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DISCUSSION OF PAPERS PRESENTED AT EARLIER REGIONAL MEETINGS

A NEW APPROACH TO THE CALCULATION OF ACTIVE LIFE DISABILITY RESERVES

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SEE PAGE 1 OF THIS VOLUME

ROBERT F. DAVIS:

I. *Introduction*

Mr. Stein is to be congratulated for making a very practical contribution to active life disability valuation theory by simplifying the derivation of active life disability net premium formulas. I predict that the new commutation functions defined in his paper will receive industry-wide acceptance. In retrospect, it is really surprising that someone in our Society had not thought of them before, because they seem so natural after one becomes familiar with them.

Mr. Stein's key contribution has been in the definition of the following two commutation functions: ${}^{w}A_x^{vr}$ and ${}_{u-k}\bar{A}^{nr}_{[u]}$. The second function appears to be much more important than the first from a formula standpoint. The first function is important, however, from a computer standpoint, because it can be used to produce values of the second function when $u - k$ is equal to or greater than the select period (in this instance, fifteen years) by using the following formula:

$$ {}_{u-k}\bar{A}^{nr}_{[u]} = {}_{14}\bar{A}^{nr}_{[u]} + \bar{N}^{i}_{u}\left({}^{w}A^{vr}_{k} - {}^{w}A^{vr}_{u-14}\right) . $$

This formula is important because it makes possible a disability valuation program which will contain all required commutation functions in about 10,000 positions of computer storage (5,000 for benefit 4 and 5,000 for benefit 5 functions). This would not be possible if the program was required to hold all possible values of ${}_{u-k}\bar{A}^{nr}_{[u]}$ instead of merely those values for $0 < u - k \leq 14$. However, I would like to see published some time soon all possible values of this function for both benefits 4 and 5 to facilitate hand checking of net premiums produced by a computer program.

I like also Mr. Stein's new commutation function ${}^{w}\bar{M}^{i}_{x}$. It is a much more useful function for the usual disability premium calculation than the standardized function ${}^{u}_{v}\bar{M}^{r}_{x}$. A lot more computer storage would be required to hold all possible values of the latter function.

I have three mildly critical comments to make about Mr. Stein's paper. They are as follows:

1. The net premium formulas require the difference between the $(t-1)$st and tth different basic plan gross premiums.
2. The tests required for selecting the applicable net premium formulas seem unduly complicated.
3. A minimum assumption of two different basic policy gross premiums is required for level premium policies.

Section III of this discussion contains a development of three general waiver disability net premium formulas which I believe will produce the same premium as Mr. Stein's formulas and which do not have to as high a degree the three defects described above.

During the development of the Section III net premium formulas, I found it convenient to use some new or redefined symbols. Such symbols are defined in Section II. My only purpose in redefining any symbols used in Mr. Stein's paper is to cut down on their size.

II. *Redefinitions*

Most of the symbols used in the formulas shown in Section III are either standard or were defined in Mr. Stein's paper. However, there are a few symbols which were either not defined in his paper or which I have redefined. I am not very pleased with the superscripts used with some of the symbols shown below, but there are only twenty-six letters in the alphabet and many of these letters had already been used with previously defined symbols. The new and redefined symbols which I have used in Section III are shown below:

$m = 6$ (months).

$D^{i}_{[t]} = D^{i}_{[t+1/2]+1/2}$.

${}_{t}\bar{N}^{i}_{u} = \bar{N}^{i}_{[t+1/2]+u-t-1/2}$.

$v^{1/2}C^{r}_{t} = k_{t}$ (I apologize for using this symbol when Mr. Stein had another definition for k in his paper).

${}_{t}\bar{A}^{r}_{u} = {}_{u-t}\bar{A}^{nr}_{[u]}$, when $u - t \leq 14$,

$= {}_{14}\bar{A}^{nr}_{[u]} + \bar{N}^{i}_{u}\,({}^{w}A^{vr}_{t} - {}^{w}A^{vr}_{u-14})$, when $u - t > 14$. Note that I have dropped the superscript n from Mr. Stein's function. I can see no useful purpose being served by it. I might make the same comment also about the superscript v in the function ${}^{w}A^{vr}_{x}$.

${}^{p}G_x^q$ = Total gross annual premium for a policy originally issued at age x which is payable at the insured's attained policy ages p through $q - 1$.

${}^{p}_{y}p^{qw}_{x:\overline{z-x}|}$ = Net annual waiver premium payable from age x through age $z - 1$ which will provide a waiver benefit in the amount of ${}^{p}G_x^q$ from attained age p through age $q - 1$, provided the insured becomes disabled between ages x and q or x and y if y is less than q.

III. *Development of General Net Premium Formulas for* ${}^{p}_{y}p^{qw}_{x:\overline{z-x}|}$

It appears that the appropriate formula to use for calculating ${}^{p}_{y}p^{qw}_{x:\overline{z-x}|}$ depends upon which of the following conditions is true:

CONDITION I. $y \geq q$.
CONDITION II. $p \leq y < q$.
CONDITION III. $y < p$.

