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FEDERAL INCOME TAXATION OF ANNUITY PAYMENTS

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ABSTRACT

This paper sets out the basic principles of federal income taxation of annuity payments, together with the general rules for determining which of two possible methods must be used and the general formulas to be used. A large number of forms of annuity are examined, with the formulas for those forms being presented. An attempt has been made to cover all the common forms of concern to annuity companies. The paper concludes with a brief discussion of variable annuities and a few miscellaneous comments.

Notation is particularly troublesome for a paper of this kind, where the same basic notation is being stretched to cover a large variety of different situations. I apologize in advance for any notational lapses that might be contained herein. I hope that the device of introducing notation as needed and summarizing it all in an appendix will make the paper easier to read than would other organizations of the material.

The object of the paper has been to collect in one place, for the use of actuaries and others concerned with such matters, an actuarial statement of the tax formulas that is as consistent as possible and as complete as reasonably practicable.

INTRODUCTION

THE theory of United States federal income taxation of annuity payments stems from the basic concept that annuity payments received by an annuitant which are in excess of the annuitant's cost arise from interest and should therefore be included in gross income. For annuities purchased in part or entirely by employer contributions, the employer contribution is also considered to be includable in gross income when paid as annuity payments, provided that it has not previously been included as income to the employee.

2. On this theory the most obvious method of taxation would be to allow the annuitant to receive all payments on an excludable basis until the total of all the payments he has received equals his cost and all subsequent payments would then be fully includable. This method, the

"cost-recovery rule," was used for all annuities at one time. It has the disadvantage of producing an uneven flow of includable income to the annuitant and the disadvantage to the government of income tax deferment for a period that may be quite long in duration. Under some forms of annuity, the average amount of tax-free recovery will be less than the annuitant's cost.

3. In the effort to spread the includable income evenly over the payment period, the "life-expectancy rule" was developed. According to this rule, a portion of each annuity payment is considered to be includable. The excludable portion of each annuity payment is in the same ratio to the total payment as the annuitant's cost is to the total amount of annuity that he can expect to receive.

4. The basic rules for federal income taxation of annuity payments are contained in Section 72 of the Internal Revenue Code and in the regulations that have been issued thereunder. The rules as stated in the law and regulations are not complete, since they do not contemplate a number of annuity forms that can and do exist. The purpose of this paper is to present an actuarial restatement of these rules and a logical extension of the rules to situations that are not contemplated by the law and regulations.

SCOPE OF THE PAPER

5. This paper is directed primarily to the formulas for so-called fixed-dollar annuities, but it includes a brief discussion of so-called variable annuities (i.e., annuities that vary in amount in some way that is substantially unpredictable at the outset of the annuity). It covers life annuities, period certain life annuities, refund and cash refund (including modified cash refund) annuities, joint and survivor annuities, annuities with amounts changing in a stipulated manner at a stipulated time, annuities scheduled to change from a single life annuity to a joint and survivor annuity at a stipulated time in the future, and annuities certain. It does not cover annuities that began before January 1, 1954, although many of the remarks apply, at least in part, to such annuities.

TWO BASIC METHODS

6. The general method of taxation is the life-expectancy rule as discussed above, under which an exclusion ratio is determined and applied to the annuity payments received to determine the portion of those payments excludable from gross income, the balance of the annuity payments being includable in gross income. There is a special rule for situations in which an employee-annuitant expects to receive, during the first three years, annuity payments equal to or greater than his cost. In this situa-

tion, the cost-recovery rule is used. There will be further comments on this "three-year rule" below.

7. The life-expectancy rule can apply only to "amounts received as an annuity." Occasionally there may be amounts received that are not received as an annuity. Examples would be dividends under participating individual annuities and commuted values or surrender values in case the annuity is canceled. Any such amounts are to be included in gross income except as they represent a return of cost. There is some provision for spreading such amounts over various tax years so as to reduce the tax impact of large distributions. Dividends received before the commencement of income are taken to represent a return of cost, and they serve to reduce the cost basis of the annuity. After commencement of income, dividends are fully includable.

8. The mortality basis for all the computations is the 1937 Standard Annuity Mortality Table with ages set back one year (six years for females). This basis certainly cannot be considered "modern." It has the effect of understating expectancies, and thus overstating exclusion ratios, for most lives. For the oldest lives, however, the reverse is true.

THE THREE-YEAR RULE

9. If contributions to the cost of the annuity were made by an employer, and if the payments to be received by the employee within the first three years will equal or exceed the employee's cost, the cost-recovery method must be used. The regulations are not clear as to whether the amounts of payments should be discounted for the probability of non-survivorship to receive them. Various private publications have furnished examples that are based on the assumption that no discount should be made. It is a fair presumption that the government acquiesces in this interpretation. Because of the words "by the employee" it is permissible to infer that it is not proper to recognize changes in amounts of annuity payable to a survivor annuitant that would occur upon the death of an employee.

10. It might be asked which rule is better for the annuitant. The question is merely academic, since there is no option as to which method should be used. Generally, if an employee's actual life expectancy were smaller than it is for others of his age, the cost-recovery method would be better for him; if it were higher than it is for others of his age, the life-expectancy rule would be better. For a person with an average life expectancy, the cost-recovery method is better because the exclusion, occurring earlier, has a greater present value.

