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ADJUSTING MULTIPLE-DECREMENT TABLES

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ABSTRACT

The analysis of a life table by cause of decrement is an important activity of those actuaries who are concerned with mortality projections. Biostatisticians frequently perform the same type of analysis for the purposes of measuring the extent of various public health problems and of estimating the value of the benefits which may arise from success in alternative health programs.

In this type of analysis there are technical problems in actuarial mathematics involved in the modification of probabilities of death under assumptions concerning changes in the forces of mortality and in performing approximate integration on the modified survival function. In addition to these problems, which tend to be mathematical in nature, there are deep questions about the interrelationships among the various causes of decrement that are in the domain of biology.

In this paper a new method of adjusting probabilities of death for a short age interval, based on the assumption of a modification in one of the forces of decrement, is developed. This method is then extended to permit the modification of all the forces of decrement. A numerical example of modifying an abridged life table for assumed changes in the forces of decrement associated with certain specified causes is presented. The example is concerned with the modification of forces of decrement associated with farm accidents.

INTRODUCTION

THE analysis of a life table by cause of decrement is a fascinating activity, but interpreting the results of this type of analysis can be treacherous. Yet, despite all the possible pitfalls, it is a topic of such importance in demography, actuarial science, and public health planning that it cannot be dismissed.

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Jenkins and Lew [5, Table 31] used cause-of-death analysis in their study of the impact of possible mortality changes on life annuity costs. Many actuaries, who are responsible for measuring the degree of risk involved in long-term group annuity rate guarantees, have done some informal analyses of annuity mortality tables by cause of death. The objective of such studies has been to measure the relative magnitudes of the investment and the mortality risks implicit in group annuity rates.

The current generation of actuarial students is introduced to cause-of-death analysis in Spiegelman's text [9, sec. 5.4]. The casual reader of the Spiegelman text, however, may miss the fact that the key estimation equation (equation 5:20) is the contribution of another actuary—T. N. E. Greville [3]. The most recent application of cause-of-death analysis to United States Census data is the work of yet another actuary—Francisco Bayo [1].

THE SCOPE OF THE PROBLEM

There are several problems in cause-of-death analysis which are primarily mathematical in nature. However, there are other, even more serious, problems that intersect both mathematics and biology. One technical actuarial problem in cause-of-death analysis is to modify the conditional probability of survival from age x to age $x + n$ to provide for the complete or partial elimination of the force of mortality associated with one or more specified causes of death. This is the problem that is reexamined in this paper. A second technical actuarial problem, of equal importance, is to develop approximate integration methods, for use with the modified life table, for the purpose of evaluating complete and partial life expectancies and other actuarial functions. This topic is discussed in several actuarial references [3, 4]. The trick to this problem is to minimize the arbitrary assumptions used in evaluating the required integrals.

If one views cause-of-death analysis as a study of two random variables—time until death and cause of death—it is clear that in general one cannot remove all the probability associated with one or more causes without forcing some redistribution of the probability. The problem that remains is to redistribute, in some reasonable fashion, the probability associated with cause of death i. It may be possible to eliminate a cause of death; given that man is mortal, however, the probability which was associated with the eliminated cause will, in fact, be taken up by some other cause.

There is simply no way to avoid this difficult problem. Nesbitt [8], in his discussion of Greville's basic paper [3], mentions that the remaining forces of mortality may not (cannot) remain unchanged when one cause is eliminated. Chiang [2] even suggests an experiment to determine which causes of death can be eliminated and not cause changes in the remaining forces over a restricted age interval. None of these authors, however, directly attacks the redistribution problem.

NOTATION

Notation in multiple-decrement theory is a perplexing matter. International actuarial notation has not been standardized in this area, and many authoritative papers have been written using different notation. In this paper we shall adopt the notation of Greville [3]; that is, a superscript without parentheses will denote that the function comes from an unmodified table and symbols whose superscripts are encased in parentheses are taken from a table which has been modified by the complete or partial reduction of one or more decrements. This convention will help to simplify our notations; for we intend to discuss cases in which causes are not entirely eliminated but simply reduced in intensity.

THE ELIMINATION OF A DECREMENT

Let μ_x^k, $k = 1, 2, \ldots, K$, be the forces of decrement in a multiple-decrement situation in which there are K decrements and x is a fixed age. It is the purpose of this paper to examine the structure of the multiple-decrement table when some or all of the K decrements are changed.

We let q_x^k be the probability that an active life age x will fall into decrement class k within one year, $k = 1, 2, \ldots, K$; and let l_x^k, $k = 1, 2, \ldots, K$, be the expected number of lives age x who will ultimately fall into decrement class k where the total lives

$$l_x^T = \sum_{k=1}^{K} l_x^k .$$

Let us consider the case when one decrement, say, i, is completely eliminated from age x to age $x + 1$. In this section we let the symbols $\mu_x^{(k)}$, $q_x^{(k)}$, $l_x^{(k)}$ denote, respectively, the force of decrement, probability of decrement, and expected number of lives for decrement k after decrement i is eliminated. Chiang [2] and Greville [3] have studied this case under the following assumptions.

1. That the forces of decrement are additive:

$$\mu_{x+t}^T = \mu_{x+t}^1 + \mu_{x+t}^2 + \ldots + \mu_{x+t}^K , \qquad 0 < t < 1 . \quad (1)$$

If μ^k_{x+t}, $k = 1, 2, \ldots, K$, is viewed as the conditional probability density at age $x + t$ for cause of death k, given survival until age $x + t$, equation (1) follows from the axioms for probability and the fact that the causes of decrement are mutually exclusive.

2. The forces are not interdependent in the sense that

$$\begin{aligned} \mu^{(k)}_{x+t} &= \mu^k_{x+t} \qquad (k = 1, 2, \ldots, K, \quad k \neq i, \quad 0 < t < 1)\,; \\ \mu^{(i)}_{x+t} &= 0\,. \end{aligned} \tag{2}$$

This implies, from equation (1), that

$$\mu^{(T)}_{x+t} = \sum_{\substack{k=1 \\ k\neq i}}^{K} \mu^k_{x+t}\,.$$

3. The key operational assumption is that

$$\frac{\mu^k_{x+t}}{\mu^T_{x+t}} = r_{x,k}\,, \tag{3}$$

a constant in the age interval $(x, x + 1)$, $k = 1, 2, \ldots, K$. To obtain the principal result of Greville, it is only necessary to assume that equation (3) holds for the particular cause of decrement to be eliminated.

