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THE VALUATION OF PENSION FUND ASSETS

PAUL H. JACKSON AND JAMES A. HAMILTON

ABSTRACT

This paper examines some of the methods used to determine the value of the assets held by a pension fund and used in actuarial valuations to determine the required contributions for a pension plan. Traditionally, most pension funds have valued such assets at book value, that is, initial cost adjusted for realized capital gains and losses. Suggested requirements for minimum funding and the introduction of less flexible accounting charges, along with a general increase in the amount of unrealized gains in typical funds, indicate that this traditional practice may require reconsideration in certain cases.

The paper explores various asset valuation methods, including book value, market value, and other measures of asset value, such as formula adjustments to book value, modifications involving both book value and market value, formula adjustments to market value, and special methods including the "present value" method. The paper then explores those characteristics of a measure of asset value that would be desirable from the standpoint of the actuarial valuation of a pension fund. Various asset valuation methods are then reviewed from the standpoint of those desirable characteristics in a general exploration of the problem of asset valuation.

The paper includes the results of testing the effect of fifteen different asset valuation methods on smoothness of contribution over the ten-year period 1957–67 for several pension plans. In addition, the same fifteen methods were also tested to determine the reasonableness of the resulting asset values throughout the period. For the plans tested, and the period studied, the methods based on a formula adjustment to market value produced more meaningful results than the traditional methods based on book value.

It was concluded that each pension plan should be considered individually in the selection of the most appropriate asset valuation method and that, in each case, the choice will depend on the relative weight assigned to the criterion of smoothness of contribution as compared with fit of asset value to market value.

THE work of the actuary in the management of pension funds has traditionally consisted of thorough analyses of the potential benefit liabilities and the various funding methods that can be used to accumulate the necessary assets. A great deal of the actuarial literature on the subject of pensions relates to the specific formulas and methods for determining liabilities and the associated required contributions.[1] On the other hand, actuaries in America have usually disclaimed investment expertise and have been prone to leave asset valuation problems to the employer, trustee, or insurance company.[2] Thus, in the process of actuarial valuation, the actuary has directed most of his time and attention to testing carefully the various decremental rates against the actual past experience and to the determination of the liabilities under the plan. As a final step in the valuation process, the actuary must compare these computed liabilities with the value of assets in evaluating the funding position of the plan and establishing the desired contribution levels, and traditionally he has accepted the asset figure given to him by the employer or trustee, usually valued at cost.

Generally, among pension plans established some twenty to thirty years ago, not much deviation occurred between the book and market values of the funds until about 1958. From that time on, however, the relative excess of market over book has increased significantly with the short-lived exception of the 1962 and 1966 market declines. This increase has brought problems, albeit pleasant ones.

THE PROBLEM

Over the last ten-year period in the United States, the traditional approach of using the book value of assets in the actuarial valuation of a pension fund has been subjected to careful scrutiny on the part of a number of entirely separate groups with widely divergent interests.[3]

[1] For example, recent papers have covered the effect of interest (Adams, *TSA*, Vol. XIX); the cost of vesting (Marples, *TSA*, Vol. XVIII; McGinn, *TSA*, Vol. XVIII); the cost of options (Hanson, *TSA*, Vol. XIII); the effect of salary increases (Marples, *TSA*, Vol. XIV); and actuarial cost methods (Taylor, *TSA*, Vol. XIX; Cooper and Hickman, *TSA*, Vol. XIX; Houseman, *TSA*, Vol. IV; Trowbridge, *TSA*, Vol. IV).

[2] British actuaries get involved quite frequently in the investment matters, due in part to the frequent use of natural person trustees in Great Britain, their responsibility for the *Financial Times*-Actuaries Index, and the fact that in many instances the actuary is a member of the investment team determining the specific investment policy as to the class of security to be emphasized and sometimes even as to particular securities to be bought and sold.

[3] One of the earliest papers to question the traditional approach, written from the viewpoint of financial and management considerations, is W. Gordon Binns, "Effects of

Where an employer is contributing maximum tax-deductible amounts, the Internal Revenue Service is more likely to question the deductibility of contributions if substantial amounts of unrealized appreciation have been ignored in the actuarial calculations. *Opinion No. 8* of the Accounting Principles Board of the American Institute of Certified Public Accountants (November, 1966) states: "The Board believes unrealized appreciation and depreciation should be recognized in the determination of the provision for pension costs on a rational and systematic basis." The Defense Contract Audit Agency and several other government procurement groups have in recent years questioned pension contributions as a reimbursable expense where unrecognized appreciation has exceeded some rule of thumb, such as 25 per cent, generally applied to the common stock portfolio alone. Finally, some experts studying rates of return on investments of pension funds and methods of measuring portfolio performance have strongly emphasized the desirability of using market value as a base for measuring investment performance, annual yield, and portfolio value. One widely recognized expert recently stated:

> Note that book value is irrelevant, totally irrelevant, that there is no distinction between income on the one hand and principal on the other, that there is no distinction between realized capital gains and unrealized capital gains. What really matters is the number of dollars that you could come up with if you sold all your assets. In brief, book value is out, distinction between principal and income, distinction between realized and unrealized capital gains, etc., are all out. One dollar is considered as good as another.[4]

(See Appendix I for a further sampling of current authoritative quotations.)