GENERAL FORMULAS AND DEFINITIONS

11. It will be helpful at this point to introduce some of the notation that will be used and to present the general formulas that apply in every case. Additional notation will be introduced as needed during the course of the paper. For convenient reference, all the notation used is presented in the Appendix.

12. For an annuity to which the life-expectancy method applies, the exclusion ratio, e, is determined as the ratio of the investment in contract, I, to the expected return, E, calculated to the nearer $\frac{1}{10}$ per cent (but not greater than 1 or less than zero):

$$e = \frac{I}{E}, \quad \ngtr 1, \quad \nless 0. \tag{12.1}$$

The formula for expected return depends on the form of the annuity and will be discussed below. The investment in contract, I, is equal to the annuitant's cost, C, reduced by the value of the refund feature, V:

$$I = C - V. \tag{12.2}$$

The annuitant's cost, C, is usually the employee's total contributions toward the cost of the annuity or the total premiums paid by the annuitant. It also includes any employer contributions that were taxed to the annuitant at the time the contributions were made or would not have been taxed if they had been paid directly to the annuitant at the time the contributions were made. For example, certain employer contributions for foreign service, or contributions by state or local governments, would not have been taxed if they had been paid directly to the employee.

The value of the refund feature, V, is obtained by multiplying the lesser of the annuitant's cost and the guaranteed return, R', by the refund feature value ratio, v:

$$V = v[\text{lesser of } (C, R')]. \tag{12.3}$$

The guaranteed return, R', is the minimum amount that is certain to be paid regardless of survivorship. Usually R' is equal to the annual amount of annuity multiplied by the exact duration of the refund feature, in years. In situations where it is not given, it must be calculated by formulas to be discussed below.

The refund feature value ratio, v, is the ratio of the expected refund to the total potential refund and is equal to the ratio of the excess of the adjusted guaranteed return, R, over the expected guarantee period annuity, $E_{\overline{n}|}$, to the adjusted guaranteed return, to the nearer whole per cent:

$$v = \begin{cases} \dfrac{R - E_{\overline{n}|}}{R} & \text{if} \quad R > 0 \,; \\ 0 & \text{if} \quad R = 0 \,. \end{cases} \tag{12.4}$$

The adjusted guaranteed return, R, is the guaranteed return, R', or an approximation to the guaranteed return that is used for computational purposes. Usually R is equal to the annual amount of annuity multiplied by the nearer whole number of years in the refund period. The formula for R depends on the form of annuity and will be discussed below.

The expected guarantee period annuity, $E_{\overline{n}|}$, is the amount of annuity that would be expected to be received during the guarantee period if there were no refund or period certain feature. It can be considered as the "present value," discounted for mortality but not for interest, of such payments. Its formula depends on the form of annuity and will be discussed below.

13. To determine whether the life-expectancy rule applies, it is necessary to compare the amount of annuity receivable by an employee during the first three years, $3P$, with the annuitant's cost, C:

$$\begin{aligned} &\text{If } 3P \geq C, \text{ the cost-recovery method applies, and} \\ &\text{if } 3P < C, \text{ the life-expectancy method applies.} \end{aligned} \tag{13.1}$$

The amount of annuity receivable by an employee during the first three years, $3P$, is usually merely three times the annual amount of annuity. There is an obvious exception to this statement where the annuity payable to the employee is scheduled to change in amount during the three-year period.

14. The annuity starting date is the first day of the first period for which an amount is received as an annuity. Generally, this day is the later of the day on which the annuity obligations become fixed and the first day of the period (month, quarter, year, etc., depending on the frequency of annuity payments) ending on the date of the first annuity payment. Although the language of the law is not crystal clear, for an immediate annuity the annuity starting date seems to be the date of purchase and for a deferred annuity (due) it seems to be the date of the first payment (i.e., the retirement date).

The age of the annuitant, x, is calculated as of the annuity starting date and is rounded to the nearer whole number of years; it is reduced by 5 if the annuitant is female.

The amount of annual annuity that is payable beginning with the annuity starting date will be denoted by A.

DISCUSSION OF SPECIFIC FORMS OF ANNUITY

15. At this point the paper proceeds to a discussion of the forms of annuity that are covered by the law or regulations and a number of common forms that are not so covered. It will by no means discuss every conceivable form of annuity. It is hoped that enough information will be presented here that it will be apparent how the formulas for the forms not presented can be devised.

Forms Covered in the Law or Regulations

16. *Single life annuity.* R', R, and $E_{\overline{n}|}$ are all zero, so $I = C$.

$$E = A\mathring{e}_x\,, \tag{16.1}$$

where $\mathring{e}_x$ denotes the complete life expectancy for (x), to the nearer one-tenth of a year. The values of $\mathring{e}_x$ are tabulated in Table I of the regulations and must be adjusted to recognize frequency of payment. (See Appendix, A2.)

Thus, $e = C/A\mathring{e}_x$. Since, in general, $e = I/E$, $\not> 1$, $\not< 0$, there does not seem to be a compelling reason to exhibit an explicit formula for e for each annuity form. Accordingly, the formula for e will not generally be shown for each annuity form.

17. *Period certain life annuity.* If we denote n' as the exact duration (in years and fractions of years) of the period measured from the annuity starting date during which annuity payments will not be contingent on survivorship, and $n = n'$ rounded to the nearer whole number of years, then

$$R' = An'\,, \tag{17.1}$$

$$R = An\,, \tag{17.2}$$

$$E_{\overline{n}|} = A\mathring{e}_{x:\overline{n}|}\,, \quad \text{and} \tag{17.3}$$

$$E = A\mathring{e}_x\,, \tag{17.4}$$

where $\mathring{e}_{x:\overline{n}|}$ denotes a complete temporary life expectancy, whose values are tabulated in Table IV of the regulations to the nearer one-tenth of a year.

