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OPTIMIZING DEBIT SIZE: AN OPERATIONS RESEARCH STUDY

DWIGHT K. BARTLETT, III

THE purpose of this paper is to make a part of the actuarial literature another example of an operations research study involving operations of particular interest to actuaries. It involves mathematical techniques which should be familiar to most actuaries and which, hopefully, will help show that a great deal of operations research can be done without having to learn many new mathematical techniques and, furthermore, that a reasonably satisfactory solution of the problem is more important than the technique used.

The study described here grew out of a wish by the author's company to develop some criteria as a guide to expanding its combination agency operations. "Combination agency operations" means that form of individual insurance marketing in which the agent is assigned exclusive responsibility for the sales and service of all individual insurance on a debit. A debit is a carefully defined geographical area. Each city in which the company does business is divided up into anywhere from five to one hundred debits. On his debit the agent sells and services industrial policies and monthly debit ordinary policies, for which the premiums are collected by the agent at the insured's home, and regular ordinary business, which is billed by mail from the home office.

When an agent terminates and a debit becomes open, the company has a choice between employing a new agent and assigning him to the debit or splitting the debit to create an additional debit in an effort to create additional selling units, or of consolidating the debit with other surrounding debits by assigning the business in force on this debit. It is a well-established fact that the size of the debit measured in terms of the average weekly service commissions—that is, the average weekly collection commissions on the home collection business and the renewal commissions on the regular ordinary business payable to an agent for servicing the debit—

greatly affects the expected performance of an agent assigned to that debit.[1] Agents on the larger debits have a higher rate of survival and production. On the other hand, the company wishes to keep its territory divided into as many debits as appropriate in order to have a large number of selling units with the attendant growth.

The most common way of combination agency operations to increase its number of selling units has been to split debits as they became open and to hire two agents in the place of the one terminated agent. However, splitting debits into new debits which are too small may actually be a step backward for the company as a result of substantially poorer rates of agents' survival, lower rates of production, and the increase in expenses resulting from the existence of the new debit.

In order to find the theoretically ideal debit size, the author reviewed a number of elements of the experience of his company's combination agency operations in 1966 to determine how they were affected by the weekly service commission content of the debit.

The object of these studies was to create a mathematical model in which the controllable or independent variable would be the average weekly service commission content of the company's debits or, equivalently, the number of debits into which the company's operations would be divided. The dependent variables in the model would be those elements of the company's operations which appear to be affected by the weekly service commission content of the company's debits and which directly or indirectly affect the profit of the company's debit operation. The mathematical form of the model would express the profitability of the company's debit operations. Various values of the independent variable would then be substituted into the model in an effort to find the value of the independent variable which maximizes the profitability.

It was decided that the mathematical form of the model should be "marginal" in the sense of marginal economic analysis. The model need not necessarily reflect accurately the total profit of combination operations as long as marginal changes in profit are reflected accurately as a result of marginal changes in the controllable variable. Therefore items of profit and expense included in the model as parameters should be included only on a marginal basis; for example, items of fixed overhead not significantly affected directly or indirectly by the weekly service commission content of the debit were ignored in the model.

Those elements of debit operations which in the author's judgment appeared to be significantly affected by the weekly service commission con-

[1] Charles T. Clayton, "The Man in the Middle," *Proceedings of the 1966 Annual Meeting of L.I.A.M.A.*, pp. 192 ff.

tent of the debit were termination rates of agents, the length of term a debit remains open between the time that one agent terminates and the time that his replacement is hired, and the production and termination rates of business assigned to the debit.

The results of the study of each of these elements follow:

1. With regard to agents' survival, after studying the termination rates of agents by size of debit and year of employment, survival expectancies of newly appointed agents were determined. A third-degree equation was fitted to the survival expectancies produced by the crude data to produce a graduated table of survival expectancy. The equation is as follows:

$$\mathring{e}_p = 0.803(10^{-5})p^3 - 0.536(10^{-3})p^2 + 1.56(10^{-2})p + 0.626\ , \quad (1)$$

where p is the debit size measured by weekly service commission content of the debit.