From an investment standpoint at least, it appears that current market value has been fairly well accepted as the only true measure of asset value. If the true value of an asset to a pension fund were significantly less than its current market value, there is an implication that the trustee would not be exercising good judgment by continuing to hold the security. If the true value were significantly greater than current market value, there is an implication that the trustee would not be exercising good judgment if he did not immediately enter the market and increase his holdings. These observations, however, relate primarily to the value of assets employed in studies of investment performance. The actuary, in determining

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Appreciation of Common Stock Investments on the Funding of Pension Costs" (thesis presented to the Graduate School of Business Administration, New York University, 1959).

[4] James H. Lorie, "NABAC Study on Pension Funds," *Financial Analysts Journal*, March–April, 1968.

pension costs, has always preferred a bit of conservatism as well as stability and comparability in any time series and indices that he uses. The book value derivative, therefore, has seemed preferable to market value for the simple reason that it is fairly stable and, at least insofar as recent years are concerned, usually lower. The strong interest of other groups, the consensus in investment circles, and the increase in market over book to substantial levels in some funds have led many actuaries to a general reappraisal of asset valuation methods. Current legislative proposals (e.g., Senate bills introduced by Senator Yarborough and by Senator Javits) to impose stricter minimum funding standards on private pension plans and the rigidity in pension costs resulting from a strict application of the rules in *Opinion No. 8* by practicing accountants may force employers to explore the possibility of changing some of the actuarial assumptions, the method of funding, and the method of valuing pension fund assets in order to minimize the impact of any required changes.

SOME OF THE METHODS NOW IN USE

In a paper submitted to the Eighteenth International Congress of Actuaries,[5] the authors discussed some of the methods then in use for determining the value of equity securities held by pension funds and used by the actuary in conjunction with a concurrent valuation of plan liabilities to determine the appropriate level of contribution to the fund. These methods may be generalized so as to apply to all the assets or to a particular type only, such as equities, bonds, mortgages, and so forth. In Appendix II a more detailed listing of specific methods has been classified according to the following:

Class I: Initial cost methods.—Value assets at initial cost with possible modifications in the rate of turnover of securities held by the fund or in the actuarial interest assumption.

Class II: Initial cost with formula modifications.—Change the asset value of equity securities each year by a factor based on long-run expected growth or on changes in the net retained earnings of the issuing corporation, earnings per share, or dividends per share; amortize premium or discount for bonds.

Class III: Modifications based on both initial cost and current market value.—Change the asset value each year by some algebraic formula based on both initial cost and current market value; use some "credibility" factor intended to reflect the likelihood of actually realizing, in dividends or gain at sale, any excess of current market value over initial cost.

Class IV: Current market value methods.—Value assets at current market value with possible modifications in actuarial assumptions or funding methods to prevent unreasonably large variations in required contributions to the plan.

[5] "The Valuation of Equity Assets of Pension Funds," submitted June 4, 1968, but written December, 1966, to meet publication requirements.

Class V: Adjusted market value methods.—Value assets at market value with a downward adjustment based on total yield each year so as to counteract market fluctuations and with book value as a rarely operative minimum.

Class VI: Present value methods.—Value assets by capitalizing current interest and dividend income, or some modification thereof, at the rate of interest used for actuarial valuation of plan liabilities.

VALUATION BY TYPE OF ASSET

The approach of valuing equity assets by one method and using another, such as initial cost or amortized value, for bonds is, again, in line with tradition. Many actuaries have an insurance company background, and insurance companies, for annual statement purposes, generally value bonds at cost, at par value, or on an amortized basis but value stocks at market value; this same approach is generally followed for insured pension plans involving separate accounts with equity investments. A number of asset valuation methods, such as the 3 per cent write-up method (Class II, *a*, in Appendix II) or the 7 per cent yield method (Class II, *b*), have been designed specifically as methods for valuing only the equity portfolio, on the assumption that a different method, such as amortized value or cost, will be employed for the bond portfolio.

If a pension fund were invested entirely in bonds, the use of amortized value would probably not be seriously questioned. It is true that bonds issued by the same corporation can be purchased at different times, and at some subsequent date a $1,000 par value unit from each particular purchase could have a different amortized value, despite the fact that the same security stands behind each of the units and thus each should have the same value. On the other hand, the smooth transition in amortized value from purchase to maturity assures stability in asset value. Also, the general narrowing of the range in market value as maturity or call date approaches implies that amortized value will probably not be so far removed from market value as to create serious difficulty given a reasonable mix in the bond portfolio.[6]

If a pension fund were invested entirely in equities, clearly the use of some special method of asset valuation would be required because of the volatility in common stock prices. Again, with only one general type of

[6] Note that long-term bonds having 4 per cent coupon rates were priced at a premium some fifteen years ago and yet might have a current market value of only 85 or 90, so that the amortized value of a particular bond may be quite unrepresentative of its current value. Even so, amortized value for an aggregate collection of bonds purchased over a long period might well constitute a satisfactory measure of their current value. Where the rate of interest employed in the amortization is equal to the actuarial valuation rate, the amortization method might more properly be classified as a present value method (such as Class VI, *a*).

asset in the fund, no internal problems would arise regarding consistency of treatment.

