## TRANSACTIONS OF SOCIETY OF ACTUARIES 1968 VOL. 20 PT. 1 NO. 58

# COSTS OF NONFORFEITURE BENEFITS ON SUBSTANDARD POLICIES 

JOHN E. HEARST


#### Abstract

Costs of nonforfeiture benefits at various levels of mortality are given in the paper. The purpose is to compare these costs and to illustrate conditions under which the extended insurance option might be offered more generally in substandard policies.

The term "costs," as used here, means the present value of the death benefits, expenses, and dividends where the present values are discounted at 3 and 5 per cent on multiples of the 1955-60 Basic Ultimate Male Table.

According to the assumptions in the paper, when expenses and dividends are considered, the costs of the extended insurance option are generally lower than those of the paid-up insurance option at the lower levels of substandard extra mortality.


Ar Lapse the standard ordinary policy can be continued in force by the automatic premium loan provision or by the extended insurance or paid-up insurance nonforfeiture options. Some automatic premium loan provisions revert to the automatic nonforfeiture option after two or three premiums have been paid by the loan provision or when the net cash value has been reduced to less than the gross premium. In the substandard policy the extended insurance option is usually omitted on the premise that it is too favorable for substandard lives.

The purpose of this paper is to compare, for substandard policies, the costs of the nonforfeiture benefits at various levels of mortality and to illustrate conditions under which the extended insurance option might be offered more generally. As used here, the term "costs" means the present value of death benefits, expenses, and dividends where the present values are discounted at 3 and 5 per cent on multiples of the 1955-60 Basic Ultimate Male Table.

The use of constant multiples of the 1955-60 Basic Ultimate Male Table was chosen as a measure of substandard mortality since, for many impairments, the incidence of extra mortality as a percentage of standard mortality is either level or slightly decreasing by duration. An ultimate
table was chosen as a convenience to limit the number of illustrations even though most lapses occur before the effects of selection have entirely worn off.

Expenses are assumed to be chargeable yearly and to be equal for $\$ 1,000$ of extended insurance and for the amount of reduced paid-up insurance purchased by the same cash value necessary to purchase the $\$ 1,000$ of extended insurance. These expenses are assumed to continue for the period of extended insurance under the extended insurance option and for the life of the reduced paid-up option. Under this assumption the cost of mortality and expenses is assumed to be $1,000 A_{x: \bar{a}}^{1}+E \cdot \vec{a}_{x:} \overline{a_{\mid}}$under the extended insurance option and $S \cdot A_{x}+E \cdot \ddot{u}_{x}$ under the amount of reduced paid-up insurance corresponding to $\$ 1,000$ of extended insurance.

Other expenses characteristic of the several types of nonforfeiture options are ignored. Dividends are calculated by the three-factor dividend formula using only the excess interest factor.

The standard nonforfeiture laws require the present value of the nonforfeiture options to be equal to the cash value at lapse according to the interest rate and mortality table used in calculating the options. However, periods of extended insurance may be calculated on a loaded table. Before the 1958 CSO Table was adopted, the same table was generally used for calculating all the nonforfeiture benefits. Now, however, the 1958 CET Table is nearly always used for calculating periods of extended insurance when the 1958 CSO Table is used for calculating the other nonforfeiture benefits. In Table 1 a comparison is made on this basis of the periods of extended insurance and amounts of paid-up whole life insurance purchased by the same cash value necessary to purchase $\$ 1,000$ extended insurance.

Even though the ages in Table 1 range from 15 to 55 and the periods of extended insurance from 2 to 20 years, lapses usually occur at the younger ages and at the early policy durations, where the periods of extended insurance are shorter. An indication of this is the Society's study of extended insurance mortality, TSA 1964 Reports, which showed 90 per cent of the deaths occurring within ten years of lapse for that part of the experience where data were segregated by duration after lapse and 68 per cent of the deaths occurring at ages under 55 at lapse.

## MORTALITY AND INTEREST

In Table 2 a comparison is shown of the present value of death benefits for the periods of extended insurance and amounts of paid-up insurance given in Table 1 at the 3 per cent nonforfeiture interest rate. The interest
rate is chosen as a compromise between the rates currently being used in participating and nonparticipating policies. The present values are discounted at 3 and 5 per cent on multiples of the 1955-60 Basic Ultimate Male Table.

