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GROSS PREMIUMS FOR TERM INSURANCE WITH VARYING BENEFITS AND PREMIUMS

RICHARD W. ZIOCK

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L. TIMOTHY GILES:

I enjoyed reading this paper. It certainly presents a comprehensive approach to the subject.

First, some minor points call for comment:

1. While simplifying the theory, the notion that "there is no inverse process to deterioration," also advanced by Levinson, does not seem realistic. There have been instances where substandard extra premiums have been removed after issue. Also, the many millions who are dieting, jogging, giving up smoking, and the like, must not all be misguided.
2. The reference to a nonzero probability of being accepted at standard rates if expected mortality is 200 per cent of standard is a bit disarming. Let us hope that not too many underwriters get that far into the paper. The reference does highlight the human influence on insured mortality rates.
3. Table 14 shows that the annual cost of convertibility of a ten-year level term is $2.87, whereas for a fifteen-year plan it is $2.76. It is felt that the cost for the fifteen-year plan should be higher because the excess mortality at the higher attained ages will not be offset by the longer premium-paying period.

A major point concerns the reliability of a preference function which assumes a rational, decision-making policyholder. There is ample evidence in the industry of widespread, apparent irrationality among policyholders:

1. The continuing growth of many companies whose products are high-priced in relation to those of other companies.
2. The people who die nonaccidentally and do not convert their decreasing term or elect guaranteed insurability options.
3. The people who buy decreasing term in their twenties and fail to get it reissued every three years or so while the attained age rate is lower than their current cost per thousand.
4. The people who have a restricted waiver of premium benefit after age 60 with a level premium but do not lapse after the restriction.

Many other examples could be cited. The purpose is not to demean the policyholder. After all, he has many concerns and may only direct his

attention to his life insurance coverage when it is time to buy. It is easier to write a check than to get out the policy and call an agent. We who work at insurance tend to forget the occasional nature of the policyholder's interest.

The theory that insured mortality is based on exposure that has been already influenced by a preference function, that is, those who lapse are likely to be healthier than those who persist, may itself overestimate policyholder rationality. Consider the comparison of 1,000 q_x in Table 1.

The lives underlying the group experience only occasionally have decisions to make, whereas those underlying the ordinary experience have at least one decision each year. The major part of the effects of

TABLE 1

Attained Age	1955–60 Ultimate Male and Female	1960 Basic Group*
25	1.00	1.05
35	1.31	1.54
45	3.73	4.29
55	10.66	11.57
65	27.52	27.50

* *TSA*, XIII, 598.

selection on the ordinary experience have worn off after fifteen years, but the group experience is still select in the sense that the lives had to be healthy enough to work. Adherents of the theory should expect the mortality rates of the ordinary experience to exceed those of the group experience, but such is not the case. To prove the theory fully, lapsed business would have to be studied.

To illustrate the recognition of irrationality, consider just one of the calculations presented in the paper—the cost of conversion. An alternate approach to pricing it is to calculate the maximum cost of the benefit and then to lower it on account of the policyholders who will not act in their own best interest.

The Part 9I study notes in the treatment of premiums for the guaranteed insurability rider describe a method that can be applied to all kinds of selection options, including term conversion. It assumes that all impaired lives select the option and identifies the impaired lives by the difference between select and ultimate mortality.

The cost estimate, however, can be reduced approximately in propor-

tion to the percentage of impaired lives who irrationally fail to convert. My guess is that most term premiums today, at least at the higher ages, assume, perhaps inexplicitly, some positive percentage. What is this percentage realistically?

An indication can be obtained from a model constructed from the 1955–60 Male Select Table and ratios from "Mortality under Term Conversions and Guaranteed Insurability Options" (*TSA, 1968 Reports Number*).