As a result of these formulas,

$$v = \frac{R - E_{\overline{n}|}}{R} = \frac{An - A\mathring{e}_{x:\overline{n}|}}{An} = \frac{n - \mathring{e}_{x:\overline{n}|}}{n}\,. \tag{17.5}$$

Values of this function are tabulated in Table III of the regulations to the nearer whole per cent; and these values for v should be used in place of the values obtained by using equation (12.4).

A natural question at this point is "Why is the exclusion ratio calculated by the complicated formula

$$e = \frac{C - [(n - \mathring{e}_{x:\overline{n}|})/n][\text{lesser of } (C, An')]}{A\mathring{e}_x} \tag{17.6}$$

rather than the apparently more natural formula

$$e = \frac{C}{A(n + {}_{n|}\mathring{e}_x)}, \tag{17.7}$$

where ${}_{n|}\mathring{e}_x$ denotes a complete deferred life expectancy equal to $\mathring{e}_x - \mathring{e}_{x:\overline{n}|}$?" The answer seems to be that the annuity continuance benefit will not be taxed as an annuity when it is received and thus should not be counted as part of the expected return as an annuity. It is easily demonstrated that the "natural formula" produces a larger exclusion ratio except in the improbable situation where $C \geq A(n + {}_{n|}\mathring{e}_x)$, where both formulas produce $e = 1$ after application of the limiting condition on e.

18. *Refund annuity.* Here and throughout this paper the term "refund annuity" should be understood to include cash refund and modified cash refund annuities. R' is known.

$$n' = \frac{R'}{A}; \tag{18.1}$$

$$n = n' \text{ rounded to the nearer whole number of years;} \tag{18.2}$$

$$R = An; \tag{18.3}$$

$$E_{\overline{n}|} = A\mathring{e}_{x:\overline{n}|}. \tag{18.4}$$

So v is obtained from Table III.

$$E = A\mathring{e}_x. \tag{18.5}$$

19. *Temporary life annuity.* If we denote the temporary period as k' and $k = k'$ rounded to the nearer integer, then

$$3P = \begin{cases} 3A & \text{if} \quad k' \geq 3; \\ Ak' & \text{if} \quad k' < 3. \end{cases} \tag{19.1}$$

R', R, and $E_{\overline{n}|}$ are all zero, so $I = C$.

$$E = A\mathring{e}_{x:\overline{k}|}. \tag{19.2}$$

20. *Single life annuity with amount of annuity changing in one step after a stipulated period.* If we denote the stipulated period as k' and the amount

of annual annuity payable after the change as B, and $k = k'$ rounded to the nearer integer, then

$$3P = \begin{cases} 3A & \text{if} \quad k' \geq 3 \, ; \\ Ak' + B(3 - k') & \text{if} \quad k' < 3 \, . \end{cases} \tag{20.1}$$

R', R, and $E_{\overline{n}|}$ are all zero, so $I = C$.

$$E = (A - B)\mathring{e}_{x:\overline{k}|} + B\mathring{e}_x \, . \tag{20.2}$$

21. *Annuity certain.* The general formulas do not apply. Instead,

$$e = \frac{C}{Ak'} \, , \tag{21.1}$$

where k' denotes the period of the annuity certain.

22. *100 per cent joint and survivor annuity.* The 100 per cent denotes the particular value of the *continuance ratio, m*, which is the ratio of the amount of annuity payable to the surviving annuitant to the amount of annuity payable initially.

If we denote as y the age of the second annuitant as of the annuity commencement date, after rounding to the nearest whole age and reducing by 5 if the life is female, we have R', R, and $E_{\overline{n}|}$ are all zero, so $I = C$.

$$E = A\mathring{e}_{\overline{xy}} \, , \tag{22.1}$$

where $\mathring{e}_{\overline{xy}}$ denotes a complete joint and last survivor life expectancy, to the nearer one-tenth of a year. Values of this function are tabulated in Table II of the regulations and must be adjusted to recognize frequency of payment. (See Appendix, A2.)

Where a joint and survivor annuity contract is purchased by an annuitant, with payments to him during his lifetime and to a surviving annuitant thereafter, a gift is deemed to have been made at the time of purchase. The income tax rules do not recognize the gift tax paid as an element of the annuitant's cost. This situation seems somewhat illogical.

23. *Joint life annuity.* R', R, and $E_{\overline{n}|}$ are all zero, so $I = C$.

$$E = A\mathring{e}_{xy} \, , \tag{23.1}$$

where $\mathring{e}_{xy}$ denotes a complete joint life expectancy, to the nearer one-tenth of a year. Values of this function are tabulated in Table IIA of the regulations and must be adjusted to recognize frequency of payment. (See Appendix, A2.)