The present values of the extended insurance death benefits are lower than those for the paid-up insurance at the younger ages and at the lower levels of mortality and interest. At 3 per cent the extended insurance present values are lower for ages $15-55$ through 150 per cent of the 195560 Basic Ultimate Table. At 5 per cent the differences are relatively small

TABLE 1

| Peplod of <br> Extiknded Insurance* | Ayounts of Padodr Whole Life Insurance Purchased by tee Case Value Necessary to Purcbase $\$ 1,000$ Extended Insuranct $\dagger$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Nonforfiture Interest Rate |  |  |  |  |  |  |  |  |
|  | $2\}$ Per Cent |  |  | 3 Per Cent |  |  | 3) Per Cent |  |  |
|  | Age at Lapse |  |  | Age at Lapse |  |  | Age at Lapse |  |  |
|  | 15 | 35 | 53 | 15 | 35 | 55 | 15 | 35 | 55 |
| 2. | 16 | 15 | 54 | 20 | 18 | 58 | 24 | 21 | 63 |
| 5. | 40 | 40 | 145 | 49 | 47 | 156 | 60 | 54 | 167 |
| 10. | 78 | 92 | 322 | 95 | 105 | 341 | 116 | 119 | 360 |
| 20. | 146 | 248 | 723 | 175 | 274 | 747 | 209 | 301 | 770 |

* Based on 1958 CET.
| Based on 1958 CSO.
through 150 per cent of this table. For the higher levels of mortality and for the older ages at lapse, the present values of the extended insurance death benefits are much larger than the paid-up insurance present values. The reason for this relationship is that, as the rate of mortality is increased, the value of the extended insurance tends toward the face amount while the value of the paid-up insurance tends toward the reduced amount.

Although the examples in Table 2 are extended to 500 per cent of the 1955-60 Basic Ultimate Male Table, most substandard issues are made at much lower ratings. For example, the digest of Mr. A. A. Windecker's report of Prudential's practices stated:

The Prudential had increased its range of acceptable substandard risks from 300 to 500 per cent about three years ago. He indicated that recent ex-

TABLE 2 :

| Period of Extended Insurance | Present Value or Deata Beneftts |  |  |  |  |  | Excess of Extended Insurance over Paid-dr Insurance Present Values |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Extended Insurance |  |  | Paid-up Insurance |  |  |  |  |  |
|  | Age at Lapse |  |  | Age at Lapse |  |  | Age at Lapse |  |  |
|  | 15 | 35 | 55 | 15 | 35 | 55 | 15 | 35 | 55 |
|  | 100 Per Cent 1955-60 Basic Ultimate Table, Males-3 Per Cent |  |  |  |  |  |  |  |  |
| 2 | 2 | 3 | 22 | 4 | 6 | 32 | - 2 | - 3 | -10 |
| 5 | 5 | 7 | 59 | 10 | 16 | 86 | - 5 | $-9$ | -27 |
| 10. | 10 | 18 | 135 | 19 | 35 | 189 | -9 | -17 | -54 |
| 20. | 18 | 58 | 319 | 35 | 93 | 414 | -17 | -35 | -95 |
|  | 150 Per Cent 1955-60 Basic Ultimate Table, Males-3 Per Cent |  |  |  |  |  |  |  |  |
| 2. | 2 | 4 | 33 | 4 | 7 | 35 | - 3 | $-3$ | - 2 |
| 5. | 7 | 16 | 88 | 11 | 18 | 94 | -4 | - 2 | - 6 |
| 10. | 15 | 27 | 194 | 22 | 39 | 205 | - 7 | -12 | -11 |
| 20. | 27 | 85 | 415 | 40 | 103 | 449 | -13 | -18 | -34 |
|  | 250 Per Cent 1955-60 Basic Ultimate Table, Males-3 Per Cent |  |  |  |  |  |  |  |  |
| 2. | 4 | 7 | 54 | 5 | 8 | 40 | -1 | $-1$ | 14 |
| 5 | 12 | 18 | 142 | 13 | 20 | 105 | $-1$ | $-2$ | 37 |
| 10. | 26 | 44 | 303 | 26 | 46 | 231 | 0 | - 2 | 72 |
| 20. | 45 | 136 | 570 | 48 | 119 | 505 | - 3 | 17 | 65 |
|  | 500 Per Cent 1955-60 Basic Ultimate Table, Males-3 Per Cent |  |  |  |  |  |  |  |  |
| 2. | 8 | 14 | 107 | 7 | 9 | 45 | 1 | 5 | 62 |
| 5. | 23 | 37 | 268 | 17 | 24 | 119 | 6 | 13 | 149 |
| 10. | 50 | 86 | 512 | 32 | 54 | 262 | 18 | 32 | 250 |
| 20. | 86 | 250 | 748 | 60 | 142 | 573 | 26 | 108 | 175 |
|  | 100 Per Cent 1955-60 Basic Ultimate Tables, Males-5 Per Cent |  |  |  |  |  |  |  |  |
| 2 | 1 | 3 | 21 | 2 | 3 | 23 | - 1 | 0 | - 2 |
| 5. | 4 | 7 | 56 | 4 | 8 | 62 | 0 | $-1$ | $-6$ |
| 10. | 9 | 16 | 121 | 8 | 19 | 135 | 1 | - 3 | -14 |
| 20. | 15 | 45 | 257 | 14 | 50 | 295 | 1 | - 5 | -38 |