We now consider an alternative to the assumption stated in equation (2). Recall that from Jordan [6, p. 16], by definition, we have

$$\mu^k_{x+t} = -\frac{1}{l^T_{x+t}} \lim_{h\to 0} \frac{\underset{h}{\Delta}\, l^k_{x+t}}{h} \qquad (k = 1, 2, \ldots, K \text{ and for all } x)\,.$$

In this expression

$$\underset{h}{\Delta}\, l^k_{x+t} = l^k_{x+t+h} - l^k_{x+t}\,.$$

We now assume that

$$\frac{\underset{h}{\Delta}\, l^{(k)}_{x+t}}{l^{(T)}_{x+t}} = \frac{\underset{h}{\Delta}\, l^k_{x+t} + \mu^k_{x+t}\, \underset{h}{\Delta}\, l^i_{x+t}}{l^T_{x+t}} \qquad (k = 1, 2, \ldots, K, \quad k \neq i)\,. \tag{4}$$

This means that the change in the expected number of lives due to decrement group k with decrement group i eliminated, divided by the total lives, equals the change in lives due to group k plus the change in lives due to group i times the force of decrement for group k, all divided by the total lives. In effect, the lives

$$\underset{h}{\Delta}\, l^{i}_{x+t}$$

have been released and are now susceptible to the force of decrement μ^{k}_{x+t}.

It follows that

$$\mu^{(k)}_{x+t} = -\frac{1}{l^{(T)}_{x+t}} \lim_{h\to 0} \frac{\underset{h}{\Delta}\, l^{(k)}_{x+t}}{h}$$

$$= \mu^{k}_{x+t}(1 + \mu^{i}_{x+t}) \qquad (k = 1, 2, \ldots, K, \quad k \neq i).$$

Then, using equation (1),

$$\mu^{(T)}_{x+t} = \sum_{\substack{k=1 \\ k\neq i}}^{K} \mu^{(k)}_{x+t} = \sum_{\substack{k=1 \\ k\neq i}}^{K} \mu^{k}_{x+t}(1 + \mu^{i}_{x+t}) = (\mu^{T}_{x+t} - \mu^{i}_{x+t})(1 + \mu^{i}_{x+t}) .$$

This equation may be interpreted much as the equation

$$q^{(-i)}_{x} = q_x - q^{i}_{x} + \tfrac{1}{2} q^{i}_{x} q^{(-i)}_{x} ,$$

which appears in Spiegelman's text [9, p. 138]. We are employing an assumption about forces of mortality, conditional probability density functions, while the equation from Spiegelman reasons from a discrete model.

Equation (4) is an alternative to equation (2). The important difference between the two is that $\mu^{(T)}_{x+t}$ of equation (4) is slightly larger than $\mu^{(T)}_{x+t}$ of equation (2), due to the exposure of lives $\Delta\, l^{i}_{x+t}$ to the forces of decrement μ^{k}_{x+t}, $k = 1, 2, \ldots, K$, $k \neq i$. This alternative is an attempt to be more realistic about the reduction in forces of decrement due to elimination of one decrement; at least the claim is not made that elimination of one decrement can be accomplished without *any* increase in the other decrements. In that sense, it is an improvement over equation (2). However, the question of the relationship between decrements remains an open one which will require further study and research.

The motivation for the development of the equation $\mu^{(T)}_{x+t}$ displayed in this section came from a desire to provide for a small increase in the remaining forces of mortality when one specified cause of death is eliminated. This was felt to be desirable because the various causes of death frequently seem to be interacting and competing rather than operating independently. It is acknowledged, however, that in the situation where $\mu^T_{x+t} - \mu^i_{x+t} > 1$ and $\mu^i_{x+t} > 0$, the equation developed in this section produces the apparently anomalous result that $\mu^{(T)}_{x+t} > \mu^T_{x+t}$. A weak rationalization for this unusual result might be provided by reasoning that, in a population subject to a very high probability of death, the elimination of a minor cause of death could serve actually to make the downward slope of the survival curve even steeper. Rather than relying on a rationalization which would not be based on biological evidence, however, we prefer simply to state that most of the suggested methodology for cause-of-death analysis may, in special situations, produce anomalous results and that the method developed in this section seems to produce perplexing results when the total probability of death is very high. To provide a more explicit warning of possible trouble, note that, if $\mu^T_{x+t} - \mu^i_{x+t} > 1$, and $\mu^i_{x+t} > 0$, $0 < t < w$, then ${}_wq^{(T)}_x > {}_wq^T_x = 1 - {}_wp_x > 1 - e^{-w}$.

CALCULATION OF $q^{(k)}_x$ USING EQUATIONS (1), (4), AND (3)

Chiang [2] and Greville [3] calculated $q^{(T)}_x$ and $q^{(k)}_x$ using the assumptions stated in equations (1), (2), and (3). Their results, written in our notation, follow:

$$q^{(T)}_x = 1 - p_x^{(q^T_x - q^i_x)/q^T_x} ; \tag{5}$$

$$q^{(k)}_x = \frac{q^k_x}{q^T_x - q^i_x} \cdot q^{(T)}_x , \tag{6}$$

where $p_x = 1 - q^T_x$. This is equation 5:20 in the Spiegelman text [9, p. 138].

If we replace the assumption stated in equation (2) with that of equation (4), we obtain

$$q^{(T)}_x = 1 - \exp\left[- \int_x^{x+1} \mu^{(T)}_s ds \right]$$

$$= 1 - \exp\left\{ - \int_x^{x+1} [\mu^T_s - \mu^i_s + \mu^T_s \mu^i_s - (\mu^i_s)^2] ds \right\} .$$

