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A RESERVE BASIS FOR GUARANTEED BENEFITS UNDER VARIABLE ANNUITY CONTRACTS

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ABSTRACT

This paper describes some of the minimum guaranteed death and maturity benefits that have been offered or proposed in conjunction with variable annuity contracts. It proposes prospective reserves for these guarantees based on change factors that are somewhat analogous to the interest factors used in typical life insurance reserves. A set of change factors is proposed and compared with both historical and simulated results. With the use of these change factors, net premium and reserve values are shown for certain illustrative cases. While this paper refers only to guarantees during the accumulation period of variable annuity contracts, the same techniques can also be applied to other variable contracts.

GUARANTEED BENEFITS UNDER VARIABLE ANNUITY CONTRACTS

For many years most individual fixed-dollar retirement annuity contracts have guaranteed that, if the annuitant died before annuity payments commenced, the death benefit would be at least as much as the total gross purchase payments made under the contract. The future cost of this guarantee can be readily determined by the use of recognized mortality and interest assumptions, as the excess of the gross purchase payments over the cash value during each year is known. Generally, this excess exists for only a few years, and the cost of the guarantee is a very small percentage of the purchase payment.

When individual variable annuity contracts were introduced, the same death benefit guarantee was sometimes included. Under a variable annuity, however, the cash value equals the number of accumulation units that have been purchased multiplied by the unit value. This unit value fluctuates from day to day, depending on the performance of the underlying investments, and cannot be accurately predicted for the future. As a result, the future cost of the guarantee on a variable annuity is far more uncertain than it is on a fixed-dollar annuity.

Currently, in the United States and Canada, attention has turned to problems connected with providing a guaranteed minimum benefit (generally return of gross purchase payments) upon a specified maturity date.

The Harleysville Mutual Insurance Company of Pennsylvania offers such a benefit in conjunction with the sale of mutual fund shares. At least one other company in the United States has attempted to offer a variable annuity with this benefit, and some Canadian companies are also moving ahead in this direction. Two papers on the subject of gross premiums required for guaranteed maturity benefits appeared in the *1969 Transactions*.[1]

As a result of the increasing variety in guaranteed minimum benefits under variable annuity contracts, the ALC-LIAA Subcommittee on Variable Contracts and Separate Accounts (Actuarial) has undertaken a study to determine a consistent approach to reserve requirements for these benefits. This paper has been submitted to the Subcommittee to suggest one such approach, although by no means the only approach; it does not necessarily reflect the views of the Subcommittee. While this paper refers only to guarantees during the accumulation period of variable annuity contracts, the same techniques can also be applied to other variable contracts.

Minimum benefits upon death or maturity could be based on either the gross purchase payment or the net purchase payment and with or without interest. (They could also be based on a cost-of-living, standard-of-living, or stock market index, but the implications of such guarantees are not included in this paper.) The considerations to provide these benefits could come either from an additional premium collected with the purchase payments for the variable annuity or from periodic charges against the assets underlying the variable annuity.

RESERVES REQUIRED FOR GUARANTEED BENEFITS

A basic problem in connection with guaranteed benefits under variable annuity contracts is the determination of net premiums and reserves. Until recently, no specific reserves were established for variable annuity death benefits, following the precedent for fixed-dollar retirement annuities, but some states now require such a reserve. Guaranteed minimum benefits at maturity will undoubtedly require reserves.

In the determination of these reserves, four major decisions are required: (1) whether reserves are located in the general or in a separate account, (2) the general method to be used in assuming future values of accumulation units, (3) the appropriate valuation methods, and (4) assumptions of future investment performance.

[1] Samuel H. Turner, "Asset Value Guarantees under Equity-Based Products," *TSA*, XXI, 459, and Frank P. DiPaolo, "An Application of Simulated Stock Market Trends to Investigate a Ruin Problem," *TSA*, XXI, 549.

Location of the Reserves

As a general rule, the reserves for the type of minimum benefits described in this paper should be held in the general account, since the reserve requirements tend to increase as the unit values of the separate account decrease. The formulas used in this paper are based on the assumption that the reserve is held in the general account.

If, however, the minimum benefits were based on the performance of a stock market index, reserves would more appropriately be held in a separate account whose performance was expected to be similar to that of the index. In such situations each company should have the right to determine which account would be most suitable.

Assumptions of Future Values of Accumulation Units

In order to establish dollar amounts of reserve, it is necessary to place a dollar value on the expected benefits. This requires an assumption of future values of the accumulation units purchased under the variable annuity contract, and that in turn requires assuming the future investment results of the separate account upon which the unit values are based.

In this discussion the expected future value of these accumulation units will be determined using "change factors," the symbol f_n representing the expected change in value during the next n-years. Thus, if U is the current value of accumulation units, the expected value of U n years in the future is assumed to be $f_n U$. These change factors are basically analogous to the interest factors used in typical life insurance reserves.

Valuation Methods: Prospective and Retrospective Reserves

Unlike most other reserves, the reserves for minimum guaranteed benefits may be very different, depending upon whether they are calculated prospectively or retrospectively. To take a simple example, a single-pay variable annuity is sold at age 40 guaranteeing a maturity value at age 60 of at least the gross purchase payment G, which includes the additional premium charged for the guarantee. The formula for the net single premium (NSP) for this guarantee (ignoring any death benefits) is

$$\mathrm{NSP}_{40} = \frac{D_{60}(G - f_{20}U_{40})}{D_{40}}\,.$$

U_{40} is defined as the value of the accumulation units at age 40, so $f_{20}U_{40}$ is the expected value of these accumulation units at age 60.

After ten years the retrospective terminal reserve will equal

$$\frac{(\mathrm{NSP}_{40})(D_{40})}{D_{50}} = \frac{D_{60}(G - f_{20}U_{40})}{D_{50}} .$$

However, the prospective terminal reserve after ten years will equal

$$\frac{D_{60}(G - f_{10}U_{50})}{D_{50}} ,$$

where U_{50} is the actual emerging value of the accumulation units at age 50, and $f_{10}U_{50}$ is in effect the updated expected value of the accumulation units at age 60. The prospective and retrospective reserves will equal each other only in the unlikely event that $f_{20}U_{40} = f_{10}U_{50}$.

Which reserve should be used? I believe that it is essential to use the prospective approach, since it makes use of the current value of the accumulation units and hence takes into account the actual investment results since issue. The retrospective reserves, on the other hand, will tend to be either redundant or insufficient, because they do not reflect the current investment situation. A combination of retrospective and prospective reserves is possible, but, if actual investment results have shown the retrospective reserves to be excessive, there is little justification in continuing to hold them. It is also difficult to justify valuing future benefits differently for two people of the same age with the same minimum guarantees and the same number of accumulation units merely because their purchases were made at different times. For these reasons, as well as for administrative simplicity, I favor using the prospective reserve rather than the retrospective reserve.

The principal difficulty with using the prospective reserve is that it may produce unnecessarily wide swings in reserves required from year to year as a result of fluctuations in accumulation unit values. This problem and proposed modifications of the prospective approach are discussed in the section "Modifications of Prospective Reserves."

It should be noted that the use of a prospective reserve unequal to the retrospective reserve renders the amount of any net premium already paid irrelevant with respect to the reserve calculations. Future net premiums do, of course, affect the amount of reserve; these net premiums would be determined as of the date of issue.

Valuation Methods: Assumptions concerning Future Premiums

While many variable annuity contracts, especially those with front-end loads, provide for level gross purchase payments each year, there are a number of other variable annuity contracts that permit a wide latitude in the amount of purchases that can be made, including no further purchases at all. In some contracts, no scheduled annual amount is ever specified. Depending upon the nature of its contracts, the insurance company may find it more appropriate to base its reserves on future periodic purchases or only on purchases already made.

For example, for a guaranteed maturity benefit (ignoring death benefits) at age 60 of the return of gross purchase payments, with purchases permitted to age 55, the terminal reserve at attained age 50, assuming future periodic payments, would be

$$\frac{D_{60}[\Sigma G + 5G - f_{10}U_{50} - (g_{10} - g_5)(N)] - P_x(N_{50} - N_{55})}{D_{50}},$$

where

ΣG = Total gross purchase payments received to date;
G = Expected annual gross purchase payment;
f_{10} = Ten-year change factor;
U_{50} = Value of accumulation units at age 50;
$g_{10} = f_1 + f_2 + \ldots + f_{10}$;
N = Expected net annual purchase payment used to purchase accumulation units;
P_x = Net annual premium for guaranteed benefit, determined at issue age x, equal to $D_{60}[(55 - x)G - (g_{60-x} - g_5)N]/(N_x - N_{55})$ appropriately adjusted if N is not constant, and never less than zero.

If no further payments were assumed, the terminal reserve would be

$$\frac{D_{60}(\Sigma G - f_{10}U_{50})}{D_{50}}.$$

Regardless of the expectation of further payments, the reserve held should not be less than the reserve assuming no further payments, nor should it ever be negative.

Valuation Methods: Net Premiums as Percentages of Gross Purchase Payments or of Assets

The net premium may be expressed as a percentage of assets in place of or in addition to a percentage of the gross purchase payment. This is more likely to occur with death benefit guarantees than with maturity benefits but could occur with either. Periodic charges against assets to cover mortality and expense guarantees are almost always made, and the portion of this charge applicable to mortality guarantees may be included in the net premium for the death benefit guarantee. In this event the terminal reserve at age y on a single purchase payment annuity with a minimum death benefit equal to the gross purchase payment would be

$$\frac{\sum_{i=1}^{m-y}[C_{y+i-1}(G-f_iU_y)-P(D_{y+i})(f_iU_y)]}{D_y},$$

where

m = The lesser of (1) age at commencement of annuity payout and (2) the highest number for which $G > f_{m-y}U_y$;
G = Gross single purchase payment;
f_i = i-year change factor;
U_y = Value of accumulation units at age y;
P = Net annual premium expressed as a fraction of assets.

For a periodic payment annuity with payments continuing to age m, when the annuity payout commences, the terminal reserve would be

$$\frac{\sum_{i=1}^{m-y}[C_{y+i-1}(\Sigma G+iG-f_iU_y-g_iN)-P(D_{y+i})(f_iU_y+g_iN)]}{D_y},$$

where

ΣG = Total gross purchase payments prior to age y;
G = Gross annual purchase payment;
f_i = i-year change factor;
U_y = Value of accumulation units at age y, excluding those acquired at age y;
$g_i = f_1 + f_2 + \ldots + f_i$;
N = Net annual purchase payment used to purchase accumulation units;
P = Net annual premium expressed as a fraction of assets.

