications later on. The exact ratio which will be used as the embodiment of the one-year outlook is discussed in the section "One-Year Outlook Ratios."

As a consequence of the risk selection process, the mortality strata distribution of recently underwritten risks (at standard rates) will show more lives in the lower mortality strata than would be the case for a random sample of lives from the population. A heuristic model for the determination of this initial distribution is presented later; however, assume for the moment that the distribution is known (see col. 4 Table 4). The term insurance model will then analyze the actions of the lives in each individual stratum. The process is repeated for each of the fourteen strata and then repeated for each duration of the term insurance contract.

To put together a picture of how the model operates, let us consider a group of people. Suppose, for example, a group is selected at standard rates and all are in stratum 2 at the beginning of the first year. A certain proportion (equal to ${}_2q$) of individuals classified in stratum 2 will die before the end of the first year. Then, at that time, a certain number will remain in stratum 2, not having deteriorated at all; a number will pass to stratum 3; a yet smaller number to stratum 4; and so on, all the way up to stratum 14. Then, at that time, after the deterioration has occurred, the remaining people will consider the possibility of converting to ordinary life insurance. There will, of course, be no reason to convert, unless we are discussing decreasing term insurance, in which case a certain amount of insurance will be lost forever unless conversion is effected at this duration or unless this is the end of the conversion period. In these cases, it is hypothesized that the policyholder looks at the increase in premium which would occur if he converted relative to the increase in benefit and the value of that benefit from his point of view.

In order to clarify this, suppose that the ordinary life premium is \$10 per thousand and that the premium under a decreasing term plan is \$2 per thousand initial amount, and suppose that the loss in coverage at a particular duration is \$100; in other words, it is a plan which decreases by \$100 at each duration per thousand initial amount. Thus if it is possible now to convert \$800, we would have a premium of \$8 per thousand initial amount of term insurance on the ordinary life insurance. This less \$2 is the increase in premium of \$6, and this would be measured against a benefit increase of \$100. Thus it would take a mortality rate of 6 per hundred to make the increase in premium and the increase in benefit equal (neglecting interest). The ratio is examined relative to the preference function. The ratio, if high, would indicate little or no desire to convert and vice versa. At this ratio a certain percentage according to the preference function would renew their insurance or, in this case, convert, and a complementary percentage would decline to convert. The latter would remain in the original body of term insurance policyholders.

The strata distribution of the lives converting can be carried forward exactly as described for lives in the general population in the section "Basic Theory of Mortality Classes." This distribution is carried forward to age 100 without recognition of any forces except the probabilities of death and deterioration. This is done because, in practice, there are very few withdrawals among converted policies. The series of mortality rates resulting can be used to calculate the present value of excess mortality on converted policies.

At this point, after the conversion possibility has been examined and is not exercised, the one-year outlook is examined relative to the preference function to determine whether lapsation or continuation of the insurance is called for. The ratio looked at in this case is the premium less the increase in (discounted) cash value, if any, relative to next year's benefit cost. Next year's benefit cost considers the expected mortality rate and the death benefit over and above the current cash value during the coming year. This ratio would be examined, as previously stated, relative to the preference function, and according to it a certain percentage of these individuals will then lapse.

To summarize, the process which continues year by year and repeats year after year is as follows: deaths occur; for the remaining persons, deterioration occurs; then the possibility of conversion is examined (if there is a loss in coverage); finally, the utility of continuing the insurance for one further year is examined.

An illustration of the calculation for the first year under a decreasing term insurance plan is shown in Table 3. The next few sections will look into some of the details of the term insurance model.

SELECTION OF RISKS

In applying the original theory of mortality classes to produce first-year select mortality, Levinson assumed that the strata distribution of new issues would be a weighted average of the strata distributions (in the population) at two ages, both younger than the age at issue. In this paper a different method was used to determine the initial distribution of standard select lives among the strata. The theory behind this method, although not unquestionable, may offer a partial insight into the actual distribution.

Let $q_{[x]}$ represent first-year mortality among lives aged x at issue. We assume that all or a given percentage of the population at each stratum apply for insurance. The justification for this is the idea that existing provision for insurance is meager and that the need is universal. Utility theory is not applied here because of the substantial influence of the agent. All the lives in strata with rates of mortality less than or equal to 115 per cent of $q_{[x]}$ would be accepted at standard rates. Let p equal the probability of being accepted at standard rates if expected mortality is 200 per cent of standard. Next, let p^2 represent the probability of being accepted at standard rates if expected mortality is 400 per cent of standard. The extension to higher ratios should be obvious. Through trial and error the value of p can be found which will give a strata distribution which has an average mortality rate of $q_{[x]}$.

Although, as is suspected, some of the most severely impaired lives will not even bother to apply for insurance, knowing that rejection or high rating is inevitable, and some of the super-healthy lives will choose not to buy insurance, the distribution of insurance applicants may be sufficiently similar to the population to avoid the necessity of determining the exact percentage at each stratum who apply. The method above also neglects to consider the chance of being misclassified and charged a substandard rate even though expected mortality is standard or less.

TABLE 3

ILLUSTRATION OF CALCULATION OF MORTALITY RATE, CONVERSION RATE, AND LAPSE RATE FOR POLICY YEAR 1 FOR DECREASING CONVERTIBLE TERM ISSUED AT AGE 37

Stratum s (1)	Rate of Mortality (2)	Units at Beginning of Year (Col. 4 of Table 4) (3)	Deaths during Year $(2)\cdot(3)$ (4)	Units at End of Year before Deterioration $(3)-(4)$ (5)	Units at End of Year after Deterioration $(6)_s=\sum_{t=1}^{s}(5)_t\,({}^t_s a)$ (6)	Conversion Dispreference Ratio* $0.3370\div(2)$ (7)	Proportion of Units Converting (Complement of Preference Function) (8)	Units Converting $(6)\cdot(8)$ (9)	Units at End of Year after Deterioration and Conversion $(6)-(9)$ (10)	Lapse Preference Ratio† $0.004379\div(2)$ (11)	Proportion of Units Lapsing (See Preference Function) (12)	Units Lapsing $(10)\cdot(12)$ (13)	Units at End of Year after Deterioration, Conversion and Lapse $(10)-(13)$ (14)
1.......	0.000122	322,863	39	322,824	276,725	2762.0	0.0	0	276,725	36.0	1.000‡	276,725	0
2.......	.000244	1,443,676	352	1,443,324	1,294,173	1381.0	0.0	0	1,294,173	18.0	0.560	724,737	569,436
3.......	.000488	2,979,403	1,454	2,977,949	2,793,470	690.5	0.0	0	2,793,470	9.0	0.150	419,021	2,374,449
4.......	.000976	3,757,585	3,667	3,753,918	3,684,820	345.0	0.0	0	3,684,820	4.5	0.000	0	3,684,820
5.......	.001952	1,208,835	2,360	1,206,475	1,505,995	172.5	0.0	0	1,505,995	2.0	0.000	0	1,505,995
6.......	.003904	245,989	960	245,029	364,241	86.0	0.0	0	364,241	1.0	0.000	0	364,241
7.......	.007808	37,079	290	36,789	62,516	43.0	0.0	0	62,516	0.5	0.000	0	62,516
8.......	.015616	4,192	65	4,127	8,022	21.5	0.335	2687	5,335	0.5	0.000	0	5,335
9.......	.031232	355	11	344	779	11.0	0.750	584	195	0.0	0.000	0	195
10.......	.062464	22	1	21	57	5.5	1.000	57	0	0.0	0.000	0	0
11.......	.124928	1	0	1	3	2.5	1.000	3	0	0.0	0.000	0	0
12.......	.249856	0	0	0	0	1.5	1.000	0	0	0.0	0.000	0	0
13.......	.499712	0	0	0	0	1.0	1.000	0	0	0.0	0.000	0	0
14.......	0.999424	0	0	0	0	0.5	1.000	0	0	0.0	0.000	0	0
Total...	....	10,000,000	9,199	9,990,801	9,990,801	....	....	3331	9,987,470	....	....	1,420,483	8,566,987

* See section on "One-Year Outlook Ratios." Whole life basic rate at age 38 is $20.20. No term conversion allowance. Amount convertible is $1,000. Coverage lost if not converted is $50. Decreasing term premium is $4 $(20.20-4.00)\cdot(1.04)\div(50_sq)$. Rounded to nearest one-half.