From the assumption stated in equation (3) we obtain

$$\frac{\mu^i_s}{\mu^T_s} = \frac{q^i_x}{q^T_x} \qquad (x \le s < x + 1) , \tag{7}$$

so that

$$q_x^{(T)} = 1 - p_x^{(q_x^T - q_x^i)/q_x^T} \exp - \left\{\left[\frac{q_x^i}{q_x^T} - \left(\frac{q_x^i}{q_x^T}\right)^2\right] \cdot a\right\}, \tag{7a}$$

where

$$a = \left[\int_x^{x+1} (\mu_s^T)^2 ds\right].$$

In order to calculate a, either assume a uniform distribution of deaths, which implies that

$$\mu_{x+t} = \frac{q_x^T}{1 - tq_x^T} \qquad (0 \leq t < 1),$$

or use the Balducci assumption, which implies that

$$\mu_{x+t} = \frac{q_x^T}{1 - (1 - t)q_x^T} \qquad (0 \leq t < 1).$$

Now recall the following approximation [6, p. 17]:

$$\text{colog } p_x \doteq \frac{q_x^T}{1 - \frac{1}{2}q_x^T}. \tag{8}$$

We obtain, under either assumption,

$$a = \frac{(q_x^T)^2}{1 - q_x^T} \doteq \text{colog } p_x \cdot \frac{(1 - q_x^T/2)}{(1 - q_x^T)} q_x^T, \tag{9}$$

and, consequently,

$$q_x^{(T)} \doteq 1 - p_x^{A+BC}, \tag{10}$$

where

$$A = \frac{q_x^T - q_x^i}{q_x^T},$$

$$B = \frac{q_x^i}{q_x^T} - \left(\frac{q_x^i}{q_x^T}\right)^2,$$

and

$$C = \frac{(1 - q_x^T/2)}{(1 - q_x^T)} \cdot q_x^T.$$

To find $q_x^{(k)}$, write, by definition,

$$q_x^{(k)} = \int_x^{x+1} \exp\left[-\int_x^s \mu_y^{(T)} dy\right]\mu_s^{(k)} ds$$

$$= \int_x^{x+1} \exp\left[-\int_x^s \mu_y^{(T)} dy\right]\mu_s^{(T)} \cdot \left(\frac{\mu_s^{(k)}}{\mu_s^{(T)}}\right) ds .$$

Recall that

$$\frac{\mu_s^{(k)}}{\mu_s^{(T)}} = \frac{\mu_s^k}{\mu_s^T - \mu_s^i} ,$$

and use equation (7) to obtain

$$q_x^{(k)} = \frac{q_x^k}{q_x^T - q_x^i} \cdot q_x^{(T)} . \tag{11}$$

Notice that this matches the previous result, equation (6).

The important difference between formulas (5) and (10) is the second term in the exponent of p_x. This is the result of the additional exposure of lives from decrement i to the other decrements. This difference carries over to $q_x^{(k)}$ through $q_x^{(T)}$. It is admitted that this adjustment will have only a small impact on estimates of $q_x^{(T)}$ for most bodies of data. We have no empirical evidence of the superiority of assumption (4). Yet the new method does face up to the problem of interactions when one cause of decrement is eliminated.

As was pointed out earlier, the method for computing $q_x^{(T)}$ suggested in this paper may produce the anomalous result that $q_x^{(T)} > q_x^T$. This possibility is apparent once more in the approximation given in equation (10). In those cases in which $q_x^T > 2 - \sqrt{2} \doteqdot 0.6$, the factor $BC > 1$ and $q_x^{(T)} > q_x^T$.

PARTIAL REDUCTION IN DECREMENTS

Let θ_x^k be a factor called an "improvement factor," which, under assumption (3) of no interdependence of decrements, will be multiplied by μ_{x+t}^k to produce a new force of decrement $\mu_{x+t}^{(k)}$. That is,

$$\mu_{x+t}^{(k)} = \mu_{x+t}^k \theta_x^k \qquad (k = 1, 2, \ldots, K, \quad 0 \le t < 1) . \tag{12}$$

This factor, θ_x^k, allows for *partial* elimination of decrement group k. Of course, if $\theta_x^k = 0$, decrement k is completely eliminated; if $\theta_x^k = 1$, the force of decrement, μ^k_{x+t}, remains unchanged; and, if $\theta^k_x > 1$, the force of decrement, μ^k_{x+t}, becomes larger.

Equation (12) is a generalization of equation (2).

It would be possible to calculate $q^{(k)}_x$, $k = 1, 2, \ldots, K$, a set of adjusted decrement probabilities corresponding to θ^k_x, $k = 1, 2, \ldots, K$, by following through with Chiang's [2] methods. Let us consider, however, the alternative assumption that $(1 - \theta^k_x)\,\mu^k_{x+t}$, $k = 1, 2, \ldots, K$, will represent a proportion of decrement k which has been eliminated and which must be re-exposed to the general population hazards. That is, we replace equation (11) with

$$\mu^{(k)}_{x+t} = \mu^k_{x+t}\theta^k_x\left[1 + \sum_{i=1}^{K}(1 - \theta^i_x)\mu^i_{x+t}\right] \qquad (k = 1, 2, \ldots, K)\,. \tag{13}$$

Then,

$$\mu^{(T)}_{x+t} = \sum_{k=1}^{K}\mu^{(k)}_{x+t} = \left[\mu^T_{x+t} - \sum_{k=1}^{K}(1 - \theta^k_x)\mu^k_{x+t}\right]\cdot\left[1 + \sum_{i=1}^{K}(1 - \theta^i_x)\mu^i_{x+t}\right].$$

Then one may find

$$q^{(T)}_x = 1 - \exp\left[-\int_x^{x+1}\mu^{(T)}_s ds\right] = 1 - p^{A'}_x e^{-B'a}\,, \tag{14}$$

where

$$A' = \frac{q^T_x - \sum_{k=1}^{K}(1 - \theta^k_x)q^k_x}{q^T_x}\,,$$

$$B' = \frac{\sum_{i=1}^{K}(1 - \theta^i_x)q^i_x}{q^T_x} - \frac{\left[\sum_{i=1}^{K}(1 - \theta^i_x)q^i_x\right]^2}{(q^T_x)^2}\,,$$

$$a = \int_x^{x+1}(\mu^T_s)^2 ds\,.$$

Then, by carrying through the same manipulations used in deriving equation (10), one arrives at the approximation

$$q^{(T)}_x \doteq 1 - p^{A'+B'C}_x \tag{14a}$$

where, as before,

$$C = \frac{(1 - q^T_x/2)}{(1 - q^T_x)}\cdot q^T_x\,.$$

The generalization of equation (11) is also easily calculated:

$$q_x^{(k)} = D\cdot q_x^{(T)}, \tag{15}$$

where

$$D = \frac{q_x^k \theta_x^k}{q_x^T - \sum_{i=1}^{K}(1-\theta_x^i)q_x^i}.$$

THREE NUMERICAL EXAMPLES

The fact that mortality probabilities are usually rather small serves to discourage the development of an elaborate methodology for estimating

