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LIFE INSURANCE BASED ON THE CONSUMER PRICE INDEX

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ABSTRACT

This paper presents an in-depth analysis of life insurance protection which rises with increases in the consumer price index. The requirements of the Standard Nonforfeiture Law with respect to such insurance are given special attention.

The paper presents two methods of calculating policy values for such insurance—the change of state method, which generates values that are fixed per policy, and the index accumulation method, which generates values that increase with each increase in the death benefit of the policy.

In addition, the paper briefly touches on gross premium determination and policy drafting as they relate to such insurance.

IN THE summer of 1968 the authors were assigned the project of developing a life insurance product which would compensate for rises in the cost of living. This paper describes the "cost-of-living" permanent insurance which was subsequently placed on the market by Life Insurance Company of Georgia. It also contains a survey of all the various forms of life insurance tied to the consumer price index which have been introduced by the life insurance industry, and it discusses other such products which seem possible. The subject is treated under the following headings:

I. NEED FOR THE PRODUCT

The values of the consumer price index, shown in Table 1, indicate the erosion in purchasing power of the dollar, caused by inflation. The insurance-buying public is aware of this eroding effect on life insurance

TABLE 1

CONSUMER PRICE INDEX FOR URBAN WAGE EARNERS AND CLERICAL WORKERS, UNITED STATES
CITY AVERAGE INDEX

For Average of the Period	Index (1957–59 = 100)	For Average of the Period	Index (1957–59 = 100)
1919	60.3	1969—*Continued*	
1929	59.7	April	126.4
1939	48.4	May	126.8
1949	83.0	June	127.6
1959	101.5	July	128.2
		August	128.7
1960	103.1	September	129.3
1961	104.2	October	129.8
1962	105.4	November	130.5
1963	106.7	December	131.3
1964	108.1	Average	127.7
1965	109.9	1970	
1966	113.1	January	131.8
1967	116.3	February	132.5
1968	121.2	March	133.2
		April	134.0
1969		May	134.6
January	124.1	June	135.2
February	124.6	July	135.7
March	125.6	August	136.0

values. Fixed-dollar life insurance is nevertheless purchased in large quantities, both in term and permanent forms. The implication is that large segments of the public are satisfied with traditional fixed-dollar products. A case can in fact be made that the life insurance industry, by using high interest assumptions in the pricing of permanent fixed-dollar life insurance and in the determination of dividends, is giving the buyer something more than "true" interest and is actually discounting the price to allow for inflation. Fixed-dollar products will no doubt continue to be purchased in very large quantities.

Products which contain a hedge against inflation are very much needed,

however, in the present climate. They will appeal especially to certain middle- and upper-class markets which are interested in a direct hedge. The products which have been proposed to bring this about are the following: (1) variable life insurance and annuities, under which the investment risk is vested in the policyholder, so that the hedge depends entirely on favorable performance of a "separate account" of investments, and (2) life insurance based on the consumer price index, such as is discussed in this paper.

Life insurance based on the consumer price index does not have one appeal attributed to the variable products (the anticipation of superior investment performance). However, it does have the following advantages:

1. It gives the cost-of-living hedge directly rather than relying on investment performance to approximate the effect of inflation.
2. No separate account is necessary.
3. No special agent licensing is involved.
4. Since this is simply a life insurance product, and not a "security," no regulation by the Securities and Exchange Commission is involved.
5. Traditional agent's commission patterns may be employed.
6. In many term and permanent forms, life insurance based on the consumer price index may be issued under existing state law.

II. SUMMARY OF PRODUCTS ON THE MARKET IN 1970

Prior to the time of writing this paper, several products, tied to a cost-of-living index, had been introduced by the life insurance industry. These products could be classified into two groups, discussed below.

Inflation Risk Assumed by Policyholder

In this first group of products, the inflation risk is borne in the main by the policyholder. The company merely undertakes to provide additional insurance, directly or indirectly at the expense of the policyholder, in event of an increase in the cost of living. This additional insurance is normally made available on a favorable cost basis (avoiding high first-year charges) and without evidence of insurability. The additional cost is sometimes not payable until the additional insurance goes into effect; in other cases it is prefunded by one means or another. Examples follow.

1. *The cost-of-living rider.*—Additional one-year term insurance is automatically issued on each anniversary, reflecting the proportionate increase in the consumer price index since the original issue date of the basic policy; the appropriate one-year term premium for such additional insurance is payable annually in addition to the basic premium for the

policy, at rates specified in the rider. These riders generally expire in fifteen years or at policy age 64, if earlier. Some conversion privilege is available during an early portion of the term.

2. *The cost-of-living dividend option.*—Under this option the policyholder is first of all required to elect his dividends as paid-up units or dividend accumulations. As each new annual dividend becomes available, it is tentatively applied in the elected fashion; if this causes the total death benefit (including all accumulated paid-up benefits or dividends) to equal or exceed the original face amount augmented proportionately by the increase since policy issue date in the consumer price index, then the dividend is so applied. Otherwise, a portion (or all) of such dividend is diverted to purchase one-year term insurance to bring about such result. If the application of even all of the dividend is still insufficient, the necessary portion of accumulated dividends, if any, is next applied. As a last resort, in some cases, the policyholder is allowed to pay any remaining term insurance premium to bring the death benefit up to the augmented amount called for by the increase in the consumer price index. Limited conversion privileges are granted for the term insurance benefits.

3. *The cost-of-living policy with side fund.*—The policyholder pays a specific level extra premium which is accumulated in a side fund. This is used to pay for cost-of-living one-year term additions as they occur, at term rates stated in the policy. If the side fund becomes inadequate, the balance of the cost is paid by the policyholder directly. Some conversion privileges are available.

4. *The policy with* ab initio *reform.*—If an increase occurs in the cost of living, the policy is changed, on an original-date basis, to the proportionately larger policy called for by such increase. The premiums, nonforfeiture values, and future dividend entitlements all increase by the same proportion as the face amount. The increase in cash value resulting from this transaction becomes the cost obligation of the policyholder and is secured by canceling paid-up dividend additions (which are compulsory). If paid-up dividend additions are inadequate, the policyholder is asked to pay the balance in cash. In the one example of this type of policy which has come to the attention of the authors, a 6 per cent limitation is placed on any annual increase, and the entire provision for increases is terminated at the high policy age of 90.

Inflation Risk Assumed by Life Insurance Company

In this group of products the inflation risk is borne by the life insurance company. A provision for this risk-bearing is included in the over-all premium paid by the policyholder. Examples follow.

1. *The cost-of-living annuity (group or individual).*—This product is not life insurance, but it deserves to be mentioned in the interests of completeness. Generally, a 3 per cent limit is placed on the annual increases in benefit payments. The product is generally of a single-premium nature.

2. *The cost-of-living term policy.*—Under this arrangement an additional term benefit is provided reflecting an increase in the consumer price index; such additional benefit expires at the end of a stated period (the two examples in the possession of the authors are ten years and four years). These provisions are themselves part of short-term policies. No additional premium is precipitated when an additional benefit goes into effect. Some renewal and conversion rights are provided. The problem of the Standard Nonforfeiture Law is apparently avoided by the use of the short-term insurance approach, and no cash values are provided.

3. *The cost-of-living permanent policy.*—The most typical version of this product is a whole life policy with the following features:

a) The death benefit increases on each anniversary, to reflect the proportionate change, since issue, in the consumer price index.
b) The only limit on the increases is a maximum death benefit of double the original face amount.
c) If the consumer price index reduces in any year, nothing is taken away (the ratchet clause).
d) Premiums are level for life.
e) Cash values are determined in accordance with the Standard Nonforfeiture Law, by a special method known as the change of state method. Cash values are fixed per policy. (They are not on a per thousand of death benefit basis.) Extended term values are based on the original face amount only. Paid-up and extended amounts do not accelerate with the cost of living. To summarize, all nonforfeiture benefits are fixed; the policyholder retains the purchasing protection given by his policy through keeping it in premium-paying status.

As a matter of interest, the authors have also studied a very similar series of policies available in Italy. The major differences are the imposition of a 3 per cent ceiling on annual increases and the lack of any necessity to conform with the Standard Nonforfeiture Law.

III. THE STANDARD NONFORFEITURE LAW—ASPECTS AFFECTING LIFE INSURANCE BASED ON THE CONSUMER PRICE INDEX

For almost a century, the Standard Nonforfeiture Law, or predecessor legislation, has been a bedrock of the life insurance industry in the United States. The authors set out to develop life insurance based on the consumer price index which would clearly conform to this law. The purpose of this discussion is not to review the law in its entirety but merely to

point out aspects of it which are particularly applicable to such life insurance.

1. For the calculations required, the law refers only to (*a*) interest, (*b*) mortality, and (*c*) specifications of the premium and guaranteed benefit structure. The law is silent with regard to any outside contingency, such as the behavior of the consumer price index.

2. The contract must provide at least one paid-up nonforfeiture benefit and a cash-surrender value; and it must contain "a table showing the cash surrender value, if any, and paid-up nonforfeiture benefit, if any, available under the policy on each policy anniversary either during the first twenty policy years or during the term of the policy, whichever is shorter. . . ."

This language leads to the following questions: Can the cash and nonforfeiture benefits be expressed "per thousand of death benefit" and thus vary with the consumer price index, in a way similar to the death benefit? Or does the law call for values that are "per policy" and thus fixed at issue? The authors feel that the law can be interpreted by a "yes" answer to the first question. However, the change of state method, described in this paper, gives values fixed per policy. The index accumulation method, also described, gives values which are tied to the consumer price index.

3. The law apparently requires that the contract have premiums which (although not necessarily level) are determinable at issue. This is required because the "adjusted premium" must be expressible as a "uniform percentage of the respective premiums specified in the policy. . . ." If there is to be an "adjusted premium," determinable at issue, the gross premium must also be determinable at issue.

4. The law requires that a paid-up benefit must have a cash value "not less than the present value of the future guaranteed benefits." This means that paid-up and extended term nonforfeiture benefits should not escalate with the consumer price index, because this would introduce an element making the determination of present value impossible.[1]

5. The law contains a specific exemption, from all nonforfeiture requirements, for *level* term policies, which expire within fifteen years or before age 66 (whichever is the earlier), for decreasing term policies with lesser adjusted premiums than such policies, and for corresponding term riders. There is no exemption for any increasing term policy or rider. This means that there is no automatic exemption for any term policy or rider tied to the consumer price index, since this could be of an increasing term nature.

[1] Use of inflation as a permissible contingency, after the fashion of the index accumulation method, might alter this situation.