24. *100 per cent joint and survivor period certain annuity.*

$$R' = An' ; \tag{24.1}$$

$$R = An ; \tag{24.2}$$

$$E_{\overline{n}|} = A\mathring{e}_{\overline{xy}:\overline{n}|} ; \tag{24.3}$$

$$E = A\mathring{e}_{\overline{xy}} . \tag{24.4}$$

$\mathring{e}_{\overline{xy}:\overline{n}|}$ denotes a complete temporary joint and last survivor life expectancy and may be calculated by

$$\mathring{e}_{\overline{xy}:\overline{n}|} = \mathring{e}_{x:\overline{n}|} + \mathring{e}_{y:\overline{n}|} - \mathring{e}_{w:\overline{n}|} , \tag{24.5}$$

where w denotes the age of the single life that is equivalent to the joint life status (xy) and, since the 1937 Standard Annuity Mortality Table is a "Gompertz" table, is equal to [the larger of (x,y)] + $\log (1 + c^{-|x-y|})/\log c$, where c is the base of the exponential term in the force of mortality underlying the 1937 Standard Annuity Mortality Table, namely, $10^{0.038}$. Values of the second term in the definition of w are tabulated in the regulations to the nearer whole number of years.

In this case, v is equal to the Table III value for x plus the Table III value for y minus the Table III value for w. Because of the severe rounding in Table III, this may result in a negative value of v. In that case, the value of v should be taken as zero.

25. *100 per cent joint and survivor refund annuity.* R' is known.

$$n' = \frac{R'}{A} ; \tag{25.1}$$

$n = n'$ rounded to the nearer integer;

$$R = An ; \tag{25.2}$$

$$E_{\overline{n}|} = A\mathring{e}_{\overline{xy}:\overline{n}|} ; \tag{25.3}$$

$$E = A\mathring{e}_{\overline{xy}} . \tag{25.4}$$

Remarks similar to those in paragraph 24 apply to v for this form of annuity.

26. *100* m *per cent joint and survivor life annuity* (where the annuity amount changes at the first death; 100 m per cent denotes the continuance ratio). R', R, and $E_{\overline{n}|}$ are all zero, so $I = C$.

$$E = (mA)\mathring{e}_{\overline{xy}} + [A - (mA)]\mathring{e}_{xy} . \tag{26.1}$$

27. *100* m *per cent joint and survivor life annuity* (where the annuity amount changes at the first death only if the annuitant predeceases the second annuitant; this is the usual arrangement and will be assumed for

all subsequent discussions of the joint and survivor annuity). R', R, and $E_{\overline{n}|}$ are all zero, so $I = C$.

$$E = A\mathring{e}_x + (mA)(\mathring{e}_{\overline{xy}} - \mathring{e}_x)\,. \tag{27.1}$$

Forms Not Covered in the Law or Regulations

28. *Temporary life annuity, period certain* (assuming $n' < k'$).

$$3P = \begin{cases} 3A & \text{if} \quad k' \geq 3\,; \\ Ak' & \text{if} \quad k' < 3\,. \end{cases} \tag{28.1}$$

n and k are equal to n' and k', rounded to the nearer integer, respectively.

$$E_{\overline{n}|} = A\mathring{e}_{x:\overline{n}|}\,; \tag{28.2}$$

$$R' = An'\,; \tag{28.3}$$

$$R = An\,; \tag{28.4}$$

$$E = A\mathring{e}_{x:\overline{k}|}\,. \tag{28.5}$$

v is obtained from Table III.

It is easily demonstrated that if k', k, n', and n are all equal, the same exclusion ratio results as for the n-year annuity certain.

29. *100* m *per cent joint and survivor annuity, period certain.* In most cases, the benefit does not reduce on the death of the annuitant before the expiry of the period certain. Assuming that this is the case, we are immediately presented with the question of whether the continuance of the annuity to the second annuitant at the original rate for the balance of the period certain contains an element of death benefit. Arguments could be made either way. Since the second annuitant will treat for tax purposes the entire amount as annuity, and none as death benefit, it is proper to include the original amount for the balance of the period certain in the calculation of the expected return. $n = n'$ rounded to the nearer integer.

$$R' = An'\,; \tag{29.1}$$

$$R = An\,; \tag{29.2}$$

$$E_{\overline{n}|} = A\mathring{e}_{\overline{xy}:\overline{n}|}\,; \tag{29.3}$$

$$E = A(\mathring{e}_x + \mathring{e}_{y:\overline{n}|} - \mathring{e}_{w:\overline{n}|}) + (mA)(\mathring{e}_{\overline{xy}} - \mathring{e}_x - \mathring{e}_{y:\overline{n}|} + \mathring{e}_{w:\overline{n}|})\,. \tag{29.4}$$

This formula for expected return is derived from the following:

$$\begin{aligned} E = {} & A\mathring{e}_x + A(\mathring{e}_{\overline{xy}:\overline{n}|} - \mathring{e}_{x:\overline{n}|}) \\ & + (mA)[(\mathring{e}_{\overline{xy}} - \mathring{e}_x) - (\mathring{e}_{\overline{xy}:\overline{n}|} - \mathring{e}_{x:\overline{n}|})]\,. \end{aligned} \tag{29.5}$$

Remarks similar to those in paragraph 24 apply to v for this form of annuity.