Specific Change Factors Proposed

In arriving at specific change factors for use in determining reserves, an important consideration is the actual historical fluctuation in common stock prices. An analysis of common stock prices from 1871–1969 has been made with the use of the Cowles Commission All Stock Price Index from 1871–1926 and Standard and Poor's 500 thereafter. The average common stock price for each year was used. Dividends were added into the change factor, and a charge of 0.25 per cent was deducted for investment expenses, such as brokerage charges. From these prices and yields, accumulation unit values were calculated for each year. These average annual values and the formulas used to obtain them are shown in Table 1. An-

TABLE 1

UNIT VALUES 1871–1969

Year	Average Value	December Value	Year	Average Value	December Value
1871	1.00000	1.03563	1921	14.97684	16.42091
1872	1.12372	1.16267	1922	19.25108	20.61001
1873	1.13696	1.08435	1923	20.70261	21.26158
1874	1.15744	1.18958	1924	23.01344	26.47351
1875	1.20064	1.21475	1925	29.57089	33.79568
1876	1.17489	1.07453	1926	34.44624	37.17955
1877	0.97767	1.03962	1927	43.55451	50.55590
1878	1.10610	1.15336	1928	58.43796	68.90128
1879	1.40163	1.70301	1929	78.24901	65.61492
1880	1.83185	2.09631	1930	66.16893	50.21031
1881	2.28261	2.24878	1931	46.34075	29.99915
1882	2.26419	2.29616	1932	26.54027	27.07187
1883	2.28147	2.22293	1933	35.79274	40.53599
1884	2.04579	1.93595	1934	40.63765	38.94753
1885	2.09820	2.42684	1935	45.20654	56.41957
1886	2.53832	2.71329	1936	67.50376	75.51848
1887	2.71103	2.63718	1937	69.87449	51.57925
1888	2.65998	2.68052	1938	55.43213	62.58786
1889	2.81764	2.86878	1939	60.60118	63.31005
1890	2.89541	2.58108	1940	58.14441	57.11177
1891	2.87841	3.15672	1941	55.27498	51.12438
1892	3.29117	3.32606	1942	52.54937	59.53791
1893	2.97031	2.81698	1943	71.55595	74.37025
1894	2.86641	2.87220	1944	82.28791	88.34183
1895	3.07167	2.99377	1945	103.54781	120.39918
1896	2.99460	3.04422	1946	120.55450	108.96077
1897	3.25693	3.54380	1947	112.06385	113.65179
1898	3.81865	4.33528	1948	120.30951	120.85812
1899	4.87027	4.74023	1949	124.98113	139.69266
1900	4.93996	5.62375	1950	158.90601	175.58637
1901	6.49106	6.70046	1951	202.78473	218.45999
1902	7.19339	7.00967	1952	233.97911	255.17996
1903	6.45600	6.00912	1953	249.15967	257.08046
1904	6.56930	7.82226	1954	311.90057	374.69863
1905	8.61353	9.29090	1955	438.65696	499.92418
1906	9.54594	9.91985	1956	521.88773	529.89349
1907	8.18412	7.05815	1957	517.52997	480.90955
1908	8.52065	10.08112	1958	559.45507	657.57789
1909	11.01635	11.90206	1959	712.97514	744.47438
1910	11.07011	10.97092	1960	716.06232	739.77114
1911	11.45015	11.56339	1961	870.96092	954.73865
1912	12.35070	12.44148	1962	845.31112	862.02292
1913	11.63041	11.28301	1963	972.33602	1,046.36968
1914	11.61064	10.85096	1964	1,159.98715	1,212.91736
1915	12.48306	14.55363	1965	1,288.87332	1,358.64003
1916	14.88867	15.79584	1966	1,284.34937	1,245.38221
1917	14.29089	11.98620	1967	1,423.99668	1,497.20435
1918	13.74098	14.85510	1968	1,568.94530	1,715.82112
1919	16.86279	17.55890	1969	1,601.87472	1,515.62979
1920	16.27192	14.36778			

NOTE TO TABLE 1

$$AV_{t+1} = AV_t\left[\frac{AI_{t+1}}{AI_t} + 0.5(i_{t+1} + i_t) - 0.0025\right],$$

$$DV_t = AV_t\left[\frac{DI_t}{AI_t} + 0.5i_t - 0.00125\right],$$

where AV_t = average value, year t; AI_t = average stock market index value, year t; DV_t = December value, year t; DI_t = December stock market index value, year t; i_t = stock market index average dividend rate, year t.

other result of this analysis was the production of ninety-eight one-year change factors (starting in years 1871–1968), ninety-seven two-year change factors (starting in years 1871–1967), ninety-six three-year change factors, and so forth, down to fifty-nine forty-year change factors. The one-year change factors are listed in Table 2. I will refer to this as the "historical approach."

In the determination of what specific change factors should be assumed for reserve purposes, two other considerations are also involved: (1) the

TABLE 2

ONE-YEAR CHANGE FACTORS 1871–1969

Year	Factor	Year	Factor	Year	Factor
1871–72	1.12372	1906–7	0.85734	1941–42	0.95069
1872–73	1.01178	1907–8	1.04112	1942–43	1.36169
1873–74	1.01801	1908–9	1.29290	1943–44	1.14998
1874–75	1.03732	1909–10	1.00488	1944–45	1.25836
1875–76	0.97855	1910–11	1.03433	1945–46	1.16424
1876–77	0.83214	1911–12	1.07865	1946–47	0.92957
1877–78	1.13136	1912–13	0.94168	1947–48	1.07358
1878–79	1.26718	1913–14	0.99830	1948–49	1.03883
1879–80	1.30694	1914–15	1.07514	1949–50	1.27144
1880–81	1.24607	1915–16	1.19271	1950–51	1.27613
1881–82	0.99193	1916–17	0.95985	1951–52	1.15383
1882–83	1.00763	1917–18	0.96152	1952–53	1.06488
1883–84	0.89670	1918–19	1.22719	1953–54	1.25181
1884–85	1.02562	1919–20	0.96496	1954–55	1.40640
1885–86	1.20976	1920–21	0.92041	1955–56	1.18974
1886–87	1.06804	1921–22	1.28539	1956–57	0.99165
1887–88	0.98117	1922–23	1.07540	1957–58	1.08101
1888–89	1.05927	1923–24	1.11162	1958–59	1.27441
1889–90	1.02760	1924–25	1.28494	1959–60	1.00433
1890–91	0.99413	1925–26	1.16487	1960–61	1.21632
1891–92	1.14340	1926–27	1.26442	1961–62	0.97055
1892–93	0.90251	1927–28	1.34172	1962–63	1.15027
1893–94	0.96502	1928–29	1.33901	1963–64	1.19299
1894–95	1.07161	1929–30	0.84562	1964–65	1.11111
1895–96	0.97491	1930–31	0.70034	1965–66	0.99649
1896–97	1.08760	1931–32	0.57272	1966–67	1.10873
1897–98	1.17247	1932–33	1.34862	1967–68	1.10249
1898–99	1.27539	1933–34	1.13536	1968–69	1.02034
1899–1900	1.01431	1934–35	1.11243		
1900–1901	1.31399	1935–36	1.49323		
1901–2	1.10820	1936–37	1.03512		
1902–3	0.89749	1937–38	0.79331		
1903–4	1.01755	1938–39	1.09325		
1904–5	1.31118	1939–40	0.95946		
1905–6	1.10825	1940–41	0.95065		

progression of factors should be smooth to avoid discontinuities in reserve requirements and (2) the factors should be conservative, so as to provide adequate reserves. This does not mean, however, that the change factor for every year of issue and duration must be adequate, as long as the reserves in total are adequate.

In light of these considerations, a series of change factors for n-years is proposed equal to the following formula:

$$f_n = 0.8(1.05)^n \qquad (n > 1) .$$

This formula does not represent a formal graduation of any specific data from the historical approach. In general, however, it is designed to be more conservative than 80–90 per cent of the results when the past experience of the stock market index is used.

In the comparison of change factors produced from the historical approach with these proposed values of f_n, it is important to remember that f_n represents the *net* appreciation after deducting any charges against assets. In this evaluation of f_n the annual charges against assets for investment management, administration, mortality and expense guarantees, and minimum benefit premiums are assumed to total 1.0 per cent. The *gross* change factor, F_n, is thus based on the formula

$$F_n = 0.8(1.06)^n \qquad (n > 1) .$$

Comparisons of f_n and F_n with values obtained from the historical approach are shown in Table 3 by using the lowest historical result, the 10th percentile (i.e., values more conservative than 90 per cent of the values for that duration), the 20th percentile, and the median values.

F_n is consistently more conservative than the 20th percentile, and at durations 1–2 and over 20 is more conservative than even the 10th percentile. At durations over three years the median value is consistently more than 130 per cent of F_n.

It is interesting to compare the results of this historical approach with the simulation approach used in DiPaolo's paper.[2] This comparison is made in Appendix I.

These results may, of course, be substantially changed by future investment results, and periodic revisions of the values of f_n may be required.

Values at Year-End

Year-end valuations require half-year values of f_n covering the period from the end of the year to the middle of some future year. We may as-

[2] Frank P. DiPaolo, *op. cit.*, p. 549.

sume $f_{n+1/2}$ equal to $0.5(f_n + f_{n+1})$, with $f_{1/2} = 0.5(1.000 + 0.840) = 0.920$. This results in $F_{1/2} = 0.925$, which is reasonable when compared with the following actual historical results:

CHANGE FACTORS FROM AVERAGE VALUE FOR DECEMBER TO AVERAGE VALUE OF FOLLOWING YEAR

Lowest	0.825
10th percentile	0.923
20th percentile	0.968
Median	1.050

TABLE 3

n-YEAR CHANGE FACTORS

n	f_n	F_n	Historical Results			
			Lowest	10th Percentile	20th Percentile	Median
1	0.840	0.848	0.573	0.916	0.969	1.076
2	0.882	0.899	0.401	0.901	0.997	1.180
3	0.926	0.953	0.339	0.912	1.016	1.231
4	0.972	1.010	0.454	0.952	1.064	1.340
5	1.021	1.070	0.519	0.996	1.137	1.466
6	1.072	1.134	0.578	1.027	1.181	1.638
7	1.126	1.202	0.773	1.072	1.207	1.729
8	1.182	1.274	0.838	1.158	1.289	1.774
9	1.241	1.351	0.708	1.255	1.374	1.906
10	1.303	1.432	0.774	1.307	1.531	2.025
11	1.368	1.518	0.743	1.345	1.643	2.260
12	1.437	1.609	0.706	1.425	1.691	2.363
13	1.509	1.706	0.672	1.585	1.779	2.488
14	1.584	1.808	0.899	1.577	1.958	2.704
15	1.663	1.917	1.052	1.633	2.120	2.681
20	2.123	2.566	1.597	2.557	2.871	3.621
25	2.709	3.433	2.898	3.699	4.197	5.340
30	3.458	4.594	3.690	5.199	5.605	7.841
35	4.413	6.147	5.791	6.614	7.703	10.279
40	5.633	8.226	6.561	8.846	11.065	14.354

Conditions Required for Use of Change Factors

The change factors described in this section should be appropriate under the following conditions:

1. The investment policy of the separate account is not significantly more speculative than the stocks constituting the Standard and Poor's index.

2. The maturity date cannot be elected in such a way that the contractholder can select a low point in unit values.
3 The annual charges made against assets are not significantly larger than 1.0 per cent.

Modifications of Prospective Reserves

In order to evaluate this approach of using specific change factors, it is useful to compare it with a theoretical approach reflecting the probabilistic nature of the changes in unit value. This is done in Appendix II. The following conclusions are reached:

1. If there were a reserve held under the specific factor approach for a given guaranteed benefit, and that guaranteed benefit is increased (by a contractual change), the increase in reserves required under the specified factor approach would normally be at least five times as much as that under the theoretical approach.
2. If a reserve is being held, a change in the value of the accumulation units will generate a change in reserves of up to approximately six times as much under the specific factor approach as that under the theoretical approach.
3. The specific factor approach undervalues the benefit when the guaranteed benefit is less than the value of the units multiplied by the change factor.