TABLE 1

DIFFERENCES IN THE ADJUSTED PROBABILITIES OF DEATH FOLLOWING THE ELIMINATION OF CAUSE OF DEATH i

q_x^T (1)	q_x^i (2)	$q_x^{(T)}$ Equation (5) (3)	$q_x^{(T)}$ Equation (10) (4)
.01	.001	.0090	.0090
	.005	.0050	.0050
.10	.01	.0905	.0914
	.05	.0513	.0540
.50	.05	.4641	.4886
	.25	.2929	.3791

or analyzing these probabilities. As indicated earlier, the method of adjusting the conditional probabilities of death for the reduction in the force of mortality associated with a specified cause of death which is suggested in this paper will not result in significantly different answers from those obtained using less complicated methods when the probability of death is small. In Table 1 this point is illustrated. Column 3 contains the adjusted probability of death after the elimination of cause i by using equation (5) of this paper. Column 4 employs the approximate computational method given by equation (10) of this paper for arriving at the adjusted probability.

In "An Index of Health and the Allocation of Health Resources," Krall [7] constructed a multiple-decrement table for eleven types of fatal accidents on farms for males, 1965 (see Table 2).

The probabilities of death, q_x^k, $k = 1, 2, \ldots, 12$, are given in Table 3 for

TABLE 2

INTERNATIONAL CAUSE OF DEATH CODES*

	Name	Code
1.	Machinery	912
2.	Drownings	929
3.	Firearms	919
4.	Falls	900–904
5.	Blows from falling objects	910
6.	Animals and insects	927–28
7.	Burns from fire and hot substances	916–18
8.	Electric current	914
9.	Poisonings	870–95
10.	Lightning	935
11.	Other	....

* Numbers after causes of death are category numbers of the Seventh Revision of the *International Lists*, 1955 (Series E).

TABLE 3

FATAL ACCIDENTS ON FARMS, MALES, 1965
MULTIPLE-DECREMENT TABLE*

(Rates per 100,000)

AGE CLASS	DECREMENT GROUP				
	912	929	919	900–904	910
0–4	5.761	7.017	0.129	0.249	0.767
5–9	5.192	6.878	1.041	0.633	0.648
10–14	6.316	8.335	4.500	0.274	0.280
15–19	10.824	11.070	7.123	0.946	1.290
20–24	13.951	5.202	7.705	1.389	1.137
25–29	9.443	4.980	2.550	2.128	1.814
30–34	15.374	3.679	3.301	1.427	0.828
35–39	14.601	1.987	3.425	1.211	3.719
40–44	14.567	2.956	3.243	0.790	3.769
45–49	14.962	1.519	3.124	1.419	2.488
50–54	19.941	1.228	3.818	2.187	2.682
55–59	23.893	1.899	3.382	3.040	2.851
60–64	25.327	1.975	3.714	5.724	4.007
65–69	35.732	2.175	3.184	6.585	4.359
70–74	26.749	4.525	2.980	6.765	2.474
75–79	23.662	2.621	1.435	12.594	1.428
80+	14.302	4.089	0.895	12.211	5.352

* Source data: *Deaths from Nontransport Accidents on Farms by Cause of Death, Age, Color, and Sex, U.S. and Each State, 1965* (furnished by the National Vital Statistics Division, United States Department of Agriculture, United States Department of Health, Education, and Welfare, 1967).
The source data were combined with appropriate population data to form the above table [7].

TABLE 3—*Continued*

Age Class	Decrement Group			
	927–28	916–18	914	870–95
0–4	0.579	0.589	0	0.715
5–9	0.469	1.194	0	0.120
10–14	0.127	0.775	1.169	0.262
15–19	0.875	0.298	2.186	0.603
20–24	0.258	0.786	3.559	1.591
25–29	0.984	0.669	2.649	1.014
30–34	0	0.336	3.428	0.682
35–39	0.560	1.431	2.911	1.159
40–44	0.975	1.737	1.967	0.503
45–49	1.126	1.528	1.298	0.775
50–54	0.404	0.619	0.699	1.251
55–59	1.877	2.151	2.433	0.241
60–64	2.789	1.136	0.322	0
65–69	3.226	4.017	0.414	0.368
70–74	2.682	4.103	0.516	0.459
75–79	3.883	5.938	0	0
80+	3.232	6.576	0	0

TABLE 3—*Continued*

Age Class	Decrement Group		
	935	Other Farm	All Other
0–4	0	0.935	619.248
5–9	0.118	1.658	32.101
10–14	0.639	3.073	26.305
15–19	1.176	1.326	98.654
20–24	0.777	1.818	148.522
25–29	1.325	2.320	154.803
30–34	0.667	1.335	180.845
35–39	0.566	1.702	268.538
40–44	0.984	1.478	431.302
45–49	0.567	1.517	712.552
50–54	1.020	1.636	1,175.14
55–59	0.474	2.843	1,830.75
60–64	0.280	0.846	2,727.67
65–69	1.084	2.175	4,143.10
70–74	0	4.066	5,785.65
75–79	0.650	3.271	8,046.22
80+	0	4.899	11,512.7

* Source data: *Deaths from Nontransport Accidents on Farms by Cause of Death, Age, Color, and Sex, U.S. and Each State, 1965* (furnished by the National Vital Statistics Division, United States Department of Agriculture, United States Department of Health, Education, and Welfare, 1967).

The source data were combined with appropriate population data to form the above table [7].

seventeen age classes. The twelfth decrement represents all mortality other than farm accidents.

Equations (10) and (11) were used to construct a modified table under the assumption $\theta_x^1 = 0$, all x (eliminate all deaths from decrement 1, machinery). The resulting modified decrement rates are given in Table 4.

