

Notation and Terminology used on Exam MLC

Version: January 15, 2013

Changes from August 1, 2012 version: Wording has been changed regarding Profit, Expected Profit, Gain, Gain by Source, Profit Margin, and lapse of Universal Life policies.

In actuarial practice there is notation and terminology that varies by country, by application, and by source. The purpose of this study note is to present notation and terminology that will be used on the MLC examination for situations where notation or terms may not be common to all sources and notation or terms that are unique to the MLC examination.

The **force of mortality** may be represented by μ_x or $\mu(x)$ or μ_{x+t} or $\mu(x+t)$ where x and $x+t$ are attained ages. The symbol $\mu_x(t)$ indicates selection at age x and attained age $x+t$. The symbols μ_x , μ_{x+t} , and $\mu_x(t)$ will be used on the examination.

The **survival function** may be represented by $s(x)$ or $S(x)$ or $S_x(t)$. The symbol $S_x(t)$ will be used on the examination.

The **number of lives** at age x can be represented by ℓ_x or l_x . The symbol l_x will be used on the examination.

The **future complete lifetime of (x) random variable** can be represented by T_x or $T(x)$. Similarly, the symbols used for joint life status can be T_{xy} or $T(xy)$, and for last survivor status can be $T_{\overline{xy}}$ or $T(\overline{xy})$. The symbols T_x , T_{xy} and $T_{\overline{xy}}$ will be used on the examination.

The **future curtate lifetime of (x) random variable** can be represented by K_x or $K(x)$. Either symbol may be used on the exam. . Similarly, the symbols used for joint life status, K_{xy} or $K(xy)$, and last survivor status, $K_{\overline{xy}}$ or $K(\overline{xy})$. The symbols K_x , K_{xy} and $K_{\overline{xy}}$ will be used on the examination.

The **present value of future losses random variable** may be represented by L or ${}_0L$ or L_0 for loss at issue and ${}_tL$ or L_t for loss from t years after issue. Superscripts may be included. When the symbol L is used to represent present value of future losses random variable the symbol including any subscripts or superscripts will be defined in the text of the question.

Duration subscripts can be used differently. For example, something happening in the first duration (between ages x and $x+1$) may be identified with a 0 or 1. The text of the question will define any notation used.

The **benefit** at time t is represented by b_t . The **benefit** at the end of period k is represented by b_k . When a benefit is payable at the beginning or end of a time period, the subscript will denote the time of the payment, not necessarily the number assigned to the time period. The text of the question will define the benefit either by formula or in words.

Actuarial present value and **expected present value** are terms used for the expectation of the random variable representing the present value of one or more contingent future payments. Either term may be used on the examination. **Actuarial accumulated value** is the term used for the expectation of the random variable representing the accumulated value of one or more

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contingent payments. For this exam, the contingency is usually the survival of one or more lives. If a single life is involved, the two concepts are related as follows:

Actuarial present value at age $x = [\text{actuarial accumulated value at age } x+n] * {}_nE_x$

Fully discrete insurance is an insurance where both the premiums and the benefits are paid only at discrete time points. **Semi-continuous** insurance is an insurance where the premiums are paid at discrete time points and the death benefits are paid at the moment of death. **Fully continuous** insurance is an insurance where the premiums are paid continuously and the death benefits are paid at the moment of death. Unless stated otherwise in the text of the question discrete time points are the beginnings of years for premium payments and the ends of years for death benefit payments.

Special insurance is an insurance that has either: non-level benefits or non-level premiums; or both. The non-level aspects of the insurance will be described in the text of the question. If an insurance is not defined as “special” then premiums and benefits are assumed to be level, unless there is explicit information in the text of the question to the contrary.

Benefit premium is the premium determined by the equivalence principle and assuming no expenses.

The **benefit premium for fully discrete insurances** will be represented by P with the appropriate symbols attached. P_x , $P_{x:\overline{n}|}$, $P_{x:\overline{n}|}^{\perp}$, and $P_{x:\overline{n}|}$ may be used on the exam.

The symbols are defined in terms of an insurance, A , and an annuity, \ddot{a} , as follows:

$$P_x = \frac{A_x}{\ddot{a}_x}, \quad P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}^{\perp}}{\ddot{a}_{x:\overline{n}|}}, \quad P_{x:\overline{n}|}^{\perp} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}, \quad P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}}$$

The symbol P will be defined within the text of the question if it is not one of the symbols shown above.

Benefit reserves are reserves based on the benefit premium assuming no expenses.

Unless stated otherwise in the text of a question all **expenses** are equal to zero. If expenses are specified in the text of a question then the expenses need to be used in the solution to the question.

In a **multi-decrement table** $q_x^{(j)}$ is the probability a life age x fails in the next year due to decrement (j) and $q_x^{(\cdot)}$ is the probability of failure due to all decrements. The associated single decrement probability of failure at age x due to decrement (j) is $q_x^{(j)'}$. The probability $q_x^{(j)}$ is also called the dependent probability. The probability $q_x^{(j)'}$ is also called the independent probability.

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In a multi-state model ${}_t p_x^{ij}$ is the probability that a life age x in state i is in state j at age $x+t$.

The symbol μ_x^{ij} is the force of transition between states i and j at age x . The symbol ${}_t \bar{p}_x^{ii}$ is the probability that a life age x in state i remains in state i through the period x to $x+t$.

The **asset share** at time t may be represented by ${}_t AS$ or AS_t . The symbol ${}_t AS$ will be used on the examination. Asset shares for universal life insurance are calculated like those for traditional life insurance.

The **reserve** at time t may be represented by ${}_t V$ or V_t . The symbol ${}_t V$ will be used on the examination.