TABLE 2-Continued

| Period of Extended Insurance | Present Value of Deate Benefits |  |  |  |  |  | Excess or Extended Lnsurance over Paid-dp Insurance Present Values |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Extended Insurance |  |  | Paid-up Insurance |  |  |  |  |  |
|  | Age at Lapse |  |  | Age at Lapse |  |  | Age at Lapse |  |  |
|  | 15 | 35 | 55 | 15 | 35 | 55 | 15 | 35 | 55 |
|  | 150 Per Cent 1955-60 Basic Ultimate Table, Males-5 Per Cent |  |  |  |  |  |  |  |  |
| 2 | 2 | 4 | 32 | 2 | 4 | 26 | 0 | 0 | 6 |
| 5. | 6 | 10 | 83 | 5 | 10 | 70 | 1 | 0 | 13 |
| 10 | 14 | 24 | 174 | 10 | 22 | 153 | 4 | 2 | 21 |
| 20 | 22 | 67 | 338 | 18 | 59 | 336 | 4 | 8 | 2 |
|  | 250 Per Cent 1955-60 Basic Ultimate Table, Males - 5 Per Cent |  |  |  |  |  |  |  |  |
| 2. | 4 | 7 | 53 | 3 | 5 | 31 | 1 | 2 | 22 |
| 5. | 11 | 17 | 134 | 6 | 13 | 84 | 5 | 4 | 50 |
| 10. | 23 | 39 | 272 | 13 | 28 | 183 | 10 | 11 | 89 |
| 20. | 37 | 107 | 472 | 23 | 74 | 402 | 14 | 33 | 70 |
|  | 500 Per Cent 1955-60 Basic Ultimate Table, Males-5 Per Cent |  |  |  |  |  |  |  |  |
| 2. | 7 | 13 | 104 | 4 | 6 | 28 | 3 | 7 | 76 |
| 5 | 22 | 34 | 253 | 9 | 17 | 102 | 13 | 17 | 151 |
| 10. | 45 | 77 | 463 | 18 | 37 | 223 | 27 | 40 | 240 |
| 20. | 71 | 198 | 643 | 33 | 97 | 489 | 38 | 101 | 154 |

perience showed that out of 1,000 cases, 888 would be issued standard, 8 would be issued with aviation extras or temporary extras, 64 would be issued substandard in the range 130 to 180 percent, 12 substandard in the range 180 to 300 percent, and 2 between 300 and 500 percent. The remaining 26 would be rejected, including those because sufficient underwriting experience was not obtainable. [TSA, IV, 789.]

In Table 2 the same rates of interest and mortality apply to the extended insurance option and to the paid-up insurance option. However, each option might be subject to different rates of interest and mortality. A comparison is shown in Table 3 of the excess of the extended insurance

TABLE 3

present values when the same mortality applies to each option and when 50 per cent higher mortality applies to the extended insurance option.