IV. PRINCIPLES OF LIFE INSURANCE BASED ON THE CHANGE OF STATE METHOD

The authors arrived at the following principles for cost-of-living life insurance products which are based on the change of state method described in Section V of this paper:

1. The premiums must be level.
2. The death benefit may be determined by the ratio of the consumer price index at approximate date of death to the consumer price index at issue. At any given duration, however, the death benefit must be in a range between a minimum guaranteed amount and a maximum guaranteed amount. Increases in the death benefit occur only on policy anniversaries.
3. There are special minimum death benefit requirements at very advanced durations. It is possible for the paid-up nonforfeiture benefit to exceed the initial face amount at such durations. For this reason the minimum guaranteed death benefit must not be less than such paid-up benefit.
4. Cash values and reserves are worked at every duration according to the change of state method. Under this method, values and reserves are fixed per policy, are determinable at issue, and are in accord with the Standard Nonforfeiture Law and the Standard Valuation Law.
5. The company must advise the policyholder of his death benefit, in writing, each time it changes, but in no event less often than once a year, during the continuation of the cost-of-living benefit.

V. THE CHANGE OF STATE METHOD

The death benefit under the Life of Georgia cost-of-living policy increases at the beginning of the second and each subsequent policy year. The amount of increase is proportionate to the increase in the consumer price index over the past year. The coverage remains level throughout each policy year and changes occur only on the policy anniversaries. Other features built into the contract are the following:

a) The death benefit shall not be less than it was for the previous year even if the index should happen to fall. This came to be known as the "ratchet clause" and is discussed in Section VIII.
b) The death benefit shall not be greater than $2,000 (the maximum guaranteed amount) for each $1,000 of initial face amount.
c) The death benefit shall not be less than $1,000 or the paid-up insurance nonforfeiture value at the beginning of the policy year, if higher (the minimum guaranteed amount).
d) Premiums are level and are payable for life.

This product is not involved with a side fund, with a separate account, or with equity investments in any way. The product is of the traditional guaranteed nature.

Since the Standard Valuation and Nonforfeiture Laws are silent on the use of an assumption regarding future performance of the CPI, the easiest and most direct solution—making an assumption with regard to future CPI behavior as one does with regard to future interest yields—is not employed for reserve and cash-value calculations. The task then becomes one of determining the "minimum values" required by law, since the cash values must be shown in the policy at the time of issue. In order to use the change of state method, it is first necessary to impose an upper limit on total coverage; therefore an upper limit on total coverage of twice the initial coverage was established. With minimum and maximum limits on the total coverage, it became possible to calculate the largest possible "minimum values" by employing the change of state method.

An early observation was that the maximum possible reserve at duration t for such a contract could be produced by using the traditional reserve formula but evaluating future benefits (future being time after duration t) at their maximum levels and past benefits (from issue up through duration t) at their minimum levels. This assumes no rise in the CPI until the point in time $x + t$, at which time the CPI rises enough to force coverage up to the maximum immediately. This produces the maximum reserve at duration t possible under the policy, and that specific reserve value can be termed the "largest possible minimum value at duration t in accordance with the law."

Hence the method makes use of traditional formulas but incorporates a change in assumptions at each duration so as to evaluate future benefits at their maximum guaranteed level and past benefits at their minimum guaranteed level. Net level and adjusted premiums are determined in accordance with these same assumptions, and they therefore decrease with duration. This method always produces a value which is not less than any conceivable unique minimum value resulting from any conceivable pattern in behavior of the CPI. The cash values are not the highest possible, because there are many ways of determining larger values. They are "the largest of the set of possible minimum values."

The following paragraphs illustrate that the method does produce the largest of the set of possible minimum values. Using the prospective approach and ignoring, for simplicity, the restriction that the coverage shall not be less than the paid-up value, we find that the analysis would be as shown in paragraphs *a*–*d*.

a) The reserve formula at duration t under the change of state method is

$$ {}_tV_x = 2A_{x+t} - \left(\frac{M_x + M_{x+t}}{N_x}\right) \ddot{a}_{x+t} ; \qquad (1) $$

the premium $(M_x + M_{x+t})/N_x$ changes with each duration t and may also be expressed as ${}_t\text{ELA}_x \cdot P_x$, where

$$ {}_t\text{ELA}_x = \frac{M_x + M_{x+t}}{M_x}; \tag{2} $$

therefore,

$$ {}_tV_x = 2A_{x+t} - {}_t\text{ELA}_x \cdot P_x \cdot \ddot{a}_{x+t}. \tag{3} $$

The formula maximizes the present value of future benefits and then, conditioned on that assumption, minimizes the present value of future premiums by minimizing past benefits in the calculation of such premium. This approach produces a possible maximum value.

b) Given this trial maximum value, how could a larger value be obtained? Making the assumption that past benefits were greater than their minimum would tend only to increase the premium and thereby the present value of future premiums. A larger result would only occur if a reduction in future benefits resulted in a corresponding reduction in the present value of future benefits which would be less than the corresponding reduction in the present value of future premiums.

c) To determine whether this would be possible, the effect of a \$1 decrease in future benefits at duration s, where s is greater than or equal to t, will be studied. This decrease will produce a change in the present value of future benefits of

$$ -\frac{C_{x+s}}{D_{x+t}} $$

and a change in the present value of future premiums of

$$ -\frac{C_{x+s}}{N_x}\left(\frac{N_{x+t}}{D_{x+t}}\right). $$

The combined effect will produce the following change in the trial maximum value:

$$ \Delta_t V_x = -\frac{C_{x+s}}{D_{x+t}} - \left[-\frac{C_{x+s}}{N_x}\left(\frac{N_{x+t}}{D_{x+t}}\right)\right], $$

$$ \Delta_t V_x = \frac{C_{x+s}}{D_{x+t}}\left(-1 + \frac{N_{x+t}}{N_x}\right) = -\frac{C_{x+s}}{D_{x+t}}\left(\frac{N_x - N_{x+t}}{N_x}\right). $$

d) The change is negative, and this illustrates that an assumption of future benefits being less than their maximum will produce lower values. The trial maximum value is, therefore, the largest of the set of possible minimum values.

The analysis for cash values would be as follows in paragraphs *a*–*e*.

a) The cash-value formula at duration t under the change of state method is

$$ {}_t\mathrm{CV}_x = 2A_{x+t} - \left\langle \left\{ (M_x + M_{x+t}) + 0.02\,{}_t\mathrm{ELA}_x\,D_x \right. \right.$$

$$ \left. \left. + 0.25 \begin{bmatrix} P^A \\ {}_LP^A_x\,{}_t\mathrm{ELA}_x \\ 0.04\,{}_t\mathrm{ELA}_x \end{bmatrix} D_x + 0.4 \begin{bmatrix} P^A \\ 0.04_t\,\mathrm{ELA}_x \end{bmatrix} D_x \right\} \Big/ N_x \right\rangle \ddot{a}_{x+t}\,, \qquad (4) $$

where ${}_LP^A_x$ is the adjusted premium for whole life insurance, P^A is the adjusted premium for the cost-of-living policy and such premium changes with each duration, t, and the smallest of the quantities in each bracket is to be used.

It can be demonstrated that the formula for P^A given in the bracket above is equivalent to ${}_t\mathrm{ELA}_x \cdot {}_LP^A_x$; therefore,

$$ {}_t\mathrm{CV}_x = 2A_{x+t} - {}_t\mathrm{ELA}_x \cdot {}_LP^A_x \cdot \ddot{a}_{x+t}\,. \qquad (5) $$

b) When we trace the effect of a \$1 decrease in future benefits at duration s, there is a change in the present value of future benefits of $-C_{x+s}/D_{x+t}$. The change in the present value of future premiums would be caused by a change in the value of ${}_t\mathrm{ELA}_x$ in expression (5). By reference to formula (2) it can be seen that such change would be $-C_{x+s}/M_x$ and the change in the present value of future premiums would therefore be

$$ -\frac{C_{x+s}}{M_x} \cdot {}_LP^A_x \cdot \ddot{a}_{x+t} = -\frac{C_{x+s}}{M_x} \cdot {}_LP^A_x \cdot \frac{N_{x+t}}{D_{x+t}}\,. $$

Now, solving for the change,

$$ \Delta_t\mathrm{CV}_x = -\frac{C_{x+s}}{D_{x+t}} - \left(-\frac{C_{x+s}}{M_x} \cdot {}_LP^A_x \cdot \frac{N_{x+t}}{D_{x+t}} \right), $$

$$ \Delta_t\mathrm{CV}_x = \frac{C_{x+s}}{D_{x+t}} \left[-1 + \left(\frac{N_{x+t}}{M_x} \right) {}_LP^A_x \right] = \frac{C_{x+s}}{D_{x+t}} \left[\frac{{}_LP^A_x N_{x+t} - M_x}{M_x} \right]. $$

c) This change will be positive where ${}_LP^A_x N_{x+t} - M_x$ is positive. Therefore, for each issue age the following can be solved so that

$$ N_{x+t} > \frac{M_x}{{}_LP^A_x}\,. $$

Then at all durations up to and including t it is possible to derive a higher value than that determined under the change of state method.

d) For each issue age, t is as follows:

Issue Age	t
15–17	3
18–32	2
33–56	1
57–65	0

e) However, since the cash values do not turn positive until *after* the durations shown above, there is no practical effect on the method, and the trial maximum value is again the largest of the set of possible minimum values.

In the determination of cash values, the change of state method, by assuming maximum future benefits, is starting with a larger expense allowance. Therefore, the negative cash value under a normal whole life policy is greater than the negative cash value under the cost-of-living policy. This is illustrated in the following tabulation for issue age 20 (1958 CSO, age last birthday, 3½ per cent interest):

t	Cash Values	
	WL	CLP
1	$−18	$−36
2	−11	−18
3	− 3	− 1
4	5	17
5	13	35

There is no practical effect, however, since these values are all negative.

It can be illustrated that values calculated with the same assumptions but with the use of a retrospective formula will be equal to those calculated by the prospective formulas used here.

The change of state method does produce high values, and, as a result, the use of a 3½ per cent interest assumption rather than the more common assumption of 3 per cent is recommended.

VI. WHY IS THE STANDARD NONFORFEITURE LAW SATISFIED?

The law is obviously satisfied in these fundamental respects:

1. The only factors included in the calculations are (i) interest, (ii) mortality, and (iii) specifications of the premium and guaranteed benefit structure. (It should be kept in mind that the "guaranteed benefit structure" at any given duration consists of a minimum guaranteed amount and a maximum guaranteed amount, both of which may enter into the calculations.)

2. Cash and paid-up nonforfeiture benefits are provided; furthermore, taking the conservative approach, they are "per policy" and are fixed and determinable at issue.
3. The premiums are determinable at issue (and are, in fact, level).