Today the assets of many pension funds consist of both common stocks and bonds, with substantial holdings in each. The use of different valuation methods for each type of asset held can, under certain circumstances, result in inconsistencies and illogical results. To illustrate, take the example of a pension fund having book and market values as shown in the following table:

	Bonds	Stocks	Total Fund
Book value............	$80,000,000	$20,000,000	$100,000,000
Market value..........	60,000,000	40,000,000	100,000,000

It is possible that an accountant, following the precepts in *Opinion No. 8* to the letter, might require a write-up of the asset value of the equity portion of the portfolio because of the relatively substantial unrealized appreciation. The Defense Contract Audit Agency, applying its rule of thumb, might similarly raise questions. In each case, it is likely that the existing depreciation in the bond portfolio would be ignored on the grounds that "bonds are held to maturity." Thus under unusual circumstances it is conceivable that separate valuation methods for each type of asset, such as a formula write-up of equity assets coupled with initial cost or amortized value for bonds, could result in an aggregate asset value in excess of the total current market value.

Over a long period of years, where the total market value of a balanced portfolio is considered, there are many occasions when the market values of stocks and bonds are moving in opposite directions.[7] Accordingly, there should be a smaller relative variation in the aggregate market value of an entire fund, stocks and bonds combined, than the variation that might be expected over a period of years in the common stock portion alone. At any rate, since the asset values adopted should never be so far removed from current market value as to be unacceptable to accountants, IRS, DCAA, and others, there is an implication that the aggregate market value of a pension fund, in some cases at least, may provide a good starting point for the asset value used in determining the cost of the plan.

[7] In about half of the years since 1900, stock and bond prices have moved in counteractive directions, although there is considerable variation by period. Such counteracting price movements have occurred in five of the last seven years and in thirteen of the twenty-one years since World War II. On the other hand, counteracting price movements occurred in only three of the thirteen years from 1923 to 1936.

STRAIGHT MARKET VALUE

The use of unadjusted market value in connection with pension fund valuations has been limited to a fairly small number of special cases because of the clearly unstable nature of market value. Extreme variations in market prices have occurred too frequently in the past to be ignored. In boom times, market values probably exceed the true worth of some of the underlying investments. Similarly, during a severe depression, market values may seriously understate the true long-run value of a portfolio.[8] The use of straight market value with an inflexible funding method can easily result in pension contributions that increase with depression and decrease with prosperity, whereas countercyclical changes in pension cost appear to be more nearly in the best interest of all concerned (although possibly in conflict with the basic tenets of the accounting profession).

The problem in using market value as the basic measure of asset value is to find some means of smoothing out the peaks and valleys which seem certain to occur in the market value of an aggregate pension fund portfolio over any lengthy period. It also seems desirable to find some mechanical means or formula whereby the distortions that may arise from temporary emotional and psychological factors can be reduced. Thus one is led quite naturally to some sort of listing as to what is really demanded of a measure of asset value to be used in pension calculations.

OBJECTIVES

A number of papers have listed those characteristics of a generalized measure of asset value that would be particularly desirable in determining the amount of pension fund assets to be used by the actuary in his calculation of required contributions.[9] For such purposes, the following characteristics would appear to be desirable:

1. The measure should produce values for the aggregate portfolio that are relatively stable from year to year.
2. The measure should produce realistic values that are acceptable to the accounting profession, IRS, DCAA, and others without modification and that are acceptable to the actuary in the sense that he does not feel impelled to "compensate" by using more liberal or more conservative actuarial assumptions, including the assumed interest rate, in the valuation of liabilities.

[8] For example, special asset valuation methods were imposed by state insurance commissioners during the early 1930's to prevent the temporary apparent insolvency of some insurance companies, which, taking a realistic long-run view, were in sound condition.

[9] J. K. Dyer, "Valuation of Pension Fund Assets," *Proceedings*, Conference of Actuaries, Vol. XII; and Paul H. Jackson and James A. Hamilton, "The Valuation of Equity Assets of Pension Funds," Eighteenth International Congress of Actuaries.

3. The measure should not entail undue expense in its application and should be readily understandable by those responsible for the management of the plan.
4. The measure should be independent of the past rate of turnover in the securities held by the fund and should not directly influence investment decisions, such as requiring the sale and repurchase of the same securities in order to increase the asset value.
5. The measure should contain a margin of safety that rises and falls in such a manner as to offset in a general way temporary aberrations in market value.