The solution of the equation gives the following illustrative values:

p	Survival Expectancy in Years of Newly Appointed Agent
\$ 45 or less	0.97
50	1.06
60	1.36
70	1.84
80	2.55
90	3.54
100	4.85
110	6.54

2. While a debit is open, it is usually the responsibility of an assistant manager in the district office to service the policies on that debit. However, because of the press of the many duties that an assistant manager has, an open debit is normally not serviced as well as it is when there is an agent employed on the debit. As a result, the debit experiences not only no sales but substantially higher lapses. The data collected on each debit which became open during 1966 were the size of the debit, how long the debit remained open, and the termination experience on the debit while it was open. Fitting a linear regression line to the amount of termination against the length of time the debit was open resulted in a linear equation which showed that each time a debit became open business lapsed equal to 1.0 per cent plus 0.38 per cent times the number of weeks the debit was open. Thus, a debit which remained open for ten weeks would be expected to lose 4.8 per cent before a new agent is assigned to it. A linear regression line was also fitted to the size of the debit, measured in terms of its weekly service commission content versus the number of weeks it remained open. The resulting linear equation showed that the best prediction of how long a debit will remain open after an agent terminated is exactly two-tenths of a year and that this is independent of the size of the debit.

3. In the author's company the most common measure of agency department performance is a measure called "premium growth." Premium growth is defined

as the annualized premium on all individual insurance business issued and placed, less the annualized premium on all controllable terminations, that is, terminations other than death terminations, maturities, and paid-ups. The data were collected on the annual rate of premium growth for agents according to the size of the debit, again measured in terms of the service commission content of the debit. The resulting linear regression line indicated that the annual rate of premium growth experienced was $2,740 plus $10 times the weekly service commission content of the debit.

In each case, a comparison of the original data with the resulting linear regression equations given above showed a large prediction interval, that is, the individual data points were scattered rather widely around the regression line. Obviously there are many influences which affect performance other than the weekly service commission content of the debit. However, to a considerable extent these other influences are not controllable. Therefore it was felt that the use of these linear regression equations was justified as a measure of what could be expected to happen on the average as a result of a decision with respect to the controllable variable, the weekly service commission content of the debit. Where it is felt that because of such particular local conditions as superior management, better local economic conditions, and so forth, better experience could be expected, adjustments could be made to the linear regression lines based either on judgment or on available supporting data before the regression equations are used in the solution of the model.

Several additional assumptions must be made with respect to the model concerning the parameters of unit profit and expenses. One is the marginal profit of each additional unit of business sold by an agent and the lost profit of each additional unit of business which terminates. In the model it was decided that the most appropriate measure of profit was the present value of future marginal profits after taxes per dollar of annualized premium either, in the former case, at the time of issue or, in the latter case, at the time of termination, assuming that the policy did not terminate. The choice of an appropriate assumption is obviously very difficult, since the present value figures differ substantially by class of business and duration at termination and each debit has its own mix of business for issue and for termination. It would not be practical to create a model for each debit according to its mix of business. After considerable testing under various model-office distributions of issue and termination, it was decided that a satisfactory approximation would be to assume that the present value of future marginal profits for each dollar of annualized premium issued was the same as the present value of future marginal profits lost for each dollar of annualized premium which terminated. This permitted a substantial simplification in the model in that it could be assumed that a dollar of premium growth had a certain present value of future marginal profits irrespective of how it was made up of placements and terminations.

The model must also include explicitly those items of expense which are affected by a decision concerning the number of debits into which company operations are divided but which are not affected by the amount of premium growth per debit and are therefore not reflected in present value of future marginal profit for each dollar of annualized premium growth.

A study of the fixed annual expenses which arise marginally as a result of the creation of an additional debit and which do not vary by the performance of the debit in terms of premium growth showed that these expenses were $2,034. This was made up mostly of the additional salaries payable to the district office manager and assistant manager to compensate them for their supervisory responsibilities. This expense is fixed on the assumption of one manager for every sixteen agents and one assistant manager for every six agents.