Are cash-surrender values, determined by the change of state method, in accordance with the law? It is required that "*any cash surrender value* available under the policy in the event of default in a premium payment due on any policy anniversary . . . shall be an amount *not less than* the excess, if any, of the present value, on such anniversary, of the future guaranteed benefits . . . over . . . the then present value of the adjusted premiums." The authors have added emphasis to the words "Any cash surrender value" and "not less than." The law deals with each cash value as a separate entity, not necessarily related to its neighbors; the change of state method is in keeping with this concept.

When considering a particular cash value, say, that at the nth duration, the reasoning at issue date is as follows:

a) There will be one and only one future behavior pattern of the consumer price index. It will yield an actual death benefit at all durations between issue and the end of the mortality table.
b) If this unique future behavior of the consumer price index were known at issue, it would be possible to determine the array of future death benefits, the equivalent uniform amount, the adjusted premium, and the unique minimum cash value at duration n.
c) The unique future behavior of the consumer price index is, however, *not known* at issue, and therefore it is impossible to determine the unique minimum cash value at duration n.
d) This presents no problem, because the law does not require that this unique minimum cash value be determined; the change of state method (as was proved in Sec. V) produces a value which is *not less than* this unique minimum value, and this is all that the law requires.
e) The change of state method produces a cash value which is the "largest of the set of possible minimum values." (It does not produce a "maximum" value, since there are many ways of producing larger values.)

Thus the change of state method is seen to produce values which are in accordance with the law.

A lesser problem concerns the matter of the "nonforfeiture factor," which is customarily referred to in policy forms either as (*a*) the ratio of the adjusted premium to the gross premium or (*b*) the adjusted premium itself. The authors believe that the statement of a nonforfeiture factor is needed only in cases where the unique minimum cash value is being used. Where higher cash values are provided, as is doubtless the case with the change of state method, the nonforfeiture factor need not

be stated. In this connection it should be pointed out that there are many methods for determining higher cash values which do not even make use of an adjusted premium and for which the term "nonforfeiture factor" is meaningless.

VII. ILLUSTRATION OF NONFORFEITURE VALUES AND RESERVE VALUES

In the explanation of the change of state method one minor point was disregarded in the interest of simplicity. The ELA at any given duration may be determined by taking the previous year's ELA and subtracting a year's death benefit evaluated at its maximum and adding that same year's death benefit evaluated at its minimum. For most durations, therefore,

$$ {}_{t}\mathrm{ELA}_x = {}_{t-1}\mathrm{ELA}_x - \frac{2C_{x+t-1}}{M_x} + \frac{C_{x+t-1}}{M_x}. $$

If the paid-up nonforfeiture benefit at duration $t-1$, ${}_{t-1}\mathrm{PU}_x$, is greater than 1, then

$$ {}_{t}\mathrm{ELA}_x = {}_{t-1}\mathrm{ELA}_x - \frac{2C_{x+t-1}}{M_x} + \frac{{}_{t-1}\mathrm{PU}_x \cdot C_{x+t-1}}{M_x}. \tag{6} $$

This is due to the imposed condition that coverage will never be less than the paid-up nonforfeiture value at the beginning of the year.

Table 2 illustrates the resulting values for the Life of Georgia cost-of-living policy ($3\frac{1}{2}$ per cent, age last birthday).

One observation from Table 2 is that the cash values are high and even exceed the initial face amount at or around attained age 65. The paid-up values also exceed the face amount before the twentieth duration. An insured, with a policy having an issue age of 35 and an initial face amount of \$10,000, can actually discontinue premium payments after sixteen years and have a paid-up policy for life of \$10,310. Furthermore, he can endow his policy at age 64 and receive a surrender value of \$10,010. Comparisons are, therefore, often made between the cost-of-living policy and 20 pay life and endowment at 65 plans. The cost-of-living plan can be priced competitively with such other plans, and the high nonforfeiture values provide very beneficial "special options." If the CPI does not rise, the insured is getting a partial return for his premium in greater nonforfeiture values. Of course, the CPI is expected to rise.

It should also be noted that since the paid-up values rise rapidly, the minimum guaranteed death benefit (which is equal to the initial face amount or the paid-up value, if higher) becomes a very important aspect of this plan. Table 3 illustrates this for a \$10,000 policy, issued at age 35.

On the basis of the 1958 CSO mortality table, a person aged 35 could expect to attain age 71. The *guaranteed minimum* death benefit at that

age is $17,060, substantially greater than the original $10,000. Of course, it is anticipated that the death benefit will actually be at the maximum, $20,000. In pricing a product such as this, it is very important to recognize the amount of actual protection being provided. It is also important, furthermore, to convey to the public the amount of actual protection, and this can be done by explaining the "minimum guaranteed death benefit."

TABLE 2

COST-OF-LIVING POLICY VALUES
DETERMINED BY THE
CHANGE OF STATE METHOD

AGE	DURATION	CASH VALUE	PAID-UP VALUE	TERMINAL RESERVES	
				Net Level	CRVM
25.....	5	$ 52	$ 193	$ 103	$ 86
	10	172	551	220	205
	15	309	858	353	339
	20	460	1,111	499	486
	30	789	1,480	818	809
	40	1,122	1,706	1,142	1,136
	50	1,411	1,836	1,424	1,420
35.....	5	97	270	153	128
	10	271	655	321	299
	15	460	975	503	484
	20	656	1,231	693	676
	30	1,037	1,577	1,062	1,051
	40	1,359	1,769	1,375	1,368
45.....	5	161	342	225	191
	10	407	764	461	432
	15	656	1,101	700	676
	20	889	1,352	924	906
	30	1,275	1,659	1,297	1,285
55.....	5	257	432	331	287
	10	592	900	649	615
	15	890	1,243	933	907
	20	1,129	1,469	1,162	1,142
	30	1,477	1,727	1,495	1,484

It might be interesting to look at what the policy values would be at certain higher interest rates, especially since there have been recent discussions surrounding the possibilities of higher statutory minimum interest rates. Table 4 illustrates values at interest rates of 4, 4½, and 5 per cent.

Change of state formulas and values for other types of cost-of-living plans are illustrated in the Appendix.

TABLE 3

COST-OF-LIVING POLICY DEATH BENEFIT

Policy Year	Minimum Guaranteed Benefit	Estimated Benefit Assuming 3% Annual Rise in CPI
1	$10,000	$10,000
2	10,000	10,300
3	10,000	10,609
4	10,000	10,927
5	10,000	11,255
10	10,000	13,048
15	10,000	15,126
16	10,000	15,580
17	10,310	16,047
18	10,850	16,529
19	11,350	17,024
20	11,830	17,535
25	13,920	20,000
Age 65	15,770	20,000
Age 71	17,060	20,000

TABLE 4

COST-OF-LIVING POLICY VALUES AT VARIOUS INTEREST RATES

Age	Duration	4% Interest			4½% Interest			5% Interest		
		Cash Value	Paid-up Value	Net Level Reserve	Cash Value	Paid-up Value	Net Level Reserve	Cash Value	Paid-up Value	Net Level Reserve
25	5	$ 43	$ 188	$ 93	$ 35	$ 179	$ 84	$ 28	$ 167	$ 76
	10	154	571	201	137	585	183	122	597	167
	20	425	1,147	464	393	1,181	431	363	1,210	400
	40	1,076	1,726	1,097	1,032	1,745	1,053	990	1,761	1,011
35	5	87	275	142	78	279	132	70	282	123
	10	252	681	301	233	700	282	217	724	264
	20	621	1,262	658	588	1,290	625	557	1,318	593
	40	1,323	1,782	1,339	1,288	1,795	1,305	1,254	1,806	1,271
45	5	151	352	214	142	363	204	133	371	194
	10	387	786	440	369	810	421	351	830	403
	20	857	1,375	892	826	1,396	861	797	1,418	831
	30	1,242	1,673	1,264	1,210	1,686	1,232	1,179	1,698	1,201
55	5	245	440	320	235	450	309	226	460	299
	10	572	918	630	553	935	611	536	954	593
	20	1,101	1,483	1,134	1,073	1,495	1,106	1,047	1,508	1,080
	30	1,452	1,733	1,471	1,428	1,740	1,447	1,404	1,746	1,424

VIII. THE RATCHET CLAUSE

The product developed by Life Insurance Company of Georgia provides that the death benefit will not be allowed to decrease if the consumer price index should happen to fall; it will remain level until such time as an increase is called for, on the basis of a change in the index since policy issue date. This clause came to be known, colloquially, as the "ratchet clause." It is equally applicable, regardless of the method used for cash-value determination—the change of state method or the index accumulation method (described in Sec. XI).

The ratchet clause is controversial; since the objective of the policy is to maintain equal purchasing power, the benefit should, as a matter of pure theory, be allowed to vary both up and down with the consumer price index. The ratchet clause was, however, adopted because of the following practical advantages:

1. The clause would encourage persistency of business at a time (such as the early 1930's) when the consumer price index is decreasing.
2. Policy loan problems are minimized if the possibility of a decrease in cash value is minimized.
3. The clause is a "plus" from the merchandising standpoint.

IX. GROSS PREMIUM DETERMINATION

Assumptions

One key assumption in developing gross premiums for guaranteed cost-of-living benefits is the assumption with respect to future increases in the consumer price index. The history of the consumer price index exhibits an average annual increase of 2.9 per cent (over the period from 1935 to 1970). The average annual increase over the past twenty years has been only 2.2 per cent, whereas the average annual increase over the past five years has been 3.4 per cent. It is interesting to note that the largest annual increase was 14.4 per cent, occurring in 1947.

It would appear that, in light of this information as well as current conditions, a select and ultimate type of assumption would be called for. For example, an assumed inflation rate of $3\frac{1}{2}$ per cent for five years and $2\frac{1}{2}$ per cent thereafter would appear to be reasonable, even conservative if war years were ignored on the assumption that the contingency charge should cover that added risk. This assumption, $3\frac{1}{2}$ per cent for five years and $2\frac{1}{2}$ per cent thereafter, will result in a death benefit for costing purposes which is increasing and is double the initial death benefit after a period of twenty-seven years.

In evaluating the risk, one should keep in mind that an upper limit on coverage of twice the initial amount is definitely more liberal than an annual limitation of the ride-up. The risk of the death benefit's doubling overnight is, however, very small. As mentioned, the largest annual in-

crease to date was 14.4 per cent, and from past history it appears unlikely that actual future experience will vary significantly from the assumed.

The persistency assumption is also important to the calculation of rates for a guaranteed cost-of-living product. Since this type of product has higher premiums and higher nonforfeiture values, withdrawal experience under similar plans (such as endowment plans) is called for. An added degree of conservatism might be called for since the nonforfeiture values at later durations are quite high. On the other hand, if inflation is rapid a higher degree of persistency might result.

Assumptions with respect to mortality and expenses would be the same as any other guaranteed product and would be based on company experience. Perhaps a small added expense is incurred in the administering of this product, mailing out annual notices of amount of coverage provided, and so forth.