These effects are caused by using a specific change factor for each duration instead of recognizing that there is a wide range of possible change factors that may actually occur, each with its own probability and its own cost consequences. The specific change factors chosen affect only the second conclusion, and even there only to a rather limited extent.

The solution is either to drop the specific factor approach altogether or to modify its effects. I have rejected the first alternative, because I believe that there is a strong need to establish reserve requirements that can be understood and computed by all companies and regulatory agencies. We may reach the point when standardized simulation approaches will satisfy these criteria, but I am not convinced that this is the case today.

I believe that the principal defects in the specific factor approach can largely be overcome using the following formula, by valuation basis, for the mean reserve:

$$\text{Mean reserve} = 0.2\,(\text{prospective reserve, end of current year}) + 0.8\,(\text{previous reserve})\,,$$

where

$$\text{Previous reserve} = (\text{mean reserve, end of previous year})\ (1+i) + (\text{gross premiums, current year})\ (1+i)^{1/2}\ .$$

In other words, each year the reserves on hand (including gross premiums, but not reduced by benefits paid) would move one-fifth of the way toward the prospective reserve, which becomes a target instead of the actual reserve. This reflects two assumptions: (1) the prospective reserves fluctuate five times as much as they should and (2) the gross premium is the best indicator at issue of the value of the benefit.

Two refinements should be added: (1) If the net premium is larger than the gross premium, it will be used instead. (2) If the previous reserve exceeds the prospective reserve, the previous reserve is decreased by any benefits paid during the year as a result of the guarantee but not below the amount of the prospective reserve. (This avoids any drain on surplus when the reserve held equals or exceeds the prospective reserve.)

Under this approach, the maximum drain on surplus in any year equals 20 per cent of the excess of the prospective reserve over the previous reserve, plus any benefits paid during the year as a result of the guarantee. The maximum contribution to surplus in any year equals 20 per cent of the excess of the previous reserve over the prospective reserve.

An example of this approach is shown in the section "Illustrative Results."

ILLUSTRATIVE RESULTS

The illustrative results below are based on the proposed change factors and on the 1958 CSO table and 3 per cent interest. Not all of the benefit and age at issue combinations shown would be practical as a plan; they merely illustrate the effect of a difference in age or benefit.

A benefit of 110 per cent of the net purchase payment might represent the guarantee of the gross purchase payment; higher percentages would generally represent a guarantee in excess of the gross purchase payment. The premiums shown below are *net* premiums for valuation purposes, not the gross premiums that would be charged.

Net Single Premiums

1. The guaranteed maturity benefit at male age 60 equals a percentage of the net purchase payment invested in the separate account. No guaranteed death benefit is included. The premium is expressed as a percentage of the net purchase payment.

NET SINGLE PREMIUM FOR GUARANTEED BENEFIT AT AGE 60
DEATH BENEFIT NOT INCLUDED
(Percentage of Net Purchase Payment)

Guaranteed Benefit as Percentage of Net Payment	Age at Issue	
	50	55
110%	0	6.3%
120	0	14.3
130	0	22.2

2. The guaranteed death benefit equals a percentage of the net purchase payment invested in the separate account. There is no guaranteed maturity benefit. Monthly annuity payments commence at age 60, terminating the death benefit. The premium is expressed as a percentage of the net purchase payment.

NET SINGLE PREMIUM FOR DEATH BENEFIT TO AGE 60
(Percentage of Net Purchase Payment)

Guaranteed Benefit as Percentage of Net Payment	Age at Issue	
	50	55
110%	0.8%	1.1%
120	1.6	1.8
130	2.6	2.5

Net Annual Premiums

3. The guaranteed maturity benefit at age 60 equals a percentage of the net purchase payments made between ages 50 and 55. No guaranteed death benefit is included. The premium is expressed as a percentage of the net purchase payments.

NET ANNUAL PREMIUM FOR GUARANTEED BENEFIT AT AGE 60
DEATH BENEFIT NOT INCLUDED

Guaranteed Benefit as Percentage of Net Payment	Percentage of Net Purchase Payment
110%	0 %
120	1.1
130	8.1

4. The guaranteed death benefit equals a percentage of the net purchase payments made between ages 50 and 60. There is no guaranteed maturity benefit. Monthly annuity payments commence at age 60, terminating the death benefit. The premium is expressed as a percentage of assets.

NET ANNUAL PREMIUM FOR DEATH BENEFIT TO AGE 60

Guaranteed Benefit as Percentage of Net Payments	Percentage of Assets
110%	0.11%
120	.30
130	0.45

Mean Reserves

5. A man age 50 buys a variable annuity with a guaranteed maturity benefit at age 60 equal to 110 per cent of the net purchase payments made annually prior to age 55. Each net purchase payment is $1,000. Accumulation units are purchased at the average value during the year, as shown in Table 1; no additional charge against assets is assumed. The gross annual premium for the guarantee is assumed to equal $20 per $1,000 of net purchase payment. The net annual premium for the guarantee is zero. One of these plans is purchased in each of the years 1922–25. Because of the years of maturity involved (1932–35) this is one of the most severe tests of the reserve approach.

The formulas used for the prospective reserve at age y per $1,000 of net annual purchase payment equal

$$\frac{D_{60}[5{,}500 - f_{59\frac{1}{2}-y}U_{y+\frac{1}{2}} - 1{,}000(g_{59-y} - g_5)] - P(N_{y+1} - N_{55})}{D_{y+\frac{1}{2}}} \qquad (y < 54)$$

$$\frac{D_{60}(5{,}500 - f_{59\frac{1}{2}-y}U_{y+\frac{1}{2}})}{D_{y+\frac{1}{2}}} \qquad (y > 54)$$

where

$f_{n-1/2} = 0.5(f_n + f_{n-1})$;

$U_{y+1/2}$ = Value of accumulation units, based on December value;

$g_n = f_1 + f_2 + \ldots + f_n$;

P = Net annual premium.

The formula used for the mean reserve is

$$\text{Mean reserve} = 0.8\,(\text{previous reserve}) + 0.2\,(\text{prospective reserve})\,,$$

where

$$\text{Previous reserve} = (\text{mean reserve, previous year})(1.03) + (\text{gross premium income})(1.015)\,.$$

RESERVES FOR 1922–25 ISSUES

End of Year	Prospective Reserve	Previous Reserve	Mean Reserve
1922......	$ 0	$ 20.30	$ 16.24
1923......	0	57.33	45.86
1924......	0	108.14	86.51
1925......	0	170.31	136.25
1926......	0	221.54	177.23
1927......	0	243.45	194.76
1928......	0	241.20	192.96
1929......	0	219.05	175.24
1930......	0	180.50	144.40
1931......	4,085.91	148.73	936.17
1932......	5,366.53	964.26	1,844.71
1933......	1,400.74	1,900.05	1,800.19
1934......	1,321.58	1,854.20	1,747.68

This reserve approach produces the following pattern of gain from operations (excluding investment income) on the 1922–25 issues:

NET GAIN FROM OPERATIONS ON 1922–25 ISSUES

End of Year	Premium Income	Increase in Reserves	Benefits Paid	Net Gain
1922.....	20.00	16.24	0	3.76
1923.....	40.00	29.62	0	10.38
1924.....	60.00	40.65	0	19.35
1925.....	80.00	49.74	0	30.26
1926.....	80.00	40.98	0	39.02
1927.....	60.00	17.53	0	42.47
1928.....	40.00	− 1.80	0	41.80
1929.....	20.00	− 17.72	0	37.72
1930.....	0	− 30.84	0	30.84
1931.....	0	791.77	0	− 791.77
1932.....	0	908.54	18.14*	− 926.68
1933.....	0	− 44.52	0*	44.52
1934.....	0	− 52.51	0*	52.51
1935.....	0	−1,747.68	269.63*	1,478.05
	400.00	0	287.77*	112.23

* Based on the excess of the guaranteed benefit over the average value of the units for the year. This approximation understates the benefits paid in years when the value of the units exceeds the guaranteed benefit during part of the year.

In retrospect, the surplus drain in 1931 and 1932 proved to be largely unnecessary. However, viewed from the perspective of 1931 and 1932, these reserve levels would certainly have appeared necessary. A larger reserve buildup during the years 1922–30 may seem desirable by hindsight, but, in view of the extreme infrequency of a depression this deep and prolonged, a larger reserve during the predepression years is difficult to justify.

APPENDIX I

A COMPARISON OF THE HISTORICAL AND SIMULATION APPROACHES

Another approach that can be used to determine change factors is the stock market simulation technique used by Frank DiPaolo in his paper "An Application of Simulated Stock Market Trends to Investigate a Ruin Problem." His simulations were based on Standard and Poor's averages for the years 1916–65, with the performance for each month being taken as a separate entity and combined with other monthly performances in random sequence to produce 1,000 simulations of fifty years' experience. Mr. DiPaolo has generously given me additional output from his simulations to produce a series of 1,000 n-year change factors for values of n for 1–50. In the comparisons below I have increased these change factors by a factor of $(1.0050)^n/(1.0025)^n$ to adjust Mr. DiPaolo's expense assumption of 0.50 per cent to my expense assumption of 0.25 per cent. I will refer to this as the "simulation approach."

Table A compares the historical and simulation approaches at the 10th percentile, 20th percentile, and median values.

The simulation approach is far more conservative at the 10th percentile, especially at the longer durations, but is much less conservative at the median point. These differences result principally from the historical approach using the actual results of years 1871–1969, while the simulation approach is based on the six hundred monthly results from 1916–65. The median one-year change factor for 1871–1969 was only 1.075, in comparison with 1.112 for 1916–65. In the determination of the 10th percentile results, however, the years with low change factors are most significant; of the five years with change factors under 0.85, four were in the years from 1929 to 1938. This effect wears off for the most part by the time the 20th percentile is reached; the results under the two approaches are remarkably similar at that point.

A second reason for the differences in 10th percentile results is that the historical approach used average annual stock market prices in determining the change factors rather than prices at the end of each year or aggregations of monthly changes. This results in a lower variance of data from the mean and hence produces higher 10th percentile values.

A third reason for the differences may be that the historical approach reflects whatever cycles or long-term trends occurred during the last ninety-eight years, while the simulation determines each monthly performance on a random basis from among the six hundred monthly factors used. There is good evidence that a long-term upward trend in the stock market indexes has occurred, as Samuel Turner described in "Asset Value Guarantees under Equity-Based Products," and this trend is more directly reflected in the historical approach than it is in the DiPaolo simulation approach.