TABLE 2

CONVERSION AT AGE 52 OF TERM POLICY ISSUED AT AGE 37
1955–60 MALE SELECT TABLE

t (1)	$l_{[52]+t-1}$ (2)	$q_{[52]+t-1}$ (3)	Unpred. Deaths (1)×(2) $d_{[52]+t-1}$ (4)	q_{52+t-1} (5)	Total Deaths (1)×(4) d_{x+t-1} (6)	Pred. Deaths (6)−(3) (7)
1	1,000.00	3.23	3.23	8.32	8.32	5.09
2	991.68	4.58	4.54	9.20	9.12	4.58
3	982.56	6.13	6.02	10.09	9.91	3.89
4	972.65	7.21	7.01	11.00	10.70	3.69
5	961.95	7.91	7.61	12.06	11.60	3.99
6	950.35	9.48	9.01	13.26	12.60	3.59
7	937.75	11.08	10.39	14.60	13.69	3.30
8	924.06	12.10	11.18	16.06	14.84	3.66
9	909.22	13.08	11.89	17.69	16.08	4.19
10	893.14	14.45	12.91	19.55	17.46	4.55
11	875.68	15.84	13.87	21.61	18.92	5.05
12	856.76	17.70	15.16	23.75	20.35	5.19
13	836.41	20.24	16.93	25.83	21.60	4.67
14	814.81	22.88	18.64	27.99	22.81	4.17
15	792.00	25.08	19.86	30.34	24.03	4.17
Total	.....	.....	168.25	.....	232.03	63.78

Table 2 shows the select and ultimate deaths resulting from the conversion of a fifteen-year term policy at attained age 52. Column (7), the impaired risks, can be considered as predictable deaths. The balance of the deaths, column (3), are the unpredictable deaths. We assume that the predictable deaths are aware of their status just as is an underwriter.

The following steps lead us to the percentage of impaired lives who fail to convert:

1. Over a fifteen-year period we can expect 23.30 per cent to die.
2. If only the 63.78 predictable deaths convert, we would expect $0.2320 \times (63.78) = 14.80$ to die. The actual to expected ratio would then be $63.78 \div$

14.80 = 431 per cent. This is the worst outcome from the company's point of view and produces the maximum cost.

3. If everyone but the predictable deaths converts, we would expect $0.232 \times (1{,}000 - 63.78) = 217.20$ to die. The actual to expected ratio would then be $168.25 \div 217.20 = 77$ per cent.
4. We base two assumptions on data in the *1968 Reports:*
   a) That 25 per cent convert at the end of the term period (p. 118).
   b) The ratio of actual to expected mortality of those who convert is 110 per cent (p. 93).
5. If a equals percentage of predictable deaths converting and b equals percentage of others converting, then $a(63.78)431 + b(936.22)77 = 110 \times [a(63.78) + b(936.22)]$ and $a = 1.509\ b$.
6. Using the assumption of a 25 per cent conversion rate, we have $63.78\ a + 936.22\ b = 0.25(1{,}000) = 250$.
7. Substituting, $63.78(1.509\ b) + 936.22\ b = 250$; $1{,}032.46\ b = 250$; $b = 24$ per cent; $a = 1.509(0.24) = 36$ per cent.
8. Hence, approximately 64 per cent of the impaired lives fail to convert. If the maximum conversion cost, before underwriting savings, is $50, a realistic cost would be $0.36(50) = \$18.00$.

In conclusion, the author is to be congratulated on his analysis, but it is felt that premiums resulting from his method can be reduced, admittedly in a somewhat arbitrary manner, to allow for the inexpert decisions of policyholders.

LOUIS LEVINSON:

The force of self-selection in the purchase of new insurance, in the renewal of existing insurance, and in the exercise of the privilege of converting term coverage to the permanent basis is a strong influence in insurance experience. Yet little effort is generally made to measure this force and to assay its effects on policy persistency and insurance mortality. In calling attention to the fact that policyholders have alternatives of varying economic value which, when exercised significantly, affect the experience of life companies, Mr. Ziock has made a valuable contribution to actuarial knowledge. The paper represents a thoughtful exploration of the general problem, using utility theory and the concept of mortality class set out in my paper, with emphasis on the decision-making process involved in the renewal and conversion options available to term policyholders.

In describing select classes of insured lives, as Mr. Ziock points out, I had utilized distributions by strata computed for the general popula-

tion for ages younger than the actual insurance ages at the formation of the classes. Mr. Ziock starts with the general population at the insurance age and introduces the selective action of underwriters in determining the stratic composition. As long as appropriate mortality rates are yielded in a class, one method may be as good as another. My method had an important advantage—uniformity; the same process, but using older population ages, was employed in the treatment of classes with heavy mortality—disabled lives and lives acquiring ordinary insurance by converting group coverage. The proportion of applicants denied standard coverage in Mr. Ziock's illustration—33 per cent—seems unduly high when general experience indicates a proportion of about 10 per cent. The high rate of nonacceptance is due partly to the reconciliation required to reduce the average census rate for white males at age 37 of 0.00236 (as adjusted for the paper) to the first-year select rate of 0.00092. One might question the assumption that the group of applicants is really a sample of the general population. The interposition of the agent, undoubtedly, weeds out many poor lives, and, of course, the substantial number of chronically ill and institutionalized individuals, who, in a way, exclude themselves must be quite effective in modifying the distribution of applicants.