The interest assumption can be the same as it is for any other guaranteed product, or it can be slightly higher than normal. One justification for using a higher rate would be the anticipation of greater investment returns as prices rise. Bond yields have exhibited a close association with price levels over the past ninety-five years (with the exception of 1931–46, a period of depression and war, when interest rates were kept artificially low by monetary and fiscal authorities).[2]

Calculations

Once the assumptions have been established regarding interest, expenses, mortality, withdrawals, and anticipated death benefit, the process of actually calculating rates is very much the same as it is for any other product. Conventional rate calculations should be performed through the use of any published method which takes persistency into account. It is especially important that the progression of the asset shares be studied by duration, since at some of the earlier durations a larger surplus strain than normally expected will result from the change of state method of calculating values.

It is very important that extensive comparisons be made between the cost-of-living plan and existing plans so as to maintain equity in the portfolio to the maximum extent possible. As mentioned earlier, the cost-of-living plan, having high paid-up values and cash values, should be compared in every aspect with existing limited pay life plans and endowment plans. Such a comparison should contain a realistic assumption with regard to the amount of death benefit at each duration under the cost-of-living plan.

[2] Charles Prather, *Money and Banking* (Homewood, Ill.: Richard D. Irwin, Inc., 1965), p. 487.

X. POLICY DRAFTING

In the drafting of the policy form, the complete definition of the consumer price index should be incorporated; for example:

> The Consumer Price Index is the figure published by the Bureau of Labor Statistics, United States Department of Labor, as the Consumer Price Index for Urban Wage Earners and Clerical Workers, United States City Average Index.

In addition, it appears advisable to incorporate some provision allowing for the use of an adjusted index figure, perhaps calculated by the insurance company subject to the approval of the insurance commissioner, should the Department of Labor discontinue the publication of the CPI. Such a discontinuance is extremely unlikely, since the CPI is embedded in many wage and rent contracts and pension plans and soon may be a part of the social security program. The index has been published for over fifty years.

The base reference period of the consumer price index is updated periodically, and provision should be made in the policy to adjust any published index to its equivalent value according to the base reference period at issue.

The policy must also contain a very explicit method for adjusting the benefits in accordance with changes in the CPI. Such adjustments, for practical reasons, are best made annually, on the policy anniversary, and depend on the index figure published for the third month (or earlier) prior to the anniversary month. Since the index for a given month is published shortly before the end of the next succeeding month, the time lag will give one complete month in which the calculations may be performed for anniversaries in the following month. Once the index is published for a month, it becomes final and is not subsequently corrected.

Of course, the policy will also indicate the CPI at issue (which is actually the CPI for the third calendar month preceding the month of issue) and the base reference period that is applicable.

One minor contract problem involves the required reinstatement provision. It is possible that selection against the company could arise in the form of numerous reinstatements shortly after a large increase in the consumer price index. This is probably not a major problem, since a very large increase in the CPI would be rare indeed. Furthermore, there are the requirement of evidence of insurability and the payment of back premiums plus interest.

The primary question raised by a few state insurance departments during the filing of the cost-of-living policy was whether or not such a policy complied with the following insurance code wording: "and shall not make

any such insurance . . . which does not distinctly state the amount of benefits payable, the manner of payment and the consideration therefor."

It is the opinion of the authors that this product does distinctly state the amount of the benefits, perhaps not in terms of actual dollars but, nevertheless, in very explicit terms. In this respect it becomes similar to other coverages which are not stated in terms of dollars, such as major medical coverages, contracts providing an additional benefit of the return of premiums or the return of the policy reserve, and so forth.

XI. THE INDEX ACCUMULATION METHOD

The life insurance described earlier in this paper has the following characteristics:

1. Death benefits escalate on each policy anniversary, in proportion to the cost of living.
2. Cash values and reserves (determined by the change of state method) are fixed per policy.
3. No assumption about the consumer price index is involved in cash-value and reserve calculations.
4. Maximum and minimum guaranteed death benefits are required at every duration.
5. Premiums are level.

The product was designed to conform beyond any question with the Standard Nonforfeiture Law and the Standard Valuation Law.

Other approaches are possible. Consider, for example, a product with the following characteristics:

1. Death benefits escalate on each policy anniversary, in proportion to the cost of living.
2. Cash values escalate in a similar fashion.
3. The maturity value, if the contract is an endowment, escalates with the cost of living.
4. An assumed increase rate in the consumer price index is included in cash-value and reserve calculations.
5. No maximum on the benefit escalation is necessary to the method (although it might be desirable on general grounds).
6. Premiums are level.

This type of product might have a great deal of appeal. The mathematical analysis for it will now be considered. The structure designed below treats such a contract, from the point of view of reserves and cash values, in a manner which is entirely typical in North America; it imposes solvency standards based on the contingencies of interest, mortality, and a new contingency—inflation.

Net Level Premiums and Reserves

The method here described is known as the index accumulation method.

Let

1 = Amount of death benefit at duration 0;

${}_tS$ = Amount of death benefit actually in force at duration t, determined a posteriori;

${}_t\bar{S}$ = Amount of death benefit at duration t, determined a priori by an escalation scale which is based on contractual restrictions, if any (such as a maximum benefit of double), and is required for the valuation of the contract (e.g., ${}_t\bar{S} = [1.03]^t$ but $\not> 2$).

It is obvious, of course, that ${}_0S = {}_0\bar{S} = 1$.

The analysis will be in terms of an m-payment n-year endowment, to illustrate a high degree of generality. The net level annual premium for a fixed-dollar assurance, of amount 1, is ${}_mP_{x:\overline{n}|}$.

At duration t the benefit will have escalated to ${}_tS$, measured in a posteriori terms. The reserve at duration t, where $t \leq m$, has two components. Component 1 is simply the net level reserve for a fixed-dollar benefit of amount ${}_tS$:

$$ {}_tS(A_{x+t:\overline{n-t}|} - {}_mP_{x:\overline{n}|}\ddot{a}_{x+t:\overline{m-t}|}) . \tag{7} $$

A reserving structure of this nature would be subject to sudden strain increases every time the consumer price index causes a benefit increase. At duration t this strain increase would be

$$ ({}_tS - {}_{t-1}S)(A_{x+t:\overline{n-t}|} - {}_mP_{x:\overline{n}|}\ddot{a}_{x+t:\overline{m-t}|}) . \tag{8} $$

In addition, the net level premium for the original face amount, 1, is insufficient to support this reserving structure. Provision must be made for the additional required premium, payable at time t:

$$ ({}_tS - 1)\ {}_mP_{x:\overline{n}|} . \tag{9} $$

A "pure endowment" must be provided at duration t for the strain increase (8) and the premium insufficiency (9). The pure endowment for plan p is ${}_tu_{p:x}$, where in this instance

$$ {}_tu_{p:x} = ({}_tS - {}_{t-1}S)(A_{x+t:\overline{n-t}|} - {}_mP_{x:\overline{n}|}\ddot{a}_{x+t:\overline{m-t}|}) + ({}_tS - 1)_m P_{x:\overline{n}|} . \tag{10} $$

The second element in the right-hand side of formula (10) becomes zero where $t \geq m$.

There is a corresponding pure endowment, ${}_t\bar{u}_{p:x}$, which is based on the a priori escalation scale ${}_t\bar{S}$ required for the valuation of the contract:

$$ {}_t\bar{u}_{p:x} = ({}_t\bar{S} - {}_{t-1}\bar{S})(A_{x+t:\overline{n-t}|} - {}_mP_{x:\overline{n}|}\ddot{a}_{x+t:\overline{m-t}|}) + ({}_t\bar{S} - 1)\,{}_mP_{x:\overline{n}|}\,. \tag{11} $$

Again, the second element in the right-hand side of formula (11) is zero where $t \geq m$.

The first element in the pure endowment, the provision for strain increase, is needed at duration 1 through n, inclusive; the second element, the provision for premium insufficiency, is needed at durations 1 through $(m - 1)$, inclusive.

The net level annual premium to provide for such pure endowments is

$$ {}_m\bar{P}_{x:\overline{n}|} = \frac{\sum_{r=1}^{n} {}_r\bar{u}_{p:x}\, D_{x+r}}{N_x - N_{x+m}}\,. \tag{12} $$

The level premium for the contract is therefore seen to be, in total, $({}_mP_{x:\overline{n}|} + {}_m\bar{P}_{x:\overline{n}|})$.

Component 2 of the reserve at duration $t (t \leq m)$ consists of the present value of the pure endowments still to be provided, less the present value of the future premiums:

$$ \frac{\sum_{s=t+1}^{n} {}_s\bar{u}_{p:x}\, D_{x+s}}{D_{x+t}} - {}_m\bar{P}_{x:\overline{n}|}\ddot{a}_{x+t:\overline{m-t}|}\,. \tag{13} $$

The reserve at duration t is seen to consist of component 1, which is based on the actual benefit in force, ${}_tS$, measured in a posteriori terms, and component 2, which is measured in a priori terms and is therefore known at issue.

The following observation is of interest. The premium and reserving structure provides for a pure endowment at duration t of ${}_t\bar{u}_{p:x}$, to allow for the expected strain increase and premium insufficiency at that duration. The pure endowment actually needed will, however, be ${}_tu_{p:x}$. The difference between these two amounts will cause either a contribution to or a drain upon surplus at duration t. The concept is exactly parallel to the normal situation in which a difference between expected and actual mortality can cause such a contribution or drain.

Term insurance plans, and situations where $t > m$, are not specifically discussed here, but the analysis of them is similar and straightforward.

Deficiency Reserves

Consideration should be given to the holding of a premium-deficiency reserve if the consumer price index should ever escalate to the point where ${}_tS\,{}_mP_{x:\overline{n}|}$ (i.e., the premium involved in expression [7]) exceeds the level gross premium for the contract, say, ${}_mG_{x:\overline{n}|}$. Such a premium-deficiency reserve might normally be thought of as

$$({}_tS\,{}_mP_{x:\overline{n}|} - {}_mG_{x:\overline{n}|})\ddot{a}_{x+t:\overline{m-t}|}\,. \tag{14}$$

This amount, however, should be fully abated by that part of the component 2 reserve (13) then being held, which is for the specific purpose of providing future premium insufficiencies.