TABLE A

n-YEAR CHANGE FACTORS

Number of Years	10th Percentile		20th Percentile		Median	
	Historical	Simulation	Historical	Simulation	Historical	Simulation
1	0.916	0.901	0.969	0.964	1.076	1.099
2	0.901	0.898	0.997	1.002	1.180	1.188
3	0.912	0.912	1.016	1.043	1.231	1.277
4	0.952	0.955	1.064	1.092	1.340	1.392
5	0.996	0.974	1.137	1.136	1.466	1.514
6	1.027	1.002	1.181	1.181	1.638	1.622
7	1.072	1.062	1.207	1.243	1.729	1.793
8	1.158	1.067	1.289	1.319	1.774	1.941
9	1.255	1.125	1.374	1.402	1.906	2.053
10	1.307	1.211	1.531	1.453	2.025	2.211
11	1.345	1.230	1.643	1.544	2.260	2.399
12	1.425	1.283	1.691	1.607	2.363	2.642
13	1.585	1.311	1.779	1.731	2.488	2.857
14	1.577	1.416	1.938	1.869	2.704	3.006
15	1.633	1.540	2.120	1.983	2.681	3.205
20	2.557	2.014	2.871	2.705	3.621	4.886
25	3.699	2.996	4.197	3.902	5.340	7.646
30	5.199	3.853	5.605	5.506	7.841	11.154
35	6.614	5.186	7.703	7.763	10.279	17.325
40	8.846	6.822	11.065	10.764	14.354	25.574

In my opinion the choice of years 1871–1969 puts both the depression years and the bull market of the last two decades in better perspective than does the narrower 1916–66 period, and the historical approach's ability to reflect both short-term cycles and long-term trends is also an advantage. The historical approach's use of average annual prices is based on the assumption that purchase payment dates and dates of death or maturity are spread evenly throughout the year and that the contractholder has no way of taking advantage of low points during the year. The principal disadvantage of the historical approach is the sparsity of data, as compared to 1,000 simulations, but this sparsity does not lead to any substantial discontinuities using standards such as the median, 10th, or 20th percentiles.

APPENDIX II

A COMPARISON OF THE SPECIFIC CHANGE FACTOR APPROACH WITH A THEORETICAL APPROACH TO RESERVES

Under the specific change factor approach described in this paper, the value of the guarantee (ignoring mortality and interest) n years in the future is

$$V_n = B_n - f_n U \quad \text{when} \quad B_n > f_n U ,$$

$$V_n = 0 \quad \text{when} \quad B_n < f_n U ,$$

where B_n is the guaranteed benefit in years, f_n is the specific n-year change factor, and U is the current value of units.

The change in V_n if B_n is changed (i.e., if a different guaranteed benefit is offered) is

$$\frac{\partial V_n}{\partial B_n} = 1 \quad \text{when} \quad B_n > f_n U ,$$

$$\frac{\partial V_n}{\partial B_n} = 0 \quad \text{when} \quad B_n < f_n U .$$

The change in V_n when U changes (i.e., when the unit value changes) is

$$\frac{\partial V_n}{\partial U} = -f_n \quad \text{when} \quad B_n > f_n U ,$$

$$\frac{\partial V_n}{\partial U} = 0 \quad \text{when} \quad B_n < f_n U .$$

Under the theoretical approach, the value of the guarantee is

$$V_n = \int_0^k (B_n - zU) h(z) dz ,$$

where $k = B_n/U$ (i.e., the n-year change factor needed to make the value of the units equal to the guaranteed benefit), z is the actual n-year change factor, and $h(z)$ is the probability function for the actual n-year change factor. (This function is assumed to be independent of k, which is not likely to create any distortion significant enough to upset the general conclusions.)

The change in V_n if B_n is changed is

$$\frac{\partial V_n}{\partial B_n} = \frac{\partial k}{\partial B_n} \cdot \frac{\partial V_n}{\partial k} = \frac{1}{U} \frac{\partial}{\partial k} \left[\int_0^k (kU - zU) h(z) dz \right]$$

$$= \frac{\partial}{\partial k} \left[k \int_0^k h(z) dz - \int_0^k z h(z) dz \right]$$

$$= \int_0^k [h(z)dz] + kh(k) - kh(k)$$

$$= \int_0^k h(z)dz \,,$$

which is the probability that the actual n-year change factor will be less than the n-year change factor needed to make the value of the units equal to the guaranteed benefit (i.e., the probability that the guaranteed benefit will exceed the value of the units).

For almost all guarantees currently under consideration, the probability at issue is less than 0.20 (i.e., the change factor needed is less than the twentieth percentile change factor) and frequently will be less than 0.10. The derivative under the specific factor approach, when $B_n > f_nU$, was 1, or at least five times as large. Thus increasing the guaranteed benefit at issue calls for an increase in reserves at least five times as much as theoretically necessary when $B_n > f_nU$, which could exercise quite a restraint on guaranteed benefits that are substantially greater than f_nU. On the other hand, the specific change factor approach obviously undervalues guaranteed benefits where $B_n \leq f_nU$, as it assigns them a value of zero. It should be remembered that B_n is established by formula at issue and is not subsequently affected by changes in unit value.

Of probably greater importance is the effect of a change in U, the value of the units, as this is constantly occurring over the lifetime of the contract.

Under the theoretical approach

$$\frac{\partial V_n}{\partial U} = \frac{\partial k}{\partial U}\cdot\frac{\partial V_n}{\partial k} = -\frac{B_n}{(U)^2}\frac{\partial}{\partial k}\left[\int_0^k (kU - zU)h(z)dz\right]$$

$$= -\frac{B_n}{U}\frac{\partial}{\partial k}\left[k\int_0^k h(z)dz - \int_0^k zh(z)dz\right]$$

$$= -k\left[\int_0^k h(z)dz + kh(k) - kh(k)\right]$$

$$= -k\int_0^k h(z)dz \,,$$

compared with $-f_n$ under the specific factor approach. If $k = f_n$, which occurs when $B_n = f_nU$, $\int_0^k h(z)dz = \int_0^{f_n} h(z)dz$, which generally varies between 0.20 and 0.10 (i.e., the 20th and 10th percentiles). If we assume a value of 0.15, the theoretical derivative equals $-0.15f_n$, or only 15 per cent of the derivative under the specific factor approach.

If $k > f_n$ (i.e., $B_n > f_nU$), which occurs under either a high guaranteed benefit or a recession that has lowered unit values after issue, the difference between the two approaches may be somewhat less.

DISCUSSION OF PRECEDING PAPER

ROBERT P. COATES:

Mr. Hickman has given us a very interesting paper that develops an actuarial technique for premiums and reserves for guaranteed benefits under variable contracts. His basic principle is the development of "change factors" as a means of arriving at an expected value of market performance on a conservative basis. The paper gives a number of examples of the application of these factors to develop premiums and reserves.

Mr. Hickman distinguishes between prospective and retrospective reserves which are based on the premiums developed at issue of the contract from the change factors. The retrospective reserves are the usual accumulation of premiums less claims, while the prospective reserves explicitly recognize the unit values existing at the date of valuation.

As indicated in the paper, one result of using these prospective reserves is that there will be substantial fluctuations in the reserves with changes in the unit values. Mr. Hickman suggests one averaging approach to the calculation of mean reserves to temper the effects of these fluctuations.

A number of calculations have been made in the Equitable to study the death benefit guarantee provided under our variable deferred annuity. These calculations review the benefits that would have been payable under our contract if it had been issued at various points in the past. Unit values were based on Standard and Poor's 425 industrial index, and we considered the period from 1871 to 1967. We thought it would be useful and illuminating to extend these studies and review the reserves during the deferred period that would have been held on the basis of Mr. Hickman's change factors. For this purpose, calculations were based on a deferred annuity with annual premiums of $1,000 invested in a separate account. The net amount invested in the separate account is 85 per cent of the first year's premium and 93 per cent of later premiums. The death benefit is equal to the greater of the sum of the premiums or the value of the units in the separate account. We made calculations for a simplified model office involving the issue of 1,000 contracts a year, beginning in 1871, at each of the ages 25, 35, 45, 52, and 57. An annual consideration of $1,000 was assumed for each contract. Contracts were assumed to be subject to mortality on the basis of our current life insurance experience to retirement at 65 (70 for issue age 57), with no other decrements.

For calculation purposes, we developed a net annual premium for the minimum death benefit[1] guarantee, payable from issue to retirement. Table 1 shows these premiums per $1,000 of annual consideration based on (1) Mr. Hickman's proposed change factors and (2) what we have called the "historic average" basis. These "historic average" premiums are simply the average of separate premiums calculated for each issue year in the period, with the premium for each calendar year of issue and issue age, sufficient, when paid to retirement, to support the death benefit guarantee emerging under the stock price index.

Table 2 shows the results using the premiums based on the change factors and the modified mean reserve structure of Mr. Hickman's paper. Columns 2 and 3 show the gross annuity premiums paid in a year and the

TABLE 1

NET ANNUAL PREMIUM FOR MINIMUM DEATH BENEFIT GUARANTEE PER $1,000 ANNUAL ANNUITY PREMIUMS

Issue Age	Based on Change Factors	Based on Historic Average
25	$ 0.92	$0.14
35	1.83	0.26
45	5.51	0.77
52	13.68	1.94
57	21.35	2.48

value of the accumulation units to the credit of the contract at the end of the year. These have been included primarily to indicate the total funds involved and to give a scale of the relative importance of the death benefit element. Note that the minimum death benefit premiums in Table 2 are less than 1 per cent of the annuity premiums.

The display of the minimum death benefit claims in relation to the premiums is an indication of the margins in the premiums. Note also the level to which the reserves build up in relation to claims. To give an indication of the impact on a company's operations from this approach, we have shown the increase in surplus year by year. This represents the premiums plus interest less claims less increase in reserve. An interest factor of $3\frac{1}{2}$ per cent was assumed.

Under this approach there are a number of years where the primary strain on the company is a decrease in surplus arising from a sharp increase in the prospective reserve. The depression years are particularly striking in this respect. In 1930 the surplus strain was $433,000 because a

[1] The term "minimum death benefit" in this discussion refers to a death benefit equal to the excess, if any, of the sum of the premiums over the value of the units in the separate account.

TABLE 2

(000 Omitted)

Year	Annuity Premiums in Year	Value of Accumulation Units at End of Year	Minimum Death Benefit			
			Premiums in Year	Prospective Reserves at End of Year	Claims of Year	Increase in Surplus of Year
1871....	$ 5,000	$ 5,217	$ 44	$ 25	$ 1	$ 20
1872....	9,990	14,758	88	83	7	25
1873....	14,966	30,151	132	157	16	46
1874....	19,923	47,747	175	311	30	− 2
1875....	24,857	74,262	218	444	42	50
1876....	29,766	84,336	261	1,051	96	− 435
1877....	34,646	119,370	303	1,596	167	− 400
1878....	39,493	172,157	345	1,739	150	61
1879....	44,305	245,138	386	1,574	46	519
1880....	49,077	351,122	427	1,202	6	808
1881....	53,806	407,954	467	1,022	11	652
1882....	58,487	449,110	507	1,075	27	445
1883....	63,115	485,083	545	1,281	55	301
1884....	66,001	574,082	553	1,187	62	604
1885....	68,870	714,927	561	923	22	823
1886....	71,718	813,169	569	728	10	773
1887....	74,543	904,881	577	623	14	687
1888....	77,344	1,153,875	584	470	10	748
1889....	80,119	1,124,920	592	526	15	540
1890....	82,865	1,163,915	599	672	38	436
1891....	84,763	1,350,890	602	621	31	643
1892....	86,649	1,193,188	604	1,086	55	105
1893....	88,521	1,165,896	607	1,780	143	− 212
1894....	90,379	1,287,221	609	2,027	152	229
1895....	92,221	1,216,230	612	2,858	200	− 402
1896....	94,046	1,351,909	614	2,988	230	272
1897....	95,852	1,692,084	617	2,380	24	1,222
1898....	97,637	2,354,460	619	1,664	2	1,355
1899....	99,400	2,240,631	622	1,247	7	1,053
1900....	101,139	2,768,255	624	887	11	995
1901....	102,052	2,729,903	625	707	15	812
1902....	102,956	2,274,415	626	978	50	325
1903....	103,850	2,131,626	627	1,528	122	− 25
1904....	104,733	3,031,405	628	1,108	32	1,037
1905....	105,604	3,547,337	628	782	2	974
1906....	106,462	2,858,926	629	961	16	456
1907....	107,306	2,841,530	630	1,211	52	350
1908....	108,135	3,817,210	631	896	18	949
1909....	108,948	3,861,774	631	740	6	804
1910....	109,744	3,768,266	632	732	17	645
1911....	109,744	3,944,857	632	639	23	724
1912....	109,744	3,391,321	632	943	43	306
1913....	109,744	3,281,473	632	1,247	75	274
1914....	109,744	3,696,379	632	1,076	40	783
1915....	109,744	4,554,957	632	773	3	954
1916....	109,744	4,210,687	632	664	7	756
1917....	109,744	3,836,463	632	699	24	595
1918....	109,744	4,802,849	632	499	12	842
1919....	109,744	4,247,169	632	533	14	607
1920....	109,744	3,241,732	632	1,254	75	− 143