The above desirable characteristics, considered in conjunction with current investment theory, accounting requirements, and so forth, led to the development of the adjusted market value methods (described in Appendix II in Class V). The rationale underlying these adjusted market value methods might be illustrated by the following logical sequence. Where a significant part of a pension fund has been invested in common stocks to secure appreciation, it seems unreasonable to ignore the appreciation that results. If, as investment experts contend, "book value is irrelevant, totally irrelevant" as a measure of current asset value, then any asset value produced by a formula that uses book value as one of its prime terms must also have some degree of irrelevancy. If current market value is the true value, but fluctuations are unreasonable from the standpoint of their influence on employer contributions, then a direct downward adjustment to market value which increases in amount with rising market values and decreases in amount with falling market values would seem to be appropriate. Since stock and bond prices frequently move in counteracting directions, full advantage should be taken of the stabilizing effect on aggregate market value. Finally, since contributions and benefit payments are not naturally allocable among particular types of securities, it is simpler to determine the total annual investment yield of a pension fund in the aggregate than it is to determine (by methods sometimes rather arbitrary) the yield for each type of investment, such as stocks, bonds, mortgages, notes, and the like. The total investment income in dollars can be readily determined by adding the benefits paid during the year to the market value at year end and by subtracting the total of the contributions paid during the year and the market value at the beginning of the year.

The adjusted market value methods depend solely on the aggregate market value of the entire fund and on the total investment income year by year. In any year where the market value of a fund has risen to such an extent that less credibility might be placed on the year-end market value, this rise will be reflected in an unusually large total yield and thus in a substantial actuarial gain from interest. All or part of such a gain, depending on the particular formula adopted, can be withheld from the current asset value by holding it in an adjustment account to be released in some subsequent year when the market turns down and there is an actuarial loss from interest. For this purpose the required interest can be approximated by taking the adjusted market value at the beginning of the year plus half the excess of contributions over benefits and multiplying the result by the actuarial interest rate.

The formulas for adjusting market value can take many forms. The particular adjustment formulas listed in Appendix II are probably representative of the possibilities and are shown in some detail, since they were actually tested for contribution smoothness and asset fit (see Appendix III and the following sections).

One theoretical approach to adjusted market value would be to predicate a specific frequency curve for the annual percentage investment yield for a particular pension fund. For example, the mean of the observed yields and the standard deviation of such yields can be determined from the actual year-by-year investment results. Then a range of possible yields can be predicated, such as the range from the mean yield less two standard deviations to the mean yield plus two standard deviations. It can then be conservatively assumed that the yield in any one future year is equally likely to fall at any point within this range, and this assumption would result in a frequency distribution having a standard deviation in yield that is greater, perhaps by 15 or 20 per cent, than the actual computed standard deviation. Based on this assumed frequency distribution and the current actuarial interest assumption, formulas similar to those employed in experience rating[10] can be used to determine the expected gain from excess interest and the expected loss from deficient interest. For example, if the assumed interest rate is less than the mean yield for the frequency distribution, then some percentage (less than 100 per cent) of the expected gains from excess interest in one year would be sufficient to cover, in probability, the expected value of the loss from deficient interest. This percentage could be directly calculated from the assumed frequency distribution and actuarial interest assumption. The adjusted market value method based on this procedure would then provide for the addition of the computed percentage of any interest gain to the adjustment account and for the deduction of 100 per cent of any interest loss from the adjustment account.

In contrast to the theoretical approach described above, the adjusted market value methods with minimum, light, and intermediate adjustment

[10] For example, the basic formulas could be similar to those used in the derivation of either the J or K factors described in "Experience Rating," *TSA*, V, 249. Note that the choice of a simple frequency function, such as the straight-line segment suggested, greatly simplifies the mathematical work.

(listed in Appendix II as Class V, *a*, *b*, and *c*, respectively) are based on the premise that modest fluctuations in market value should be permitted to affect contribution requirements but that extreme market movements from one year to the next should be absorbed by the adjustment account to the extent possible. The actual formulas employed assume that interest gains due to yields in excess of the mean yield plus some multiple of the mean yield would be sufficient, in probability, to cover interest losses in other future years due to yields falling below the mean yield less that same multiple of the mean yield, thus implying symmetry in the frequency function of yield.

The adjusted market value method with full adjustment (Class V, *e*) is based on the premise that asset value and contributions should properly be insulated from any market fluctuation, however minor, so long as the resulting asset value falls within the prescribed range (85–100 per cent of market value). The full adjustment formula would thus tend to insulate the fund, at least partially, from the effects of longer-term market movements, such as a sequence of years in each year of which the market moved up (or down) and yet the change in market value in each of the years might not be sufficiently great to require any adjustment under, say, the minimum or light adjustment methods.

The adjusted market value methods with heavy and variable adjustment (Class V, *d*, *f*, and *g*) are based on the premise that, as a practical matter, it is probably more important to provide reasonably adequate protection against a sharp drop in asset value than against a sharp rise.