30. *100 m per cent joint and survivor refund annuity.* In this form of annuity, the annuity amount usually changes on the death of the annuitant regardless of when it occurs. If it occurs during the refund period, the remaining refund period is increased because of the smaller annuity then payable. R' is given.

$$n' = \frac{R'}{A}\ ; \tag{30.1}$$

$n = n'$ rounded to the nearer integer;

$$R = An\ ; \tag{30.2}$$

$$E_{\overline{n}|} = A\overset{\circ}{e}_{x:\overline{n}|} + (mA)\int_0^n {}_tp_{xy}\mu_{x+t}\overset{\circ}{e}_{y+t:\overline{(n-t)/m}|}\,dt\ . \tag{30.3}$$

Extensive investigation has failed to uncover a practical method of precisely evaluating the above integral, so that approximations are in order. A satisfactory approximation, derived from the "trapezoidal rule," would be $\frac{1}{2}n\mu_x\overset{\circ}{e}_{y:\overline{n/m}|}$ (where n/m is taken to the nearer whole integer). Another approximation that has been used is ${}_nq_x \cdot {}_{n/2}p_y \cdot \overset{\circ}{e}_{y+n/2:\overline{n/2m}|}$ (where $n/2$ and $n/2m$ are each taken to the nearer integer).[1] This approximation results from replacing the portion of the integrand relating to y with its central value.

$$E = A\overset{\circ}{e}_x + (mA)(\overset{\circ}{e}_{\overline{xy}} - \overset{\circ}{e}_x)\ . \tag{30.4}$$

For the evaluation of $\mu_x, {}_nq_x$ and ${}_{n/2}p_y$, recourse must be had to the underlying mortality table, since the regulations do not contain the required tables. A satisfactory approximation to μ_x would be

$$\mu_x = \frac{l_{x-1} - l_{x+1}}{2l_x}\ . \tag{30.5}$$

31. *Conversions from single life annuities to joint and survivor annuities.* Frequently under group annuities, because of notice period requirements for the election of optional forms of annuity, it is necessary to provide that the annuity begin on the normal form and that it then be converted, at some stipulated time in the future, to a joint and survivor form of annuity, usually of a smaller amount, if both prospective annuitants should survive to that stipulated time. Since the conversion in form is entirely automatic, it is really an integral part of the annuity contract from the beginning, and thus it should be recognized in calculating the exclusion ratio. Accordingly, the following formulas have been developed to accomplish this recognition.

In these formulas, r' denotes the period to conversion, $r = r'$ rounded

[1] See author's supplemental note, paragraph A4.

to the nearer whole number of years, and B denotes the annual amount of annuity payable after conversion.

In any case,

$$3P = \begin{cases} 3A & \text{if} \quad r' \geq 3\,; \\ 3A - {}_{r'}p_y(3 - r')(A - B) & \text{if} \quad r' < 3\,. \end{cases} \tag{31.1}$$

It will be necessary to refer to the underlying mortality table to calculate ${}_{r'}p_y$.

32. *Single life annuity converting to 100 per cent joint and survivor life annuity.* R', R, and $E_{\overline{n}|}$ are all zero, so $I = C$.

$$E = A\mathring{e}_x + {}_rp_{xy}(B\mathring{e}_{\overline{x+r:y+r}} - A\mathring{e}_{x+r})\,. \tag{32.1}$$

33. *Single life annuity converting to 100* m *per cent joint and survivor life annuity.* R', R, and $E_{\overline{n}|}$ are all zero, so $I = C$.

$$E = A\mathring{e}_x + {}_rp_{xy}[B\mathring{e}_{x+r} + (mB)(\mathring{e}_{\overline{x+r:y+r}} - \mathring{e}_{x+r}) - A\mathring{e}_{x+r}]\,. \tag{33.1}$$

34. *Period certain life annuity converting to 100 per cent joint and survivor life annuity, balance of period certain.*

$$R' = \begin{cases} An' - {}_{r'}p_{xy}(A - B)(n' - r') & \text{if} \quad n' > r'\,; \\ An' & \text{if} \quad n' \leq r'\,. \end{cases} \tag{34.1}$$

$$R = \begin{cases} An - {}_rp_{xy}(A - B)(n - r) & \text{if} \quad n > r\,; \\ An & \text{if} \quad n \leq r\,. \end{cases} \tag{34.2}$$

$$E_{\overline{n}|} = \begin{cases} A\mathring{e}_{x:\overline{n}|} + {}_rp_{xy}(B\mathring{e}_{\overline{x+r:y+r}:\overline{n-r}|} - A\mathring{e}_{x+r:\overline{n-r}|}) & \text{if} \quad n > r\,; \\ A\mathring{e}_{x:\overline{n}|} & \text{if} \quad n \leq r\,. \end{cases} \tag{34.3}$$

$$E = A\mathring{e}_x + {}_rp_{xy}(B\mathring{e}_{\overline{x+r:y+r}} - A\mathring{e}_{x+r})\,. \tag{34.4}$$

35. *Period certain life annuity converting to 100* m *per cent joint and survivor life annuity, balance of period certain.*

$$R' = \begin{cases} An' - {}_{r'}p_{xy}(A - B)(n' - r') & \text{if} \quad n' > r'\,; \\ An' & \text{if} \quad n' \leq r'\,. \end{cases} \tag{35.1}$$

$$R = \begin{cases} An - {}_rp_{xy}(A - B)(n - r) & \text{if} \quad n > r\,; \\ An & \text{if} \quad n \leq r\,. \end{cases} \tag{35.2}$$

$$E_{\overline{n}|} = \begin{cases} A\mathring{e}_{x:\overline{n}|} + {}_rp_{xy}(B\mathring{e}_{\overline{x+r:y+r}:\overline{n-r}|} - A\mathring{e}_{x+r:\overline{n-r}|}) & \text{if} \quad n > r\,; \\ A\mathring{e}_{x:\overline{n}|} & \text{if} \quad n \leq r\,. \end{cases} \tag{35.3}$$