In this paper fourteen strata are employed in establishing mortality classes. I used six, and I have wondered if advantages of significance are gained by a larger number. Differences in distribution are accompanied by compensating death rates in the several strata, so that results ordinarily come close. I was surprised to find that the conclusions in Table 3 were quite different if six strata were used rather than fourteen. I believe this is due to the fact that the ratios measuring conversion dispreference and lapse preference are not dependent on the number of strata or on the rates of death operating within them and consequently should be applicable for any number of strata. However, the relative proportions of the units at the end of the year after conversion and/or deterioration (Table 3, cols. [6] and [10]) to which the ratios apply are materially altered by the difference in the number of strata. The consequences in experience on lapse, conversion, and the like, are substantially changed. If my supposition is correct, the preference ratios used in the paper, by being dependent on the assumptions employed, may be limited in their application to the particular parameters underlying the data of the paper.

ANNA MARIA RAPPAPORT:

Mr. Ziock should be congratulated for making a very worthwhile contribution to actuarial science. Several points seemed particularly important and significant to me:

1. The handling of conversion costs. Conversions are distinguished into two types—those involving selection and those not involving selection. There is no conversion cost associated with conversions not involving selection. This provides a sensible basis for the handling of conversion costs in rate calculation.
2. The use of utility theory and preference functions to measure policyholder behavior. This provides new flexibility in dealing with potential antiselection and its effects on premium rates.
3. The paper explicitly recognizes that the premium calculation is a model and in so doing should help broaden the viewpoint of the actuary when he thinks of rate making.

The methods used by Mr. Ziock could logically be extended to substandard life insurance. Such an extension would require some additions to the model, including the following:

1. Use of rates of improvement as well as deterioration so that a policyholder could go to a lower as well as a higher mortality class.
2. Modification of utility curves to reflect the changed attitude of the insured who has previously been rated.

This is exciting to me, since I believe that traditional methods are not very helpful for development of substandard premium rates.

(AUTHOR'S REVIEW OF DISCUSSION)

RICHARD W. ZIOCK:

I greatly appreciate the time and effort taken by Mr. Giles, Mr. Levinson, and Mrs. Rappaport in presenting written discussions to the paper. I will review their discussions in alphabetical order.

In Mr. Giles's first minor point he states that "there have been instances where substandard extra premiums have been removed after issue" as justification of the contention that there is an inverse process to deterioration. On this I would like to point out that it is not necessary that there be an improvement in the mortality stratum in which the person finds himself in order that the extra premium be removed. It is quite possible that the general level of mortality could move past that in which the insured finds himself, while the insured remains at the same

level. Thus, since he is now better than the substandard class, he can have his extra premium removed. I agree that the contention that there is no inverse process to deterioration is not quite realistic; however, I do not believe that the effect of this simplifying assumption on Levinson's theory of mortality classes is a significant one.

In regard to Mr. Giles's second minor point, it seems to me quite reasonable to have a nonzero probability of being accepted at standard rates if expected mortality is 200 per cent. We see many claims which occur shortly after issue which cannot be rescinded and in which the insured was in a very ill state at the time the policy was issued. This happens because the condition is undiscoverable without the most extensive medical tests available and perhaps not even with them.

In his third minor point Mr. Giles complains that the difference between the convertible and nonconvertible premium is larger for the ten-year plan than it is for the fifteen-year plan. This is due to the availability of money out of the eleventh through the fifteenth premiums to pay for the benefit and more than pay for the increased cost of the benefit. Premiums in the eleventh through the fifteenth year have much less expense assessed against them, and the proportion of these premiums which can be used to offset the cost of convertibility is higher than it is for most of the premiums under the ten-year plan.

Mr. Giles questions the reliability of the assumption of a rational decision-making policyholder, and he cites four examples of irrationality among policyholders. The four examples cited seemed to be all examples of irrationality where the benefit to be gained by the policyholder if he were rational would be pennies. The second of these four points seems to assume that everyone knows when he is going to die. It should be obvious that this is not correct. I grant that all policyholders may not act rationally. This, in fact, is the point of view behind the preference function and the derivation of the preference function from actual experience. This function represents those who do lapse or those who do not convert and takes into account, since it is derived from actual figures, those who do not act rationally and do not take advantage of the options offered and the economics of their situation. What we are mainly concerned with in the derivation of our preference function and what we are attempting to predict is antiselection and rational decision-making policyholders when the benefit to be gained by such rational action is significant.