The adjusted market value methods can be started off with an initial adjustment account. Any such initial adjustment will depend in large part on the degree of conservatism considered desirable by the actuary, the employer, and the investment adviser, but it will also depend on the range in asset values acceptable to the accountant, in the sense that future modifications will not be required based on *Opinion No. 8*. It is, of course, possible to start off at book value and write up asset values to the desired "starting" point as a percentage of market value over a period of several years. Similarly, where book value is close to market value, it can be continued as the measure of asset value under rising market conditions until, expressed as a percentage of market value, it falls below the desired level, at which time asset valuation can be converted to the adjusted market value method.

BOOK VALUE

While it is accepted that book value is of little meaning in determining the extent to which a pension fund could meet its obligations if terminated (i.e., the degree of benefit security provided) or in measuring investment results over a period of time, book value is a fundamental part of the accounting procedure used by trustees and serves a very useful purpose in any audit of the fund. It is customary for trustees to maintain records for the securities they hold on an initial cost basis, with realized gains, interest, dividends, contributions, and benefit payments all handled in an internally consistent accounting system which enables the trustee to demonstrate that every dollar received has been properly held, invested, or applied to pay benefits, in accordance with the terms of the trust agreement. Annually (or perhaps more frequently) market values of the securities held are also tabulated, but these do not enter directly into this basic accounting system.

During any period of time with a rising market and with reasonable turnover in fund investments, book value provides a stable measure of asset value that normally possesses an element of conservatism in the form of unrecognized appreciation. It is a simple asset measure to employ, and certainly in the early years of a pension fund, when assets are small relative to total liabilities, it provides a reasonable measure of asset value for use in pension valuation. Accordingly, book value should continue to serve as a valuable measure of asset value for many newer pension funds and for many funds where, in the aggregate, book value is close enough to market value to provide a realistic measure. And book value can also be included as an element of an adjusted market value method as a minimum asset value intended to put a temporary floor on asset values following an extreme drop in market value. The desirability of using book value as such a minimum will depend, of course, on the initial relationship of book to market and on a continuation of the turnover in fund investments at a reasonable rate.

While book value serves many purposes, the method was not designed primarily for use in pension cost determinations; rather, it developed as a natural accounting device. When used in determining pension costs, there is a tacit consensus that recognition of capital gains should occur at the time these gains are realized through actual sale of assets. The logic of recognizing such profits while ignoring unrealized appreciation in value is by no means clear. As Binns observed:

In the first place, there is no assurance that particular assets will ever be sold. In fact, it would seem that such sales would be most likely to occur in the case of securities whose future performance is in question. If a trustee has made a long term commitment in a certain common stock, it is entirely possible that it will never seem desirable to dispose of it. The realization of a gain might be more likely to occur in the case of a speculative security which has rapidly risen in price subsequent to its purchase, than in the case of a long term investment-grade issue which continues to show reasonable non-speculative capital appreciation. Just why the pension fund should receive credit for profits of the former sort and no credit for profits of the latter sort is not explained in pension literature.[11]

The use of any particular asset valuation method, including book value, should be based on a rational analysis of the various alternatives rather than on mere adherence to tradition. Thus, for example, corporate management's decision to use book value might involve many of the same considerations that are involved in the decision to fund a pension plan more rapidly than minimum requirements. While the continued ignoring of unrealized appreciation of common stocks in pension funds would seem to heighten the security of employees generally, the older employees and those already pensioned might well argue that the unaccounted for appreciation should be used to improve pension benefits immediately. Finally, the decision to ignore unrealized appreciation might result in the penalizing of the present generation of stockholders through higher pension costs than necessary, while tending to favor those who happen to be stockholders when eventual recognition must be given to unrealized appreciation, if it continues.

CREDIBILITY

Approaching asset valuation from the standpoint of "credibility," as that term is used in experience rating, provides further insight into some of the problems involved. First, there is a fundamental difference between the use of statistical time series to forecast (1) the future value of a particular constant of physics, say, the force of gravity at sea level in 1980; (2) the rate of mortality over some future period; and (3) the market value of a particular security at a specified future date of sale or the investment yield thereon up to that date.[12] The first two can be determined currently within reasonable limits and projected with whatever future trends may seem appropriate, but the last is subject to human influence and in reality attempts to assess just how an anticipated future buyer will go about deciding how much to pay. Utility theory and other abstract approaches can, of course, add to our understanding of the process but have yet to come up with the numbers we want. A prominent economist has said, "If we have learned anything about economics, it is that the future will not duplicate the past."[13] It is not any inadequacy in the volume of our statistics that makes our future forecast of asset values so uncertain; it is, rather, a very basic characteristic of econometrics.

[11] W. Gordon Binns, *op. cit.*, p. 18.

[12] See, for example, Ludwig Von Mises, *Theory and History* (New Haven, Conn.: Yale University Press, 1957), pp. 11–23.

[13] Gardner Ackley, chairman of the Council of Economic Advisors, February, 1966.

