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Pricing Surface

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Pricing an insurance product requires assumptions, actuarial models and professional judgments; the pricing results are usually accomplished by a set of finite numbers deemed as the best estimate of certain profitability measures, and they are also accompanied by a list of sensitivity testing results to help actuaries better understand any potential deviation from the pricing target due to misestimates, misjudgments or other uncertainties.

This paper suggests expanding the current approach by constructing a pricing surface, or capturing the joint distribution of interested pricing measure driven by pricing variables. In this paper, we will discuss why we use pricing surface, how to construct the surface and what the benefits of using pricing surface are; we also provide an example to illustrate the idea and draw a conclusion based on the discussion.

WHY PRICING SURFACE

The pricing results are driven by pricing variables. What value we assign to a variable is based on the assumption. Some assumptions can be obtained directly from the market such as interest rate or from a company's experience of similar products (such as mortality or lapses). Other assumptions may require professional judgment if the experience is relevant but not directly applicable.

Due to various degrees of uncertainty of the assumptions, a point estimate (usually labeled as best estimate) is not sufficient to provide the complete picture of pricing results even with a list of sensitivities, let alone to support the decision-making process. Here are a few examples.

It is a challenge to reflect economy of scale.

For an insurance company, it is common to see a fight between sales force and pricing actuaries. The sales force wants to lower the price to make the product more competitive or easy to sell; they argue that as long as the marginal profitability (where only policy-driven expenses are included, no overhead expenses) is positive, additional policies sold will make a positive contribution to the company. On the other hand, pricing actuaries feel that pricing should reflect the true cost to the company. Each side has its own argument. This assumption is driven by sales volume. If more policies were sold, the cost per policy would go down, and the economy of scale can be partially achieved. The profitability could converge to pricing results with marginal expense assumptions to a certain degree.

When pricing actuaries develop the expense assumption, they usually have a certain sales target in mind and use it to spread out the overhead expenses. Once determined, it won't change. Although actuaries have tested two extreme cases, it is difficult to reflect profitability with the actual sales level. This fight is usually resolved in front of the CEO and/or CFO with a reasonable balance between profitability and growth for the company.

Cross-terms among the pricing variables are usually ignored.

Sensitivity tests are commonly performed at one dimension (or one variable) and one dimension only. The interactions between two pricing variables (or cross-terms) are usually ignored. For some products, the cross effect can be significant, especially at the tail. For example, for single premium immediate annuity (SPIA) product pricing, the company performs sensitivity tests on interest rate and longevity, respectively, but did not test the combined changes of interest rate and longevity at the same time. Some actuaries found that the impact of the cross-term can be greater than the two individual sensitivity results combined at the tail. The reason is that the change of one pricing variable magnifies the impact of the change on the other variable. In this case, the longevity extends the duration and makes the profitability more sensitive to the interest rate. Although the effect may not be significant with moderate changes of assumptions, it should not be overlooked until tested. Of course, some cross effects can go the opposite way, where the changes of two pricing variables can be off-set to each other to certain extent. This would be good news for the company. When this is observed, pricing actuaries or risk managers need to know as well.

More sensitivity tests may not be enough.

To price an innovative product, it is a challenge to get comfortable with actuarial assumptions because of lack of experience (if we assume experience is relevant). Actuaries usually rely on the experience of similar products, or competitors' experience (usually indirectly from consulting firms), or simply rely on their own professional judgment. No matter where the assumptions landed, they are still actuaries' best guess. The high level of uncertainty leads to more sensitivity tests to help understand the results that could potentially deviate significantly from the mean. However, these sensitivity tests may not be enough to cover all possibilities for certain assumptions, especially at the extremes, where human judgement has its limitations. As an example, when interest rates were above 10 percent the in

1980s, probably no pricing actuaries at that time would have thought the rate can go down to today's level.

Despite the issues of the current approach, pricing exercises are usually complete before the product is launched. After the products sold turn into in force and are passed on to in-force managers, there are no further follow-ups in the pricing area. This can be dangerous as the pricing assumption may change from time to time; and the actual profitability may significantly deviate from pricing target.

Pricing surface can help address these issues by selecting the right pricing variables and building the joint distribution of the pricing results with pricing variables chosen.

HOW TO CONSTRUCT A PRICING SURFACE

Because the joint multivariate distribution is usually unknown, it makes constructing a surface a challenge. However, there are a few simplified approaches.

One approach is the so-called curve fitting, which requires multiple point estimates to help look for a statistical distribution that best fits these points. Once the distribution is identified, actuaries can use the distribution to find other pricing points they are interested in.

Another approach is to apply multiple-variate Taylor expansion using a few observed points. Here we use Taylor expansion to illustrate the process.