Life insurance is often referred to as an aleatory contract, which means that a great gain can be secured by the policyholder for a small outlay—his premium. No one would argue that a policyholder who knows cer-

tainly that he will die next year would lapse. A certain number of those who will die next year will not be aware of that fact. Some of these may lapse and some may not. This is all part of the preference function as derived from practical experience. Looking at it another way, those who have a probability of dying next year of 0.95 and know that probability will be very much inclined not to drop their insurance. Those who have a probability of dying next year of 0.02, in contrast, and are aware of the probability will be more inclined to let their insurance lapse, especially if the premium is quite high in comparison to 0.02. The purpose of the paper was to predict the actions of decision-making policyholders using functions which contain the degree of rationality which they have heretofore exercised.

Mr. Giles attempts to illustrate that antiselection on withdrawal does not occur by using as an example thereof a comparison of 1955/60 Ultimate Male and Female mortality compared to 1960 Basic Group mortality. It seems to me that so many factors or points of difference exist between these two experiences that any conclusions drawn would be completely meaningless. A very partial list of such influences or differences between the two experiences is (1) group experience contains lives who cannot qualify for standard ordinary insurance or could not have fifteen years previously; (2) the difference between medical and nonmedical standard ordinary experience after fifteen policy years certainly illustrates that the effect of the medical has not completely worn off in fifteen years; (3) the proportions of male and female in the two experiences differ; (4) those lives too unhealthy to work will leave the group experience; and (5) ordinary insurance is generally sold to a higher socioeconomic class than is group insurance.

It would seem that such an amalgamation of factors would make the conclusion that Mr. Giles attempts to make somewhat doubtful.

Mr. Giles attempts to derive a percentage of impaired lives who irrationally fail to convert by using a method based on the difference between select and ultimate mortality which is described in the Part 9I study notes. This is quite an ingenious method to modify the conversion costs calculated in accordance with the difference in select and ultimate mortality. I have not seen this method before. However, applying the conclusions of this analysis to the conversion costs and the general procedure for obtaining results under converted policies contained in my paper is completely erroneous. I did not use the method of difference between select and ultimate mortality in deriving conversion costs. I used a conversion dispreference function, the complement of which determines the number of lives converting. Within the philosophy of

the conversion dispreference function is contained the idea that not all those for whom conversion would be profitable actually convert. A certain number remain irrational. This again is part of the derivation of the preference function.

In actuality, I derived the lapse preference function from actual withdrawal data and assumed that the complement of the lapse preference function would predict the proportion willing to convert. That is, I derived a function which might be described as a keep-or-get-new-insurance or don't-convert-or-lapse-old-insurance type function. Actually, I believe that the conversion dispreference function should have been derived separately, because there seem to be other considerations than those present at the time of lapse in the choice whether or not to convert. It is my considered opinion that the conversion rates shown in the paper are perhaps somewhat too high. For this reason and not for the reason Mr. Giles mentioned, the use of lower conversion rates would result in somewhat lower term premiums than those shown in the paper.

In writing the paper, I had in mind the construction of a comprehensive model of term insurance and the results which I knew existed under the various types of term insurance policies. I did not have in mind just the construction of a model which predicted conversion rates and conversion costs. However, looking at the model from the point of view of the conversion cost it develops and comparing it with the known methods, we have three methods of producing conversion costs. (1) The first is the difference between select and ultimate mortality. This method assumes that everyone will convert, and it is only applicable to level term. It may not be used with decreasing term. (2) The second method was presented by Mr. Frank L. Griffin, Jr., in "A New Approach to the Problem of Term Insurance Conversion Costs." This method predicts the conversion costs which would result if the policyholders converted in such a manner as to cost the company the most money. This method, again, does not treat decreasing term. (3) The third is my method, which bases conversion cost on a percentage converting and mortality after conversion which is dependent upon the number converting where the number converting depends upon a demand curve or preference function which reflects the amalgamation of rational and irrational desires and actions of policyholders. This method is applicable to decreasing term as well as to level term.

In the second paragraph of his discussion, Mr. Levinson takes me to task on my method of producing a strata distribution representative of select lives. He states, "As long as appropriate mortality rates are yielded in the class, one method may be as good as another. My method

has an important advantage—uniformity." It may be true that his method offers the advantage of uniformity; however, my method offers the advantage of reality. In developing this new method for deriving the distribution of select lives, my intention was to compare the forces of desire for insurance and the selection influences of the underwriter and the agent opposing the interest of the insureds. These forces do not exist with disabled lives and lives converting their group coverage.