† See section on "One-Year Outlook Ratios." Decreasing term premium at beginning of year is $4. No cash values on plan. Benefit per unit throughout year 2 is $950 $(4.00\cdot1.04)\div(950_sq)$. Rounded to nearest one-half.

‡ Note that, while this implies that all of the 276, 725 units left in stratum 1 lapse, of the 322,863 originally in stratum 1, 39 died and 46,099 deteriorated to higher strata.

Table 4 shows the application of the above theory in derivation of the initial distribution at age 37.

ONE-YEAR OUTLOOK RATIOS

Let (*subscript* [x] *omitted for convenience*)

${}_t\text{BR}$ = Basic rate per \$1,000 initial amount payable on the term insurance at the beginning of policy year t.

${}_tB$ = Benefit on term insurance of \$1,000 initial amount throughout policy year t. Amount of term insurance which can be converted at the end of policy year t if the term insurance is convertible for t or more years.

${}_t\text{CV}$ = Cash value on term insurance of \$1,000 initial amount at the end of policy year t.

$\Delta_t\text{CV} = {}_{t+1}\text{CV} - {}_t\text{CV}$.

${}_t\text{WLBR}$ = Basic rate per \$1,000 of whole life converted at end of term insurance policy year $t-1$, payable at beginning of policy year t.

${}_t\text{CP} = {}_t\text{WLBR}\cdot{}_{t-1}B/1{,}000$ = Conversion premium payable at beginning of policy year t if term insurance converted in full at end of policy year $t-1$.

TABLE 4

STRATA DISTRIBUTIONS OF LIVES APPLYING FOR STANDARD INSURANCE, LIVES RATED OR REJECTED, AND LIVES PAYING FOR STANDARD INSURANCE AT AGE 37

Stratum s (1)	Applied for Standard ${}_sp_{37}\times 15{,}067{,}448$ (2)	Rated or Rejected (3)	Paid for Standard* (2)−(3) (4)
1........	322,863	0	322,863
2........	1,443,676	0	1,443,676
3........	2,979,403	0	2,979,403
4........	3,757,585	0	3,757,585
5........	3,231,116	2,022,281	1,208,835
6........	2,000,476	1,754,487	245,989
7........	917,450	880,371	37,079
8........	315,570	311,378	4,192
9........	81,409	81,054	355
10........	15,549	15,527	22
11........	2,140	2,139	1
12........	196	196	0
13........	15	15	0
14........	0	0	0
Total....	15,067,448	5,067,448	10,000,000

* Assuming no not-takens among those accepted at standard rates

${}_t\mathrm{TCA}$ = Term conversion allowance over and above the cash value at the end of policy year t.

${}_sq$ = Rate of mortality for lives in stratum s.

i = Rate of interest policyholder can realize in safe investments.

Ratio for lapse preference:

$$\frac{{}_{t+1}\mathrm{BR} - (1 - {}_sq)\left(\frac{1}{1+i}\right)(\Delta_t\mathrm{CV})}{({}_{t+1}B - {}_t\mathrm{CV})({}_sq)\left(\frac{1}{1+i}\right)}.$$

This ratio expresses the incremental cost (numerator) at the end of policy year t of continuing the insurance for one year versus the death benefit protection (denominator) which will be received as a result of continuation. An increase in cash value during the ensuing year will decrease cost. The cash value on the policy at the end of year t is considered to be an asset of the policyowner which is payable in the event of death as part of the death benefit under the policy.

Ratio for conversion dispreference:

$$\frac{\text{Increase in cost caused by conversion}}{\text{Increase in benefit value resulting from conversion}}$$

$$= \frac{{}_{t+1}\mathrm{CP} - {}_t\mathrm{TCA} - \left[{}_{t+1}\mathrm{BR} - (1 - {}_sq)\left(\frac{1}{1+i}\right)(\Delta_t\mathrm{CV})\right]}{[({}_tB) - ({}_{t+1}B - {}_t\mathrm{CV})]({}_sq)\left(\frac{1}{1+i}\right)}$$

$$= \frac{{}_{t+1}\mathrm{CP} - {}_t\mathrm{TCA} - {}_{t+1}\mathrm{BR} + (1 - {}_sq)\left(\frac{1}{1+i}\right)(\Delta_t\mathrm{CV})}{({}_tB - {}_{t+1}B + {}_t\mathrm{CV})({}_sq)\left(\frac{1}{1+i}\right)}.$$

This is called the "ratio for conversion dispreference" because a high value would indicate a small percentage of conversions and a low value, a large percentage. It is exactly opposite in operation to the ratio for lapse preference. In accordance with our later definition of selective conversion, ${}_tB$ will always be greater than ${}_{t+1}B$ when this ratio is used. It is also assumed that the first-year cash value and dividend on the converted whole life are zero. No term is included for the fact that the cash value on the term policy at duration t would be payable as an offset to the initial cost of conversion, since this is already an asset of the policyowner. At the end of the coverage period ${}_{t+1}\mathrm{BR}$, ${}_{t+1}B$, and ${}_t\mathrm{CV}$ would, of course, be zero.

PRACTICAL VARIATIONS IN PREFERENCE FUNCTIONS

In the section entitled "Utility Theory," I stated that risk-aversion attitudes and hence utility functions and preference functions were a reflection of the individual's security needs, marital status, income, projected and present needs of dependents, and awareness or acceptance of insurance. In practice, in looking at a preference function which has been computed for a particular company, we may expect variations by age chiefly as a result of differences in marital status and dependency status by age.

Also, we might expect differences by length of coverage period, since the people buying longer-term coverage with slightly higher premiums will be giving some consideration to guarantees for the future in paying the higher premium and thus would not be that much more prone to lapse than the people buying short-term insurance. Since a higher premium will produce a higher preference for lapse with the same preference function, a difference preference function is required to reflect plan differences.

Another substantial source of variation in the preference function, as among different companies or as among plans, would result from the purpose of the insurance. A good example of this variation is, in my company, an informal study of purpose of term insurance by plan, which showed that a renewable term plan had 41.3 per cent of the amounts purchased being purchased as business insurance, that is, key-man protection, buy-sell agreements, or loans, whereas on a decreasing term insurance plan only 34.2 per cent were for business purposes. I believe that it is conjecturable that preference functions would be somewhat steeper for business insurance; that is, the probability of lapse, for the same probability of death, would be somewhat higher for business insurance than it is for insurance purchased for burial purposes or replacement of income.

Interesting in this regard is the following quotation of C. O. Shepherd in *RAIA*, *Volume XXXII* (June, 1943):

> We can also expect a more severe selection on large policies, particularly in the case of business insurance where there are less likely to be neutralizing influences to prevent exercise of an option advantageous to the beneficiary. I question if it isn't a misuse of the conversion option to include it in large policies, particularly business insurance contracts.

One other substantial variation in preference function occurs when the insurance was purchased with the intention of lapsation at a certain duration. It is expected here that the preference function will show a much larger percentage lapsing at the same mortality rate than in the ordinary preference function. One example of this is the lack of, or largely practical lack of, shorter-term coverage in the past, which caused five-year renewable and convertible term to be bought frequently for business situations in which the need for insurance protection was known to exist for only three, four, or five years, it being planned that at the end of three, four, or five years the insurance would be lapsed. Thus we see the necessity for somewhat steeper preference function at these durations on five-year renewable and convertible term.