TABLE 4

MODIFIED MULTIPLE-DECREMENT TABLE

(Rates per 100,000)

Age Class	Decrement Group				
	912*	929	919	900–904	910
0–4	0	7.017	0.129	0.249	0.767
5–9	0	6.879	1.041	0.633	0.648
10–14	0	8.336	4.500	0.274	0.280
15–19	0	11.072	7.124	0.947	1.290
20–24	0	5.203	7.706	1.389	1.137
25–29	0	4.981	2.550	2.128	1.815
30–34	0	3.680	3.302	1.428	1.828
35–39	0	1.987	3.426	1.211	3.720
40–44	0	2.957	3.244	0.790	3.770
45–49	0	1.520	3.125	1.419	2.488
50–54	0	1.228	3.819	2.187	2.682
55–59	0	1.900	3.383	3.041	2.852
60–64	0	1.976	3.715	5.727	4.008
65–69	0	2.176	3.185	6.588	4.362
70–74	0	4.527	2.982	6.767	2.475
75–79	0	2.622	1.436	12.599	1.428
80+	0	4.090	0.895	12.213	5.353

TABLE 4—*Continued*

Age Class	Decrement Group			
	927–28	916–18	914	870–95
0–4	0.579	0.589	0	0.715
5–9	0.469	1.194	0	0.120
10–14	0.127	0.775	1.169	0.262
15–19	0.876	0.298	2.186	0.603
20–24	0.258	0.786	3.559	1.591
25–29	0.984	0.669	2.649	1.014
30–34	0	0.336	3.429	0.682
35–39	0.560	1.431	2.911	1.160
40–44	0.975	1.737	1.968	0.503
45–49	1.126	1.529	1.298	0.775
50–54	0.404	0.618	0.699	1.252
55–59	1.878	2.151	2.434	0.241
60–64	2.790	1.136	0.323	0
65–69	3.227	4.019	0.414	0.368
70–74	2.683	4.104	0.517	0.459
75–79	3.884	5.940	0	0
80+	3.232	6.578	0	0

TABLE 4—*Continued*

Age Class	Decrement Group		
	935	Other Farm	All Other
0–4	0	0.935	619.302
5–9	0.118	1.658	32.103
10–14	0.639	3.073	26.307
15–19	1.176	1.326	98.670
20–24	0.777	1.819	148.553
25–29	1.325	2.320	154.825
30–34	0.667	1.335	180.887
35–39	0.566	1.703	268.597
40–44	0.984	1.478	431.396
45–49	0.567	1.518	712.712
50–54	1.020	1.637	1,175.49
55–59	0.474	2.844	1,831.41
60–64	0.280	0.846	2,728.71
65–69	1.084	2.176	4,145.33
70–74	0	4.067	5,787.99
75–79	0.650	3.272	8,049.10
80+	0	4.900	11,515.2

A second modified table was constructed with partial improvements for each decrement. These factors were derived [7] to be the most optimistic improvements which would be possible for one year with a maximum effort. The actual improvement factors used were: $\theta_x^1 = .25$, $\theta_x^2 = .93$, $\theta_x^3 = .96$, $\theta_x^4 = .70$, $\theta_x^5 = .90$, $\theta_x^6 = .90$, $\theta_x^7 = .70$, $\theta_x^8 = .97$, $\theta_x^9 = .83$, $\theta_x^{10} = .96$, $\theta_x^{11} = .96$, $\theta_x^{12} = 1.00$, all x. Formulas (14) and (15) were used to produce the modified decrement rates in Table 5.

CONCLUSION

The structure of the multiple-decrement table has been examined when some or all decrement rates have been changed. A new assumption for the relationship between decrement rates, given that one or more rates have changed, is introduced, and formulas for $q_x^{(k)}$ and $q_x^{(T)}$, probabilities of decrement for the new situation, are calculated. These formulas could be especially useful when it is assumed that the force of mortality associated with specified causes of death is changed but not necessarily eliminated. It is suggested that this is a more appropriate assumption than the assumption of complete elimination.

ACKNOWLEDGMENTS

The help of Mr. L. W. Knapp, Jr., and Mr. W. H. McConnell of the Institute of Agricultural Medicine of the University of Iowa, who provided

TABLE 5

Modified Multiple-Decrement Table*

(Rates per 100,000)

Age Class	Decrement Group				
	912	929	919	900–904	910
0–4........	1.440	6.526	0.123	0.174	0.690
5–9........	1.298	6.397	0.999	0.443	0.583
10–14.......	1.579	7.752	4.320	0.192	0.252
15–19.......	2.706	10.297	6.839	0.663	1.161
20–24.......	3.488	4.839	7.398	0.972	1.023
25–29.......	2.361	4.632	2.448	1.490	1.633
30–34.......	3.844	3.422	3.170	0.999	1.646
35–39.......	3.651	1.848	3.288	0.848	3.348
40–44.......	3.642	2.750	3.114	0.553	3.393
45–49.......	3.741	1.413	3.000	0.993	2.239
50–54.......	4.986	1.142	3.666	1.531	2.414
55–59.......	5.975	1.767	3.248	2.129	2.567
60–64.......	6.334	1.838	3.566	4.008	3.607
65–69.......	8.937	2.024	3.058	4.611	3.925
70–74.......	6.690	4.210	2.862	4.737	2.227
75–79.......	5.918	2.439	1.378	8.819	1.285
80+........	3.577	3.804	0.859	8.550	4.818

TABLE 5—*Continued*

Age Class	Decrement Group			
	927–28	916–18	914	870–95
0–4........	0.521	0.412	0	0.593
5–9........	0.422	0.836	0	0.100
10–14.......	0.114	0.542	1.134	0.217
15–19.......	0.788	0.208	2.120	0.500
20–24.......	0.232	0.550	3.453	1.320
25–29.......	0.886	0.469	2.569	0.842
30–34.......	0	0.235	3.326	0.566
35–39.......	0.504	1.002	2.824	0.962
40–44.......	0.877	1.216	1.909	0.417
45–49.......	1.013	1.070	1.259	0.643
50–54.......	0.363	0.433	0.678	1.039
55–59.......	1.690	1.506	2.361	0.200
60–64.......	2.511	0.795	0.313	0
65–69.......	2.905	2.813	0.402	0.306
70–74.......	2.415	2.873	0.501	0.381
75–79.......	3.496	4.158	0	0
80+........	2.909	4.605	0	0

* Partial reduction in all decrements.