Step 1: Define Pricing Variables and Sensitivity Levels

Taking SPIA pricing as an example, we assume the pricing result is a function of two pricing variables, namely interest rate and mortality rate, because we assume they drive the pricing results. We also assume that function meets the certain mathematical assumptions such that we can apply Taylor series to this function.

We then define the sensitivity levels so that we calculate the first and second orders of the derivatives. In Table 1, we choose the following:

Table 1
Sensitivity Levels of Pricing Variables

Pricing Variables	Changes of Pricing Variable	# of Tests
Best Estimate	None	1
Interest rate ("R")	+/−1% parallel shift	2
Mortality ("M")	+/−10% of base mortality table	2
Interest rate × Mortality	+/−1% parallel shift × +/−10% Mortality	4

Step 2: Calculate the First and Second Derivatives

After obtaining nine actual testing results, including the best estimate, we calculate the first order of derivatives, the second order of derivatives and the second order of derivatives for the cross term.

The notations used in the formulas are as follows:

ΔR = the change in interest rate as defined

ΔM = the change in mortality as defined

V_0 = the baseline value with pricing assumptions

V_{R-} = the ending value when interest rate declined by ΔR

V_{R+} = the ending value when interest rate increased by ΔR

V_{M-} = the ending value when mortality declined by ΔM

V_{M+} = the ending value when mortality increased by ΔM

V_{R+M+} = the ending value when interest rate and mortality increased

V_{R+M-} = the ending value when interest rate increased and mortality decreased

V_{R-M+} = the ending value when interest rate declined and mortality increased

V_{R-M-} = the ending value when interest rate and mortality decreased

To calculate first order of derivatives with respect to interest rate, we have the following formula:

For rate up, the formula becomes

$$\left(\frac{\partial V}{\partial R}\right)_{+} = \frac{V_{R+}/V_0 - 1}{\Delta R}$$

Similarly, for when the rate goes down, we have

$$\left(\frac{\partial V}{\partial R}\right)_{-} = \frac{V_{R-}/V_0 - 1}{\Delta R}$$

The first order of derivatives with respect to mortality can be done in the same fashion.

Similarly, for second order of derivatives, we take the calculated first order of derivatives and calculate them using the following formulas:

For interest rate move,

$$\left(\frac{\partial^2 V}{\partial R^2}\right)_+ = \frac{V_{R+}/V_0 - 1 - \left(\frac{\partial V}{\partial R}\right)_+ \Delta R}{(\Delta R^2)} \times 2$$

For mortality move,

$$\left(\frac{\partial^2 V}{\partial M^2}\right)_+ = \frac{V_{M+}/V_0 - 1 - \left(\frac{\partial V}{\partial M}\right)_+ \Delta M}{(\Delta M^2)} \times 2$$

To calculate second orders of derivatives for cross items, we need to specify the directions of the movement of pricing variables.

For interest and mortality rates' upward movement, we have

$$\left(\frac{\partial^2 V}{\partial R \partial M}\right)_{++} =$$

$$\frac{V_{R+M+}/V_0 - 1 - \left(\frac{\partial V}{\partial R}\right)_+ \Delta R - \left(\frac{\partial V}{\partial M}\right)_+ \Delta M - \frac{1}{2}\left(\frac{\partial^2 V}{\partial R^2}\right)_+ (\Delta R)^2 - \frac{1}{2}\left(\frac{\partial^2 V}{\partial M^2}\right)_+ (\Delta M)^2}{(\Delta R \Delta M)}$$

Similarly, we can calculate the following

$\left(\frac{\partial^2 V}{\partial R \partial M}\right)_{+-}$, $\left(\frac{\partial^2 V}{\partial R \partial M}\right)_{-+}$ and $\left(\frac{\partial^2 V}{\partial R \partial M}\right)_{--}$.

Step 3: Estimate the Impact Using Taylor Series

When the derivatives are calculated, we estimate the final movement in the target value that is driven by pricing variables using the following formula. As an example, if we want to estimate the final value with a rate increase of Δr and a mortality increase of Δm, we will have

$$\Delta V_{++} = \left(\frac{\partial V}{\partial R}\right)_+ \times \Delta r + \left(\frac{\partial V}{\partial M}\right)_+ \times \Delta m$$

$$+ \frac{1}{2}\left[\left(\frac{\partial^2 V}{\partial R^2}\right)(\Delta r)^2 + \left(\frac{\partial^2 V}{\partial M^2}\right)(\Delta m)^2 + 2 \times \left(\frac{\partial^2 V}{\partial R \partial M}\right)_{++} \times \Delta r \times \Delta m\right]$$

Other combinations of moves will be estimated in similar fashion.