In the second paragraph of his discussion, Mr. Levinson states that the proportion of applicants denied standard coverage in my illustration, 33 per cent, seems high when compared to the industry average of 10 per cent. The reason for this is that my illustration was intended to reflect all the filters which occur before a group of standard insurance applicants is accepted out of a group of the population. These filters constitute the interposition of the agent, those chronically ill and institutionalized individuals who do not apply, and, finally, the influence of the underwriter. It is my error that I did not label the section "Selection of Risks" and Table 4 in another manner and avoid using the word "applicants."

In the third paragraph of his discussion, Mr. Levinson asked whether advantages of significance are gained by a large number of strata. It is my impression that the more strata that are used, the better the results will be. This is true because the strata constitute a point distribution intended to approximate what is, in reality, a continuous distribution.

Also in the third paragraph of his discussion, Mr. Levinson questions whether the ratios measuring conversion preference and lapse preference are independent of the number of strata used. My answer is that they are; I have not been able, however, to determine whether the preference function itself is independent or dependent upon the number of strata used. I do not personally believe that this would be the case, especially when the number of strata being used is large. The interdependence of the preference function and the number of strata should be almost nil as long as the number of strata is large, for instance, from fourteen to twenty-five strata. However, since six strata are a very small number, fairly crude results might be expected. There may be some discontinuities as a result of this; thus, I can well see why different results were obtained using six strata instead of fourteen. It is not my opinion, however, that the results would be greatly different with, say, one hundred strata. It would perhaps be well to mention my reason for using fourteen strata at this point. It is very simple; it is the number my computer would hold.

I wish to thank Mrs. Rappaport for her discussion of my paper. Her

list of three particularly important and significant points seems to me to be a very valuable summary of the main features of the paper.

Her suggestion to extend my methods to substandard life insurance is a useful idea which had not occurred to me. Relevant to such an extension of the model she made the point that perhaps rates of improvement as well as rates of deterioration could be used. On this point (which was also mentioned by Mr. Giles) I would like to say that my knowledge of Mr. Levinson's paper does not go beyond the confines of his paper itself. I applied the methods of his paper with very few modifications to get a cohesive system of mortality strata among which transfers would take place. As to whether rates of improvement should be included, logically I would guess that they should. As to the effect of introducing rates of improvement into his model, I think such an effect would be extremely small, particularly since, as time goes on, mortality with respect to a particular person always increases. Therefore, what you might regard as an improvement in mortality for a certain individual might actually be only a stationary state. In other words, an improvement of a given individual relative to his colleagues from one year to the next may mean only that his mortality has not changed at all or that he has not moved from his present stratum. Whether a modification of Levinson's model to include rates of improvement will be meaningful or not, I do not know. A modification of that model to include rates of improvement in mortality would most likely have to be done by Mr. Levinson himself, certainly not by me.

In her second point on the extension of the model to substandard life insurance, Mrs. Rappaport mentioned the modification of utility curves to reflect the changed attitude of the insured who has previously been rated. It seems to me that the change in attitude would be directly proportional to the change in price to the insured; thus the improvement in attitude or improved persistency among this class of policyholders would be directly proportional to their premium before the removal of rating and the premium after removal of rating. Thus I do not think that this requires a modification of the utility curves themselves but is directly incorporated into them.

Mrs. Rappaport's suggestion of extending the model to substandard life insurance brings to mind the extensions of the model I had in mind when I stated the following in the last portion of the paper: "It is to be hoped that actuaries will pursue and develop the model described in this paper and apply it to other situations in which expected mortality, utility functions, and premiums and benefits interconnectedly determine

or, rather, predetermine the results to be expected." When I made this statement, I had in mind the extension of the model to the pricing of the guaranteed insurability option, to the pricing of the move-over option on decreasing term, to the pricing of the options whereby the amount convertible under the policy is greater than the amount in force, and any and all situations where options or conversion privileges are allowed under whatever circumstances.

After the paper was written, I discovered a reference in the *Transactions* that is relevant to and generally supportive of the conclusions of the paper and that may be useful to those further interested in term insurance experience. This reference is the remarks on term insurance mortality made by Mr. Edward A. Lew on pages 777 and 778 and the remarks of Mr. Hillary J. Fisher on pages 779 and 780 of Volume XII.