Commissioners Reserve Valuation Method

Application of the Commissioners Reserve Valuation Method will not be described here in detail, but the following observations are of interest:

1. As is true in the case of the net level approach, premiums and reserves consist of two components.
2. The method provides for a full preliminary term or "first year cost only" approach in circumstances where the full preliminary term renewal premium is $\leq {}_{19}P_{x+1}$; such a renewal premium contains two elements (which might be denoted $\beta^F + \bar{\beta}^F$).
3. Determination of $\bar{\beta}^F$ will involve pure endowment values ${}_r\bar{u}_{p:x}$, and it will be seen that these pure endowments are *different* from their net level equivalents (since they are designed to support strain increases and premium insufficiencies appropriate to Commissioners Reserves rather than to level reserves). The symbol p in ${}_r\bar{u}_{p:x}$ is intended to refer both to the plan of insurance and the reserve method.
4. In circumstances where $(\beta^F + \bar{\beta}^F) > {}_{19}P_{x+1}$, i.e., where the full preliminary term approach is not available, it will be desirable to take the available allowance in the component 1 portion of the reserve, adopting a net level approach in the case of component 2.

Minimum Cash Values

For purposes of determining minimum cash values, the analysis is similar. The adjusted premium can be thought of as the sum of an adjusted premium for a fixed-dollar assurance, ${}_mP^A_{x:\overline{n}|}$, and an additional amount, ${}_m\bar{P}^A_{x:\overline{n}|}$. Based on the usual Standard Nonforfeiture Law requirement,

$$({}_mP^A_{x:\overline{n}|} + {}_m\bar{P}^A_{x:\overline{n}|})\ddot{a}_{x:\overline{m}|} = A_{x:\overline{n}|} + \frac{\sum_{r=1}^{n} {}_r\bar{u}_{p:x}\, D_{x+r}}{D_x} + 0.02\,\overline{\mathrm{ELA}}_x \tag{15}$$

$$+ 0.40({}_mP^A_{x:\overline{n}|} + {}_m\bar{P}^A_{x:\overline{n}|}) + 0.25\,{}_LP^A_x(\overline{\mathrm{ELA}}_x)\,.$$

The second element on the right side of this equation is the value at issue of all the future pure endowments. The quantity $\overline{\text{ELA}}_x$ is "the equivalent level amount" for age x, based on a priori considerations:

$$\overline{\text{ELA}}_x = \frac{1}{A^1_{x:\overline{n}|}} \left(\frac{\sum_{r=0}^{n-1} {}_r\bar{S} \cdot C_{x+r}}{D_x} \right). \tag{16}$$

On the right side of equation (15) the expressions $({}_mP^A_{x:\overline{n}|} + {}_m\bar{P}^A_{x:\overline{n}|})$ and ${}_LP^A_x(\overline{\text{ELA}}_x)$ (which is the adjusted premium for a fixed-dollar whole life assurance) may not exceed $0.04(\overline{\text{ELA}}_x)$. There are three cases, depending on the size of these quantities. In the case where both are less than $0.04(\overline{\text{ELA}}_x)$, expression (15) may be rearranged as follows:

$$\begin{aligned} {}_mP^A_{x:\overline{n}|} + {}_m\bar{P}^A_{x:\overline{n}|} = \frac{1}{\ddot{a}_{x:\overline{m}|} - 0.40} \Bigg[& A_{x:\overline{n}|} + 0.02 + 0.25\, {}_LP^A_x \\ & + \frac{\sum_{r=1}^{n} {}_r\bar{u}_{p:x}\, D_{x+r}}{D_x} + 0.02(\overline{\text{ELA}}_x - 1) + 0.25\, {}_LP^A_x(\overline{\text{ELA}}_x - 1) \Bigg]. \end{aligned} \tag{17}$$

This will be satisfied if

$${}_mP^A_{x:\overline{n}|} = \frac{A_{x:\overline{n}|} + 0.02 + 0.25\, {}_LP^A_x}{\ddot{a}_{x:\overline{m}|} - 0.40} \tag{18}$$

and

$${}_m\bar{P}^A_{x:\overline{n}|} = \frac{1}{\ddot{a}_{x:\overline{m}|} - 0.40} \left[\frac{\sum_{r=1}^{n} {}_r\bar{u}_{p:x} D_{x+r}}{D_x} + 0.02(\overline{\text{ELA}}_x - 1) + 0.25\, {}_LP^A_x(\overline{\text{ELA}}_x - 1) \right]. \tag{19}$$

There are two other cases. Determination of adjusted premium formulas, equivalent to (18) and (19), is straightforward and will not be detailed here. Some trial and error appears necessary to determine which of the three cases applies in a particular situation. This analysis ensures that only the statutory expense allowances are being taken.

The following observation is of importance. The pure endowments, ${}_r\bar{u}_{p:x}$, which are taken into account in cash-value computations, are *not the same* as their counterparts in reserve computations. They must provide for sudden strain increases in cash values (rather than in reserves) and for insufficiencies in adjusted (rather than valuation) premiums. This viewpoint is necessary to ensure that cash values are independent of the particular reserving method followed.

The required pure endowments, ${}_r\bar{u}_{p:x}$, are determined by expression (11), but making use of the adjusted premium (${}_mP^{A}_{x:\overline{n}|}$) rather than the valuation premium. The second adjusted premium (${}_m\tilde{P}^{A}_{x:\overline{n}|}$) is then obtained by expression (19) or its counterpart in the event that one of the other two cases applies. Trial and error is required to see which of the three cases applies.

Finally, the cash values are obtained by formulas parallel to (7) and (13), but making use of appropriate pure endowments and adjusted premium factors. As in the case of reserves, cash values consist of component 1 (varying with ${}_kS$) and component 2 (fixed at issue).

For policy drafting and ratebook purposes, the alternative is available of expressing cash values as the sum of (*a*) a factor multiplied by the number of thousands of initial amount plus (*b*) a factor multiplied by the number of thousands of increase in amount since issue, i.e. $({}_kS - 1)$ (initial amount). The factor referred to in (*a*) is the sum of the per thousand values for component 1 and component 2; the factor referred to in (*b*) is the per thousand value for component 1. This method may have some advantages in situations where one component is positive but the other is negative.

Tables 5 and 6 illustrate reserves and cash values determined by the index accumulation method.

Paid-up and Extended Term Values

It is clear that the entire cash value (i.e., the total of the two components) can be applied to purchase level paid-up insurance or level extended term insurance for the attained amount, ${}_kS$. It is also possible to divide the available cash value into two portions, which would represent component 1 and component 2 reserves for paid-up insurance (of smaller amount) or extended term insurance (of shorter term) which would continue to escalate with the consumer price index.

Further Plan Variations

The index accumulation method calls for premiums of the form $(P + \tilde{P})$ and for cash values which have a component escalating with the cost of living and a second fixed component associated with the extra premium $\tilde{P}$. This leads to the possibility that the cost-of-living feature could be attached by rider, designed to convert any fixed-dollar plan into one whose benefits are tied to the consumer price index; such a rider would have a separate net premium $\tilde{P}$ and separate fixed cash values (those for component 2).

Another variation provides for premiums (on the basic contract) which escalate in the same fashion as the benefits instead of remaining

TABLE 5

POLICY VALUES DETERMINED BY THE INDEX ACCUMULATION METHOD FOR A WHOLE LIFE PLAN PROVIDING COVERAGE THAT INCREASES WITH THE CPI

$\bar{S} = (1.03)^t$ but ≥ 2

(1958 CSO, 3½ Per Cent Interest, Age Last Birthday, Level Premiums)

AGE	DURATION	NET LEVEL RESERVE FACTORS		CASH-VALUE FACTORS		ILLUSTRATED TOTAL CASH VALUE	
		Component 1: Value per M of Attained Coverage	Component 2: Value per M of Initial Coverage	Component 1: Value per M of Attained Coverage	Component 2: Value per M of Initial Coverage	Actual Inflation Rate of 3%	Actual Inflation Rate of 5%
25...	5	$ 47	$ 42	$ 21	$ 22	$ 47	$ 49
	10	102	72	78	57	162	185
	15	165	80	142	71	293	355
	20	235	55	214	51	438	479
	30	391	6	374	6	754	754
	40	554	1	541	1	1,083	1,083
	50	698	0	690	0	1,380	1,380
35...	5	71	59	42	38	87	92
	10	148	99	122	85	249	284
	15	233	111	209	103	429	521
	20	322	77	301	74	618	676
	30	503	9	488	10	986	986
	40	664	1	654	1	1,309	1,309
45...	5	100	79	66	62	139	147
	10	204	133	175	123	359	409
	15	311	150	285	146	591	716
	20	417	110	395	112	826	902
	30	606	19	591	22	1,204	1,204
55...	5	135	97	95	92	203	214
	10	268	162	234	164	479	546
	15	392	186	364	193	761	921
	20	505	148	482	158	1,029	1,122
	30	691	32	677	39	1,393	1,393

TABLE 5—*Continued*

AGE	PREMIUMS			
	P_x	$\bar{P}_x$	P^A	$\bar{P}^A_x$
25.....	$10.35	$ 9.23	$11.56	$10.35
35.....	15.36	13.01	16.88	14.33
45.....	23.90	17.70	26.03	19.11
55.....	38.64	22.16	41.97	23.03

TABLE 6

POLICY VALUES DETERMINED BY THE INDEX ACCUMULATION METHOD FOR A 20-YEAR ENDOWMENT PLAN PROVIDING ENDOWMENT COVERAGE THAT INCREASES WITH THE CPI

$_t\bar{S} = (1.03)^t$ but ≥ 2

(1958 CSO, 3½ Per Cent Interest, Age Last Birthday, Level Premiums)

Age	Duration	Net Level Reserve Factors		Cash-Value Factors		Illustrated Total Cash Value	
		Component 1: Value per M of Attained Coverage	Component 2: Value per M of Initial Coverage	Component 1: Value per M of Attained Coverage	Component 2: Value per M of Initial Coverage	Actual Inflation Rate of 3%	Actual Inflation Rate of 5%
25...	5	$ 188	$118	$ 157	$115	$ 298	$ 316
	10	412	181	389	182	705	816
	15	679	152	667	154	1,194	1,488
	20	1,000	0	1,000	0	1,807	2,000
35...	5	190	117	157	113	296	314
	10	412	179	389	181	704	815
	15	678	151	665	153	1,190	1,483
	20	1,000	0	1,000	0	1,807	2,000
45...	5	191	115	156	112	293	312
	10	410	177	385	179	697	807
	15	671	151	657	154	1,178	1,468
	20	1,000	0	1,000	0	1,807	2,000
55...	5	193	112	155	111	291	309
	10	405	173	378	177	685	793
	15	655	151	639	115	1,151	1,433
	20	1,000	0	1,000	0	1,807	2,000

TABLE 6—*Continued*

Age	Premiums			
	$P_{x:\overline{20}\rceil}$	$\bar{P}_{x:\overline{20}\rceil}$	$P^A_{x:\overline{20}\rceil}$	$\bar{P}^A_{:\overline{20}\rceil}$
25.....	$35.49	$27.45	$38.14	$28.08
35.....	36.59	27.35	39.40	28.06
45.....	39.95	27.10	43.08	27.88
55.....	48.49	26.55	52.28	27.17

level. The premiums are then of the form $({}_{t}S \cdot P + \bar{P})$. This type of contract generates extra premiums, $\bar{P}$, and cash values, which are much smaller than the fixed premium counterparts, because the required pure endowments, ${}_{t}\bar{a}_{p:\overline{x}|}$, need provide only for the sudden strain increases and not for any premium insufficiencies. Calculations are, however, parallel to those already illustrated.