TABLE 5—*Continued*

Age Class	Decrement Group		
	935	Other Farm	All Other
0–4	0	0.898	619.298
5–9	0.113	1.591	32.103
10–14	0.613	2.951	26.307
15–19	1.129	1.273	98.669
20–24	0.746	1.746	148.550
25–29	1.272	2.227	154.824
30–34	0.641	1.281	180.880
35–39	0.544	1.635	268.590
40–44	0.945	1.419	431.384
45–49	0.544	1.457	712.691
50–54	0.979	1.571	1,175.43
55–59	0.455	2.730	1,831.31
60–64	0.269	0.813	2,728.58
65–69	1.041	2.089	4,145.05
70–74	0	3.905	5,787.79
75–79	0.624	3.141	8,049.17
80+	0	4.705	11,515.8

* Partial reduction in all decrements.

data for the example "Fatal Accidents on Farms," is gratefully acknowledged by the authors. Dr. Paul E. Leaverton, Associate Professor, Department of Preventive Medicine and Environmental Health, the University of Iowa, also participated in the research project which is partially described in this paper.

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DISCUSSION OF PRECEDING PAPER

LESTER R. MC CRACKEN:

The problem of modifying multiple-decrement tables also arises in connection with service tables used to evaluate pension plans. A plan may have several modes of retirement, which may differ by benefit formula, postretirement mortality, valuation method, and type of associated decrement. This last distinction calls for a modification technique which not only preserves the forces of decrement for the existing decrements but also allows the actuary to introduce (or replace) both absolute rates of decrement, such as mortality, while reflecting the underlying force of decrement, and empirical probabilities, such as early retirement, which must be reproduced in the final $q_x^{(k)}$'s for the new table.

Many things can make a study appear to be inconsistent with a previous valuation based on the same data. Consider a study of the financial implications of liberalizing guidelines for disability retirement, perhaps to align them with those of the Social Security Administration. The higher postretirement mortality assumed for retired lives could produce lower costs, although more employees were retiring and more total benefits were payable to retirees. (This outcome would suggest either that the previous mortality assumption for disability was ultra-conservative or that some improvement in postretirement mortality should be introduced for the new "redefined" disabled groups, but such questions are beyond the scope of this discussion.) It is important that each step in the process be internally consistent and easy to explain.

The following approach is an extension of the C-A-P table, a problem-solving technique taught by the late Harry Gershenson in classes sponsored by the Actuarial Club of New York. Since it uses the formula $m_x = q_x/(1 - q_x/2)$, which assumes a uniform distribution of decrement throughout the year, it may not be universally appropriate. In cases where uniform distribution may be assumed, however, this technique appears to be completely general and to produce consistent results. It is useful because it applies only simple algebraic functions and can be performed on either a small computer or a desk calculator.

In order to identify the various methods of decrement modification, it is necessary to introduce a second superscript j, with a range of 1–4 where:

a) $j = 1$ implies that the decrement remains unchanged;
b) $j = 2$ implies a new absolute rate of decrement;
c) $j = 3$ implies an algebraic modification of the absolute or central rate of dec-

rement, the type of modification discussed in "Adjusting Multiple Decrement Tables."

d) $j = 4$ implies a new probability of decrement.

For a given decrement, the j's are mutually exclusive. For instance, if $j = 4$, a new probability, all functions involving other values of j would be zero.

If it is desired to introduce a new mode of exit, not contemplated by the table to be modified, a new set of decrements having a value of zero should be appended initially to the existing table.

Two formulas will be derived: The first develops $q_x^{(T)}$ directly; the second first develops the new $q_x^{(k)}$'s and then sums to arrive at $q_x^{(T)}$.

Let

$$H_x = \sum_{k=1}^{K} q_x^{k,4}$$

and

$$G_x = \sum_{k=1}^{K} \sum_{j=1}^{3} m_x^{k,j} ,$$

where

a) $m_x^{k,1} = q_x^k/(1 - q_x^T/2)$, where q_x^k is the old probability associated with decrement k and the underlying rate is to remain unchanged, or

b) $m_x^{k,2} = q'^k_x/(1 - q'^k_x/2)$, where q'^k_x is the new absolute rate of decrement being introduced for decrement k, or

c) $m_x^{k,3} = F(q_x^T, q_x^k)$ some algebraic function of the old probabilities q_x^T and q_x^k.

Since

$$1_x^{(T)} H_x = \sum_{k=1}^{K} d_x^{k,4}$$

and

$$[1_x^{(T)} - d_{x/2}^{(T)}]/G_x = \sum_{k=1}^{K} \sum_{j=1}^{3} d_x^{k,j} ,$$

one may write

$$d_x^{(T)} = \sum_{k=1}^{K} \sum_{j=1}^{4} d_x^{k,j} = 1_x^{(T)} H_x + [1_x^{(T)} - d_x^{(T)}/2] G_x ,$$

finding

$$q_x^{(T)} = d_x^{(T)}/1_x^{(T)} = (H_x + G_x)/(1 + G_x/2) .$$

The individual $q_x^{(k)}$'s are often needed to develop the various $C_x^{(k)}$'s used to evaluate different modes of retirement.

Since

$$m_x^{k,j} = d_x^{k,j}/[1_x^{(T)} - d_x^{(T)}/2]$$

and

$$q_x^{k,j} = d_x^{k,j}/1_x^{(T)}$$

one can see that

$$
\begin{aligned}
q_x^{k,j} &= \{[1_x^{(T)} - d_x^{(T)}/2]/1_x^{(T)}\}\, m_x^{k,j} \\
&= [q_x^{(T)}/m_x^{(T)}]m_x^{k,j} \\
&= [(1 - H_x/2)/(1 + G_x/2)]m_x^{k,j} \,.
\end{aligned}
$$

It is often more convenient to use the preceding formula to compute $q_x^{k,j}$ where $j \neq 4$ and sum the $q_x^{(k)}$'s to get $q_x^{(T)}$. The following examples demonstrate this second approach.

The original table follows:

	Decrement			
	1	2	3	4
$q_x^k(q_x^T = .140632)$	.013572	.022060	.030000	.075000
m_x^k	.014598	.023728	.032269	.080673

Example 1.—Replace decrement 2 with a new set of decrements based on absolute rates. The absolute rate at age x is .025795.

	Decrement			
	1	2	3	4
$m_x^{k,1}$	.014598		.032269	.080673
$m_x^{k,2}$		.026132		
$G_x = .153672$				
$H_x = 0$				
q_x^k (new values)	.013556	.024267	.029967	.074917
$q_x^{(T)} = .142707$				

Example 2.—Replace decrement 4 with a new set of decrements based on empirical probabilities. The probability at age x is .100000.