A. Condition I Formula

From the definition given for ${}^{p}_{y}p^{qw}_{x:\overline{z-x}|}$ in Section II, it should be obvious from general reasoning that the formula for ${}^{p}_{q}p^{qw}_{x:\overline{z-x}|}$ is as shown below:

$$ {}^{p}_{q}p^{qw}_{x:\overline{z-x}|} = ({}^{p}G_x^q)\left\{\sum_{t=x}^{p-1} \frac{k_t({}_t\bar{N}^i_p - {}_t\bar{N}^i_q)}{D^i_{[t]}} + \sum_{t=p}^{q-1} k_t\left[\frac{({}_t\bar{N}^i_{t+1} - {}_t\bar{N}^i_q)}{D^i_{[t]}} + \tfrac{1}{2}\right]\right\} \Big/ (N_x - N_z) . \tag{Ia} $$

The first summation term

$$ \sum_{t=x}^{p-1} $$

in formula (Ia) can be rewritten in terms of previously defined commutation functions as shown below:

$$ {}_x\bar{A}^r_p - {}_x\bar{A}^r_q + {}_p\bar{A}^r_q . \tag{Ib} $$

Similarly, the second summation term

$$ \sum_{t=p}^{q-1} $$

can be rewritten in terms of commutation functions as shown below:

$$ {}^{w}\bar{M}^i_p - {}^{w}\bar{M}^i_q - {}_p\bar{A}^r_q . \tag{Ic} $$

By substituting the commutation functions of formulas (Ib) and (Ic) for the appropriate summation terms in formula (Ia), we arrive at the following general Condition I formula for ${}^{p}_{q}p^{qw}_{x:\overline{z-x}|}$:

$$ {}^{p}_{q}p^{qw}_{x:\overline{z-x}} = \frac{{}^{p}G^{q}_{x}({}^{w}\bar{M}^{i}_{p} + {}_{x}\bar{A}^{r}_{p} - {}^{w}\bar{M}^{i}_{q} - {}_{x}\bar{A}^{r}_{q})}{N_x - N_z} . \tag{Id} $$

B. Condition II Formula

The only difference between the Condition II ($p \leq y < q$) formula is in the upper limit of the second summation term in formula (Ia). The upper limit under Condition II is $y - 1$ and not $q - 1$. The second summation term with an upper limit of $y - 1$ can be rewritten in terms of commutation functions as shown below:

$$ {}^{w}\bar{M}^{i}_{p} - {}^{w}\bar{M}^{i}_{y} - {}_{p}\bar{A}^{r}_{q} + {}_{y}\bar{A}^{r}_{q} . \tag{IIa} $$

By substituting the commutation functions of formulas (Ib) and (IIa) for the summation terms in formula (Ia), we arrive at the following general Condition II formula for ${}^{p}_{y}p^{qw}_{x:\overline{z-x}|}$:

$$ {}^{p}_{y}p^{qw}_{x:\overline{z-x}|} = \frac{{}^{p}G^{q}_{x}({}^{w}\bar{M}^{i}_{p} + {}_{x}\bar{A}^{r}_{p} - {}^{w}\bar{M}^{i}_{y} - {}_{x}\bar{A}^{r}_{q} + {}_{y}\bar{A}^{r}_{q})}{N_x - N_z} . $$

C. Condition III Formula

The difference between the Condition III ($y < p$) formula for ${}^{p}_{y}p^{qw}_{x:\overline{z-x}}$ and the Condition I formula (Ia) is that there is no second summation term and the upper limit for the first summation term is $y - 1$ rather than $p - 1$. The first summation term in formula (Ia) with an upper limit of $y - 1$ can be rewritten in terms of commutation functions as shown below:

$$ {}_{x}\bar{A}^{r}_{p} - {}_{y}\bar{A}^{r}_{p} - {}_{x}\bar{A}^{r}_{q} + {}_{y}\bar{A}^{r}_{q} . \tag{IIIa} $$

By substituting the commutation functions of formula (IIIa) for the first summation term in formula (Ia) and dropping the second summation term, we arrive at the following general Condition III formula for ${}^{p}_{y}p^{qw}_{x:\overline{z-x}|}$:

$$ {}^{p}_{y}p^{qw}_{x:\overline{z-x}|} = \frac{{}^{p}G^{q}_{x}({}_{x}\bar{A}^{r}_{p} - {}_{y}\bar{A}^{r}_{p} - {}_{x}\bar{A}^{r}_{q} + {}_{y}\bar{A}^{r}_{q})}{N_x - N_z} . \tag{IIIb} $$

It should be very easy to use the three general waiver disability net premium formulas shown above for level premium policies. For such policies $p = x$ and $q = u$. The Condition III formula, of course, is not applicable for these plans because y cannot be less than x.

(AUTHOR'S REVIEW OF DISCUSSION)

MEL STEIN:

Mr. Davis is to be congratulated for his scholarly discussion of this paper.

The functions that he put forth provide an interesting and clever alternative way of viewing the material put forth in the paper.

If we look a little deeper into Mr. Davis' discussion, it can be seen that to use his functions requires the following choices: (1) to store a full two-dimensional table of ${}_t\bar{A}^r_u$ functions in the computer or its mass storage media (e.g., tapes or disks) or (2) to store ${}_t\bar{A}^r_u$ in two separate classes (e.g., ${}^wA^{vr}_x$ and ${}_{u-k}\bar{A}^{nr}_{[u]}$), as is done in the paper.

The first choice will, of course, require a substantially greater amount of computer storage, while the second will result in an approach analogous, if not identical, to that taken in the paper.

The author again wishes to thank Mr. Davis for the time he put into his excellent discussion.