Still another variation provides for total premiums which escalate, being of the form ${}_{t}S(P + \bar{P})$. Although the extra premium will actually be ${}_{t}S \cdot \bar{P}$, this is estimated as ${}_{t}\bar{S} \cdot \bar{P}$ for purposes of the analysis. In a sense this is the "purest" form of contract, since both the benefits and premiums

TABLE 7

Duration	Level Premium Contract		Fully Escalating Premium Contract	
	P^A $39.40	$\bar{P}^A$ $28.06	P^A $39.40	$\bar{P}^A$ $12.81
	Cash Values per M		Cash Values per M	
	Component 1	Component 2	Component 1	Component 2
5	$ 157	$113	$ 157	$52
10	389	181	389	93
15	665	153	665	90
20	1,000	0	1,000	0

are oriented to the index. This type of contract gives extra premiums and component 2 cash values which are substantially lower than the level premium counterparts. This is illustrated in Table 7, applicable to twenty-year endowment coverage issued at age 35, last birthday.

Inflation Rate Assumed for Valuation Purposes

A study was made of the history of the consumer price index from 1919 to the present. The average annual rate of increase in the CPI for the fifty-year period from 1919 to 1969 was revealed to be slightly greater than 1.5 per cent. The average annual rates of increase in the CPI have been derived for each ten-year period (1919–29, 1920–30, 1921–31, etc.), each twenty-year period, each thirty-year period, and each forty-year period of the era 1919–69; these are exhibited in Table 8.

One of the primary criteria in selecting a valuation rate is that the rate should prove adequate (according to past experience) for all the very long-term situations, for almost all the medium-term situations,

and for a definite majority of the shorter-term situations. Table 9 illustrates how well various assumed rates stand up to this criterion.

A 3 per cent rate would appear to be a suitable assumption for long-term coverages (covering 91 per cent of thirty-year cases) and performs fairly well for short-term coverages (covering 73 per cent of ten-year cases). The $3\frac{1}{2}$ per cent rate is perhaps too conservative, whereas the $2\frac{1}{2}$ per cent rate covers less than one-half of the twenty-year cases. No attempt was made to exclude or to adjust the CPI for certain years. The rate of inflation during and after World War II is complicated by

TABLE 8

RATE OF INCREASE IN THE CPI, 1919–69

Average Annual Rate of Increase for Period (Per Cent)	Number of Periods Having Indicated Rate of Increase							
	10-Year Periods		20-Year Periods		30-Year Periods		40-Year Periods	
	No.	Per Cent	No.	Per Cent	No.	Per Cent	No.	Per Cent
Less than 1	13	32%	8	26%	1	5%	1	9%
1–2	10	24	5	16	11	52	10	91
2–2½	4	10	2	6	1	5	0	0
2½–3	3	7	5	16	6	29	0	0
3–3½	2	5	5	16	2	9	0	0
3½ or greater	9	22	6	20	0	0	0	0
	41	100%	31	100%	21	100%	11	100%

TABLE 9

ADEQUACY OF VARIOUS ASSUMED INFLATION RATES

Assumed Rate (Per Cent)	Number of Periods in Which Assumed Rate Would Not Be Adequate							
	10-Year Periods		20-Year Periods		30-Year Periods		40-Year Periods	
	No.	Per Cent	No.	Per Cent	No.	Per Cent	No.	Per Cent
2	18	44%	18	58%	9	43%	0	0%
2½	14	34	16	52	8	38	0	0
3	11	27	11	36	2	9	0	0
3½	9	22	6	20	0	0	0	0

price-stabilization techniques which were used in the early 1940's. For these reasons it was felt that a contingency loading on top of the 3 per cent would be unnecessary.

Conformity with the Statutes

The index accumulation method provides cash values which are completely equitable, since they return, with interest, all the net premiums not needed for expected benefits and allowable expenses. The reserves provide full funding for expected benefits. Paid-up nonforfeiture benefits are equivalent in value to cash values.

It is the opinion of the authors that the index accumulation method provides a sound method for funding the obligations assumed, is equitable in terms of nonforfeiture values provided, and conforms with the Standard Valuation Law and the Standard Nonforfeiture Law.

The inflation rate assumed for reserve and cash-value purposes (3 per cent being used in this paper) would be filed at the time of policy-form submission and would, along with other matters, be subject to approval by regulatory authorities.

The Method Reviewed

The index accumulation method, unlike the change of state method, is applicable to any plan of insurance. It produces generally lower cash values and reserves than that method. A cash-value interest assumption higher than 3½ per cent would nevertheless seem desirable, as is true of the change of state method.

This paper is being written at a time of quite rapid inflation, accompanied by very high interest rates on new investments. Interest yields, on a new high plateau, are expected to continue for many years. For policy value and gross premium determination, interest assumptions which are higher than traditional seem desirable. In this way it seems possible to achieve a pricing and cost structure for life insurance based on the consumer price index which is in the range of traditional structures for fixed-dollar products.

XII. GENERAL NATURE OF THE METHODS

Products discussed in this paper have been based on the consumer price index. The methods described are of a general nature, however; any suitable index can be used. To be "suitable," an index would no doubt require the following characteristics:

1. Its performance must be determinable promptly.
2. Once determined, the performance must be final, and beyond any doubt.

3. It must be of a permanent nature, so that its availability can be relied on over a lengthy period of time.
4. Its performance must be effectively beyond control of the policyholder.
5. Insurance supervisory authorities must agree to its use, since it will be referred to in policy forms, which must be approved.

In addition to government-published price indexes, suitable bases might include widely published investment indexes, such as the Dow-Jones industrial index.

Furthermore, it should be pointed out that the change of state method does not actually require the use of an index. All that it requires is an irrefutable means of determining the death benefit for any policy year, in a range between the minimum guaranteed benefit and the maximum guaranteed benefit for that year.

XIII. MARKETING RESULTS

Life Insurance Company of Georgia has marketed a cost-of-living policy since November 22, 1968. The product was initially available only in one state; approval was gradually obtained, until in April, 1970, the product had been introduced throughout the company's eleven-state territory.

Cost-of-living life insurance seems particularly suited to the middle-class market. Agents must be thoroughly trained in the product; it is markedly different from the familiar fixed-dollar insurance.

Evidence of the sales appeal of the product was gathered in a special fifteen-week test conducted by the company's two district offices in Charleston, South Carolina. The test was accompanied by television advertising. Each agent was asked to submit a report on every cost-of-living sales interview, with the following results:

Sales interviews	163
Cost-of-living sales	56 (closing ratio 34.3%)
Other products sold	29 (closing ratio 17.8%)
Total sales	85 (total closing ratio 52.1%)

Diversion to other products is a familiar phenomenon in life insurance selling; that aspect of the results is not a surprise to the authors. In fact, the agents involved were highly complimentary about the "door-opening" capabilities of the product.

A survey of this type can be expected to contain underreporting of interviews.[3] Even making allowance for this, the closing ratios, both for

[3] For comparative purposes, see page 63 of Volume XX of *TSA*, which gives closing ratio information for all lines of the same company at an earlier date.

the cost-of-living product and in total, seem to be very high. The authors believe that the cost-of-living concept has appeal to the public.

XIV. CONCLUSION

Inflation erodes the value of the life insurance dollar. The authors believe that there is a need for products which directly protect against this erosion. The methods described are attempts to design such products in a sound manner, fully in keeping with practices on the North American continent. Many other methods, and many variations, seem possible.

It is hoped that this paper will spur the development of life insurance based on the consumer price index.

APPENDIX

CHANGE OF STATE FORMULAS FOR OTHER COST-OF-LIVING PLANS

The change of state method may be used to develop reserves and surrender values for various types of cost-of-living plans (other than the whole life version which was described earlier). The necessary modifications in the formulas are illustrated here.

Example 1.—This is a term to 65 plan which provides that the face amount will ride up with the CPI each year. It is also provided that coverage will be at least equal to the paid-up nonforfeiture value at the beginning of the year but that coverage may not exceed double the initial amount. Premiums are payable to policy age 65.

Formulas for developing values are as follows:

$$ {}_tV_x = 2A^1_{x+t:\overline{65-x-t}|} - {}_t\mathrm{ELA}_x \cdot P^1_{x:\overline{65-x}|}\ddot{a}_{x+t:\overline{65-x-t}|} . $$

$$ {}_t\mathrm{CV}_x = 2A^1_{x+t:\overline{65-x-t}|} - {}_t\mathrm{ELA}_x \cdot P^A_{x:\overline{65-x}|}\ddot{a}_{x+t:\overline{65-x-t}|} . $$

$P^1_{x:\overline{65-x}|}$ = Net level premium for term to 65 insurance.

$P^A_{x:\overline{65-x}|}$ = Adjusted premium for term to 65 insurance.

$$ {}_t\mathrm{ELA}_x = \frac{M_x + M_{x+t} - 2M_{65}}{M_x - M_{65}}, \quad \text{if} \quad {}_{t-1}\mathrm{PU}_x > 1, \quad \text{then} $$

$$ {}_t\mathrm{ELA}_x = {}_{t-1}\mathrm{ELA}_x - \frac{(2 - {}_{t-1}\mathrm{PU}_x)C_{x+t-1}}{M_x - M_{65}}, $$

where

$$ {}_t\mathrm{PU}_x = \frac{{}_t\mathrm{CV}_x}{A^1_{x+t:\overline{65-x-t}|}} . $$

The following per thousand values are produced ($3\frac{1}{2}$ per cent, age last birthday):

x	t	${}_tV_x$	${}_t\mathrm{CV}_x$	${}_t\mathrm{PU}_x$
20	5	\$ 42	\$ 0	\$ 0
	10	88	49	375
	20	187	157	991
	30	259	241	1,411
	40	189	182	1,663
50	5	97	57	366
	10	122	104	951

Example 2.—This is a twenty-year income-protection plan providing a basic benefit upon the death of the insured of \$10 monthly income until the end of the twenty-year period. It further provides that the \$10 will ride up with the CPI each year the insured remains alive. It is provided that coverage will be at least equal to the paid-up nonforfeiture value at the beginning of the year but that coverage may not exceed double the initial amount. Premiums are payable for twenty years.