$$E = \begin{cases} A\mathring{e}_x + {}_rp_{xy}[B(\mathring{e}_{x+r} + \mathring{e}_{y+r:\overline{n-r}|} - \mathring{e}_{w+r:\overline{n-r}|}) + (mB)(\mathring{e}_{\overline{x+r:y+r}} \\ \qquad - \mathring{e}_{x+r} - \mathring{e}_{y+r:\overline{n-r}|} + \mathring{e}_{w+r:\overline{n-r}|}) - A\mathring{e}_{x+r}] \quad \text{if} \quad n > r\,; \\ A\mathring{e}_x + {}_rp_{xy}[B\mathring{e}_{x+r} + (mB)(\mathring{e}_{\overline{x+r:y+r}} - \mathring{e}_{x+r}) - A\mathring{e}_{x+r}] \\ \qquad \text{if} \quad n \leq r\,. \end{cases} \tag{35.4}$$

36. *Refund annuity converting to 100 per cent joint and survivor, balance of refund certain.*

$$n' = \frac{R'}{A} ; \tag{36.1}$$

$$R = An ; \tag{36.2}$$

$$E_{\overline{n}|} = \begin{cases} A\mathring{e}_{x:\overline{n}|} + {}_r p_{xy}[B\mathring{e}_{\overline{x+r:y+r}:\overline{(n-r)A/B}|} - A\mathring{e}_{x+r:\overline{n-r}|}] & \text{if } n > r ; \\ A\mathring{e}_{x:\overline{n}|} & \text{if } n < r . \end{cases} \tag{36.3}$$

Note that the remaining term for the new annuity amount, namely, $(n - r)A/B$, should be rounded to the nearer whole number.

$$E = A\mathring{e}_x + {}_r p_{xy}(B\mathring{e}_{\overline{x+r:y+r}} - A\mathring{e}_{x+r}) . \tag{36.4}$$

37. *Refund annuity converting to 100* m *per cent joint and survivor, balance of refund certain.*

$$n' = \frac{R'}{A} ; \tag{37.1}$$

$$R = An ; \tag{37.2}$$

$$E_{\overline{n}|} = \begin{cases} A\mathring{e}_{x:\overline{n}|} + {}_r p_{xy}[B\mathring{e}_{x+r:\overline{(n-r)A/B}|} + (mB) \int_0^{(n-r)A/B} {}_t p_{\overline{x+r:y+r}} \\ \qquad \times \mu_{x+r+t}\mathring{e}_{y+r+t:\overline{[(n-r)A/B-t]/m}|}\, dt - A\mathring{e}_{x+r:\overline{n-r}|}] \\ \qquad \text{if} \quad n > r ; \\ A\mathring{e}_{x:\overline{n}|} \quad \text{if} \quad n \leq r . \end{cases} \tag{37.3}$$

The integral may be approximated by $\frac{1}{2}[(n-r)A/B]\mu_{x+r}\mathring{e}_{y+r:\overline{(n-r)A/(mB)}|}$.[2] In the above formula, $(n - r)A/B$ and $(n - r)A/(mB)$ should each be rounded to the nearer whole number.

$$E = A\mathring{e}_x + {}_r p_{xy}[B\mathring{e}_{x+r} + (mB)(\mathring{e}_{\overline{x+r:y+r}} - \mathring{e}_{x+r}) - A\mathring{e}_{x+r}] . \tag{37.4}$$

VARIABLE ANNUITIES

38. Variable annuities are taxed in much the same way as fixed-dollar annuities are taxed. An exclusion ratio is determined as if the annuity were a fixed-dollar annuity, where the annual amount of the annuity is the amount received in the first tax year (adjusted to an annualized basis, if necessary). The exclusion ratio is applied to the (annualized) amount received in the first tax year to determine the annual excludable amount. The excess of amounts received in any tax year over the annual excludable amount (prorated, if appropriate) is includable.

[2] See author's supplemental note, paragraph A4.

39. If a year occurs in which the annuity payments received are less than the excludable amount, the taxpayer has the option of recalculating the excludable amount. This recalculation is done by increasing the existing excludable amount by the amount of the deficiency divided by the appropriate expected return factor based on the attained age of the annuitant.

ADDITIONAL MISCELLANEOUS COMMENTS

40. Occasionally there will be two or more annuity elements provided for a single consideration. The law and regulations are quite clear that a single exclusion ratio applicable to all the annuities is to be determined. At various points it is necessary to allocate the amount of cost to the various annuity forms, and this allocation must be in proportion to the expected returns for the various forms. There are serious questions as to when annuities must be combined, particularly where annuities are provided by the same employer under two different paying agencies. It is difficult to make a categorical statement as to when annuities must be combined.

41. A surviving annuitant is taxed by continuing the same exclusion ratio that applied before the death of the annuitant. If the value of the annuity is taxed to the surviving annuitant as part of the estate of the deceased, there is a small additional deduction permitted for the estate tax that was paid. This deduction is limited in duration to the life expectancy of the surviving annuitant at the time of death of the annuitant.

42. A refund beneficiary is taxed on the cost-recovery rule regardless of the method by which the annuitant was taxed. The refund beneficiary can exclude the amount of benefit that he receives to the extent that it represents a return of the annuitant's cost that was not previously returned tax-free to the annuitant, and he must include the remainder of the benefit as taxable income.