Additional assumptions have to be made in the model concerning the marginal tax rate with respect to expenses, since the present value of future profit figures for premium growth were on an after-tax basis. Furthermore, an assumption has to be made concerning the average rate of service commission per dollar of premium in force on the debit.

The next step was to create the mathematical form of the model to reflect the profit of company debit operations as a function of total company premium growth, which in turn is a function of the variables previously described and is adjusted by the fixed expenses per debit of the operation. Symbolically, the model can be expressed as follows, if we assume that the company has total business in force which provides $1 of weekly service commission:

$$f(p) = \frac{1}{p}\left[\left(1 - \frac{\bar{o}_p}{\mathring{e}_p + \bar{o}_p}\right)(C \cdot {}_1g_p) + \left(\frac{\bar{o}_p}{\mathring{e}_p + \bar{o}_p}\right)(C \cdot {}_2g_p - 52mp) - mE\right], \qquad (2)$$

where

$f(p)$ = Present value of future marginal contribution to profit resulting from one year's premium growth per $1 of weekly service commission in force for the whole company ($1/p$ is the number of debits per $1).

p = Weekly service commission size of debit.

C = Present values of future marginal profits resulting from each additional $1 of premium growth with fixed expenses not affected by an incremental unit of premium growth not being charged against the profit.

$\bar{o}_p$ = Average time in years debit remains open between termination of old agent and assignment of new agent to debit.

$\mathring{e}_p$ = Survival expectancy in years of newly appointed agents.

${}_1g_p$ = Expected premium growth performance of debit while agent is assigned to it.

${}_2g_p$ = Expected premium growth performance of debit while debit is open.

m = 1 minus the marginal income tax rate with respect to expenses.

E = Fixed annual expenses resulting from the creation of each additional debit.

The term $52\,mp$ is in the formula to cover the expense savings while a debit is open due to the fact that service commissions are not paid during this period. Using the findings of the study of agency operations, described previously, results in the following substitutions:

$\bar{o}_p = 0.2.$

$C = \$1.$

${}_1g_p = \$2{,}740 + 10p$ per year.

${}_2g_p$ = Annualized premium content of debit times 0.2476 per year, equals $128.8p$ on the assumption that service commissions average 10 per cent of premium.

$m = 0.76.$

$E = \$2{,}034$ per year.

After substituting in the model and putting it in its simplest form, we have the following expression:

$$f(p) = (1/p)[\mathring{e}_p(1{,}194 + 10p) - 17.9p - 309]/(\mathring{e}_p + 0.2). \quad (3)$$

The model at this point is theoretically solvable for its maximum value by using the classical methods of differential calculus. However, it is easier to solve the model for particular values of p with the help of the computer, and furthermore it is instructive to see how the values of $f(p)$ change as p changes. If we solve for p first, using equation (1), and then solve for $f(p)$, we arrive at the results shown in Table 1.

It can also be instructive to do some sensitivity analysis with respect to some of the assumptions made. The period of time during which the study of agency operations was made in the author's company was felt to be an unusually trying one for combination operations and not necessarily representative of normal experience. It was a time of unusually full employment in the economy and of rapidly increasing labor costs. This resulted in the problems that many combination companies had in hiring and retaining debit agents. To reflect what might be felt to be a more normal experience, in one assumption the model calculations were rerun with the single change that the average time the debit remains open, $\bar{o}_p$, was assumed to be 0.1 year instead of 0.2 year. The results are shown in Table 2.