TABLE 2—*Continued*

Year	Annuity Premiums in Year	Value of Accumulation Units at End of Year	Minimum Death Benefit: Premiums in Year	Minimum Death Benefit: Prospective Reserves at End of Year	Minimum Death Benefit: Claims of Year	Minimum Death Benefit: Increase in Surplus of Year
1921	$109,744	$3,965,770	$632	$1,057	$ 65	$ 785
1922	109,744	3,965,134	632	943	19	749
1923	109,744	4,014,829	632	836	14	747
1924	109,744	4,921,762	632	596	6	888
1925	109,744	5,468,657	632	450	4	797
1926	109,744	6,546,877	632	358	3	743
1927	109,744	8,400,128	632	264	0	748
1928	109,744	9,988,698	632	205	1	712
1929	109,744	7,278,197	632	355	14	490
1930	109,744	4,498,141	632	1,346	94	− 433
1931	109,744	2,285,891	632	5,384	554	−3,948
1932	109,744	3,180,532	632	5,828	864	− 668
1933	109,744	3,609,370	632	5,198	295	985
1934	109,744	3,880,333	632	4,330	72	1,448
1935	109,744	5,370,616	632	3,231	1	1,753
1936	109,744	5,225,324	632	2,421	4	1,460
1937	109,744	3,806,891	632	2,072	25	978
1938	109,744	3,775,037	632	1,778	62	885
1939	109,744	3,330,124	632	1,871	91	470
1940	109,744	2,981,689	632	2,341	160	22
1941	109,744	2,668,362	632	3,235	254	− 498
1942	109,744	3,415,714	632	2,569	92	1,226
1943	109,744	3,545,770	632	2,013	5	1,206
1944	109,744	4,063,177	632	1,514	6	1,148
1945	109,744	4,346,282	632	1,118	6	1,045
1946	109,744	3,770,666	632	938	17	817
1947	109,744	3,791,368	632	793	31	767
1948	109,744	3,648,398	632	752	38	656
1949	109,744	4,421,555	632	502	11	893
1950	109,744	5,355,984	632	310	2	845
1951	109,744	5,653,133	632	262	5	698
1952	109,744	5,465,746	632	297	11	609
1953	109,744	6,385,611	632	247	9	695
1954	109,744	8,479,891	632	160	0	742
1955	109,744	9,360,747	632	154	2	658
1956	109,744	8,416,576	632	222	9	577
1957	109,744	8,196,714	632	300	20	556
1958	109,744	9,532,619	632	247	11	696
1959	109,744	8,589,041	632	331	12	558
1960	109,744	9,395,112	632	289	13	684
1961	109,744	8,160,560	632	427	21	494
1962	109,744	8,498,219	632	437	26	619
1963	109,744	9,277,033	632	349	11	731
1964	109,744	9,345,814	632	308	8	687
1965	109,744	8,475,130	632	395	17	550
1966	109,744	8,586,995	632	424	25	600

reserve increase of nearly a million dollars was called for. Both of these figures far outweigh the actual claims of $94,000. The same effect is present in the two succeeding years. The reserves built up during this period were then reduced with significant additions to surplus in the succeeding years. These swings in surplus and the fluctuations in reserves seem to me to be undesirable characteristics of this approach.

These characteristics led us to wonder whether a different approach might not better serve the interest of adequate funding of the death benefit reserve with a less irregular and smaller impact on surplus. The approach tested involved the concept of accumulating a more conservative premium than our "historic average" premium in a reserve which would be charged with claims as they arose and which would be held to a maximum limit. Table 3 shows such a calculation where the premiums were set at three times the "historic average" premiums and where the reserves were restricted not to rise beyond ten years' premiums. The impact on surplus is shown in the last column. From the years 1871 to 1888 there is no contribution to surplus. The reserves built up from the premiums proved sufficient to pay all claims, and the reserve at this point is checked by the maximum of ten years' premiums. Over the succeeding years there is a return to surplus every year with the exception of the 1931–35 depression period. During this five-year stretch the reserve is more than adequate to take care of the claims, and, because it is permitted to decrease, there is no surplus strain to add to the company's problems. From the viewpoint of surplus, the transition to the depression is limited to the dropping-out of what had been a fairly steady contribution to surplus equal to the excess of premiums and interest over claims.

These calculations are, of course, merely a rather crude model, and we plan to extend them to a wider variety of possible market performance by simulation techniques. At this point, we are inclined to believe that it is possible to develop a conservative retrospective approach which will involve less likelihood of undue surplus fluctuations than Mr. Hickman's method.

It may be interesting to note that this approach is somewhat analogous to the Mandatory Securities Valuation Reserve, which builds a fund intended to buffer variations in security values by contributing somewhat arbitrary amounts to the fund until it reaches a maximum point. It also appears to be consistent with the suggested requirement in the published draft version of Regulation 47 in New York which required the following:

> A reserve liability for any such incidental benefit in excess of the accumulated value of the contract shall be accumulated and maintained in the general account of the company pursuant to a plan for such accumulation which specifies a reasonable maximum target for such reserve and is approved by the Superin-

TABLE 3

(000 Omitted)

Year	Minimum Death Benefit			
	Premiums in Year	Claims of Year	Retrospective Reserves at End of Year	Increase in Surplus in Year
1871	$ 17	$ 1	$ 17	$ 0
1872	33	7	44	0
1873	50	16	80	0
1874	67	30	120	0
1875	83	42	166	0
1876	99	96	176	0
1877	116	167	130	0
1878	131	150	115	0
1879	147	46	223	0
1880	163	6	390	0
1881	178	11	574	0
1882	193	27	763	0
1883	208	55	945	0
1884	211	62	1,130	0
1885	214	22	1,366	0
1886	218	10	1,625	0
1887	221	14	1,892	0
1888	224	10	2,176	0
1889	227	15	2,209	259
1890	230	38	2,241	242
1891	231	31	2,268	255
1892	233	55	2,291	237
1893	234	143	2,310	153
1894	235	152	2,325	150
1895	236	200	2,336	106
1896	237	230	2,347	78
1897	238	24	2,358	289
1898	239	2	2,369	313
1899	240	7	2,380	309
1900	241	11	2,390	307
1901	242	15	2,400	305
1902	242	50	2,407	272
1903	242	122	2,414	201
1904	243	32	2,419	293
1905	243	2	2,423	326
1906	243	16	2,426	312
1907	244	52	2,430	277
1908	244	18	2,434	311
1909	244	6	2,437	324
1910	245	17	2,441	313
1911	245	23	2,443	308
1912	245	43	2,445	288
1913	245	75	2,447	257
1914	245	40	2,447	293
1915	245	3	2,447	332
1916	245	7	2,447	327
1917	245	24	2,447	311
1918	245	12	2,447	322
1919	245	14	2,447	320
1920	245	75	2,447	258

TABLE 3—*Continued*

Year	Minimum Death Benefit			
	Premiums in Year	Claims of Year	Retrospective Reserves at End of Year	Increase in Surplus in Year
1921	$245	$ 65	$2,447	$269
1922	245	19	2,447	315
1923	245	14	2,447	320
1924	245	6	2,447	329
1925	245	4	2,447	331
1926	245	3	2,447	331
1927	245	0	2,447	334
1928	245	1	2,447	334
1929	245	14	2,447	320
1930	245	94	2,447	239
1931	245	554	2,218	0
1932	245	864	1,666	0
1933	245	295	1,673	0
1934	245	72	1,907	0
1935	245	1	2,222	0
1936	245	4	2,447	97
1937	245	25	2,447	309
1938	245	62	2,447	271
1939	245	91	2,447	242
1940	245	160	2,447	172
1941	245	254	2,447	76
1942	245	92	2,447	241
1943	245	5	2,447	330
1944	245	6	2,447	329
1945	245	6	2,447	329
1946	245	17	2,447	317
1947	245	31	2,447	303
1948	245	38	2,447	296
1949	245	11	2,447	324
1950	245	2	2,447	333
1951	245	5	2,447	330
1952	245	11	2,447	324
1953	245	9	2,447	326
1954	245	0	2,447	335
1955	245	2	2,447	333
1956	245	9	2,447	326
1957	245	20	2,447	314
1958	245	11	2,447	324
1959	245	12	2,447	322
1960	245	13	2,447	322
1961	245	21	2,447	313
1962	245	26	2,447	309
1963	245	11	2,447	324
1964	245	8	2,447	326
1965	245	17	2,447	317
1966	245	25	2,447	310

tendent as otherwise reasonable. The amount of any death claim in excess of the accumulated value of the contract at the time of death paid pursuant to such incidental death benefit provision shall be charged against such reserve.

The premiums used here, and in Mr. Hickman's paper, are collectible over the full deferred period of the annuity. One refinement which we expect to consider is to shorten the premium-paying period for this benefit, since the death benefit under normal conditions will be payable only in the first few policy years. The historical tests under our form of contract indicated that death benefits were seldom payable after the tenth year, with the extreme case being a benefit in the sixteenth year.

One additional thought that may be of interest is a comparison of these approaches with a viewpoint generally held on interest and mortality factors. For the usual forms of insurance and annuities, the reserve structure is based on a mortality table and an interest rate which are believed to be conservative representations of future experience. When the actual experience differs from the expected, the change in surplus emerging in a given year can be expressed in the usual form as

$$\Delta S = (i' - i)({}_{n-1}V + \pi) + (q - q')(DB - {}_nV) ,$$

where ΔS is the contribution to surplus and the primed symbols involve actual experience. Interest or mortality assumptions are not usually changed unless circumstances have departed so far from the original assumption that a change is deemed necessary. Interest rates below the reserve assumption or less favorable mortality in a particular year would not necessarily call for a change of reserve basis. They would, however, tend to produce a charge to surplus in the particular year.

This suggests the possibility of a similar viewpoint for the minimum guaranteed death benefit, that is, it would be possible to develop a reserve structure consistent with the assumptions made at issue without changing them unless experience indicated that the assumptions for market performance, as well as interest or mortality, required alteration. In such a structure the reserves would be consistent with the assumed premiums and prospective and retrospective reserves would retain their usual equality. The gain or charge to surplus would be as follows:

$$\Delta S = (i' - i)({}_{n-1}V + \pi) + (q - q')(DB - {}_nV) - q'(DB' - DB) ,$$

where DB' equals the actual minimum death benefit and DB equals the "tabular" minimum death benefit projected at issue.