	Decrement			
	1	2	3	4
$m_x^{k,1}$	.014598	.023728	.032269	
$q_x^{k,4}$				.100000
$G_x = .070595$				
$H_x = .100000$				
q_x^k (new values)	.013395	.021773	.029610	.100000
$q^{(T)} = .164778$				

FRANCISCO BAYO:

It is satisfying to see that some theoretical work is being done in the important and complex area of multiple decrements. This subject has not received in the past the attention that it deserves. In the field of human mortality the possible effects in national demography due to elimination or partial control of some causes of death have not been fully investigated. Also, as the authors indicate, the question of the relationships between the decrements remains an open one which requires further study and research.

In the case of human lives, one can easily conceive the existence of some kind of interrelation among the various causes of deaths. Practically nothing has been done to attempt to discover what these relations are or their persistency in regard to time or among different population groups. We could think of a disease of the respiratory system creating some stress to the cardiovascular system and therefore increasing the mortality from failure of that system. We do not, however, know what the relationships among these causes are.

To a large extent the situation seems to be somewhat in a state of confusion. I believe that most of the causes of death are positively correlated. I would say that in the example indicated above an increase in the diseases of the respiratory system would put a heavier stress on the cardiovascular system and would, therefore, increase even more the mortality from cardiovascular causes. Or, in the reverse case, a decrease in the respiratory diseases would result in a lower mortality from cardiovascular causes. But the authors seem to have proved the reverse. They tell us that a reduction in the mortality rates from one cause produces an increase in the mortality rates from the other causes.

I find it hard to accept the idea that, on an instantaneous basis, the reduction in the force of mortality from one cause could result in an increase in the force of mortality from other causes. I believe that it would be possible to assume that such a reduction would eventually increase the force from other causes at a later age (although I am not convinced that it would be the best assumption); but I cannot see how they could occur simultaneously. I find it impossible to believe that, as I reduce my mortality from automobile accidents by driving more carefully, I would be simultaneously increasing my susceptibility to cancer.

Naturally my chances of eventually dying from a cause other than automobile accidents will increase, but this is due to the fact that we all have to die sooner or later, regardless of the interrelationship among the causes of death.

In theory it can be concluded that a change in mortality at a given age should have some effect on the mortality at all subsequent ages. In this instance it would be helpful to think of mortality at any given age as an average of the expected experience of those who survive to that age. This means that mortality at age 85, for example, is associated with those who survive to that age. If it were assumed that there are significant changes in mortality before age 85, the surviving group would also be different, and, therefore, it would also follow a different mortality pattern at higher ages.

It is entirely possible that, as we acquire the means of prolonging life and as more impaired lives survive to older ages, the mortality of the whole population at those ages could be increasing, but this is far from the instantaneous effect that the authors are suggesting. In fact, I cannot see how it would be possible to assume, as the authors do, that there is only an instantaneous effect and that the mortality at later ages would not be altered.

Equation (4), which is the key equation in the paper, is presented without any discussion of the type of interrelationship among causes that is being assumed by the authors. I believe that an explanation should have been offered for, at least, the last term in the numerator:

$$\mu^{k}_{x+t} \underset{h}{\Delta} l^{i}_{x+t} ,$$

since it is the origin of the correction factor that is developed later in the paper. This term seems to imply that the authors intended to allow the lives

$$\underset{h}{\Delta} l^{i}_{x+t}$$

that have been released to be exposed to the remaining forces of mortality. However, in that case the term would involve a factor h, since the needed death rate is related to the period over which it is supposed to apply. If this correction were adopted, the adjustment factor that is developed later in the paper would vanish, leaving us with the conclusion that $\mu^{(k)}_{x+t} = \mu^{k}_{x+t}$, which is precisely what the authors are trying to avoid. In fact, it can be concluded that, under appropriate assumptions of the type adopted by the authors in equation (4), the forces of mortality for the various causes are independent of one another.

I must add that I fail to see what would be the value of adopting equation (4) without the correction that I am suggesting.

Another point that I would like to discuss is the statement by the authors that in equation (3), which is the key operational assumption, $r_{x,\,k}$ is assumed constant within the interval. I do not believe that such is the case. In the work that I have done in this field it has been assumed that for each interval the average $r_{x,\,k}$ in the life table can be approximated by ${}_nD_x^k/{}_nD_x^T$ from the actual population deaths. This assumption is related to an average $r_{x,\,k}$ and does not imply any particular distribution of that function within the interval.

EDWARD A. LEW:

The mathematical exposition presented by Messrs. Krall and Hickman rests on assumptions about the nature of the components of the force of mortality and on a concept of an independent component. These underlying considerations invite discussion inasmuch as the authors have sought "to provide for a small increase in the remaining forces of mortality when one specified cause of death is eliminated."

A component should not be regarded as independent when its elimination produces changes in death rates from other causes, even in later age intervals. If there is a limit on the human life span—and I believe available evidence points in that direction—then partial elimination of the major components of mortality at some ages must eventually increase the remaining forces of mortality at the older ages. Hence the major components of mortality—currently the chronic diseases—are probably not independent.

On the other hand, many types of accidents have behaved very much as independent components, in that their decrease has not perceptibly affected mortality from other causes. For instance, on-the-job accidents in large-scale industries have been sharply reduced over the years, and there is no evidence that this has increased death rates from other causes, either at the working or the retirement ages.

Where mortality from the major chronic diseases has declined, it has not been because we succeeded in bringing major components of the force of mortality under control but because we learned to mitigate the impact of infections or of acute episodes. In effect, we have managed only to blunt or postpone some of the processes which render us liable to death. The experience of persons with medical histories indicates that medical intervention usually tends to give greater scope to other components of mortality, more commonly in the period immediately following intervention but sometimes also at longer range. Such intervention rarely extirpates a major element of the force of mortality.

More attention needs to be given to the cumulative results that can be achieved by eliminating two or more independent components of mortality. Some thirty years ago Dublin and Lotka demonstrated that the effect of simultaneous elimination of two or more independent causes of death was greater than the sum of the years of life gained from each of the individual causes. Thus when poliomyelitis and some types of farm accidents are both eliminated at the same time, not only are those persons saved who would have died from poliomyelitis but also an additional number who, not having succumbed to the fatal farm accidents, would otherwise have died of poliomyelitis; similarly, an additional number of those who did not die of poliomyelitis are saved from the fatal farm accidents. This illustrates how the elimination of some causes of death releases and exposes additional lives to death rates from the remaining causes; whether mortality from these remaining causes remains unchanged or rises depends on the nature of the causes of death eliminated.