For purposes of determining the present value of the monthly income amount upon death, the following formulas employ the assumption that death occurs in the middle of the year

$$ {}_tV_x = \frac{20}{D_{x+t}} \sum_{s=t+1}^{20} \left\{ 12 \left[\frac{1 - v^{20-s}}{d^{(12)}} \right] + \frac{(1+i)^{1/12}[1 - (1+i)^{1/2}]}{1 - (1+i)^{1/12}} \right\} C_{x+s-1} $$

$$ - {}_t\mathrm{ELA}_x \cdot P^{1}_{x:\overline{20}|} \cdot \ddot{a}_{x+t:\overline{20-t}|} \,. $$

$$ {}_t\mathrm{CV}_x = \frac{20}{D_{x+t}} \sum_{s=t+1}^{20} \left\{ 12 \left[\frac{1 - v^{20-s}}{d^{(12)}} \right] + \frac{(1+i)^{1/12}[1 - (1+i)^{1/2}]}{1 - (1+i)^{1/12}} \right\} C_{x+s-1} $$

$$ - {}_t\mathrm{ELA}_x \cdot P^{A}_{x:\overline{20}|} \cdot \ddot{a}_{x+t:\overline{20-t}|} \,. $$

The first term in each of the above formulas represents the present value of future benefits (at their maximum, \$20 per month). The second part of the numerator of that term represents the series of monthly payments beginning upon death and continuing to the end of the year of death, or $(1+i)^{6/12} + (1+i)^{5/12} + \ldots + (1+i)^{1/12}$.

$P^{1}_{x:\overline{20}|}$ = Net level premium for twenty-year term.

$P^{A}_{x:\overline{20}|}$ = Adjusted premium for twenty-year term.

$$ {}_tELA_x = \frac{10}{M_x - M_{x+20}} \Bigg\langle \sum_{s=1}^{t} \left\{ 12 \left(\frac{1 - v^{20-s}}{d^{(12)}} \right) + \frac{(1+i)^{1/12}[1-(1+i)^{1/2}]}{1-(1+i)^{1/12}} \right\} C_{x+s-1} $$

$$ + 2 \sum_{s=t+1}^{20} \left\{ 12 \left[\frac{1 - v^{20-s}}{d^{(12)}} \right] + \frac{(1+i)^{1/12}[1-(1+i)^{1/2}]}{1-(1+i)^{1/12}} \right\} C_{x+s-1} \Bigg\rangle, $$

if ${}_{t-1}PU_x > 10$, then

$$ {}_tELA_x = {}_{t-1}ELA_x - \frac{(20 - {}_{t-1}PU_x) \left\{ 12 \left[\frac{1 - v^{20-t}}{d^{(12)}} \right] + \frac{(1+i)^{1/12}[1-(1+i)^{1/2}]}{1-(1+i)^{1/12}} \right\} C_{x+t-1}}{M_x - M_{x+20}}, $$

where

$$ {}_tPU_x = \frac{{}_tCV_x D_{x+t}}{\sum_{s=t+1}^{20} \left\{ 12 \left[\frac{1 - v^{20-s}}{d^{(12)}} \right] + \frac{(1+i)^{1/12}[1-(1+i)^{1/2}]}{1-(1+i)^{1/12}} \right\} C_{x+s-1}}. $$

The following values are produced ($3\frac{1}{2}$ per cent, age last birthday):

x	t	${}_tV_x$	${}_tCV_x$	${}_tPU_x$
20.....	5	\$ 3	\$ 0	\$0
	10	0	0	0
	15	0	0	0
35.....	5	18	0	0
	10	17	0	0
	15	3	0	0
50.....	5	70	29	2
	10	70	46	4
	15	14	3	1

As may be noted in the tabulation, cash-surrender values are generated only at the higher ages.

Example 3.—This is a twenty-year term rider to be attached to a permanent plan. The rider provides that each dollar of coverage under the base plan will ride up with the CPI each year. Total coverage may not exceed double the initial amount, and coverage provided by the rider must be at least equal to the rider's paid-up nonforfeiture value at the beginning of the year. At the end of twenty years the rider drops off; however, the insured is allowed to maintain the attained amount of coverage through conversion rights.

Formulas for developing values are as follows:

$$ {}_tV_x = A^1_{x+t:\overline{20-t}|} - {}_t\mathrm{ELA}_x \cdot P^1_{x:\overline{20}|} \cdot \ddot{a}_{x+t:\overline{20-t}|} \cdot $$

$$ {}_t\mathrm{CV}_x = A^1_{x+t:\overline{20-t}|} - {}_t\mathrm{ELA}_x \cdot P^{\mathrm{AR}}_{x:\overline{20}|} \cdot \ddot{a}_{x+t:\overline{20-t}|} \cdot $$

$P^{\mathrm{AR}}_{x:\overline{20}|}$ = Adjusted premium for a twenty-year term rider.

$$ {}_t\mathrm{ELA}_x = \frac{M_{x+t} - M_{x+20}}{M_x - M_{x+20}}, $$

if ${}_{t-1}\mathrm{PU}_x > 0$:

$$ {}_t\mathrm{ELA}_x = {}_{t-1}\mathrm{ELA}_x - \frac{(1 - {}_{t-1}\mathrm{PU}_x)C_{x+t-1}}{M_x - M_{x+20}}, $$

where

$$ {}_t\mathrm{PU}_x = \frac{{}_t\mathrm{CV}_x}{A_{x+t}}. $$

The following values are produced when the rider is attached to a whole life base plan of \$1,000 ($3\frac{1}{2}$ per cent, age last birthday):

x	t	${}_tV_x$	${}_t\mathrm{CV}_x$	${}_t\mathrm{PU}_x$
20.....	5	\$ 9	\$ 6	\$26
	10	13	12	45
	15	11	11	36
35.....	5	25	19	53
	10	40	37	90
	15	37	36	77

The examples given here are for purposes of illustrating the method only. There are, of course, numerous other types of plans or riders that could be devised. One such variation would be a plan providing for a limit on each year's ride-up of 5 per cent rather than the limitation of double the initial amount. The change of state method can easily be adapted to such a situation.

DISCUSSION OF PRECEDING PAPER

ANNA MARIA RAPPAPORT:

Mr. Bragg and Mr. Stonecipher are to be congratulated on making a substantial contribution to our literature and on working within the framework of our legal structure to provide a product that will better fit the needs of consumers.

In preparation for a workshop at which this paper is to be discussed, I have done some investigation of cost-of-living benefits and of apparent marketing success. I find that in general results have been very disappointing. This is in marked contrast to the results cited in the paper. It is my opinion that these results are not contradictory but rather that the following is true about introduction of new-product concepts.

The agent is slow to show interest in different concepts. The buyer does not have a chance to evaluate a new concept, no matter how good it may be for him, until the agent with whom he is doing business is completely sold on the concept himself. Strong company support for the sales effort, combined with a determined effort to interest the agent in the product, is necessary for successful introduction of different product concepts. Such support was present in the marketing test cited in the paper. (Such support was also present when the family plan became a marketing success, etc.) Marketing success or failure of a new or different type of product is as much a function of the company's approach to the agent and of the support given him as it is of the merit of the product.

Many people in our industry seem to feel that cost-of-living products cannot be sold. The industry is moving toward variable life insurance as the answer to the insurance needs of the consumer in the face of inflation. I feel that neither solution provides the complete answer, and therefore we must try to find a solution which better meets consumer needs.

The sale of equity-linked benefits on the grounds that they will tend to protect against inflation because inflation raises equity prices is incorrect. The experience of the last year or two is typical. When prices are going up, stocks go down, so that when the policyholder really needs the protection it is taken away from him.

Equity-based products are better than debt-based products only because investing has been more profitable than lending. If we intend to sell to people who trust us, we must have products that perform in a way related to our claims. A combination may do this, but a pure equity-based product does not.

The insurance program of an individual can be used to provide two kinds of dollars—death benefit dollars and cash value dollars. It seems to me rather inconsistent with the needs of the policyholder and with the entire tradition of life insurance (and the stability of the guarantees that it has always provided) to provide a death benefit which fluctuates markedly with security prices on a short-term basis. Death occurs at once and not at the time of the policyholder's choosing, so that fluctuating death benefits are rather unfair (unless they are to be applied under variable settlement options). On the other hand, cost-of-living-linked death benefits can meet this problem of inflation on a stable long-term basis. I realize that this is a rather unpopular viewpoint. I believe that this is due to a reluctance on the part of many to think honestly about death benefit dollars and to a rather incredible optimism about the future performance of the stock market and the ability to realize spectacular results by investing in the market.

Cash value dollars, on the other hand, are rather different from death benefit dollars. The insured can choose whether to use them and when. Furthermore, it is logical that the earnings be as great as possible and that the insured have the choice of taking the risk of getting more or less than he would under traditional programs. Investment in equities, therefore, is quite logical.

It is my opinion that it should be possible to combine these two things—death benefits varying with the cost of living and cash values varying with equity performance. I realize that this requires modification in nonforfeiture law and regulation. Now that such law is being modified to make variable life insurance possible, it should also be changed to make possible products which will do the best job for the consumer.

NAFTALI H. TEITELBAUM AND LOUIS WEINSTEIN:

The authors are to be congratulated on their excellent paper. This discussion reflects our experience in developing a four-year renewable and convertible cost-of-living term insurance policy for the Bankers Security Life Insurance Society.

As proud parents (in the posture of staff actuary and consulting actuary) of a policy using the term insurance approach, we believe a more detailed description of our policy would be of interest to other actuaries.

Our policy provides for a death benefit and a renewal amount for any four-year period based upon the initial death benefit for that period adjusted by the relative change in the consumer price index (CPI), up to a

maximum of 125 per cent of the amount previously renewed (issued if never previously renewed). Since the renewal premium is the guaranteed renewal rate shown in the policy multiplied by the renewal amount, the policyholder pays for increased benefits due to inflation every fourth anniversary, and the company risk is limited to the four-year period. No over-all maximum during the policy lifetime is necessary, which is a key advantage over the permanent form. Death benefits could exceed twice the initial amount after thirteen years.

Our policy did not contain an annual ratchet clause, but the death benefit and renewal benefit for any four-year period may never be less than the amount at previous renewal (or issue, if never previously renewed) regardless of CPI performance.

It may be noted that the state of Georgia requires an annual ratchet clause and will not approve a policy which provides for a changing death benefit dependent upon an "outside factor" which produces a value less than the preceding death benefit.

The development of cash values under the change of state method is a brilliant solution to a vexing actuarial problem. The authors state, however, "The only factors included in the calculations are (i) interest, (ii) mortality, and (iii) specifications of the premium and guaranteed benefit structure." Since the formula is developed by selecting that assumption regarding CPI performance which produces the maximum-minimum value, the authors cause the CPI assumption to be a factor in the nonforfeiture calculation in their formula selection.

Despite this comment, we agree with the authors' conclusion. The law is satisfied since it is not violated, and nothing in the law excludes the use of a CPI assumption. We recognize that this argument may evoke more regulatory criticism than the authors' argument.