This expression differs from the usual form by the addition of a term covering the departure of the amount of death benefit from that assumed in the premium and reserve structure.

We have not had an opportunity to investigate the possible consequences of this approach, but it may offer alternate possibilities for smoothing out surplus fluctuations within a theory which contains a reasonably conservative allowance for investment fluctuations.

I should like to acknowledge the assistance of my associates Paul M. Kahn and Jeffrey K. Tilford in developing this discussion.

LAWRENCE MITCHELL:

I agree with Mr. Hickman's observation that a prospective reserve based upon the *actual* value of accumulation units and *assumed* rates of mortality and interest will produce wide fluctuations in reserves. Perhaps such a valuation method, whether modified or not, is appropriate in a gross premium valuation, but only there.

In my opinion, it is inappropriate as a reserve for either the annual convention blank or the adjusted earnings annual report to stockholders (policyowners).

Historically, policy reserves established by companies have been based upon average amounts and have not concerned themselves with the real life fluctuations which occur in any year. To the extent that experience differs from assumed, such gains or losses will be noted in the summary of operations.

Therefore, in order to be consistent with the valuation of other liabilities in the convention blank, and in any adjusted earnings, we should use an assumed average benefit, to go along with the assumed rate of interest and rates of mortality (the 3 per cent and 1958 CSO in the paper). Until such time as we are required to value all assets and liabilities on a "real world basis," we should maintain the consistency of "assumeds."

We have prepared a study of normal reserves compared with assumed benefit payments for a benefit which may be described as a ten-year variable endowment. If the insured dies within ten years after purchase, and the value of his account is less than the gross premium, the benefit is equal to the difference in value between the gross premium and his account. The benefit contemplates single premiums (the benefit and its history are described in my paper in Volume XIX of the *Proceedings*, Conference of Actuaries in Public Practice).

Tables 1–3 assume that 1,000 persons will invest $1,000 each, subject to a 9 per cent load, and their value will follow the Standard and Poor closing averages for each of the ten years. Table 1 shows the results assuming that the ten-year investment started on December, 1919. Columns headed Accum 1 and Accum 2 are equal to an initial gross premium of 2.25 and 3.0 per cent, respectively, accumulated at $4\frac{1}{2}$ per cent with benefits assumed to be paid at the end of each year. The columns headed

VD, VW, and VT are the reserves released at death (assuming male aged 47 at entry and U.S. 59–61 white male mortality rates), reserves released by withdrawal (assuming a 5 per cent per year redemption rate), and the terminal reserve (assuming a net premium of 3 per cent).

Table 2 is equivalent to Table 1 but combines each of the forty ten-year periods (40,000 lives) beginning with 1918.

Table 3 shows the effect over the fifty-year period of the overlapping ten-year periods beginning with the year 1918.

TABLE 1

(Assuming Male Aged 47; Year 1919)

Duration	Payout	Accum 1	Accum 2	VD	VW	VT
1	1,998	21,513	29,351	231	1,500	29,618
2	3,127	19,353	27,543	250	1,480	29,220
3	2,309	17,915	26,474	270	1,461	28,803
4	1,748	16,972	25,916	291	1,440	28,367
5	1,248	16,487	25,834	313	1,418	27,912
6	0	17,229	26,996	338	1,395	27,434
7	0	18,005	28,211	364	1,371	26,932
8	0	18,815	29,481	391	1,346	26,406
9	0	19,662	30,807	419	1,320	25,855
10	0	20,546	32,194	447	1,292	25,278
Endowment	0	20,546	32,194	0	25,278	0

Net Premiums/M at 0%, 10.43; at 3%, 9.77; at 6%, 9.19

NOTES.—Standard and Poor closing averages. Premiums at 0, 3, and 6 per cent. Reserve accumulated at 4.5 per cent. Reserve 1 at 2.25 per cent; reserve 2 at 3 per cent. Redemption rate 5 per cent. Load 9 per cent. 1,000 lives. Initial investment, $1,000.

TABLE 2

(Assuming Male Aged 47)

Duration	Payout	Accum 1	Accum 2	VD	VW	VT
1	22,941	917,558	1,231,058	9,251	60,000	1,184,747
2	27,909	930,939	1,258,547	10,022	59,237	1,168,801
3	30,801	942,031	1,284,380	10,823	58,440	1,152,134
4	31,095	953,326	1,311,082	11,659	57,606	1,134,713
5	30,785	965,441	1,339,295	12,549	56,735	1,116,490
6	29,086	979,799	1,370,477	13,531	55,824	1,097,376
7	26,020	997,869	1,406,128	14,573	54,868	1,077,316
8	25,266	1,017,507	1,444,137	15,653	53,865	1,056,276
9	19,082	1,044,212	1,490,041	16,763	52,813	1,034,231
10	15,397	1,075,804	1,541,695	17,902	51,711	1,011,157
Endowment	980,979	94,824	560,715	0	1,011,157	0

Net Premiums/M at 0%, 30.98; at 3%, 23.89; at 6%, 18.67

NOTES.—Standard and Poor closing averages. Premiums at 0, 3, and 6 per cent. Reserve accumulated at 4.5 per cent. Reserve 1 at 2.25 per cent; reserve 2 at 3 per cent. Redemption rate 5 per cent. Load 9 per cent. 1,000 lives. Initial investment $1,000.

TABLE 3

Year	Payout	Accum 1	Accum 2	VD	VW	VT
1918	0	23,512	31,350	231	1,500	29,618
1919	2,230	45,852	61,879	481	2,980	58,838
1920	5,050	66,377	90,964	752	4,441	87,642
1921	2,796	90,081	123,611	1,043	5,882	116,009
1922	2,452	115,194	158,071	1,357	7,300	143,922
1923	1,524	142,366	195,009	1,695	8,696	171,356
1924	0	172,284	235,135	2,060	10,067	198,289
1925	462	203,087	276,603	2,451	11,414	224,696
1926	0	235,739	320,400	2,870	12,734	250,552
1927	0	269,859	366,168	3,318	14,027	275,831
1928	1,343	304,172	412,653	3,318	39,306	275,831
1929	5,619	335,753	456,953	3,318	39,306	275,831
1930	18,769	355,605	490,096	3,318	39,306	275,831
1931	40,457	354,663	503,043	3,318	39,306	275,831
1932	197,046	197,088	359,983	3,318	39,306	275,831
1933	20,847	208,622	386,685	3,318	39,306	275,831
1934	13,947	227,575	421,488	3,318	39,306	275,831
1935	6,353	254,975	465,452	3,318	39,306	275,831
1936	10,798	279,163	506,949	3,318	39,306	275,831
1937	208,789	106,449	352,322	3,318	39,306	275,831
1938	234,337	− 99,585	165,189	3,318	39,306	275,831
1939	192,809	−273,363	11,164	3,318	39,306	275,831
1940	124,573	−386,725	− 81,556	3,318	39,306	275,831
1941	17,332	−397,948	− 71,209	3,318	39,306	275,831
1942	10,676	−403,020	− 53,740	3,318	39,306	275,831
1943	6,373	−404,016	− 31,182	3,318	39,306	275,831
1944	1,701	−400,386	− 2,936	3,318	39,306	275,831
1945	2,395	−397,286	25,885	3,318	39,306	275,831
1946	100,464	−492,116	− 42,063	3,318	39,306	275,831
1947	2,698	−493,447	− 15,304	3,318	39,306	275,831
1948	2,210	−494,351	13,145	3,318	39,306	275,831
1949	0	−493,084	45,087	3,318	39,306	275,831
1950	68	−491,828	78,398	3,318	39,306	275,831
1951	310	−490,758	112,965	3,318	39,306	275,831
1952	1,184	−490,514	148,215	3,318	39,306	275,831
1953	0	−489,075	186,234	3,318	39,306	275,831
1954	0	−487,571	225,965	3,318	39,306	275,831
1955	575	−486,574	266,908	3,318	39,306	275,831
1956	2,212	−487,169	308,057	3,318	39,306	275,831
1957	953	−486,533	352,316	3,318	39,306	275,831
1958	0	−484,915	399,520	3,086	37,806	246,212
1959	0	−483,224	448,848	2,836	36,325	216,992
1960	0	−481,456	500,397	2,565	34,864	188,189
1961	0	−479,609	554,264	2,274	33,424	159,821
1962	0	−477,679	610,556	1,960	32,006	131,908
1963	0	−475,662	669,381	1,622	30,610	104,474
1964	0	−473,555	730,854	1,257	29,238	77,541
1965	0	−471,352	795,092	866	27,892	51,134
1966	0	−469,051	862,221	447	26,571	25,278
1967	0	−466,645	932,371	0	25,278	0

NOTES.—Standard and Poor closing averages. Premiums at 0, 3, and 6 per cent. Reserve accumulated at 4.5 per cent. Reserve 1 at 2.25 per cent; reserve 2 at 3 per cent. Redemption rate 5 per cent. Load 9 per cent. 1,000 lives. Initial investment $1,000.

FRANK P. DI PAOLO:

Mr. Hickman has given us an excellent insight into the valuation of variable products. I am afraid, however, that his method does not properly reconcile the equities of separate account contractholders, on the one side, and regular participating policyholders (if a mutual company) or stockholders (if a stock company), on the other side. In effect, the former receive certain guarantees that are backed by assets owned by the latter. Thus it would seem to me that a valuation method suitable for variable products should reconcile and protect the interest of all parties involved in the risk-transfer process.

One way to protect the interest of regular participating policyholders or stockholders would be to require the build-up of a "risk reserve" within the general funds of the company through the accumulation, at interest, of a minimum "risk premium." I am aware that a retrospective reserve of this type may be distasteful to some actuaries, but there are three basic reasons why I believe such a reserve is necessary:

1. Participating policyholders or stockholders are assuming a risk and should therefore be compensated accordingly—hence the need for a minimum risk premium.
2. The risk assumed is of a long-term nature. Thus it seems logical that this risk premium be used to build up a risk reserve that could be released into surplus only after the risk has ceased to exist.
3. If the risk premium were to be released immediately into surplus, a company in a phase II tax position would have to pay income tax on it without being able to offset losses that may later materialize.

I am somewhat happy to see that Mr. Hickman's modified reserve formula allows for a limited accumulation of the "gross premium," which, I believe, is a kind of risk premium to cover the investment guarantees. Mr. Hickman does not define his gross premium, however, and I presume that he leaves it up to each company to decide what gross premium to use. Furthermore, Mr. Hickman does not go far enough to avoid taxable releases in the early years.

I would like to see the risk premium defined in such a manner that the probability of different levels of investment performance of the separate account can be properly recognized. In view of the smallness of the theoretical risk premium—especially if the accumulation period is fairly large—I would also like to see a statutory minimum risk premium set at about 1 per cent of the contract premium. Even though the risk premium may be very small, its standard deviation could be significant; hence the need for an adequate security loading.