DERIVATION AND PRACTICAL USE OF THE PREFERENCE FUNCTION

The ratio can be described as the multiple of cost the insured is willing to pay. Thus, in practice, we would expect that at a ratio of unity (cost equaling benefits) there would be 100 per cent retaining their insurance. It is also to be expected that there would be some very large ratio at

TABLE 5

PREFERENCE FUNCTION

Ratio	Per Cent Lapsing	Ratio	Per Cent Lapsing
6 and under	0%	18	56%
7	5	20	62
8	10	24	74
10	20	28	86
12	30	32	98
14	40	36 and over	100
16	50		

which almost 100 per cent would lapse their insurance. In order to derive a preference function, one-year outlook ratios were calculated for several plan, age, and duration cells, using the 1955–60 select male mortality rates in place of $_sq$ in the equation for the one-year outlook ratio for lapse preference. These ratios were correlated with lapse-rate experience in the corresponding cells. The correlations were very high in most cases. Therefore least-squares lines were fitted to the data with the one-year outlook ratio as the independent variable. For ratios of 2–6 (the practical range of the experience) the slope of the least-squares line was 4–8. For the final preference function a slope of 5 was used for ratios below 16 and of 3 for above 16. The level or starting point for the function was determined to produce the desired first-year lapse rate. Characteristic values of the function are shown in Table 5. In order to reflect variations in the preference function, in the actual calculations a constant was added to or subtracted from the calculated ratio before the above preference function was used. In this manner variations were recognized approximately without excessive detail. In general an addition to the calculated ratio will increase the lapse rate. This adjustment was employed in the five-year renewable and convertible term calculations by adding 3 to the calculated ratios at duration 5 only. This is in recognition of the tendency to buy five-year renewable and convertible term as a substitute for shorter-term coverage. This was the only case where it was necessary to modify the preference function by duration. In all other cases modification by issue age alone was sufficient.

Considerable care in the ratio calculation and choice of preference function is necessary at the younger ages. This is true because the first few select mortality rates among male insureds aged 20–24 are largely affected by the comparative rate of accidental fatalities among single and married insureds and by the proportion of single and married insureds. An informal study and government statistics show the following:

ACCIDENTAL FATALITIES RATES

(Males Aged 20–24)

	Per 1,000
Single	1.59
Married	0.49
Combined (in proportion existing in general population)	1.07
Combined (in proportion to amounts of insurance sold)	0.73

Newly insured males aged 20–24 are thus seen to be two distinct groups: (1) the single insureds, holding 21 per cent of the insurance and experiencing an accidental death rate of 1.59 per 1,000, and (2) the married insureds, holding 79 per cent of the insurance and experiencing an accidental death rate of 0.49 per 1,000 (31 per cent of the rate among single males). The single males will also have a different preference function at the same ratio. These problems of application are negligible at age groups 25–29 and above, since nearly all insureds will be married at these ages.

CONVERSIONS

The rate of conversion as used in the model refers solely to selective conversions, that is, conversions which occur just before a drop in coverage or at the end of the conversion period under the term insurance contract. Selective conversions preserve the benefit amount currently in force and thus include a high percentage of impaired lives. Nonselective conversions, in contrast, occur uniformly across the mortality spectrum. The mortality on converted policies converted as a result of nonselective conversions will be standard when measured by the duration from issue of the term insurance contract. Nonselective conversions occur when the in-force amount is level from year to year and before the end of the conversion period. Although there may be, theoretically, a small increase in death benefit, since the cash value is returned at time of conversion, we will include conversions which involve a term insurance cash value which otherwise qualify as nonselective in nonselective conversions. Selective conversions, although some healthy lives may be included, usually occur just prior to a drop in coverage or at the end of the conversion period.

Nonselective conversions, being, in essence, a continuation of the present coverage on another plan, are not included in the lapse rates derived from the one-year outlook ratios and the preference function. They can be considered excess lapses. There will be a gain from underwriting savings on the converted policy but a loss on surrender as a result of these conversions. To the extent that the gain does not offset the loss, the calculated gross premium will be deficient. This factor would be very small in practice and has been neglected in our calculations.

All selective conversions to plans having a higher premium and smaller net amount at risk (than whole life) would result in a gain because of the smaller conversion single premiums required. Direct reflection in the premium calculation of this factor as well as of the gain or loss on nonselective conversions would require extensive actual statistics.

In a consideration of the probability of conversion at the end of the conversion period when it precedes the end of the coverage period, theoretically there will be some severely impaired lives who will choose to remain with the term insurance rather than convert, since they feel certain that they will die before the end of the term insurance coverage period. Practically, however, where the interval between the end of the two periods in question is five years or less (especially at the younger ages), this effect would be small. Therefore, this analysis treats such conversions as if the end of the coverage period coincided with the end of the conversion period.

A PRACTICAL SHORTCOMING OF THE MODEL AND A REMEDY THEREFOR

On the commonly sold, uniformly decreasing term plans with premiums payable for about 80 per cent of the coverage period, the one-year outlook one year before the end of the premium payment period would be cause for high lapsation (according to the model). The one-year outlook shows a modest benefit (for example, $200–$300 per $1,000 initial amount) compared to a large premium (same as was paid in policy year 1). A high lapse rate is hardly to be expected, because with the payment of just one more premium three or more years of paid-up coverage are obtained. The chief reason for this anomaly is the exemption of these plans from the requirement of provision for cash values. Were cash values present, the benefits would be larger relative to the premium less increase in cash value. One solution would be to use cash values or natural reserves (asset share with profit deducted) in the computation of lapse rates by the method of this paper but not in the premium calculation. Another solution would be to grade the last few lapse rates into zero at the end of the premium-payment period, neglecting those developed by the model.

THE TERM INSURANCE MODEL REVISITED

In Tables 6–13, summaries of the results of the term insurance model for various plans are shown. A multiple decrement table shows units persisting and units terminating by death, conversion, and withdrawal. Also, the conversion single premium is shown, which is the cost per unit converting. Also, for convenience, the independent rates of decrement have been calculated. It should be noted that the basic rate shown is not necessarily the final basic rate, but only a trial basic rate for use in the calculation of the one-year outlook ratios.

In Table 6 we have shown the results under five-year term renewable to 70 and convertible to 65 issued at age 37. It is interesting to compare the results in Table 6 with those exhibited in the second section of this paper. In order to facilitate this comparison, calculations have been made employing a model office using three different ages on this plan and assuming an annual growth rate in new business of 10 per cent per year. Characteristic A, 2, that is, higher mortality on policies converted at the last possible moment, is verified by our model office, which shows that we could expect a mortality ratio of 482.8 per cent in the first year of insurance on converted policies converted from five-year renewable and convertible term at attained age 65. This percentage is relative to that expected on newly underwritten risks at age 65. This compares with a ratio of 404 per cent shown in Table 4 (Part A) on page 69 of the most recent study of "Mortality under Term Conversions and Guaranteed Insurability Options" (*TSA, 1968 Reports*, the Society of Actuaries). Also, in Part B (on expected mortality according to duration from term insurance issue) of that same study (on p. 98), Table 13 shows a ratio of 150 per cent (during the first fifteen policy years) for conversions at the end of the conversion period where such conversions are from renewable and convertible term. This compares with a ratio of 130-133 per cent shown in our model on last-minute conversions (at age 65) from five-year renewable and convertible term issued at age 57 only. Next, looking at characteristic A, 3, which stated that a higher lapse rate would be expected in a year just before a premium increase, we see that our lapse rate jumps to 14.55

TABLE 6

ANALYSIS OF EXPERIENCE ON TERM INSURANCE
(Five-Year Renewable and Convertible Term)