(AUTHORS' REVIEW OF DISCUSSION)

JOHN M. KRALL AND JAMES C. HICKMAN:

In replying to the discussions of our paper, we feel it appropriate to call the attention of the Society to a recent paper by Kimball[1] devoted to a critical review of the method of adjusting a life table for the elimination of one cause of death that is given by equation (5) in our paper. Kimball is critical of the proportionality assumption, stated in equation (3) in our paper, that is used in evaluating the integrals that appear in the exponents of the modified mortality probabilities. He is even more critical, however, when he discusses the assumption stated in equation (2):

> The assumption that risks are mutually independent, while perhaps less tenable than any others which are made, is an essential component of all generally useful procedures developed so far. The assumption implies that the risk of death from one cause is independent of and unaffected by changes in the risk of death from other causes. The patent falsity of this assumption when incorporated in models for populations of living organisms is recognized by most authors, but in theoretical or mathematical developments the implications of this premise are often obscured and its importance is understressed.

Although we did not articulate our concern with the eloquence of Kimball, we started on the study of adjusting multiple-decrement tables with views that were close to his. We owe Mr. McCracken a debt of gratitude for reminding us of what should have been a perfectly obvious observation: the problem of the interaction between causes of decrement

[1] A. W. Kimball, "Models for the Estimation of Competing Risks from Grouped Data," *Biometrics*, XXV (1969), 329–37.

in a pension service table is just as perplexing as it is in a demographic or biostatistical life table.

Unlike Kimball, who sought to avoid the multiple-decrement or competing-risk model directly, we followed earlier authors by adopting the basic framework already constructed by Greville and Chiang. Our objective was to replace the assumption stated in equation (2) with a more general one of the form

$$\mu_{x+t}^{(k)} = \mu_{x+t}^{k}[1 + a(x + t, i, k)] , \quad k = 1, 2, \ldots , k,$$
$$k \neq i, 0 \leq \tau < 1, \mu_{x+t}^{(i)} = 0 .$$

In this general expression, the symbol $a(x + t, i, k)$ denotes an adjustment factor which is to incorporate for the age interval x to $x + 1$ the interrelation between the removal of cause of death i and cause of death k.

In selecting values for $a(x + t, i, k)$, we agreed that they might be less than zero where both causes i and k are members of the same family of diseases. Thus, for example, in the case of cardiovascular diseases, success against one manifestation of this general family of diseases may reduce the probabilities of death from other members of the family. One of us is already on record as predicting that any significant future progress in reducing mortality will depend on additional knowledge of the aging process, and, when this success comes, the favorable impact will be discernible within a large group of diseases.[2]

The contrary view may also be entertained, however. Hickman has observed in the study of mortality among cancer patients that, given partial success against one type of cancer, the patients under study frequently continue to die from other types of cancer with high probability.

Before leaving this topic, we should comment that our thinking was influenced by reading "On the Methodology of Studying Aging in Humans," by William F. Taylor.[3] In section 5.2 of this far-reaching survey paper Taylor discusses "Correlation among Death Rates." Mr. Bayo states his belief that most of the causes of death are positively correlated, and there is certainly some empirical evidence to support his belief. Part of this evidence is reviewed in Taylor's paper. At the risk of quoting Taylor out of context, but with the hope that more actuaries will be

[2] J. C. Hickman and R. T. Estell, "On the Use of Partial Life Expectancies within a Large Group of Diseases in Setting Health Goals," *American Journal of Public Health*, 1969.

[3] *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, Vol. IV, University of California Press, 1961.

motivated to survey this complicated literature, we would like to quote from his paper two sentences that seem to give the essence of his position: "The resulting impression was that there is no positive association between high cardiovascular disease death rates and all-other-cause rates." However, Taylor points out the problems in this type of analysis: "It appears that the impressions obtained from correlation studies on death rates are difficult to evaluate and should be treated with critical care."

It seemed to us that, in making mortality adjustments for the purpose of health planning, an effective upper bound on $a(x + t, i, k)$ is

$$\mu^{i}_{x+t} \Big/ \sum_{k=1, k\neq i}^{K} \mu^{i}_{x+t} ,$$

which will produce $\mu^{(T)}_{x+t} = \mu^{T}_{x+t}$, no change in the total force of mortality. For the practical application that we had in mind, we had no firm evidence about the values of $a(x + t, i, k)$, except that we had some feeling that they would be positive. The general methodology of the paper might be employed for estimates of $a(x + t, i, k)$ derived from any source. Indeed, in the second part of the paper, the generality of these methods is partially exploited.

The modification of multiple-decrement tables is usually done on an a priori basis. In biostatistics work the objective of this adjustment is the setting of health goals, and in employee welfare plans the objective is to estimate the financial impact of changed benefit definitions. That is, in adjusting multiple-decrement tables, although ancillary information may be helpful, the adjustments must always involve subjective elements. In our special study we felt that our particular adjustment factors $a(x + t, i, k) = \mu^{i}_{x+t}$ were conservative and established our point that it is necessary to consider the interactions.

Mr. Lew makes an excellent point when he discusses the possibility that, if two or more causes of death are removed, there may be additional interaction terms. That is, for example, the adjustment factor associated with cause k if causes i and j are removed or reduced would be a function of $x + t$, i, j, and k.

Mr. Lew also raises the issue of adjusting mortality at high ages. At high ages not only is there the problem of redistributing the probability in some reasonable fashion that was moved by assumed health successes at earlier ages, but the assumptions used in the adjustment processes at younger ages are less likely to be satisfied. One of us in the paper mentioned earlier has catalogued some of the pitfalls in using cause-of-death analysis at high ages.

Although we have quoted with approval Kimball's comments on cause-of-death analysis, we do not expect that he would agree with our modification of the multiple-decrement model. Both his simple multinomial model and our method, equation (10), produce higher modified probabilities of death than does the method given by equation (5). Our two methods are not identical, however. The method suggested in this paper would undoubtedly fail Kimball's requirement that the adjustment method be simple. It does appear, however, that we approached the interdependency problem more directly than did Kimball.