Comparing Tables 1 and 2 indicates how sensitive the model is to changes in $\bar{o}_p$. Under this assumption the optimum point for the model is to have an average debit size of $45. The model was not solved for average debit size of less than $45, since during 1966 the author's company had a

TABLE 1

Weekly Service Commission per Debit (p)	Newly Appointed Agent's Survival Expectancy ($\mathring{e}_p$)	Present Value of Future Profit per $1 of p In Force for Entire Company [$f(p)$]
45	0.97	9.22
50	1.06	9.57
55	1.19	10.35
60	1.36	11.37
65	1.58	12.46
70	1.84	13.50
75	2.16	14.43
80	2.55	15.21
85	3.01	15.84
90	3.54	16.32
95	4.15	16.67
100	4.85	16.92
105	5.65	17.07
110	6.54	17.16
111	6.73	17.17
112	6.93	17.18
113	7.13	17.19
114	7.33	17.19
115	7.54	17.19
116	7.75	17.19
117	7.97	17.19
118	8.19	17.19
119	8.42	17.19
120	8.65	17.18
125	9.88	17.14
130	11.23	17.07
135	12.72	16.99
140	14.33	16.89

minimum weekly service commission of $45 for debits that would otherwise pay less than $45. This factor was felt to make the experience on debits smaller in size than $45 not meaningful for the model in its present form. It is also interesting to note in Table 2 that the profit for the entire combination force seems not to be particularly sensitive to the number of debits into which the company's business in force is divided, assuming a constant in force for the company as a whole. The model has found an optimum size debit, but sensitivity analysis shows that if we divide up the company's in force into a different number of debits from the optimum, thereby arriving at an average weekly service commission content per debit different from the optimum, we do not change total company profits radically. The total present value of future profits if the average weekly service commission per debit were $135 is only 17.2 per cent less than it is when the average weekly service commission per debit is $45, even though in the latter case there would be 3 times as many debits and consequently substantially more total premium growth. The greater

TABLE 2

Weekly Service Commission per Debit (p)	Present Value of Future Profit per $1 of p In Force for the Entire Company [$f(p)$]
45	21.64
50	20.72
55	20.24
60	20.03
65	19.96
70	19.95
75	19.94
80	19.90
85	19.82
90	19.71
95	19.56
100	19.38
105	19.19
110	18.98
115	18.77
120	18.55
125	18.34
130	18.12
135	17.91
140	17.70

premium growth coming from the greater number of debits is apparently offset, to a great extent, by the higher fixed expenses.

Sensitivity analysis could and should also be done with respect to other assumptions in the model before decisions are made with the help of the model.

One of the questions addressed to the author by the management of his company, which led to the development of this model, was whether the cost of paying subsidies to agents on smaller debits could be justified as a result of hoped-for better survival rates of agents on these debits. Specifically, the form of the subsidy considered was a minimum weekly service commission. The author's company had been paying a minimum of $45 on those few debits where the actual service commission fell below $45. It was suggested that perhaps $65 would be a more appropriate figure. A review of the company's actual debits indicated that this would affect a substantial number of debits and would involve costs running well in excess of $100,000.

In the examination of the desirability of this proposal, essentially the same model was used as that previously described except that the cost of the subsidies was built into the model. It was assumed that a new agent's survival expectancy was the same as it was in the previous model except that it was now a function of the total service compensation, including the subsidy. However, it was assumed that the average premium growth performance of an agent on a debit was still a function of the basic service commission and would not be affected by the payment of the subsidy on the theory that the agent's production comes primarily from the homes he services and the referred leads he receives from them, which would not be changed by the payment of the subsidy. No other changes were made in the model, either in its basic form or in the experience assumptions used in its solution. Symbolically the model can be expressed as follows:

$$f(p, p') = \frac{1}{p}\Big[\Big(1 - \frac{\bar{o}_{p'}}{\mathring{e}_{p'} + \bar{o}_{p'}}\Big)(C \cdot {}_1g_p) + \Big(\frac{\bar{o}_{p'}}{\mathring{e}_{p'} + \bar{o}_{p'}}\Big)(C \cdot {}_2g_p - 52mp') - mE - 52m(p' - p)\Big], \tag{4}$$

where p' = weekly service commission per debit, including subsidy.