The trend in current market values can be projected with some accuracy for a day, or a week, because tomorrow's market price starts where today's stops (or perhaps in extreme cases, such as "Black Tuesday" in 1929, one minute's price starts where the last minute's transactions left off). The longer the projection, however, the more effectively new influences arise and old ones diminish so as to render the very process of projection less meaningful; yet, strangely enough, projections of investment yields over very long periods, by minimizing cyclical influence on sales prices, provide results that are probably within acceptable limits. To the extent that disability rates, withdrawal rates, early retirement rates, average age at entry, rates of pay, size of work force, and so forth, are also influenced by economic factors, they too are not as susceptible to forecasting accurately as are rates of mortality.

One intriguing aspect of the credibility concept is that it can be used, in a general way, in ferreting out the rationale underlying some of the asset valuation formulas. For example, the observation, probably unprovable, that the longer an excess of market value over initial cost has existed, the greater the credibility that should attach to it appears to be the basis for formulas III, *a*, *b*, *c*, and *d*, in Appendix II. Similarly, formula II, *d*, implies that greater credibility can be attached to increases in market value that result from increased current corporate earnings than to increases in market value that result in a corresponding rise in price/earnings ratios.

THE PERPETUITY METHOD[14]

Capitalizing the current amount of annual interest and dividends as a perpetuity has the disadvantage as to any given security that the asset value can exceed or be less than both initial cost and current market value. Once again, however, this method possesses characteristics that provide further insight into asset valuation. To begin with, the perpetuity method involves a measure of asset value that is not independent of the assumptions and methods used to determine liabilities. The funding of a pension plan involves an estimation with regard to both the present value of future benefits and the present value of assets held, with the difference between these items being provided for through future contributions. Under the perpetuity method an increase in the actuarial interest rate serves to decrease both the liabilities and the value of assets, and this reduces the effect of a given change in the actuarial interest assumption on

[14] See G. Heywood and M. Lander, "Pension Fund Valuations in Modern Conditions," *Journal of the Institute of Actuaries*, LXXXVII, Part 3, 377.

Also see J. G. Day and K. M. McKelvey, "The Treatment of Assets in the Actuarial Valuation of a Pension Fund," *Journal of the Institute of Actuaries*, XC, Part 2, 104.

the level of required contributions. As pension funds approach full funding, this matter of interdependence of asset and liability measures becomes much more important, because the value of assets and the value of liabilities, for a mature, well-funded plan, may each be many times the size of the difference between the two. Using an asset valuation method that is completely independent of, and perhaps even inconsistent with, the actuarial assumptions can, therefore, distort the results to a far greater extent under a pension plan approaching full funding than it can during the early years of a plan where the unfunded liabilities are many times the size of the assets.

The perpetuity method is based on the reasonable premise that, if future estimated benefit payments are going to be discounted at the valuation interest rate to a present value basis, the future estimated cash income generated by the present fund should also be discounted at the same rate. Put another way, the future cash income derived from the investments now on hand is assumed to pay part of the estimated benefit outgo in each future year. Thus the future contributions to the fund can be determined so as to have a present value that is equal to the present value of that portion of the estimated benefit outgo in each future year that is not expected to be covered by the cash income in that year generated by the present investment holdings.

FORMULA METHODS FOR WRITING UP BOOK VALUE

If a rational and systematic method is adopted for recognition of unrealized gains by writing up book value, it must clearly tie into the underlying accounting system on which book value is based. Any unrealized gains that are "recognized" at one point in time will eventually make their way into the book value as securities are sold and the gains realized. This implies that any "fictitious" current or prospective contribution credit or offset (i.e., one consisting of the recognition of some unrealized appreciation as opposed to actual cash) should be accounted for separately, carried forward at the actuarial interest rate, and made up out of actual realized gains. This process is also necessary where continuing annual adjustments are made, based on the amount of unrealized appreciation, since only the remaining balance of unrealized appreciation that has previously been "recognized" but has not yet gotten into the book value via realized gains should be added to the book value in determining the asset value before adjustment.

In the practical application of the 3 per cent write-up method, the write-up takes the form of a special "appreciation account" which in effect recognizes a portion of the appreciation that is as yet unrealized on the book value basis. The amount in the "appreciation account" would simply be added to the book value of equity securities at any point in time in determining the asset value. As appreciation in value is realized through the sale of securities, however, the realized gains must then be subtracted from the appreciation account, since they increase the book value and, if they were not deducted from the appreciation account, it would be possible, given substantial turnover in the portfolio, to end up with an asset value well in excess of market value. One alternative approach, frequently used, is to average the aggregate book value at the beginning and end of a given year and reduce the beginning-of-year appreciation account by the proportion that the book value of all disposals during the year bears to the average aggregate book value. Another modification is to impose some maximum on the appreciation account; for example, limit the adjusted asset value of common stocks to, say, 80 per cent of their aggregate market value (possibly adjusted downward for any current unrealized depreciation in the bond portfolio).