Because current yields on investments are high, it might be assumed that a higher rate of interest applies to the extended insurance option than to the presumably longer-lasting paid-up insurance option. A comparison is shown in Table 4 of the excess of the extended insurance present values

TABLE 4

| Period of Extended Insurance | Excess of Extended Insurance over Paid-tp Insurance Present Values |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Discounting Present Values of Both Options at 5 Per Cent |  |  | Discounting Present Values of Extended Insurance Option at 5 Per Cent and of Paid-up Insurance Option at 4 Per Cent |  |  |
|  | Age at Lapse |  |  | Age at Lapse |  |  |
|  | 15 | 35 | 55 | 15 | 35 | 55 |
| 251020 | 100 Per Cent 1955-60 Basic Oltimate Table, Males |  |  |  |  |  |
|  | -1 | 0 | $-2$ | - 2 | - 2 | $-7$ |
|  | 0 | - 1 | $-6$ | - 3 | - 5 | -18 |
|  | 1 | - 3 | -14 | -4 | -11 | -41 |
|  | 1 | -5 | -38 | -10 | -26 | -98 |
|  | 250 Per Cent 1955-60 Basic Ultimate Table, Males |  |  |  |  |  |
| 2. | 10 | 2 | 22 | 0 | 1 | 17 |
| 5. |  | 4 | 50 | 1 | 1 | 40 |
| 10. |  | 11 | 89 | 4 | 2 | 65 |
| 20. |  | 33 | 70 | 1 | 11 | 19 |

when the same interest rate is used in discounting each option and when an interest rate 1 per cent higher is used in discounting the extended insurance option.

Although the relationship is not constant, the change in present values when the extended insurance mortality is increased 50 per cent is roughly equal to the change in present values when the interest rate used in discounting the extended insurance present values is increased 1 per cent.

## DIVIDENDS

In participating policies the paid-up insurance option is participating, whereas the extended insurance option is nearly always nonparticipating. Since most of the dividend on the paid-up insurance arises from interest earnings, the dividends used here are equal to the excess interest factor of 2 per cent applied to the paid-up insurance single premium. The present values of the dividends are discounted at 5 per cent on 100 per cent of the 1955-60 Basic Ultimate Male Table. Dividends under this assumption are given in Table 5 per $\$ 1,000$ paid-up life insurance.

TABLE 5

> DIvidends and Present Value of
> Dividends per $\$ 1,000$ Paid-UP
> LIfe Insurance

| Age | Dividend* | Present Value of Dividends |
| :---: | :---: | :---: |
| 15. | 4 | 119 |
| 25. | 5 | 143 |
| 35 | 7 | 157 |
| 45. | 9 | 169 |
| 55. | 11 | 159 |

[^0]In Table 6 the present values of the dividends are given for the amounts of paid-up insurance at the 3 per cent rate of interest in Table 1. The excess of the present value of the extended insurance death benefits over the paid-up insurance death benefits is also shown for 150 and 250 per cent of the 1955-60 Basic Ultimate Male Table at 5 per cent. Under these assumptions, if the present value of the dividends exceeds the difference of the present value of the death benefits, the extended insurance option costs less than the participating paid-up insurance option.

The present values of the dividends shown in Table 6 exceed the excess of the present value of the extended insurance death benefit over the present value of the paid-up insurance death benefits at 150 per cent of the 1955-60 Basic Ultimate Male Table at 5 per cent. This relation holds true at 250 per cent of this table if lapse occurs at age 35 or younger.

For simplicity this example assumes dividends are paid in cash. In practice dividends are usually applied to purchase additional insurance on the basis of a net single premium. The preceding example could be ad-
justed to reflect this by increasing the dividends at each age by the ratio of the life insurance single premiums at the substandard level of mortality to that at the standard level of mortality and then discounting the adjusted dividends. This has not been done here because the adjustment would not have made a significant difference in the comparison. For example, the ratios of the life insurance single premiums at 250 per cent mortality to those at 100 per cent mortality are 137 per cent at age 15,128 per cent at age 35, and 122 per cent at age 55.

TABLE 6

| Period of Extended Insurances | Present Value of Dividends on Paid-up Insorance* |  |  | Excess of Persent Value of Extended Insurance Deaty Benefits over Paid-up Insurance Death Benefits: <br> (1) $\dagger$ <br> (2) $\ddagger$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Age at Lapse |  |  | Age at Lapse |  |  | Age at Lapse |  |  |
|  | 15 | 35 | 55 | 15 | 35 | 55 | 15 | 35 | 55 |
| 2. | 2 | 3 | 9 | 0 | 0 | 6 | 1 | 2 | 22 |
| 5. | 6 | 7 | 25 | 1 | 0 | 13 | 5 | 4 | 50 |
| 10. | 11 | 16 | 54 | 4 | 2 | 21 | 10 | 11 | 89 |
| 20. | 21 | 43 | 119 | 4 | 8 | 2 | 14 | 33 | 70 |

* Based on 100 per cent 1955-60 Basic Ultimate Male Table.
$\dagger$ Based on 150 per cent 1955-60 Basic Ultimate Male Table.
$\ddagger$ Based on 250 per cent 1955-60 Basic Ultimate Male Table.