Attained Age	t	Plan Characteristics per Unit in Force: Benefit ${}_tB$	Basic Rate ${}_t\text{BR}$	Cash Value ${}_t\text{CV}$	Multiple Decrement Table: Units Persisting $l_{[x]+t-1}$	Units Terminating by: Death $d^{d}_{[x]+t-1}$	Conversion $d^{c}_{[x]+t}$	Withdrawal $d^{w}_{[x]+t}$	Independent Rates of Decrement*: $q^{d}_{[x]+t-1}$	$q^{55\text{–}60\ \text{male}}_{[x]+t-1}$	$q^{c}_{[x]+t}$	$q^{w}_{[x]+t}$	Conversion Single Premium $\text{CSP}_{[x]+t}$
37..........	1	1,000	5.68	0.0	10,000,000	9,201	0.0	2,072,454	0.92	0.92	0.00	20.74	....
38..........	2	1,000	5.68	0.0	7,918,346	9,636	0.0	730,426	1.22	1.18	0.00	9.24	....
39..........	3	1,000	5.68	0.0	7,178,283	10,614	0.0	359,230	1.48	1.50	0.00	5.01	....
40..........	4	1,000	5.68	0.0	6,808,439	11,904	0.0	197,197	1.75	1.74	0.00	2.90	....
41..........	5	1,000	5.68	0.0	6,599,338	13,413	0.0	958,057	2.03	1.99	0.00	14.55	....
42..........	6	1,000	7.63	0.0	5,627,868	14,337	0.0	223,330	2.55	2.27	0.00	3.98	....
43..........	7	1,000	7.63	0.0	5,390,201	16,087	0.0	149,206	2.98	2.61	0.00	2.78	....
44..........	8	1,000	7.63	0.0	5,224,908	18,078	0.0	104,942	3.46	2.94	0.00	2.02	....
45..........	9	1,000	7.63	0.0	5,101,888	20,344	0.0	76,221	3.99	3.34	0.00	1.50	....
46..........	10	1,000	7.63	0.0	5,005,323	22,884	0.0	152,026	4.57	3.83	0.00	3.05	....
47..........	11	1,000	10.87	0.0	4,830,413	25,692	0.0	90,549	5.32	4.37	0.00	1.88	....
48..........	12	1,000	10.87	0.0	4,714,172	29,023	0.0	53,617	6.16	4.94	0.00	1.14	....
49..........	13	1,000	10.87	0.0	4,631,533	32,908	0.0	31,612	7.11	5.46	0.00	0.69	....
50..........	14	1,000	10.87	0.0	4,567,014	37,291	0.0	18,700	8.17	6.22	0.00	0.41	....
51..........	15	1,000	10.87	0.0	4,511,022	41,975	0.0	111,562	9.31	7.26	0.00	2.50	....
52..........	16	1,000	16.02	0.0	4,357,485	46,570	0.0	75,245	10.69	8.32	0.00	1.75	....
53..........	17	1,000	16.02	0.0	4,235,670	51,162	0.0	53,166	12.08	9.20	0.00	1.27	....

* $q^{d}_{[x]+t-1}$ and $q^{55\text{–}60\ \text{male}}_{[x]+t-1}$, rate per 1,000; $q^{c}_{[x]+t}$ and $q^{w}_{[x]+t}$, rate per 100.

TABLE 6—*Continued*

Attained Age	t	Plan Characteristics per Unit in Force: Benefit ${}_tB$	Basic Rate ${}_t$BR	Cash Value ${}_t$CV	Multiple Decrement Table: Units Persisting $l_{[x]+t-1}$	Units Terminating by: Death $d^{d}_{[x]+t-1}$	Conversion $d^{c}_{[x]+t}$	Withdrawal $d^{w}_{[x]+t}$	Independent Rates of Decrement*: $q^{d}_{[x]+t-1}$	$q^{55-60\text{ male}}_{[x]+t-1}$	$q^{c}_{[x]+t}$	$q^{w}_{[x]+t}$	Conversion Single Premium CSP$_{[x]+t}$
54	18	1,000	16.02	0.0	4,131,342	55,713	0.0	38,541	13.49	10.09	0.00	0.95	....
55	19	1,000	16.02	0.0	4,037,088	60,319	0.0	28,266	14.94	11.00	0.00	0.71	....
56	20	1,000	16.02	0.0	3,948,502	65,078	0.0	77,091	16.48	12.06	0.00	1.99	....
57	21	1,000	24.05	0.0	3,806,333	70,237	0.0	47,230	18.45	13.26	0.00	1.26	....
58	22	1,000	24.05	0.0	3,688,866	76,124	0.0	29,748	20.64	14.60	0.00	0.82	....
59	23	1,000	24.05	0.0	3,582,994	82,713	0.0	19,422	23.08	16.06	0.00	0.55	....
60	24	1,000	24.05	0.0	3,480,859	89,671	0.0	13,217	25.76	17.69	0.00	0.39	....
61	25	1,000	24.05	0.0	3,377,971	96,581	0.0	52,993	28.59	19.55	0.00	1.61	....
62	26	1,000	35.94	0.0	3,228,396	103,044	0.0	34,462	31.92	21.61	0.00	1.10	....
63	27	1,000	35.94	0.0	3,090,890	109,185	0.0	22,800	35.32	23.75	0.00	0.76	....
64	28	1,000	35.94	0.0	2,958,906	114,884	2,724,855.0	8,569	38.83	25.83	95.81	7.19	224.39
65	29	1,000	35.94	0.0	110,599	708	0.0	5,680	6.40	27.99	0.00	5.17	....
66	30	1,000	35.94	0.0	104,210	830	0.0	9,480	7.96	30.34	0.00	9.17	....
67	31	1,000	49.88	0.0	93,901	953	0.0	5,212	10.15	33.04	0.00	5.61	....
68	32	1,000	49.88	0.0	87,736	1,115	0.0	2,965	12.71	35.92	0.00	3.42	....
69	33	1,000	49.88	0.0	83,656	1,312	0.0	0	15.68	39.27	0.00	0.0	....

* $q^{d}_{[x]+t-1}$ and $q^{55-60\text{ male}}_{[x]+t-1}$, rate per 1,000; $q^{c}_{[x]+t}$ and $q^{w}_{[x]+t}$, rate per 100.

per cent in the fifth year, reflecting the probability of nonrenewal. With regard to the characteristic A, 4, we can see quite clearly that the mortality rates for most durations are considerably above those shown by the 1955–60 male select table. Under the model office we would expect a mortality ratio of 123.9 per cent. This compares quite favorably with the percentages shown in my company and cited earlier in this paper. It is interesting to note the drop in mortality relative to the 1955–60 rates at attained age 66, just after a large percentage of the unhealthy lives have converted.