The interest of separate account contractholders can be properly safeguarded by means of existing valuation laws. For example, let us consider the reserve for a variable single-premium, deferred-annuity contract which promises to pay the value of the accumulated units at maturity, prior death, or surrender, with the guarantee that the minimum amount paid at maturity or prior death will not be less than the original single premium. This reserve should be equal to the higher of (1) the value of the accumulated units held with respect to this contract in the separate account or (2) the regular single-premium endowment reserve (which, in the case of a New York company, would be calculated in accordance with section 205). If the endowment reserve is larger, an "additional reserve" may have to be set up within the general funds of the company. The size of the additional reserve would depend on the size of the risk reserve described previously.

The valuation method suggested in this discussion can be summarized analytically in the following manner:

Let

${}_t\mathrm{RV}_{x:\overline{n}|}$ = The tth terminal risk reserve.

${}_t\mathrm{RP}_{x:\overline{n}|}$ = The risk premium paid at the beginning of the tth contract year; then

$$ {}_t\mathrm{RV}_{x:\overline{n}|} = \sum_{s=1}^{t} {}_s\mathrm{RP}_{x:\overline{n}|}(1+i)^{t-s+1} . $$

${}_tV_{x:\overline{n}|}$ = The tth terminal reserve calculated in accordance with standard valuation laws.

${}_tU_{x:\overline{n}|}$ = The book value of assets held in the separate account on behalf of a given contractholder at the end of the tth contract year.

$$ {}_t\mathrm{DV}_{x:\overline{n}|} = {}_tV_{x:\overline{n}|} - {}_tU_{x:\overline{n}|} , \quad \text{if} \quad {}_tV_{x:\overline{n}|} > {}_tU_{x:\overline{n}|} , $$

$$ = 0 , \quad \text{if} \quad {}_tV_{x:\overline{n}|} \le {}_tU_{x:\overline{n}|} . $$

The additional reserve, AV_t, which should be carried in the general funds of the company, would be calculated in the following manner:

$$ \mathrm{AV}_t = \sum_n \sum_x \sum_t {}_t\mathrm{DV}_{x:\overline{n}|} - \sum_n \sum_x \sum_t {}_t\mathrm{RV}_{x:\overline{n}|} , \quad \text{if positive} , $$

$$ = 0 , \quad \text{if} \quad \sum_n \sum_x \sum_t {}_t\mathrm{RV}_{x:\overline{n}|} \ge \sum_n \sum_x \sum_t {}_t\mathrm{DV}_{x:\overline{n}|} . $$

It should be noted that the additional reserve would be calculated on an aggregate basis and may fluctuate from year to year generating strains or releases in accordance with fluctuations in stock prices. The nature of the risk with which we are dealing is such that we must accept this type of fluctuation. On the other hand, if the level of guarantees is not excessive, the additional reserve is likely to be zero, except during periods of serious depression, when it may be a noticeable amount. The risk reserve, instead, should be calculated on a policy-by-policy basis, and releases would occur whenever contracts terminate by death, withdrawal, or maturity. Mean reserves would be calculated by means of the usual averaging formulas.

I find rather interesting the statement made by Mr. Hickman to the effect that his change factors would be appropriate only if "the investment policy of the separate account is not significantly more speculative than the stocks constituting Standard and Poor's index." Indeed, not only do I find myself in full agreement with Mr. Hickman, but I wish he had gone further and said that an essential requirement for the applicability of his change factors would be to invest the separate account in common stocks proportionately weighted to the components of Standard and Poor's index. As a sequel to the simulation study we made last year at Confederation Life, we became interested in studying how the risk inherent in the investment guarantee changes when the volatility of the separate account changes in relation to that of the index used to quantify the risk. Our recent study is based on the volatility of the Toronto Stock Exchange index and each of its components. We found, for example, that if the separate account is invested in those stocks with the highest degree of volatility, the investment risk increases substantially and so do the related risk premium and risk reserve.

Certainly, it would be downright impossible to require that a given separate account follow a very narrow investment policy. Before any legislation dealing with variable products is formulated, however, the question of how volatility affects the cost of investment guarantees should be studied very carefully.

One last comment. I feel very flattered that Mr. Hickman has chosen to use in his study the one thousand stock market trends we simulated last year. In producing the cumulative distribution function of monthly percentage changes in stock price indexes, we thought of including the data given in the Cowles Commission stock price index from 1871 to 1915. We rejected this idea for two reasons:

1. We doubted if stock price changes during the period from 1871 to 1915 would have any statistical value. The "cornering" practices so widespread in the twentieth century must have seriously affected this index.

2. Inflation, which has become part of our economic folklore since World War I, has a definite effect on the drifting movement of stock prices. The Cowles index is almost unaffected by inflationary pressures.

Indeed, we even considered limiting our study to Standard and Poor's index from 1936 to 1965. This thirty-year period is certainly more attuned to our times. Nowadays we are living in a Friedmanesque-Keynesian economy where central banks, with their power to control the money supply, can easily interfere with a wide variety of business activities. The Federal Reserve Board and the Bank of Canada did not have such power prior to 1935. We rejected the idea of using a thirty-year index, however, because we felt it would be more conservative to base our study on a fifty-year period which included the Great Depression as well as two world wars and two postwar periods.

GERALD A. LEVY:

The author has approached this problem by using a classical determination of expected benefits and from this calculating net level premiums and then net level premium reserves. It is a useful way to develop premiums for guaranteed benefits under variable contracts. The formulas required to develop reserves, however, appear to be complex and costly to administer. The benefit is generally small, and the dollar amount of reserves produced should not be significant. Also, death benefits under expected investment performance will generally decrease with time. These can be seen from Table 1. An alternative reserve calculation that uses the property that the net cost of the guaranteed death benefit tends to decrease with time is suggested—that is, a one-year-term method. Thus we can

TABLE 1

(1958 CSO 3 Per Cent)

t	Total Premiums Paid through Year t	Death Benefit per $100 Gross Premium	One-Year-Term Cost	
			Age 40	Age 55
1	100	11.60	.04	.15
2	200	19.66	.07	.27
3	300	24.05	.10	.36
4	400	24.61	.11	.41
5	500	21.20	.10	.38
6	600	13.65	.07	.27
7	700	1.79	.01	.04

Notes.—One-year-term cost, variable annuity death benefit.

Assumptions: Death benefit equals excess of gross premiums over fund; separate account earnings equal 4 per cent compounded annually. Of the gross premium, 85 per cent is invested in the separate account.

assume that each year's premium covers the risk of that policy year and therefore that the reserve is the unearned premium of that year's actual risk. The actual net one-year-term cost of the benefit at age 45 is

$$(\Sigma G - U_{45})C_{45}/D_{45}\,, \tag{1}$$

where ΣC is gross premiums paid to date, U_{45} is the total fund value as of the valuation date at age 45, C_{45}/D_{45} is one-year-term cost of \$1 death benefit, and the unearned premium reserve is $\frac{1}{2}$.

Table 1 displays the risk for issue ages 40 and 55 under a hypothetical variable annuity contract. It is of interest to note how insignificant the reserve is under these assumptions. For example, at age 40, \$1,000,000 of premium only produces a maximum of \$1,100 of reserve. It hardly warrants the calculation of even a one-year-term approach.

What will be appropriate to calculate the reserve for guaranteed minimum death benefits under variable life insurance (death benefit equals the excess of the initial sum insured over the current variable amount of insurance)? In this instance the guaranteed death benefit may never become positive due to favorable investment experience, whereas in the variable annuity in the early years it is always positive. Also, this minimum benefit risk with time will be expected to decrease unless total accumulative investment experience in the separate account is less than the valuation interest assumptions. The expected situation in variable life insurance is not much different from that in the variable annuity. The minimum death benefit risk fluctuates; however, with time it is expected to decrease. Hence we may expect its annual claim cost to decrease. This again suggests the appropriateness of a one-year-term approach to reserve calculations. That is, only for those policies where the variable death benefit falls below the initial death benefit should a one-year-term insurance reserve be held. This appears to be a reasonable approach, since it establishes a reserve only for those deaths expected in the next year whose death benefit is below the guaranteed amount. It also recognizes the true risk of the company, which is of a temporary nature.

Our neighbors to the North have sold variable endowment insurance contracts for several years. The approach their regulators have taken with regard to reserve requirements on guaranteed benefits may be of interest. The variable endowment insurance policies sold in Canada usually include a minimum death benefit and a minimum maturity value.

The following quotation is taken from the covering memo of the Canadian Federal Insurance Department's released Guidelines on Reserves for Equity Linked Contracts:

There are two matters of concern to supervisory authorities in addition to that of adequate disclosure, namely,

(1) the effect of such contracts and their funding on company solvency, and
(2) the question of "equity" as it concerns company policyholders who bear a risk, unrelated to their own policies, arising in respect of the guaranteed benefits included in equity-linked insurance contracts.

From the guidelines, we quote only those items which are pertinent to reserves:

5. For equity-linked contracts with a guaranteed death benefit, the level portion of the gross premium required to provide for the decreasing amount at risk over the term of the contract shall be allocated to a life insurance fund and a reserve for this insurance shall be set up within that fund.

The amount of "insurance reserve" shall be determined in accordance with mortality and interest bases that comply with the provisions of the applicable insurance Act but the amount of the reserve at the end of any year shall not be less than the aggregate of one-half of the yearly net insurance premiums (with the usual deferred premium deductions) subject to allocation to the life insurance fund in respect of all equity-linked insurance contracts in force at the end of that year.

The amount of term insurance to be used for the purposes of determining the premium to be allocated and of calculating the reserve shall not be less than the difference between the guaranteed death benefit and the cash surrender values applicable to a non-participating policy of similar form issued by the company with face amount equal to the guaranteed death benefit. (If such a non-participating policy is not issued by the company, the cash surrender values used for establishing the amount at risk for the noted purposes shall be calculated on mortality and interest bases that are reasonable for non-participating business.)

6. For equity-linked contracts with a guaranteed maturity value, a "risk premium," not less than 1% of the gross premium for the contract, shall be allocated to a "security reserve" within a life insurance fund, and the security reserve shall be accumulated at a rate of interest not less than that used in the calculation of the minimum reserve determined in accordance with Guideline 7.

The amount of the security reserve at the end of any year shall not be less than

(*a*) 60% of the aggregate of the guaranteed maturity values of contracts maturing in the following calendar year, reduced by 40% of the aggregate value at the end of the current year of the units applicable to those contracts, or
(*b*) 10% of the aggregate of the guaranteed maturity values of contracts maturing in the following calendar year,

whichever is the greater.

7. The total reserve held in respect of equity-linked contracts, i.e., the sum of the market value of the assets in the separate fund and any reserve or reserves held in respect of such contracts in a life insurance fund, shall not at any time

be less than the reserve for the guaranteed benefits under those contracts calculated in accordance with mortality and interest bases that comply with the provisions of the applicable insurance Act.

8. Transfers may be made from the surplus of a life insurance fund to insurance or security reserves to meet the requirements set out in Guidelines 5 to 7, and the amount of such transfers together with interest thereon shall be released back to such surplus as soon as conditions permit.

9. Provided that the total amount of any transfers from the surplus of a life insurance fund pursuant to Guideline 8, together with interest thereon, has been returned to such surplus, a portion of the security reserve not needed from time to time to meet the requirements set out in these Guidelines may be allocated to the separate fund to which the reserve is applicable and to the surplus of the life insurance fund in which the reserve is held, with not less than one-half of such portion being allocated to the latter, but

(*a*) no such allocation shall be made before the first calendar year in which maturities occur, and

(*b*) an allocation in any one calendar year shall not exceed 10% of the amount in the security reserve at the end of the preceding calendar year.