The handling of the adjustment under the 7 per cent yield method would be similar to that under the 3 per cent write-up method, except that both dividends and realized gains as received would be subtracted from the special account. The 7 per cent yield method could be applied to the total portfolio rather than to the equity portion only and would then represent the extreme case of adjusted market value in the sense that the adjustment account at the end of any year would simply be set at whatever amount was necessary to make the yield on adjusted market value turn out to be the 7 per cent for that year. This method produces extremely stable asset values, but its chief disadvantage is that it would be based on an asset value at a single date (usually book value, but market value seems equally feasible). Following that initial determination of asset value, the subsequent asset values would depend solely on the cash contributions and benefit payments year by year and on the 7 per cent factor employed. Since this method is not self-adjusting, it could be expected, under extreme market conditions, to produce asset values that differ considerably from both market and book values. When adjustments are required under the 7 per cent yield method, they could take two forms: (1) if the asset value gets too far out of line at some future date, a one-shot adjustment, up or down, could be made with the 7 per cent yield method being continued from the new starting point; (2) it would be possible to adjust the 7 per cent figure itself upward or downward so as to speed up or slow down the growth in asset values in order to produce values closer to the desired percentage of actual market value. This method, therefore, would seem to require continual review and the recurrent exercise of judgment in establishing bench marks. On the other hand, if 7 per cent, or whatever other factor is initially selected, should turn out to be the true long-term yield, the method would, of course, work very well.

The 7 per cent yield method carries with it the implication that the actuarial interest assumption ought to be 7 per cent. If an actuarial interest rate of 4 per cent were assumed, for example, the 7 per cent yield method would automatically produce a 3 per cent gain from excess interest for each and every future year, except one in which a one-shot adjustment is made. When the actuarial interest rate is set at the same level as that used for growth of asset values, however, some flexibility is lost. For example, if asset values should get seriously out of line with current market values, the only practical adjustment would seem to be the one-shot type of adjustment already mentioned, since a change in the 7 per cent figure to speed up or slow down the growth in asset values could result in a substantial change in contribution requirements because of the accompanying required change in the actuarial interest rate assumption.

Some of the other write-up methods also involve fairly simple, straightforward treatment insofar as the handling of realized gains is concerned. For example, where asset value is initially set at book value plus 50 per cent of the difference between market and book, half of any realized gain or loss has already been anticipated in the asset value and should properly be deducted from the write-up account. (This would result in a gradual change in the 50 per cent factor, however, and so is usually ignored.) Under other write-up methods, however (such as Class III, *b*, and Class III, *c*, in Appendix II), it becomes quite complicated to determine the proper amount of realized gains to be used with the asset valuation method from the realized gains derived on the book value basis. Thus in some cases it has been found more understandable to write up the value of each security held in the portfolio in order to permit the ready determination of the realized gains under the particular asset valuation method being used. In any event, the design of a method for writing up book value by some formula will usually involve the most careful consideration of the treatment of realized gains and losses if the method is to operate satisfactorily.

Under the 10 per cent offset method (Class III, *c*), the net effect of an extra dollar of unrealized appreciation is to reduce the employer's current contribution by 10¢, whereas an extra dollar going into the pension fund in other ways, such as contribution, actuarial gain, or realized gain from the sale of securities (if not used directly to reduce contributions), might decrease subsequent contribution requirements only through the application of the regular amortization factor, which would generally be less than 10 per cent. There seems little reason to give greater current weight to a dollar of unrealized appreciation that is being recognized currently than that given to an actual dollar of cash contribution in terms of its effect on subsequent contribution requirements. (Note, however, that even *Opinion No. 8* appears to support such an approach, since it suggests the spreading of actuarial gains over a period that can occasionally be shorter than the average remaining period for amortizing unfunded past service.) The 10 per cent offset method has been used to recognize a portion of initial unrealized appreciation, such as the excess of 75 per cent of market value over book value, with another method, such as the 3 per cent write-up method, being used as the basis for recognizing the remaining initial unrealized appreciation and any future increases in unrealized appreciation. The 10 per cent offset method also has considerable practical value in special cases, where the recognition of all or a large part of the excess of market over book might result in a plan's being currently overfunded so that the employer would be denied a tax deduction on current contributions. In such special cases, the 10 per cent offset method permits the current recognition of only 10 per cent of the unrealized appreciation and yet maximizes the reduction in contribution requirement, which appears to be satisfactory to both IRS and DCAA. In the absence of such tax complications, it is generally preferable, where some or all of the unrealized appreciation is recognized, to include recognized appreciation as part of the fund in the determination of the past-service cost included in future contribution requirements.