Table 7 shows the results under a twenty-year convertible annually decreasing term insurance issued at age 37. We have also compared the results on this plan under a model office at three different ages, assuming a 10 per cent increase in new business. In characteristic B, 1, we said that we would expect substandard mortality on converted policies bought before the end of the conversion period. The model office calculations yield a mortality ratio of 507.5 per cent on conversions from this decreasing term policy before the end of the (twenty-year) conversion period. This compares with a ratio of 421 per cent on page 73 of the report alluded to earlier. To reiterate, these ratios are on the first year of experience under converted policies relative to the mortality expected on policies underwritten at the attained age (Part A of the study). On conversions at the end of the conversion period from twenty-year decreasing term, our model office shows an aggregate expected mortality ratio of 197 per cent. This compares with 357 per cent shown on page 73 of the *TSA 1968 Reports*. While the agreement of these two ratios with those found from experience is not exact, they are comparable considering the variations possible. For instance, in the actual experience data used in the construction of the mortality ratios in the *1968 Reports* there may be some data on decreasing term policies which contained a conversion privilege allowing conversion of only 80 per cent of the amount in force. Also, there would be decreasing term plans of various terms of coverage in the experience. I believe these sources of diversity could account for the differences. In characteristic B, 2, was mentioned the tendency on decreasing term plans to experience somewhat more level lapse rates by duration. This characteristic is quite evident in Table 7. The fact of lower mortality on the active lives or the lives remaining within the original group of term policyholders (characteristic B, 3) is evident on comparison of the rates of mortality with those of the 1955–60 male select table. On the model office we obtained a mortality ratio of 83.5 per cent, which compares favorably with that shown in the second section of this paper. You may notice that the multiple decrement table of the first line of Table 7 is not in exact

TABLE 7

ANALYSIS OF EXPERIENCE ON TERM INSURANCE

(Twenty-Year Convertible Decreasing Term)

Attained Age	t	Plan Characteristics per Unit in Force: Benefit $_tB$	Basic Rate $_tBR$	Cash Value $_tCV$	Multiple Decrement Table: Units Persisting $l_{[x]+t-1}$	Units Terminating by: Death $\overset{d}{d}_{[x]+t-1}$	Conversion $\overset{c}{d}_{[x]+t}$	Withdrawal $\overset{w}{d}_{[x]+t}$	Independent Rates of Decrement*: $\overset{d}{q}_{[x]+t-1}$	$\overset{55\text{-}60\text{ male}}{q}_{[x]+t-1}$	$\overset{c}{q}_{[x]+t}$	$\overset{w}{q}_{[x]+t}$	Conversion Single Premium $CSP_{[x]+t}$
37	1	1,000	4.00	0.0	10,000,000	9,201	3,332	1,420,482	0.92	0.92	0.03	14.22	248.68
38	2	950	4.00	0.0	8,566,985	9,833	5,028	656,922	1.15	1.18	0.06	7.68	240.68
39	3	900	4.00	0.0	7,895,202	10,811	8,586	407,909	1.37	1.50	0.11	5.18	237.44
40	4	850	4.00	0.0	7,467,896	11,985	13,990	270,610	1.60	1.74	0.19	3.64	235.58
41	5	800	4.00	0.0	7,171,311	13,253	21,963	198,823	1.85	1.99	0.31	2.79	233.03
42	6	750	4.00	0.0	6,937,272	14,647	32,491	128,314	2.11	2.27	0.47	1.86	230.79
43	7	700	4.00	0.0	6,761,820	16,073	46,456	137,768	2.38	2.61	0.69	2.06	227.13
44	8	650	4.00	0.0	6,561,523	17,405	61,598	139,177	2.65	2.94	0.94	2.15	224.52
45	9	600	4.00	0.0	6,343,344	18,650	80,897	135,744	2.94	3.34	1.28	2.17	220.80
46	10	550	4.00	0.0	6,108,052	19,727	107,586	150,611	3.23	3.83	1.77	2.52	211.73
47	11	500	4.00	0.0	5,830,128	20,588	218,096	144,292	3.53	4.37	3.75	2.58	166.70
48	12	450	4.00	0.0	5,447,152	20,597	336,356	122,368	3.78	4.94	6.20	2.40	145.80
49	13	400	4.00	0.0	4,967,831	19,596	437,300	101,728	3.94	5.46	8.84	2.26	127.72
50	14	350	4.00	0.0	4,409,207	17,712	493,938	139,841	4.02	6.22	11.25	3.59	113.65
51	15	300	4.00	0.0	3,757,717	15,053	488,913	177,210	4.01	7.26	13.06	5.45	98.76
52	16	250	4.00	0.0	3,076,541	12,175	661,217	194,732	3.96	8.32	21.58	8.10	56.17
53	17	200	4.00	0.0	2,208,417	8,471	0	119,360	3.84	9.20	0.0	5.43	0.0
54	18	200	4.00	0.0	2,080,586	9,518	0	76,324	4.57	10.09	0.0	3.69	0.0
55	19	200	4.00	0.0	1,994,744	10,731	0	49,639	5.38	11.00	0.0	2.50	0.0
56	20	200	4.00	0.0	1,934,374	12,130	1,554,355	0	6.27	12.06	80.86	0.0	31.81

* $\overset{d}{q}_{[x]+t-1}$ and $\overset{55\text{-}60\text{ male}}{q}_{[x]+t-1}$, rate per 1,000; $\overset{c}{q}_{[x]+t}$ and $\overset{w}{q}_{[x]+t}$, rate per 100.

agreement with that of the totals for Table 3. The reason for the discrepancy is rounding. Table 8 shows for comparative purposes the experience under a twenty-year annually decreasing term with the same benefits as those in Table 7 but without the conversion privilege. You will note how much higher the mortality rates are, especially at the later durations. This, of course, results from a locking-in of the impaired lives.

Table 9 shows the results under a fifteen-year level term for comparative purposes. The results in this case are particularly significant, since neither the benefits nor the premiums vary. The results show that the mortality rates for policy years 1–15 are very close to the 55–60 male select rates and that the lapse rates are close to a typical pattern of lapse rates, although they seem a little low for durations 7–15. It is well to keep in mind that all rates—lapse, mortality, and conversion—were in a sense generated on the assumption of first-year lapse and mortality rates only.

Tables 10, 11, 12, and 13 show the experience under the special decreasing term plan which was mentioned in the introduction. Table 10 shows this plan without convertibility; Table 11, with five-year convertibility; Table 12, with ten-year convertibility; and Table 13, with fifteen-year convertibility.

In order to analyze the financial effects of these various types of expected experience, gross premiums have been computed for various plans of insurance. These computations have been based on a hypothetical set of assumptions. These assumptions are not necessarily representative of my company or any other company. The assumptions used are shown in the Appendix. Table 14 shows the gross premiums for several plans of term insurance.

CONCLUSION

In summary, it has been shown how the term insurance model reflects the decisions that the average group of policyholders will make when faced with various benefit and premium levels as measured against their utility functions. Analysis such as this can be used to determine premiums for new and experimental plans which heretofore could not have been offered without the aid of this tool. It is to be hoped that actuaries will pursue and develop the model described in this paper and apply it to other situations in which expected mortality, utility functions, and premiums and benefits interconnectedly determine or, rather, predetermine the results to be expected.

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TABLE 8

Analysis of Experience on Term Insurance

(Twenty-Year Nonconvertible Decreasing Term)