## EXPENSES

In the following equations the present value of the death benefits and expenses of the extended insurance option are equal to those of the paidup insurance option, according to the assumptions given earlier. $E$ denotes the annual expense per $\$ 1,000$ extended insurance, and $S$ denotes the amount of paid-up life insurance given in Table 1 at 3 per cent.

$$
\begin{gathered}
1,000 A_{x: \overline{1}}^{1}+E \cdot \bar{a}_{x: \bar{\eta}}=S \cdot A_{x}+E \cdot \bar{a}_{x}, \\
E=\frac{1,000 A_{x: \overline{\mid}}^{1}-S \cdot A_{x}}{a_{x}-\vec{a}_{x: \bar{a}}} .
\end{gathered}
$$

If the expense is larger than the amount shown in the second equation, the costs, ignoring dividends, of the extended insurance option will be lower than those of the paid-up insurance option. Table 7 shows the expenses meeting this condition.

TABLE 7
Annual Expense per $\$ 1,000$ Extended Insurance for Which Extended Insurance Costs Equal

Paiddup Insurance Costs

| Perion of Extended Lisubance | Present Values Discounted at: |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 3 Per Cent Interest |  |  | 5 Per Cent Interest |  |  |
|  | Age at Lapse |  |  | Age at Lapse |  |  |
|  | 15 | 35 | 55 | 15 | 35 | 55 |
| 2.5.10.20. | 100 Per Cent 1955-60 Basic Ultimate Male Table |  |  |  |  |  |
|  | $*$$*$$*$$*$ | * | * | 000 | * | * |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | 150 Per Cent 1955-60 Basic Ultimate Male Table |  |  |  |  |  |
| $\begin{array}{r} 2 . \\ 5 . \\ 10 . \\ 20 . \end{array}$ | * | * | * | 0001 | 0002 | 1252 |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  | 250 Per Cent 1955-60 Basic Ultimate Male Table |  |  |  |  |  |
| 2. | * | * | 0 | 0 | 0 | 3 |
| 5. | * | * | 6 | 0 | 0 | 9 |
| 10. | 0 | * | 20 | 1 | 2 | 33 |
| 20. | * | 3 | $\dagger$ | 3 | 11 | $\dagger$ |
|  | 500 Per Cent 1955-60 Basic Ultimate Male Table |  |  |  |  |  |
| 2. | 0 | 0 | 10 | 0 | 1 | 14 |
| 5. | 0 | 1 | 39 | 1 | 2 | 47 |
| 10. | 1 | 4 | $\dagger$ | 3 | 7 | $\dagger$ |
| 20. | 3 | 38 | $\dagger$ | 8 | 58 | $\dagger$ |

[^1]According to Table 7 , if expenses are as high as $\$ 0.50$ per year per $\$ 1,000$ extended insurance, the costs of the extended insurance are lower than those of the paid-up insurance for most periods of extended insurance at 150 per cent of the 1955-60 Basic Ultimate Male Table. This relation also holds true at 250 per cent of this table at ages 15 and 35 .

The examples in this paper, although necessarily general, compare the costs of the nonforfeiture options at various levels of mortality. When dividends and expenses are considered, the costs of the extended insurance option are lower than those of the paid-up insurance option for many levels of substandard mortality according to the assumptions used here.

These conclusions may, or may not, be true for a particular company. However, by adjusting the assumptions to fit its experience, the relative costs of the nonforfeiture benefits can be found for its substandard policies.


[^0]:    - Basis: $0.02 \times 1,000 A_{x-1} 1958$ CSO, 3 per cent.
    $\dagger$ Discounted at 100 per cent 1955-60 Basic Ultimate Male Table, 5 per cent.

[^1]:    - Less than 0.
    $\dagger$ Greater than 100.