If a **table of select and ultimate values** is presented in a question the format of the table will follow the convention of reading across the row of select rates and then down the column of ultimate rates for the values corresponding to each age at selection.

On the examinations the **transition probabilities** for a multi-state model may be presented in a matrix. For example, for a model with two states, 0 and 1, the transition probabilities would be presented in a matrix as follows:

$$\begin{bmatrix} p_x^{00} & p_x^{01} \\ p_x^{10} & p_x^{11} \end{bmatrix}$$

For a time period where all cash flows occur only at the beginning and end of the time period:

Profit for the time period occurs at the end of the time period and is (a) minus (b) where:

- (a) is the accumulated value of the sum of the reserve at the end of the previous period and the cash flows that occur at the beginning of the period; and
- (b) is the sum of the value of the reserve at the end of the period and the cash outflows that occur at the end of the period.

Expenses at the inception of a contract may be classified as negative profit at time 0 or part of period 1 cash flow and included in the profit calculation for period 1. Any initial expenses that are not part of period 1 cash flow will be identified in the question.

Expected profit is the profit calculated using the gross premium, expected cash flows at the beginning and end of the period, and accumulating beginning of year values. The assumptions for expected cash flows and the expected interest rate may or may not be the assumptions in the reserve model.

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Actual profit is calculated using the gross premium, actual cash flows at the beginning and end of the period, and accumulating beginning of year values using the actual investment rate earned during the period.

Gain is the actual profit minus the expected profit for the period. **Gain by source** is the gain calculated where the effect of the difference between the observed values and the expected values in the profit calculations from one source is reflected, while the differences for the other sources are not. Examples of sources are: expenses, interest, mortality and withdrawal. Often, gains from multiple sources are calculated sequentially. For example, the gain from mortality might be calculated first, reflecting the difference between the observed mortality and the assumed mortality, and the gain from interest calculated second, reflecting the difference between the observed and assumed interest, while using only the observed mortality.

Profit margin is the ratio of the actuarial present value at issue of profits divided by the actuarial present value at issue of gross premiums.

Internal rate of return is the interest rate such that the actuarial present value at issue of profits is equal to 0.

Spot rates are the yield rates on zero-coupon bonds. Normally, spot rates vary by the maturity of the bond. In the examination, spot rates will always be stated as annual effective rates.

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Other terms	
Terms used on the examination	Equivalent or similar terms (not used on the examination)
annuity-due	due annuity
annuity-immediate	immediate annuity
temporary life annuity	term annuity
certain period	guarantee period
premium paying period	premium paying term
face amount	sum assured, sum insured
net amount at risk	death strain at risk, sum at risk, amount at risk
benefit premium	net premium
gross premium	contract premium, expense-loaded premium, expense-augmented premium
benefit reserve	net premium policy value, net premium reserve
gross premium reserve	gross premium policy value
cost of insurance, COI	mortality charge
μ_x^{ij}	$\mu_{ij}(x), \lambda_{ij}(x)$
${}_tP_x^{ij}$	${}_tP_{ij}^{(x)}$
Terms used to describe payments by policyholders on universal life policies	
premium	contribution, deposit
To describe a universal life insurance death benefit:	
a specified amount (e.g. "1000")	Option A, Type A, fixed failure benefit
a specified amount plus the account value (e.g., "1000 plus the account value")	Option B, Type B, variable failure benefit

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While all universal life insurance policies have similar structures, there are variations, especially with respect to cost of insurance calculations. The following structure applies to universal life policies on the exam unless otherwise specified. If variations to this structure occur on the exam the variation will be stated in the applicable question.

Account values are calculated at regular intervals. The question will indicate the calculation period.

For each calculation period the account value is calculated as follows:

$$AV_{end} = [AV_{start} + P(1-f) - e - COI](1+i^c), \text{ where } COI = \frac{DB_{end} - AV_{end}}{1+i^q} (\text{coi rate})$$

The symbols in the equations above will not be used on the exam unless the symbols are also defined in the question.

AV_{start} is the account value at the start of the period.

AV_{end} is the account value at the end of the period.

P is the premium paid (unless otherwise stated in the question, all premiums are paid)

f is the percent of premium charge.

e is the expense charge.

DB_{end} is the projected death benefit at the end of the period, consistent with AV_{end} , reflecting the policy's death benefit description and the corridor factor if specified.

COI is the cost of insurance charge.

$coi\ rate$ is the cost of insurance rate.

i^c is the credited interest rate per period.

i^q is the interest rate per period for discounting the net amount at risk in the COI calculation (unless otherwise stated in the question $i^q = i^c$)

Death benefits and surrender benefits are paid at the end of the period, after the account value at the end of the period has been calculated.

The **cash surrender value** is the account value at the end of the period minus the applicable surrender charge, if any.

A universal life policy will lapse if the account value is 0. Except, a universal life policy may remain in force, even if the account value is below 0, if the policy has a no-lapse

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guarantee provision and satisfies the conditions of the **no-lapse guarantee** provision. You should assume that a universal life policy does not lapse unless you are told in the question to check if it lapses.

Corridor factors:

- a. Universal life policies may contain a provision that the death benefit will be the greater of: (1) the specified amount; and (2) the account value at the end of the period times a corridor factor. For example, if the specified amount is 50,000, the account value is 30,000, and the corridor factor is 200%, then the death benefit would be 60,000 which is the greater of 50,000 and 200% of 30,000.
- b. If a corridor factor provision would increase the death benefit, the cost of insurance charge calculation should reflect the increased death benefit in the calculation of the cost of insurance charge.
- c. If no corridor factors are stated in the question, assume the policy has none.
- d. If corridor factors are stated in the question, you should consider the corridor factors in any calculations.