The authors' assertion that the statement of a nonforfeiture factor is needed only in cases where the unique minimum cash value is being used is not valid in New York State, and the exclusion of such factors from the policy would violate New York State law and possibly the laws of other states.

The authors have imposed a logical minimum death benefit equal to the paid-up value. The consequences of this complication could be lessened by having the paid-up values contain a guaranteed 3 per cent linear increase. The formula would be:

$$ {}_tPU_x = {}_tCV_x \cdot \frac{D_{x+t}}{M_{x+t} + .03R_{x+t+1}}, $$

which would produce a paid-up value of \$836 at $t = 20$ for $x = 35$ instead of the \$1,231 based on the formula

$$ {}_tPU_x = {}_tCV_x \cdot \frac{D_{x+t}}{M_{x+t}} . $$

Table 1 contains the assumptions of CPI performance used in pricing our four-year renewable and convertible term policy. It is a conservative assumption based upon historical trends but liberal in light of recent experience. The average death benefit is \$1,093.75, which is quite low

The authors have effectively argued that the benefits in the policy are distinctly stated. This argument was accepted in New York State, but in other jurisdictions a preliminary position was that the policyholder should

TABLE 1

AVERAGE ANNUAL DEATH BENEFIT

(Per \$1,000 In Force at Beginning of Any Four-Year Term Period)

Year	End of Year	During Year
1.....	\$1,050	\$1,025
2.....	1,100	1,075
3.....	1,140	1,120
4.....	1,170	1,155

not have to look outside the four corners of the contract in order to determine the benefits. In one jurisdiction the regulators demanded that the specific cost for the cost-of-living feature be stated in the contract. Companies wishing to market this type of product on a national basis will find it difficult to comply with the requirements of all United States jurisdictions by using only one policy form.

The regulators were also concerned about the comprehensibility of the policy, and we think that the use of term insurance and the resulting elimination of cash values would be extremely useful in securing state approval.

New York State required reserves to be established upon the assumption that death benefits during any four-year-term period were always 125 per cent of the actual amount during such period, since 125 per cent was the maximum value of our CPI ratio. Even on this basis, no deficiency reserves resulted.

ROBERT C. TOOKEY:

The authors have presented an elegant and exhaustive analysis on the matter of coping with the requirements of the standard nonforfeiture law

and the standard valuation law when designing an ordinary life policy under which the amount of insurance is indeterminate at issue and is defined by a formula with constraints that assure a maximum and a minimum amount. The cash values determined by the change of state method are truly the maximum statutory minimum values that could possibly arise in any pattern of gyrations in the consumer price index. In other words, the conformity to the standard nonforfeiture law cannot be questioned.

But what price conformity!

One must agree with the authors that we need a statutory maximum valuation interest rate considerably in excess of the $3\frac{1}{2}$ per cent presently

TABLE 2

ASSUMPTIONS USED IN PROFIT MARGIN STUDY

(Present Value of Future Profits Discounted 10 Per Cent over Twenty-Year Period)

1. Mortality	15-year select modification of intercompany experience 1955–60
2. Lapse rate persistency	WL policy, Linton B COL policy, .20; .10; .05; and .025 thereafter
3. Interest rate	6 per cent
4. Commission and taxes	92.35 per cent first year, 7.85 per cent next nine years, 5.35 per cent thereafter
5. Per policy expense	$75.00 first year, $5.00 thereafter
6. Average size policy	WL policy, $20,000 COL policy, $10,000 increasing to $20,000 in twenty years
7. Reinsurance	None

in effect to enable this elaborately developed product to compete effectively in the market place.

It appeared that an interesting profit-margin comparison could be made between this cost-of-living plan and an ordinary life policy, with minimum cash values, issued for twice the initial amount of the cost-of-living plan. We assumed a uniform 5 per cent annual increase in amount of insurance under the cost-of-living policy, thus resulting in the doubling of the initial amount of insurance in twenty years. The cash values and reserves shown in the paper were used at the indicated durations with second difference interpolation providing the intervening values. The CRVM renewal valuation premium for a corresponding ordinary life policy with double the initial amount of insurance was tested. Table 2, statement of assumptions, reveals that the same parameters were used in both cases except for a lower withdrawal rate that applied to the cost-of-living policy, and, of course, there is a slight difference in cash values and reserves, which are higher under the cost-of-living policy.

In Table 3 we show the test premiums (renewal valuation net) and the twenty-year profit margins for ages 25, 35, 45, and 55 for the two policies. Since the margins ran somewhat lower above age 30 for the whole life policy, we computed the required premium to produce the same profit as under the cost-of-living policy. The premium difference and percentage difference are shown in the far-right column. Question: Wouldn't one pay from 1 to 7 per cent more premium for twice as much protection?

Because of the very high cash values required under the present standard nonforfeiture law for the whole life cost-of-living policy (cash values that even exceed the face amount after a point), the policy possesses the

TABLE 3

(Figures per $2,000 of Ultimate Amount of Insurance)

Age	Test Premium	Profit Margin		Additional Whole Life Premium Required to Produce Same Margin as under COL Policy	
		COL	WL	Amount	Percentage of COL Premium
25	$21.58	$11.78	$13.05	$−0.32	−1.5%
35	32.04	17.54	14.57	0.76	2.4
45	50.06	33.46	25.43	2.09	4.2
55	81.26	76.63	47.70	7.80	7.6

same objectionable features that apply to endowment plans and limited pay life plans in these days of inflation and high interest rates. The consumer is simply paying on the nose for high cash values.

It is felt that the 5 per cent inflation assumption is closer to a "most probable" condition than a "disaster" condition. This nation has been warned by economic prognosticators to expect an annual inflation rate of not less than 4 per cent throughout this decade. The cork of stability is out of the bottle, and there is no forcing it back in.

It is true that persistency should be encouraged as the amount of benefit increases because of the almost certain increases in the CPI that may be anticipated. In fact, this approach has been used in accident and sickness policies to discourage twisting. The amount of benefit is increased annually to an ultimate level of as much as 25 per cent of the initial amount to reward persistent policyholders. For this reason we assumed a $2\frac{1}{2}$ per cent ultimate lapse rate from the fourth policy year on in the profit margin studies applicable to the cost-of-living policy. Our experience has

shown that the ultimate irreducible minimum lapse rate is around 2½ per cent because of the so-called unavoidable lapses (e.g., those caused by divorce, business failure, partnership dissolution, and a radical change in a person's financial affairs).

I was intrigued with some of the alternative plans suggested by the authors. In particular, the family income policy that provides for an annual cost-of-living benefit seemed especially intriguing. We are working on such a product, a decreasing term to 65 plan that provides for a uniform annual increase in monthly benefit. We have not yet decided upon a name for this little jewel but are seriously considering calling it "increasing level decreasing term to 65."

JOHN E. HEARST:

The authors are to be commended for their imaginative solution to a complex and thorny problem. Although they demonstrate that the cash values and reserves generated by their change of state method satisfy existing standard valuation and nonforfeiture laws, their task would have been much simpler had the laws explicitly provided for benefits which varied with changes in the consumer price index or with changes in some other index. In my opinion the laws should be amended to provide for such changes.

One simple scheme would be to express the reserves and nonforfeiture values as a percentage of the reserves and nonforfeiture values developed for the traditional life insurance products under existing laws. The reserves and cash values shown in Table 2 of the authors' paper are almost a constant percentage of those for a whole life policy without a varying death benefit.

Another scheme would be to specify a minimum rate of change in the index in the same way that the rate of interest or a rate of mortality is specified in the standard valuation and nonforfeiture laws. Cash values and reserves would then be calculated under the assumption that the death benefit would increase exactly by this rate in all future years. Such a plan would require the use of only one nonforfeiture factor, whereas the authors' change of state method requires as many nonforfeiture factors as there are policy durations. The paid-up insurance and extended insurance would continue to vary with the index rather than remaining fixed. Moreover, it would not be necessary to have a maximum limit on the death limit.

The limit on the death benefit which the authors had adopted may be unduly restrictive. For example, in Table 3 the maximum death benefit

payable at the expected age at death is $20,000. The benefit, however, should have been $28,139, if it were to compensate completely for a 3 per cent annual increase in the CPI.

The authors cite several advantages of their ratchet clause as arguments for its adoption. Its adoption may be required by existing laws because the statutes of most states prohibit any provision which limits settlement at maturity of a policy for less value than the amount insured on the face of the policy.

The authors stated that their policy did not have the appeal attributed to variable products because it provided for the cost of living directly rather than relying on investment performance to approximate the effect of inflation. Investment performance, at least for many mutual funds, has generally been much worse this year than the performance of unmanaged portfolios. The mutual fund investment performance index prepared by the Arthur Lipper Company shows a percentage decline for 461 mutual funds of 17.81 per cent for the year ending November 19, 1970, while the Dow-Jones industrial average declined 5.57 per cent in the same period.

(AUTHORS' REVIEW OF DISCUSSION)

JOHN M. BRAGG AND DAVID A. STONECIPHER:

The authors wish to express their appreciation to those persons who have submitted discussions of the paper.

Our experience is in agreement with Mrs. Rappaport's comment that new products have to be properly promoted to the agent. Her discussion of a contract in which the death benefits are cost-of-living-related but the cash values are tied to equities is interesting to all of us; as she points out, however, such a product is yet to be developed from a theoretical or regulatory standpoint. This product might give the ideal marriage of need and incentive. On the other hand, there are some who believe that it would be better to give the buyer purchasing-power protection on both the death benefit and the cash value, a feature which the authors' index accumulation method does provide.

We thank Messrs. Teitelbaum and Weinstein for the more detailed description of their cost-of-living term policy than is included in the paper. We believe that their suggestion for a 3 per cent linear increase in the paid-up values would be helpful in delaying the time when the guaranteed face amount would exceed the initial amount.

Mr. Tookey's comments clearly bring out the need for cash values based on a higher rate of interest than the present $3\frac{1}{2}$ per cent legal maximum—a point with which the authors agree most emphatically. Mr. Tookey's

profit study is most interesting, but we would disagree with the implication that *twice* the protection may be received for only 1–7 per cent more premium. The cost-of-living policy provides protection which *increases* significantly after the first policy year, and, using our actual rates, we find a premium difference of 16–21 per cent.

Age	Additional Premium Required to Provide for $2,000 Whole Life
25	16.4%
35	17.7
45	16.6
55	21.1

We are indebted to Mr. Hearst for pointing out the poor performance of "managed accounts" in 1970 in comparison with unmanaged accounts. If the present standard nonforfeiture and valuation laws cannot be interpreted as allowing for contracts that vary with an index and are fully guaranteed, then we would agree that they should be changed so as to provide more flexibility for innovative ideas.