Attained Age	t	Plan Characteristics per Unit in Force			Multiple Decrement Table				Independent Rates of Decrement*				Conversion Single Premium $CSP_{[x]+t}$
		Benefit $_tB$	Basic Rate $_tBR$	Cash Value $_tCV$	Units Persisting $l_{[x]+t-1}$	Units Terminating by: Death $d^{d}_{[x]+t-1}$	Conversion $d^{c}_{[x]+t}$	Withdrawal $d^{w}_{[x]+t}$	$q^{d}_{[x]+t-1}$	$q^{55-60\ \text{male}}_{[x]+t-1}$	$q^{c}_{[x]+t}$	$q^{w}_{[x]+t}$	
37	1	1,000	4.00	0.0	10,000,000	9,201	0.0	1,420,482	0.92	0.92	0.0	14.22	....
38	2	950	4.00	0.0	8,570,317	9,897	0.0	656,922	1.15	1.18	0.0	7.67	....
39	3	900	4.00	0.0	7,903,498	10,972	0.0	407,909	1.39	1.50	0.0	5.17	....
40	4	850	4.00	0.0	7,484,617	12,311	0.0	270,610	1.64	1.74	0.0	3.62	....
41	5	800	4.00	0.0	7,201,695	13,857	0.0	198,823	1.92	1.99	0.0	2.77	....
42	6	750	4.00	0.0	6,989,015	15,697	0.0	128,314	2.25	2.27	0.0	1.84	....
43	7	700	4.00	0.0	6,845,004	17,797	0.0	137,768	2.60	2.61	0.0	2.02	....
44	8	650	4.00	0.0	6,689,439	20,106	0.0	139,177	3.01	2.94	0.0	2.09	....
45	9	600	4.00	0.0	6,530,157	22,688	0.0	135,744	3.47	3.34	0.0	2.09	....
46	10	550	4.00	0.0	6,371,724	25,557	0.0	150,611	4.01	3.83	0.0	2.37	....
47	11	500	4.00	0.0	6,195,557	28,801	0.0	144,292	4.65	4.37	0.0	2.34	....
48	12	450	4.00	0.0	6,022,464	32,599	0.0	122,368	5.41	4.94	0.0	2.04	....
49	13	400	4.00	0.0	5,867,497	37,028	0.0	101,728	6.31	5.46	0.0	1.74	....
50	14	350	4.00	0.0	5,728,741	42,040	0.0	139,841	7.34	6.22	0.0	2.46	....
51	15	300	4.00	0.0	5,546,861	47,302	0.0	177,210	8.53	7.26	0.0	3.22	....
52	16	250	4.00	0.0	5,322,348	52,569	0.0	194,732	9.88	8.32	0.0	3.70	....
53	17	200	4.00	0.0	5,075,047	57,727	0.0	119,360	11.37	9.20	0.0	2.38	....
54	18	200	4.00	0.0	4,897,960	62,923	0.0	76,324	12.85	10.09	0.0	1.58	....
55	19	200	4.00	0.0	4,758,712	68,247	0.0	49,639	14.34	11.00	0.0	1.06	....
56	20	200	4.00	0.0	4,640,826	73,797	0.0	0	15.90	12.06	0.0	0.0	....

* $q^{d}_{[x]+t-1}$ and $q^{55-60\ \text{male}}_{[x]+t-1}$, rate per 1.000 $q^{c}_{[x]+t}$ and $q^{w}_{[x]+t}$, rate per 100.

TABLE 9

ANALYSIS OF EXPERIENCE ON TERM INSURANCE

(Fifteen-Year Level Nonrenewable Convertible Term)

ATTAINED AGE	t	PLAN CHARACTERISTICS PER UNIT IN FORCE			MULTIPLE DECREMENT TABLE				INDEPENDENT RATES OF DECREMENT*				CONVERSION SINGLE PREMIUM $CSP_{[x]+t}$
						Units Terminating by:							
		Benefit $_tB$	Basic Rate $_tBR$	Cash Value $_tCV$	Units Persisting $l_{[x]+t-1}$	Death $d^{d}_{[x]+t-1}$	Conversion $d^{c}_{[x]+t}$	Withdrawal $d^{w}_{[x]+t}$	$q^{d}_{[x]+t-1}$	$q^{55-60\text{ male}}_{[x]+t-1}$	$q^{c}_{[x]+t}$	$q^{w}_{[x]+t}$	
37.........	1	1,000	7.39	0.0	10,000,000	9,201	0	2,029,843	0.92	0.92	0.00	20.32	....
38.........	2	1,000	7.39	0.0	7,960,956	9,681	0	624,850	1.22	1.18	0.00	7.86	....
39.........	3	1,000	7.39	0.0	7.326,425	10,706	0	320,744	1.46	1.50	0.00	4.38	....
40.........	4	1,000	7.39	0.0	6,994,976	12,024	0	192,420	1.72	1.74	0.00	2.76	....
41.........	5	1,000	7.39	0.0	6,790,531	13,554	0	119,746	2.00	1.99	0.00	1.77	....
42.........	6	1,000	7.39	0.0	6,657,232	15,378	0	75,168	2.31	2.27	0.00	1.13	....
43.........	7	1,000	7.39	0.0	6,566,686	17,451	0	47,231	2.66	2.61	0.00	0.72	....
44.........	8	1,000	7.39	0.0	6,502,004	19,774	0	29,572	3.04	2.94	0.00	0.46	....
45.........	9	1,000	7.39	0.0	6,452,658	22,401	0	18,465	3.47	3.34	0.00	0.29	....
46.........	10	1,000	7.39	0.0	6,411,793	25,336	0	11,445	3.95	3.83	0.00	0.18	....
47.........	11	1,000	7.39	0.0	6,375,011	28,688	0	7,002	4.50	4.37	0.00	0.11	....
48.........	12	1,000	7.39	0.0	6,339,322	32,617	0	4,232	5.15	4.94	0.00	0.07	....
49.........	13	1,000	7.39	0.0	6,302,472	37,189	0	2,541	5.90	5.46	0.00	0.04	....
50.........	14	1,000	7.39	0.0	6,262,743	42,360	0	1,530	6.76	6.22	0.00	0.02	....
51.........	15	1,000	7.39	0.0	6,218,853	47,917	4,659,711	0	7.71	7.26	75.51	0.0	74.98

* $q^{d}_{[x]+t-1}$ and $q^{55-60\text{ male}}_{[x]+t-1}$, rate per 1,000; $q^{c}_{[x]+t}$ and $q^{w}_{[x]+t}$, rate per 100.

TABLE 10

Analysis of Experience on Term Insurance

(Fifteen-Year Special Decreasing Term Nonconvertible)

Attained Age	t	Plan Characteristics per Unit in Force			Multiple Decrement Table				Independent Rates of Decrement*				Conversion Single Premium $CSP_{[x]+t}$
		Benefit $_tB$	Basic Rate $_tBR$	Cash Value $_tCV$	Units Persisting $l_{[x]+t-1}$	Units Terminating by: Death $d^{d}_{[x]+t-1}$	Conversion $d^{c}_{[x]+t}$	Withdrawal $d^{w}_{[x]+t}$	$q^{d}_{[x]+t-1}$	$q^{55-60\ male}_{[x]+t-1}$	$q^{c}_{[x]+t}$	$q^{w}_{[x]+t}$	
37..........	1	1,000	5.66	0.0	10,000,000	9,201	0.0	2,072,454	0.92	0.92	0.0	20.74	....
38..........	2	1,000	5.66	0.0	7,918,346	9,636	0.0	730,426	1.22	1.18	0.0	9.24	....
39..........	3	1,000	5.66	0.0	7,178,283	10,614	0.0	359,230	1.48	1.50	0.0	5.01	....
40..........	4	1,000	5.66	0.0	6,808,439	11,904	0.0	197,197	1.75	1.74	0.0	2.90	....
41..........	5	1,000	5.66	0.0	6,599,338	13,413	0.0	580,736	2.03	1.99	0.0	8.82	....
42..........	6	667	5.66	0.0	6,005,189	14,808	0.0	348,819	2.47	2.27	0.0	5.82	....
43..........	7	667	5.66	0.0	5,641,562	16,517	0.0	226,457	2.93	2.61	0.0	4.03	....
44..........	8	667	5.66	0.0	5,398,588	18,500	0.0	153,601	3.43	2.94	0.0	2.85	....
45..........	9	667	5.66	0.0	5,226,487	20,783	0.0	106,818	3.98	3.34	0.0	2.05	....
46..........	10	667	5.66	0.0	5,098,887	23,355	0.0	523,135	4.58	3.83	0.0	10.31	....
47..........	11	333	5.66	0.0	4,552,396	25,628	0.0	281,175	5.63	4.37	0.0	6.21	....
48..........	12	333	5.66	0.0	4,245,593	28,564	0.0	166,178	6.73	4.94	0.0	3.94	....
49..........	13	333	5.66	0.0	4,050,851	32,100	0.0	105,157	7.92	5.46	0.0	2.62	....
50..........	14	333	5.66	0.0	3,913,595	36,137	0.0	69,907	9.23	6.22	0.0	1.80	....
51..........	15	333	5.66	0.0	3,807,551	40,463	0.0	0	10.63	7.26	0.0	0.00	....