(AUTHOR'S REVIEW OF DISCUSSION)

HERBERT W. HICKMAN:

Messrs. Coates, DiPaolo, Levy, and Mitchell have raised a number of important issues. Perhaps the most fundamental is the question raised by Messrs. Coates and Mitchell of why the estimated future value of the accumulation units changes unpredictably from year to year, when the practice for annual statement purposes is to have reserves based on mortality and interest assumptions which do not vary from year to year.

Basically, the estimated future value of the accumulation units is an offset to the amount at risk. While reserve calculations are indeed based on mortality and interest assumptions that only approximate reality, the amounts at risk in practically all situations are not approximations but the actual amounts. There is a good reason for this, in addition to the fact that the actual amounts are usually easy to determine.

Consider, for example, a pure endowment policy. If the amount of the maturity value is precisely stated, the reserves required from year to year will gradually converge toward that amount even if the mortality and interest assumptions are incorrect. To the extent that these assumptions differ from actual experience, a relatively small gain or loss from operations will be reflected each year. On the other hand, if the amount of pure endowment is incorrectly stated, the reserve will not converge toward the correct amount, and the entire effect on gain or loss will occur in the year of maturity. As a result, I think that the analogy between interest assumptions and the future value of accumulation units is not applicable in this context and that there is a need to revalue the units annually in the light of actual investment experience.

Mr. Coates notes that the net annual premiums resulting from the change factors are much larger than the historic average annual premiums that would actually have been required. This result is not surprising, considering the fact that the change factors are based below the 20th percentile of the historic results. Whether conservatism should be introduced in this manner, or by taking 300 per cent of the historically adequate premium, as Mr. Coates does in Table 3 in his discussion, is a matter of individual judgment.

TABLE 1

INCREASE IN SURPLUS*

Year	Coates, Table 1, Modified Reserve	Prospective Reserve ≥0, Modified Reserve	Coates, Table 2, Retrospective
1893......	− 212	− 9	153
1894......	229	402	150
1895......	− 402	− 229	106
1896......	272	381	78
1930......	− 433	− 195	239
1931......	−3,948	−3,625	0
1932......	− 668	− 442	0
1933......	985	1,003	0
1934......	1,448	1,347	0
1935......	1,753	1,349	0

* Premiums (1.035) − claims (1.0175) − increase in reserves. Interest on the reserves held is not included.

Mr. Coates illustrates in Table 2 the fluctuation in surplus from year to year that would result from using my modified reserve approach. His calculations were based on the assumption that the prospective reserves could go below zero, which was not my intention. Setting a minimum value of zero on the prospective reserve tends to increase the level of the reserves held and slightly reduce the fluctuations. The results, based on additional data supplied by Mr. Coates, are shown in Tables 1 and 2 for two of the most important depressions.

The only year in which there would have been a significant decrease in surplus was in 1931. Even that would have been equal to only .16 per cent of the value of the accumulation units at the end of the year, which is much less than the usual asset charge.

In the determination of the modified reserve in these calculations, the "previous reserve" is reduced by claims incurred during the year before moving one-fifth of the way toward the "target" of the prospective reserve. This approach would appear consistent with the new Regulation 47 in New York, which provides that claims shall be charged against the reserve, provided the concept of a moving "reasonable maximum target" was acceptable. I find it difficult, however, to understand the concept that reserves should be reduced because high claims have been experienced, when the cause of the high claims may also cause high claims in the future.

TABLE 2

RESERVES HELD

Year	Coates, Table 1, Modified Reserve	Prospective Reserve ≥ 0, Modified Reserve	Coates, Table 2, Retrospective
1893	1,780	2,671	2,310
1894	2,027	2,745	2,325
1895	2,858	3,404	2,336
1896	2,988	3,425	2,347
1930	1,346	2,833	2,447
1931	5,384	6,548	2,218
1932	5,828	6,765	1,666
1933	5,198	6,116	1,673
1934	4,330	5,350	1,907
1935	3,231	4,654	2,222

In comparing the results of retrospective and prospective reserve approaches, it is necessary to keep in mind that there are two conflicting objectives—to maintain sufficient funds to pay claims and to keep fluctuations in gain and loss to a minimum. A pure prospective reserve tends to emphasize the first objective, while a pure retrospective reserve emphasizes the second objective. In determining the proper balance between these objectives, I doubt whether it is useful to use the Mandatory Securities Valuation Reserve as a precedent, because that reserve is concerned only with the stabilization of gain and loss and is not involved with the actual payment of claims. The principal difficulty I see in Mr. Coates' retrospective method is that no minimum reserve is required; the reserve is most likely to reach zero (or half the annual premium) at the time when the claims are greatest.

I have a few general comments on model offices for guaranteed benefits, not all of which are directed specifically at the models of Messrs. Coates and Mitchell.

1. If the variable annuity contract involves a higher first-year load and level annual premiums for the guarantee, failure to assume a high lapse rate may substantially understate the risk, especially if a lapse does not terminate the guarantee. (If a lapse does terminate the guarantee, there may be an interesting public relations problem.)

2. Even if the variable annuity involves a level load, the assumption that premium income in depression years will come close to equaling premium income in prosperous years is very dubious and will probably lead to understating the amount at risk.

3. If a retrospective approach is used and reserves are not released upon death or maturity, the model office should cover fairly short periods of time only. Otherwise one generation is likely to leave a large reserve to the next generation, and the adequacy of premiums is no longer being effectively tested.

4. The purpose of using historical data is principally to test the methods being used over a wide variety of economic conditions; there is no real expectation that the future will closely resemble the past. As a result, a series of short model offices are generally more revealing than an amalgamation of experience over a long period of years.

Mr. DiPaolo gives three reasons why he believes a retrospective reserve is necessary.

1. To compensate policyholders in the general account, or stockholders, because they are assuming a risk.

Whether or not general account policyholders or stockholders are compensated for the risk assumed depends on the gross premium collected for the minimum benefit and what is ultimately done with it. Whether the reserve is prospective or retrospective may affect the timing of the distribution of the gross premium but does not affect whether it ultimately goes to general account policyholders, separate account policyholders, or stockholders.

2. The risk is long term, so no reserve should be released into surplus until the risk has ceased to exist.

Reserves for long-term risks are normally valued on a prospective basis, with gain or loss resulting each year. While the incidence of risk may be more heavily weighted to the long term with a guaranteed endowment, this difference does not seem significant enough to require the retention of all reserves regardless of the level of the unit value.

3. The release of reserves may lead to a taxable gain.

This is quite true, but piling up potential future gains also has its hazards. The prospective approach more clearly ties in the reserves to both anticipated future claims and the effect of life contingencies, which is useful in establishing them as life insurance reserves.

The gross premium referred to in my paper is indeed a risk premium which would be determined by each company. I agree that this gross premium should be determined by use of approaches similar to those described in the DiPaolo and Turner papers. I do not agree that this premium should be the valuation net premium or be subject to a statutory minimum, such as 1 per cent of the contract premium. The size of the premium should depend on many factors, such as the guaranteed benefit, the level and timing of the load on the annuity contract, the minimum period required from purchase to maturity, and the nature of the separate account investments. As long as the proper regulatory authorities can specify the minimum reserve, it does not appear necessary to establish a minimum gross premium.

Mr. DiPaolo describes a reserve approach which is in effect the greater of a retrospective and a prospective reserve. His prospective reserve equals the excess of the regular statutory reserve over the current value of the accumulation units. Considering a single-premium pure endowment, for example, this means a reserve equal to

$$\frac{D_m(G)}{D_y} - U_y = \frac{D_m(G - f_{m-y}U_y)}{D_y}$$

where

$$m = \text{Age at maturity},$$

$$y = \text{Attained age},$$

$$G = \text{Minimum benefit},$$

$$U_y = \text{Value of units at age } y, \text{ and}$$

$$f_{m-y} = \frac{l_y(1+i)^{m-y}}{l_m} = \left[\frac{(1+i)}{p_y}\right]\left[\frac{(1+i)}{p_{y+1}}\right]\cdots\left[\frac{(1+i)}{p_{m-1}}\right],$$

which at most ages 40–60 is an accumulation from .5 to 1.5 per cent above the valuation interest rate. This is generally less conservative than my proposed formula $f_n = .8(1.05)^n$.

A prospective approach of this nature can lead to wide swings in reserves; I believe that a modified reserve approach leveling out the reserves is as desirable with his approach as with mine.

I agree with Mr. DiPaolo that my change factors would not be appropriate for a separate account invested in those stocks on the Toronto Stock Exchange having the highest degree of volatility, but the similarity over the years between the Dow Jones and Standard and Poor's indexes indicates to me that a close relationship between the stocks held in the separate account and the stocks "held" in Standard and Poor's 500 is not necessary to justify the use of the change factors.

I believe the period of time used in analyzing stock market indexes is principally a matter of taste, since we do not expect the past to repeat itself. I favor using the full period 1871–1969 because it provides a wider range of stock market situations. I agree that the monetary and manipulatory practices of the late nineteenth century are unlikely to be repeated, but I would say the same about World War II, which has probably had more impact on the stock market over the last thirty years than either central banks or Friedmanesque-Keynesian theory.

Mr. Levy suggests that the reserve methods described are more complex than really necessary if the only benefit is the return of gross purchase payments upon death; the magnitude of the resulting reserves is simply too small. I basically agree with this position but favor a different solution than his one-year-term method.

The one-year-term method does not appear suitable for variable annuity contracts under which premiums can be discontinued without losing the minimum death benefit. It is also not clear to me that variable life insurance contracts with a $3\frac{1}{2}$ per cent assumed investment return will generally have decreasing guaranteed death benefits. In addition, variable life insurance contracts on a reduced paid-up basis will probably still have the guaranteed minimum death benefit. In other words, there seem to be a number of cases where a one-year-term premium will either be unavailable or insufficient.

My own preference for variable annuity contracts having only minimum death benefits would be to determine, using the methods described in the paper, an average percentage charge against assets, and to use that percentage charge for a period of three to five years, after which another study would be made. This charge would be taken from the asset charge guaranteeing mortality and expense assumptions. In effect this has been the practice since the inception of variable annuity contracts, and I see no reason to change the practice other than to specify a specific percentage as the reserve. This specific percentage would vary from company to company, depending on the age distribution and the difference between the gross and net purchase payments under the annuity contract. A flat asset charge may leave something to be desired from the standpoint of equity among policyholders, but the amounts involved are sufficiently small that simplicity seems more desirable than strict equity. This approach should, of course, not be applied to maturity guarantees.

My principal concern with the Canadian Guidelines on Reserves for Equity Linked Contracts is that the guidelines are not sufficiently related to the value of the benefits to be appropriate over a wide range of products or economic conditions. For example, guideline 6 specifies a minimum risk premium of 1 per cent of the gross premium for the contract regardless of the amount of the guaranteed maturity value or the minimum period between purchase and maturity. A minimum reserve of at least 10 per cent of guaranteed maturity value must be held for contracts maturing within one year, even if the accumulation units are worth twice the guaranteed maturity value; on the other hand, no minimum is specified for maturities in two years, regardless of the value of the accumulation units in relation to the guaranteed maturity value. I think that more benefit-related guidelines will be needed in the future.