We then apply the Taylor expansion formula to construct a pricing surface so that we can estimate the pricing results for any combination of mortality and interest rate changes.

To illustrate, the pricing surface in Figure 1 was plotted to show the joint distribution of profitability (as percentage of baseline or best estimate) by interest rate and mortality changes (relative to best estimate assumptions).

Here we not only see the relationship between profitability and each individual pricing variables while holding the other variable constant, but also see the cross effect of the two variables. The change of the steepness of the slope tells us that the cross effect is not even across the spectrums, because it would be, otherwise, a flat surface tilted at an angle.

Of course, the estimates can be improved if using third order of derivatives. Also, the users can easily expand the exercise to include more than two pricing variables. For example, selecting number of policies sold as the pricing variable can address the expense assumption issues we discussed earlier; it could also influence the results driven by volatility of mortality or lapses if those variables are modeled stochastically.

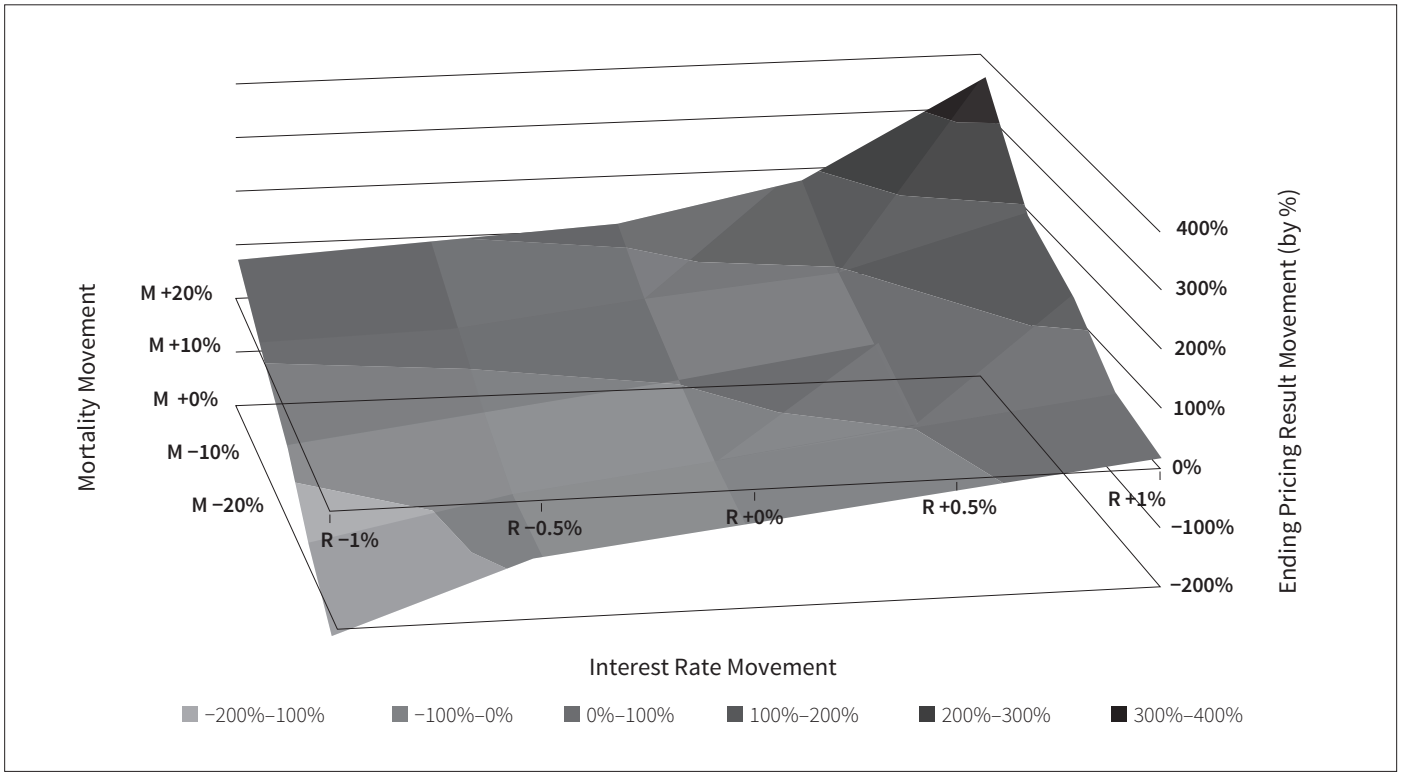
THE BENEFITS OF PRICING SURFACE

From the example, one can see that by constructing the pricing surface, we are able to resolve issues mentioned before. Here are the benefits:

- The pricing surface provides a joint distribution of the pricing results; actuaries not only get the mean and variance, but also its relationship with all pricing variables (interaction among these variables or cross effects). Diversification or magnification between two or more pricing variables is observable. So here we suggest, even with no plan for actuaries to construct pricing surface, to perform sensitivity testing on cross term for better pricing.