43. It appears that the surviving annuitant or refund beneficiary is bound by whatever determinations were made by the annuitant as to excludable amount, even if he made an error. On its face this appears to be an improper result, since the annuitant's estate should be liable for unpaid taxes or should receive the benefit of tax overpayments. The surviving annuitant or refund beneficiary should be entitled to calculate for himself the amounts to be excludable on the presumption that the annuitant did his calculations correctly, regardless of whether he did in fact perform the calculations correctly.

SUMMARY

44. In this paper there have been set out the basic principles of federal income taxation of annuity payments, together with the general rules

for determining the method to be used, the general formulas to be used, and the specific formulas to be used for a large number of annuity forms. A number of comments have been introduced, particularly where questions of interpretation arise. A number of provocative questions have been left unanswered. It is hoped that presentation of this paper will generate discussions that might lead to the development of some of these answers.

APPENDIX

RÉSUMÉ OF NOTATION

(Parenthetic References Are to the Paragraph in Which the Item First Appears.)

A1. The following items denote information that is usually available before beginning any calculations. The annuity starting date is the date as of which ages are measured (par. 14).

n' = Exact duration (in years and fractions of years) of a period measured from the annuity starting date (*a*) during which annuity payments will not be contingent on survivorship or (*b*) during which a death benefit is payable (par. 17).

k' = Exact duration (in years and fractions of years), from the annuity starting date of a temporary annuity either certain or contingent (par. 19).

r' = Period (in years and fractions of years) from the annuity starting date to the date as of which the annuity is scheduled to be changed from a single life annuity to a joint and survivor annuity (par. 31).

C = Annuitant's cost (par. 12).

R' = Minimum amount that is certain to be paid regardless of survivorship (par. 12).

A = Amount of annual annuity that is payable beginning with the annuity starting date (par. 14).

(mA) = Amount of annual annuity that is payable after a death that causes a change in the amount of annuity (par. 26).

B = Amount of annual annuity that is payable beginning with the conversion from a single life form to a joint and survivor form or after a scheduled change in the amount of a single life annuity (par. 20).

(mB) = Amount of annual annuity that is payable after a death occurring after a conversion to a joint and survivor form, where the death results in a change in the amount of annuity (par. 33).

A2. The following actuarial functions are referred to in the course of this paper:

$\mathring{e}_x$ = Complete life expectancy for a person aged x. This function is tabulated in Table I of the regulations to one decimal place (par. 16).

$\overset{\circ}{e}_{\overline{xy}}$ = Joint and survivor life expectancy of two persons aged, respectively, x and y; this function is tabulated in Table II of the regulations to one decimal place (par. 22).

$\overset{\circ}{e}_{xy}$ = Joint life expectancy for two lives aged, respectively, x and y; this function is tabulated in Table IIA of the regulations to one decimal place (par. 23).

(An adjustment to the three foregoing functions is required whenever the annuity is payable less frequently than twelve times per year. The amount of the adjustment depends on the frequency of payment and the duration in months from the annuity starting date to the date of the first payment. Values of the adjustment are tabulated in the regulations to one decimal place.)

$(n-\overset{\circ}{e}_{x:\overline{n|}})/n$ = Ratio of the value of the refund feature of the annuity to the total minimum guaranteed amount under the annuity. Values of this function are tabulated in Table III of the regulations, in the form of percentages, to the nearest whole per cent. The tabulated function is commonly called "the percentage value of the refund feature" (par. 17).

$\overset{\circ}{e}_{x:\overline{n|}}$ = A temporary life expectancy; values of this function are tabulated in Table IV of the regulations to one decimal place (par. 17).

${}_{n|}\overset{\circ}{e}_x$ = A deferred life expectancy and is equal to $\overset{\circ}{e}_x - \overset{\circ}{e}_{x:\overline{n|}}$ (par. 17).

$\overset{\circ}{e}_{\overline{xy}:\overline{n|}}$ = A temporary joint and survivor expectancy, equal to $\overset{\circ}{e}_{x:\overline{n|}} + \overset{\circ}{e}_{y:\overline{n|}} - \overset{\circ}{e}_{w:\overline{n|}}$ (par. 24).

$\overset{\circ}{e}_{xy:\overline{n|}}$ = A temporary joint life expectancy, taken as equal to $\overset{\circ}{e}_{w:\overline{n|}}$, as discussed in par. 24 (par. A2).

l_x = Number of survivors at age x in accordance with the 1937 Standard Annuity Mortality Table, with ages set back one year (six years for females) (par. 30).

${}_rp_x$ = Probability of survivorship of (x) to age $x + r$ and is equal to l_{x+r}/l_x (par. 30).

${}_rp_{xy}$ = Probability of survivorship of the joint life status (xy) for r years and is equal to ${}_rp_x \cdot {}_rp_y$ (par. 32).

${}_nq_x$ = Probability of death of (x) within n years and is equal to $(l_x - l_{x+n})/l_x$ (par. 30).

μ_x = Force of mortality at age x. While this function can be evaluated exactly, it is sufficiently accurate to use $(l_{x-1} - l_{x+1})/2l_x$ as an approximation to its value (par. 30).