* $q^{d}_{[x]+t-1}$ and $q^{55-60\ male}_{[x]+t-1}$, rate per 1,000; $q^{c}_{[x]+t}$ and $q^{w}_{[x]+t}$, rate per 100.

TABLE 11

ANALYSIS OF EXPERIENCE ON TERM INSURANCE

(Fifteen-Year Special Decreasing Term Convertible for Five Years)

Attained Age	t	Plan Characteristics per Unit in Force: Benefit ${}_tB$	Basic Rate ${}_tBR$	Cash Value ${}_tCV$	Multiple Decrement Table: Units Persisting $l_{[x]+t-1}$	Units Terminating by: Death $d^{d}_{[x]+t-1}$	Conversion $d^{c}_{[x]+t}$	Withdrawal $d^{w}_{[x]+t}$	Independent Rates of Decrement*: $q^{d}_{[x]+t-1}$	$q^{55-60\text{ male}}_{[x]+t-1}$	$q^{c}_{[x]+t}$	$q^{w}_{[x]+t}$	Conversion Single Premium $CSP_{[x]+t}$
37	1	1,000	5.66	0.0	10,000,000	9,201	0	2,072,454	0.92	0.92	0.00	20.74	....
38	2	1,000	5.66	0.0	7,918,346	9,636	0	730,426	1.22	1.18	0.00	9.24	....
39	3	1,000	5.66	0.0	7,178,283	10,614	0	359,230	1.48	1.50	0.00	5.01	....
40	4	1,000	5.66	0.0	6,808,439	11,904	0	197,197	1.75	1.74	0.00	2.90	....
41	5	1,000	5.66	0.0	6,599,338	13,413	1,254,916	580,736	2.03	1.99	19.05	10.89	73.47
42	6	667	5.66	0.0	4,750,273	7,962	0	348,819	1.68	2.27	0.00	7.36	....
43	7	667	5.66	0.0	4,393,492	8,868	0	226,457	2.02	2.61	0.00	5.16	....
44	8	667	5.66	0.0	4,158,167	9,975	0	153,601	2.40	2.94	0.00	3.70	....
45	9	667	5.66	0.0	3,994,591	11,295	0	106,818	2.83	3.34	0.00	2.68	....
46	10	667	5.66	0.0	3,876,478	12,822	0	505,801	3.31	3.83	0.00	13.09	....
47	11	333	5.66	0.0	3,357,855	13,968	0	268,781	4.16	4.37	0.00	8.04	....
48	12	333	5.66	0.0	3,075,105	15,615	0	157,407	5.08	4.94	0.00	5.14	....
49	13	333	5.66	0.0	2,902,084	17,694	0	98,987	6.10	5.46	0.00	3.43	....
50	14	333	5.66	0.0	2,785,403	20,142	0	65,557	7.23	6.22	0.00	2.37	....
51	15	333	5.66	0.0	2,699,704	22,832	0	0	8.46	7.26	0.00	0.00	....

* $q^{d}_{[x]+t-1}$ and $q^{55-60\text{ male}}_{[x]+t-1}$, rate per 1,000; $q^{c}_{[x]+t}$ and $q^{w}_{[x]+t}$, rate per 100.

TABLE 12

ANALYSIS OF EXPERIENCE ON TERM INSURANCE

(Fifteen-Year Special Decreasing Term Convertible for Ten Years)

Attained Age	t	Plan Characteristics per Unit in Force: Benefit $_tB$	Basic Rate $_t$BR	Cash Value $_t$CV	Multiple Decrement Table: Units Persisting $l_{[x]+t-1}$	Units Terminating by: Death $d^{d}_{[x]+t}$	Conversion $d^{c}_{[x]+t}$	Withdrawal $d^{w}_{[x]+t}$	Independent Rates of Decrement*: $q^{d}_{[x]+t-1}$	$q^{55\text{–}60\text{ male}}_{[x]+t-1}$	$q^{c}_{[x]+t}$	$q^{w}_{[x]+t}$	Conversion Single Premium CSP$_{[x]+t}$
37	1	1,000	5.66	0.0	10,000,000	9,201	0	2,072,454	0.92	0.92	0.00	20.74	....
38	2	1,000	5.66	0.0	7,918,346	9,636	0	730,426	1.22	1.18	0.00	9.24	....
39	3	1,000	5.66	0.0	7,178,283	10,614	0	359,230	1.48	1.50	0.00	5.01	....
40	4	1,000	5.66	0.0	6,808,439	11,904	0	197,197	1.75	1.74	0.00	2.90	....
41	5	1,000	5.66	0.0	6,599,338	13,413	1,254,916	580,736	2.03	1.99	19.05	10.89	73.47
42	6	667	5.66	0.0	4,750,273	7,962	0	348,819	1.68	2.27	0.00	7.36	....
43	7	667	5.66	0.0	4,393,492	8,868	0	226,457	2.02	2.61	0.00	5.16	....
44	8	667	5.66	0.0	4,158,167	9,975	0	153,601	2.40	2.94	0.00	3.70	....
45	9	667	5.66	0.0	3,994,591	11,295	0	106,818	2.83	3.34	0.00	2.68	....
46	10	667	5.66	0.0	3,876,478	12,822	2,549,514	371,937	3.31	3.83	65.99	28.30	36.98
47	11	333	5.66	0.0	942,205	1,876	0	177,622	1.99	4.37	0.00	18.89	....
48	12	333	5.66	0.0	762,707	1,955	0	94,533	2.56	4.94	0.00	12.43	....
49	13	333	5.66	0.0	666,218	2,156	0	55,344	3.24	5.46	0.00	8.33	....
50	14	333	5.66	0.0	608,719	2,443	0	35,005	4.01	6.22	0.00	5.77	....
51	15	333	5.66	0.0	571,271	2,788	0	0	4.88	7.26	0.00	0.0	....

* $q^{d}_{[x]+t-1}$ and $q^{55\text{–}60\text{ male}}_{[x]+t-1}$, rate per 1,000; $q^{c}_{[x]+t}$ and $q^{w}_{[x]+t}$, rate per 100.

TABLE 13

ANALYSIS OF EXPERIENCE ON TERM INSURANCE

(Fifteen-Year Special Decreasing Term Convertible for Fifteen Years)

Attained Age	t	Plan Characteristics per Unit in Force: Benefit $_tB$	Basic Rate $_tBR$	Cash Value $_tCV$	Multiple Decrement Table: Units Persisting $l_{[x]+t-1}$	Units Terminating by: Death $\overset{d}{d}_{[x]+t-1}$	Conversion $\overset{c}{d}_{[x]+t}$	Withdrawal $\overset{w}{d}_{[x]+t}$	Independent Rates of Decrement: $\overset{d}{q}_{[x]+t-1}$	$\overset{55\text{–}60\ \text{male}}{q}_{[x]+t-1}$	$\overset{c}{q}_{[x]+t}$	$\overset{w}{q}_{[x]+t}$	Conversion Single Premium $CSP_{[x]+t}$
37	1	1,000	5.66	0.0	10,000,000	9,201	0	2,072,454	0.92	0.92	0.00	20.74	....
38	2	1,000	5.66	0.0	7,918,346	9,636	0	730,426	1.22	1.18	0.00	9.24	....
39	3	1,000	5.66	0.0	7,178,283	10,614	0	359,230	1.48	1.50	0.00	5.01	....
40	4	1,000	5.66	0.0	6,808,439	11,904	0	197,197	1.75	1.74	0.00	2.90	....
41	5	1,000	5.66	0.0	6,599,338	13,413	1,254,916	580,736	2.03	1.99	19.05	10.89	73.47
42	6	667	5.66	0.0	4,750,273	7,962	0	348,819	1.68	2.27	0.00	7.36	....
43	7	667	5.66	0.0	4,393,492	8,868	0	226,457	2.02	2.61	0.00	5.16	....
44	8	667	5.66	0.0	4,158,167	9,975	0	153,601	2.40	2.94	0.00	3.70	....
45	9	667	5.66	0.0	3,994,591	11,295	0	106,818	2.83	3.34	0.00	2.68	....
46	10	667	5.66	0.0	3,876,478	12,822	2,030,228	432,573	3.31	3.83	52.55	23.59	48.06
47	11	333	5.66	0.0	1,400,854	3,116	0	216,422	2.22	4.37	0.00	15.48	....
48	12	333	5.66	0.0	1,181,316	3,334	0	120,352	2.82	4.94	0.00	10.22	....
49	13	333	5.66	0.0	1,057,629	3,724	0	72,920	3.52	5.46	0.00	6.92	....
50	14	333	5.66	0.0	980,985	4,245	0	47,183	4.33	6.22	0.00	4.83	....
51	15	333	5.66	0.0	929,557	4,858	733,934	0	5.23	7.26	79.37	0.00	37.05