- The pricing surface helps monitor the profitability of sales that may derivate from initial pricing target. Ideally, once priced, the profitability of a product does not change. In reality, this may not always be the case; the market environment changes over time. Assumptions change as the experience emerges. A company's pricing team or in-force management team cannot afford to keep up with the changes and conduct repricing exercise as frequently as they want to, or to monitor the actual profitability of the new sales bring to the table due to real-time changes at point of sale from original pricing. With pricing surface, one can either confirm the pricing results for recent sales, or quantify the gap between the actual and pricing results, and pinpoint the drivers. This helps the company make the right decision with respect to encouraging sales when the environment is favorable or put a limit of sale when otherwise. For example, using the preceding chart and taking SPIA, when experiencing persistent low interest rate (e.g., 0.5 percent lower than pricing) and seeing mortality improves overtime (e.g., mortality is reduced by 5 percent), the pricing surface would tell us that the profitability would be reduced by 55 percent. If the company feels they are missing pricing target by a margin, they may choose to slow down or stop the sales or conduct repricing if the market demand persists. On the other hand, if the interest rate and mortality movement are exactly opposite, the pricing surface shows the profitability would increase by 71 percent. This better-than-expected profitability could make the company

Figure 1
Pricing Surface with Interest Rate and Mortality Movement



create additional incentive to encourage sales or consider reducing the premiums (one could build the surface with premium as a variable) to boost sales. Nevertheless, when it comes to repricing, if there is no change in product design, the surface should contain all the results already, no repricing exercise is necessary.

- Finally, the pricing surface can facilitate communications both within and outside a company. If a number of policies sold is selected as a pricing variable and pricing surface is constructed accordingly, it will capture the relationship between the pricing results and number of policies sold. As a result, there is no need to argue between using marginal expense pricing or using fully allocate expense assumptions, because it is baked in pricing and the surface will show how the pricing results vary as number of policies sold change. If only one policy is sold, the surface will tell us the product is expensive or the profitability is low because all the overhead expense has to be allocated to one policy. At the other extreme, if huge amount of policies are sold (up to certain extent or high end of economy of scale under current service capacity), the surface will say that the profitability is close to the one when marginal expense assumption is used under traditional pricing. The actuals profitability is probably somewhere in between. This surface would facilitate the communication with fields or senior management by bringing everything to the table. If a certain sales goal is met by the sales force, marginal expense assumption can be achieved, the product can be cheaper, on the other hand, if sales are lagging, the product has to be expensive to meet the profitability target. This tool could also help the company to communicate with regulators or rating agencies if used properly.

CONCLUSION

While current best estimate pricing results provides information for decision making, a pricing surface offers a comprehensive view of the pricing results throughout the spectrum of each driver that might alter the pricing results. Although constructing the joint distribution is a challenge, there are simplified approaches to make it happen. Furthermore, it is worth the effort to obtain the pricing surface. It helps make an informed decision and facilitate the pricing conversation within and outside a company. ■

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Universal Life with Secondary Guarantees Survey Summary

Through its Policyholder Behavior in the Tail workgroup, the Society of Actuaries has published a new report[1] summarizing the results of its most recent assumption survey for Universal Life Insurance with Secondary Guarantees. Highlights are as follows:

- 25 companies participated in the survey up from 20 last time, covering $740 billion of insurance inforce.
- Capital requirements are highly dependent on assumptions for lapse rates and investment returns.
- A wide range of assumptions is evident across companies, particularly for "tail" scenarios and elderly insureds, only some of which is explained by product design differences.

This is the latest in a series of surveys[2] covering Universal Life Insurance with Secondary Guarantees and Variable Annuities, respectively, started in 2007. The motivation for these surveys is the high degree of sensitivity that these products have to elective policyholder behavior, and the emergence and changes in these behaviors in recent years. The reports from these surveys should be of interest to actuaries in product development, pricing, inforce management, and valuation roles, and should aid in the development of prudent policyholder behavior assumptions for these important product lines.

Anyone interested in more information or learning about how to participate in future surveys should contact Barbara Scott at *bscott@soa.org*. ■

ENDNOTES

1 *https://www.soa.org/research-reports/2017/2017-ul-second-guarantee-survey/*

2 *https://www.soa.org/research/topics/risk-mgmt-res-report-list/*