A3. The following are the items of data that may usually be derived from the above:

x = Age of the annuitant as of the annuity starting date, rounded to the nearer whole number of years and reduced by 5 if the life is female (par. 14).

y = Age of the second annuitant as of the annuity starting date, rounded to the nearer whole number of years and reduced by 5 if the life is female (par. 22).

w = Age of the single life that is equivalent to the joint life status (xy) (par. 24).

n = Adjusted guarantee period and is equal to n' if n' is known or R'/A if R′ is known, rounded to the nearer whole number of years (par. 17).

k = Adjusted temporary annuity period and is equal to k' rounded to the nearer whole number of years (par. 19).

r = Adjusted period to conversion to a joint and survivor form and is equal to r' rounded to the nearer whole number of years (par. 31).

R = Adjusted guaranteed return, which is used for computational purposes (par. 12).

m = Ratio of the annuity payable to the surviving annuitant to the amount of annuity payable to the annuitant (the "continuance ratio") and is equal to $(mA)/A$ or $(mB)/B$, whichever is applicable (par. 22).

$E_{\overline{n}|}$ = Amount of annuity that would be expected to be received during the actual guarantee period if there were no refund or period certain feature (par. 12).

v = Ratio of the expected refund to the total potential refund and is equal to $(R - E_{\overline{n}|})/R$ if R is greater than zero and to zero if R equals zero (par. 12).

V = Value of the refund feature and is equal to v [lesser of (C,R')] (par. 12).

I = Investment in the contract and is equal to $C - V$ (par. 12).

E = Expected return (par. 12).

e = Exclusion ratio and is equal to I/E; e can in no event be greater than one or less than zero (par. 12).

$3P$ = Amount of annuity receivable by an employee during the first three years (par. 13).

AUTHOR'S SUPPLEMENTAL NOTE

A4. Further research on the integral contained in formula (30.3) has led to another alternate formula for the evaluation of this integral, namely,

$$\mathring{e}_{y:\overline{n}|} - \mathring{e}_{xy:\overline{n}|} + {}_nq_x \cdot {}_np_y \cdot \mathring{e}_{y+n:\overline{(n/2m)(1-m)}|}\,.$$

This formula produces exactly correct values for $m = 1$ and in the limit as m approaches zero. For intermediate values of m the approximation should be better than the approximations given in the text.

The comparable formula for evaluating the integral in formula (37.3) would be

$$\mathring{e}_{y+r:\overline{n_1}|} - \mathring{e}_{x+r:y+r:\overline{n_1}|} + {}_{n_1}q_{x+r} \cdot {}_{n_1}p_{y+r} \cdot \mathring{e}_{y+r+n_1:\overline{(n_1/2m)(1-m)}|}\,,$$

where $n_1 = (n - r)A/B$. n_1 and $(n_1/2m)\,(1 - m)$ should each be taken to the nearer integer.

DISCUSSION OF PRECEDING PAPER

ROBERT H. JORDAN:

We are deeply indebted to Mr. Crosson for providing us with his very thorough examination and exposition of the formulas that are involved.

Some further comment seems in order with regard to the theory supporting the development of the present method of taxing annuity payments (or, more properly, the method of determining the portion of such payments to be included in gross income).

It is my impression that the theory underlying the present method follows essentially the following rationale: The government recognized that previous methods were inequitable in excluding from the gross income of the annuitant never more than his cost basis and, frequently, a good deal less. It was also recognized that, because of the nature of an annuity contract, the government cannot give every annuitant an exclusion of his full cost basis (some annuitants die "early"). With these facts in mind, it was evidently felt that it would be satisfactory if a method could be devised under which, on the average, the excludable amount would equal the cost basis. Excluding considerations of health at the annuity commencement date, such a method would give to all annuitants, of the same age with the same type of annuity, the same expected total excludable annuity benefits relative to their cost bases.

Looking at the problem another way, we find that the government's objective is to require the inclusion in gross income of only that part of the annuity payments that can be attributed to interest (or investment) earnings of the funds involved, all other amounts being considered a return of cost. The annuity method of taxing annuity payments achieves the desired result by "equalizing" both the interest earnings and the return of the cost so that each annuity payment is considered to be composed of two parts: (*a*) the equalized interest earnings of the contract, which are to be included in gross income, and (*b*) the equalized return of cost, which is to be received tax-free by the annuitant. Under this arrangement, provided that a suitable mortality table has been used to perform the equalizing, the aggregate amount of excludable portions received by annuitants will in the long run prove to be equal to the cost basis of the annuities, the remainder of the annuity payments being includable and being attributable to interest earnings on the funds involved. In practice, some annuitants will receive an exclusion much larger than their cost, and others will receive an exclusion much lower than their cost; on the average, however, the exclusion will be equal to the cost.

(AUTHOR'S REVIEW OF DISCUSSION)

WILLIAM H. CROSSON:

Mr. Jordan gives a good exposition of the theoretical foundation of the "life-expectancy rule," which foundation was referred to quite indirectly in paragraphs 1 and 3 of the paper. The discussion illuminates the point made in the last sentence of paragraph 2.

The proviso in Mr. Jordan's discussion that "a suitable mortality table has been used to perform the equalizing" is important to the essential equity of the life-expectancy rule. Paragraph 8 of the paper has some comments on this issue and implicitly suggests that the government ought to consider the substitution of a more modern mortality table that would be fairer to the various classes of annuitants and to the government.

I wish to thank Mr. Jordan for his significant addition to the value of the paper.