* $\overset{d}{q}_{[x]+t-1}$ and $\overset{55\text{–}60\ \text{male}}{q}_{[x]+t-1}$, rate per 1,000; $\overset{c}{q}_{[x]+t}$ and $\overset{w}{q}_{[x]+t}$, rate per 100.

TABLE 14

GROSS PREMIUMS FOR VARIOUS PLANS AT AGE 37

Plan Description	Gross Premium
1. Initial period on five-year level term, renewable to 70, convertible to 65 (Table 6)	$ 5.84
2. Initial period on five-year level term, renewable to 70, nonconvertible	4.93
3. Twenty-year decreasing term, twenty-year premium period, convertible for twenty year (Table 7)	4.04
4. Twenty-year decreasing term, twenty-year premium period, nonconvertible (Table 8)	2.93
5. Fifteen-year increasing term, convertible for fifteen years (convertible for $2,000 at end of fifteen years, benefit increases by $67 per $1,000 per year)	11.01
6. Fifteen-year increasing term, nonconvertible (benefits same as item 5)	6.02
7. Ten-year level term, convertible for ten years	6.71
8. Ten-year level term, nonconvertible	3.84
9. Fifteen-year level term, convertible for fifteen years (Table 9)	7.26
10. Fifteen-year level term, nonconvertible	4.50
11. Fifteen-year special decreasing term, convertible fifteen years (benefits 1–5, $1,000; 6–10, $667; 11–15, $333) (Table 13)	6.00
12. Fifteen-year special decreasing term, convertible for ten years (benefits same as item 11) (Table 12)	5.98
13. Fifteen-year special decreasing term, convertible for five years (benefits same as item 11) (Table 11)	4.68
14. Fifteen-year special decreasing term, nonconvertible (benefits same as item 11) (Table 10)	3.40

APPENDIX

FORMULA AND ASSUMPTIONS USED IN GROSS PREMIUM CALCULATION

Notation

$l_{[x]+t-1}$ = Number surviving and persisting at beginning of policy year t among $l_{[x]}$ entrants at age x.

$d^{d}_{[x]+t-1}$ = Number dying in policy year t among $l_{[x]}$ entrants at age x.

$d^{w}_{[x]+t}$ = Number withdrawing at end of policy year t among $l_{[x]}$ entrants at age x.

$d^{c}_{[x]+t}$ = Number converting (selective conversions only) at the attained age at the end of policy year t among $l_{[x]}$ entrants at age x.

f = Additional first-year expenses per $1,000 of initial death benefit (including per policy expenses converted to this form).

c_t = The ratio to be applied to premium in the tth policy year to obtain the "per premium" expenses for that year.

$\mathrm{CSP}_{[x]+t}$ = Conversion single premium per \$1,000 of term insurance converted at attained age at the end of policy year t.

${}_t\mathrm{TCA}_{[x]}$ = Credit allowed (if any) per \$1,000 of term insurance converted at the attained age at the end of policy year t.

${}_t\mathrm{CV}_{[x]}$ = Cash value (if any) per \$1,000 of initial death benefit, at the end of policy year t.

${}_tB_{[x]}$ = Average death benefit in policy year t per \$1,000 of initial benefit.

${}_t\pi_{[x]}$ = Ratio of basic rate (premium) payable at beginning of policy year t to the basic rate payable at beginning of policy year 1.

n = Length of benefit period in years.

m = Length of premium payment period in years.

Formula

$$P = \frac{\left\{\begin{array}{l} \sum_1^n {}_tB_{[x]}\cdot d^d_{[x]+t-1}\cdot v^{t-1/2} + \sum_1^n {}_t\mathrm{CV}_{[x]}(d^w_{[x]+t} + d^c_{[x]+t})\cdot v^t \\ + \sum_1^n ({}_tB_{[x]}/1{,}000)({}_t\mathrm{TCA}_{[x]} + \mathrm{CSP}_{[x]+t})d^c_{[x]+t}v^t + f \end{array}\right\}}{\sum_1^m (1 - c_t)(l_{[x]+t-1}\cdot v^{t-1})({}_t\pi_{[x]})}.$$

Assumptions

1. f = \$6.63 per \$1,000 of initial amount at issue age 37.
2. $i = 0.04$.
3. ${}_t\mathrm{CV}$ are always minimum cash values.
4. ${}_t\mathrm{TCA}$ are zero for all t.
5. c_t:

	Year	c_t
Decreasing term	1	0.733
	2–10	0.095
	11–15	0.055
	16–m	0.040
Level or increasing term ($t \leq 10$)	1	0.654
	2–10	0.095
Level or increasing term ($t > 10$)	Same as decreasing term	
Five-year renewable term	1	0.654
	2–5	0.095
	6	0.654
	7–10	0.095
	11–15	0.055
	16–m	0.040

6. Policy fee includes:
   a) Maintenance expense.
   b) Claim expense.
   c) Surrender and lapse expense.
   d) Conversion expense.
   e) Any other expense which is uniform by duration.

FORMULA AND ASSUMPTIONS USED IN THE CALCULATION OF CONVERSION SINGLE PREMIUMS

Additional Notation

$q'_{[z]+n-1}$ = Rate of mortality per 1,000 on lives converted at attained age z at duration n on the converted policy.

$\overset{55-60}{q}_{[z]+n-1}$ = Rate of mortality per 1,000 on 1955–60 male select table, where n is duration from selection.

$(1 - {}_nV_z)$ = Net amount at risk on whole life computed according to the 1958 CSO at 3 per cent interest, curtate functions.

${}_np'_{[z]}$ = Persistency rate among lives converted at age z, i.e., those which do not terminate voluntarily or by death, within n years, the converted coverage relative to lives originally converting.

$\text{PVXM}_{[z]}$ = Present value of excess mortality per $1,000 converted at age z.

$\text{US}_{[z]}$ = Underwriting expense provision in the whole life premium rate per $1,000 at age z.

Formulas

$$\text{PVXM}_{[z]} = \sum_{n=1}^{\infty} v^n_{n-1}p'_{[z]}(q'_{[z]+n-1} - \overset{55-60}{q}_{[z]+n-1})(1 - {}_nV_z)$$

$$\text{CSP}_{[z]+t} = \text{PVXM}_{[z+t]} - \text{US}_{[z+t]}\,.$$

Assumptions

1. $i = 0.04$.
2. $\text{US}_{[z]}$ = $5 plus 15 per cent of the whole life basic rate at age z.
3. Lapses among converted lives..........

	n	${}_nw_{[z]}$
3. Lapses among converted lives..........	1.............	5.8%
	2.............	3.3
	3–5...........	2.7
	6 and over.....	2.1