Substituting the experience assumptions previously described and simplifying results in the following formula:

$$f(p, p') = \frac{(1{,}194\,\mathring{e}_{p'} + 49.5p\,\mathring{e}_{p'} - 39.5p'\mathring{e}_{p'} - 17.9p - 309)}{p(\mathring{e}_{p'} + 0.2)}. \tag{5}$$

This equation was solved for all integral values of p' from $45 to $144 and for all values of p equal to or less than p'. Table 3 shows some of the results of this calculation.

The conclusion was that under the experience assumptions used the payment of subsidies could not be justified. Further sensitivity analysis revealed that no reasonable change in the assumption would justify the subsidy. As a result, the company did eliminate a substantial number of smaller debits which would have received large subsidies if the $65 minimum service compensation had been adopted.

The model described up to this point represents the current premium growth situation of the company at a moment in time and its effect on the present value of future profits, assuming a fixed amount of business in force for the company. A decision which maximizes present value of future profits from the current premium growth of the company at its present amount of business in force may not be the best decision of the company if a different amount of business in force is assumed. Most companies, furthermore, are willing to forgo some current profit in the hope of achieving greater future profits. It was felt, therefore, that it would be an improvement in the model if it could be made to reflect not only profits from current premium growth at the company's present amount of business in force but also the change in profit as a result of the change in the amount of business in force for the company over time.

TABLE 3

PRESENT VALUE OF FUTURE PROFIT PER $1 OF WEEKLY SERVICE COMMISSION IN FORCE FOR THE ENTIRE COMPANY [$f(p,p')$]

Weekly Service Commission per Debit (p)	Minimum Service Compensation (p')					
	$45	$65	p	p+$5	p+$10	p+$20
$ 10	$−48.89	$−105.38	...	...	...	...
20	−11.53	− 35.75	...	...	...	...
30	+ 0.91	− 12.53	...	...	...	$− 2.43
40	7.14	− 0.93	...	$ 7.14	$ 5.07	1.18
45	9.22	2.93	$ 9.22	7.57	6.07	2.93
50	...	6.03	9.57	8.43	7.29	4.54
60	...	10.67	11.37	10.67	9.77	7.15
65	...	12.46	12.46	11.78	10.84	8.14
70	...	...	13.50	12.76	11.76	8.96
80	...	...	15.21	14.27	13.10	10.15
90	...	...	16.32	15.20	13.91	10.89
100	...	...	16.92	15.69	14.43	11.35
110	...	...	17.16	15.89	14.53	11.62
120	...	...	17.18	15.91	14.57	11.77
130	...	...	17.07	15.82	14.52	...
140	...	...	16.89	...	...	...

One method of implementing this desire to include in the model the change in profitability of the operation arising from the change in the amount of business in force over time is to use the model in the previous form to determine what the change in present value of future profits is as the company's in force changes. This can be done by calculating the marginal change in $f(p)$ as the amount of business in force changes and then adding this to the original form of the model, $f(p)$, in a linear combination with an appropriate weighting factor.

There are several assumptions that can be made about how the company's agency force is going to be affected by the growth of the company. One assumption could be that the company intends to find the optimum debit size and to hold its debits to this size as far as possible as time passes, letting the growth in the company's in force result in additional debits.

The second assumption could be that the company intends to hold the number of its debits constant. In the formulas which follow, it is assumed that the distribution of business resulting in premium growth is the same as the distribution of business in force, so that the growth in total service commissions on the company's business is proportional to the growth in premiums in force.

Introducing considerations concerning the rate of change in in force requires the determination of the rate of uncontrollable terminations. It will be recalled that premium growth as used in prior formulas was previously defined as the annualized premium on all individual insurance business issued and placed, less the annualized premium on all controllable terminations, that is, terminations other than death terminations, maturities, and paid-ups. If we are to change our model to reflect changes in the company's in force, we will have to net against the rate of premium growth the average rate of uncontrollable terminations.

If we plan to let the number of debits increase in proportion to the increase in the company's in force, the form of the model will be

$$f(p)\left[1+a\left(\frac{dp}{p\,dt}-r\right)\right], \qquad (6)$$

where r is the average rate of uncontrollable terminations, t is time, a is the linear combination coefficient relating a dollar of increase in present value of future profits to a dollar in present value of future profits, $f(p)$ is as defined in equation 1, and $[(dp/pdt) - r]$ is the rate of growth in service commissions for the company.

If we plan to keep the number of debits constant and let growth occur in the average weekly service commission content of debits, then symbolically the model is as follows:

$$f(p)+\frac{a\,d[p\cdot f(p)]}{dp}\left(\frac{dp}{p\,dt}-r\right), \qquad (7)$$

where $dpf(p)/dp$ is the change in profit per debit per unit increase in the weekly service commission content of the debit,

$$=f(p)+\left[a f(p)+a p\,\frac{df(p)}{dp}\right]\left(\frac{dp}{p\,dt}-r\right).$$

Under the experience assumptions previously used values of dp/pdt are as shown in Table 4.

Since $f(p)$ is a complex function, it is difficult to solve for $df(p)/dp$ analytically. However, it can be approximated by the computer, using the regular methods of numerical analysis.

If a high value is given to increase earnings so that the linear combination coefficient a equals, let us say, 10, it is possible to justify with the model very much smaller debits.

In summary, the author recognizes that the model described in this paper in its various forms has substantial limitations. For example, it does not consider explicitly the capacity of particular managers to recruit and train additional men if the model should indicate that debit-splitting seems to be warranted in a particular office. Judgment must still be the main element in making decisions about agency operations. Some elements of judgment can be reflected in the model by changes in the experience assumption. Others no doubt cannot be so reflected. However, in spite of the limitations, it is felt that the model is a substantial aid to judgment, as are other operations research models. Perhaps others can suggest further refinements to the model to reflect factors which the author has chosen to ignore. In any case, this paper will have served its purpose if it encourages members of the actuarial profession to use the techniques of operations research in their professional work.

TABLE 4

Weekly Service Commission (p)	dp/pdt
$ 50	0.066
60	.061
70	.058
80	.061
90	.059
100	.059
110	0.055

DISCUSSION OF PRECEDING PAPER

RALPH E. EDWARDS:

Mr. Bartlett quite rightly emphasizes sensitivity analysis in studies of the sort presented by this paper. An example is his assumption, based on appropriate data, that more new business tends to be written on a larger debit. I have not checked into the matter, but it is logical that the extra time spent collecting a larger debit is more than offset by the additional sales provided by a larger pool of captive prospects. On the other hand, it would also seem logical that the smaller amount of time spent in collecting a smaller debit would provide additional sales time and more sales. If the latter were the case, it might not appear to be true. What is involved is that in each locality larger debits would have fewer new sales, but for the company as a whole the regression line could indicate the contrary because of larger debits being more numerous in the more successful field offices. To illustrate, if ${}_1g_p = \$3,740 - 10_p$, the yields corresponding to Table 1 are the following: $f(45) = 10.96$, $f(100) = 7.31$, and $f(140) = 4.21$. Conclusions based on these results would be quite misleading.

As one who is still not entirely at ease in the field of operations research, I am grateful to Mr. Bartlett for his fine illustration of a practical application. This latest of his valuable papers is especially appropriate for this particular subject to those of us who are rusty and out of date in our mathematics, since we can follow his full development instead of blindly using his results.

(AUTHOR'S REVIEW OF DISCUSSION)

DWIGHT K. BARTLETT, III:

I would like to thank Mr. Edwards for his kind comments about my paper.

Unfortunately, I have no data to show whether or not his point holds true for my own company. I intuitively feel, however, that even within an office the performance by an agent improves with the increase in the size of his debit. I base this intuitive feeling on the fact that approximately 40 per cent of the sales made by our agents are in homes in which there is already existing business.

Furthermore, I suspect that a substantial portion of the remaining 60 per cent of sales is to referred leads. It is also known that our agents, on the average, spend only about one day a week servicing their debits.

I therefore conclude that even large debits do not prevent a typical agent from having adequate time to devote to selling but that his main need is for prospects, which, in fact, are supplied by a large debit.