TEST OF SMOOTHNESS OF CONTRIBUTIONS

Fifteen asset valuation methods were tested for their effect on the pattern of the required contributions of several pension plans over the ten-year period 1957–67. In each case, asset valuation over that period had been based on book value, with the entry age normal cost method used to determine contribution requirements for each year. The plans selected were those where market value, book value, and realized gains or losses for the stock and bond portfolios were separately available. The benefits included flat benefits as well as pay-related benefits, and the plans covered either or both union and salaried employees. The test results did not vary significantly from plan to plan, and the results shown herein represent, as to each asset valuation method, the total for all plans and thus constitute an average result, of sorts. In any event, the results of the tests are perhaps not as important, from an actuarial viewpoint, as the methodology employed.

The basic data for each plan consisted of the actual asset values, benefit payments, amortization factors, and contribution requirements, all of which include the effect of actual changes in level of benefits, employment and pay, and the actual investment results of the funds over the period studied. Since a change from book value to another asset valuation method would have resulted in higher (or lower) contributions, it was assumed, in adjusting subsequent asset values, that the additional amounts of contributions were invested in (or obtained from the sale of) bonds yielding 5 per cent, with their market values being equal to their book values throughout the period. The fifteen methods studied were as follows (references are to Appendix II):

1. Book Value, I
2. Market Value, IV, *a*
3. Adjusted Market Value—Minimum Adjustment, V, *a*
4. Adjusted Market Value—Light Adjustment, V, *b*
5. Adjusted Market Value—Intermediate Adjustment, V, *c*
6. Adjusted Market Value—Heavy Adjustment, V, *d*
7. Adjusted Market Value—Full Adjustment, V, *e*
8. Adjusted Market Value—Variable Adjustment I, V, *f*
9. Adjusted Market Value—Variable Adjustment II, V, *g*
10. 10 Per Cent Offset, III, *c*
11. Perpetuity Method, VI, *a*
12. 3 Per Cent Write-up (Starting at Book), II, *a*
13. 3 Per Cent Write-up (Starting at Market), II, *a*
14. 7 Per Cent Yield (Starting at Book), II, *b*
15. 7 Per Cent Yield (Starting at Market), II, *b*

In testing these asset valuation methods as to their effect on required pension contributions, we assumed that the variations in required contributions from year to year, with assets valued at book value, would be due primarily to changes, over the ten-year period, in (*a*) the level of employment and salaries, (*b*) the benefits provided by the plans, and (*c*) the actuarial assumptions. Because of the existence in some of the plans of substantial amounts of unrealized gains, it was recognized that a change in asset valuation method from book value to, say, market value, could result in a sharp drop in the absolute level of contributions and that other "natural" changes in contribution levels, resulting from plan amendment or changes in level of employment, salaries, or actuarial assumptions, should not properly be smoothed out. Thus the differences in dollar contribution requirements year by year or for the whole ten-year period would not provide a meaningful test for smoothness of contribution. The test actually employed was based on a comparison of the percentage change in contribution from one year to the next under a given asset valuation method with the corresponding percentage change under the book value method. Appendix III shows these percentage changes year by year, along with the mean absolute deviation and the root-mean-square deviation. Table 1 summarizes the results of this test, by method, showing first the mean absolute deviation and the ranking of the methods in order of lowest deviation, and second the root-mean-square deviation together with the ranking by method.

The book value method, of course, ranks first, since it was chosen as the standard for smoothness of contributions. The 3 per cent write-up methods, starting from either book or market, rank close behind for the simple reason that over the particular ten-year period the adjustment was minimal, as shown in Table 2.

TABLE 1

Method	Mean Absolute Deviation		Root-Mean-Square Deviation	
	Per Cent	Rank	Per Cent	Rank
1. Book Value, I	0.0%	1	0.0%	1
2. Market Value, IV, *a*	10.1	14	12.6	14
3. Minimum Adjustment, V, *a*	9.6	13	11.7	13
4. Light Adjustment, V, *b*	7.0	12	8.4	12
5. Intermediate Adjustment, V, *c*	6.0	11	7.1	11
6. Heavy Adjustment, V, *d*	3.2	6	5.1	6
7. Full Adjustment, V, *e*	1.8	4	2.8	4
8. Variable Adjustment I, V, *f*	4.0	8	5.2	7
9. Variable Adjustment II, V, *g*	2.4	5	3.5	5
10. 10 Per Cent Offset, III, *c*	14.7	15	17.3	15
11. Perpetuity Method, VI, *a*	4.7	10	5.5	9
12. 3 Per Cent Write-up (Book), II, *a*	0.4	2	0.6	2
13. 3 Per Cent Write-up (Market), II, *a*	1.3	3	2.2	3
14. 7 Per Cent Yield (Book), II, *b*	3.7	7	5.3	8
15. 7 Per Cent Yield (Market), II, *b*	4.3	9	7.1	10

The 10 per cent offset method showed the greatest deviation and ranked last, being even poorer than straight market value. The adjusted market value method with minimum adjustment shows almost the same deviation as straight market value, while as heavier adjustments are introduced the deviation drops rather sharply. Both 7 per cent yield methods show up well with modest deviations.

ACCEPTABILITY OF ASSET MEASURE

The same fifteen methods tested for smoothness of contribution were also subjected to a test to determine the relative acceptability of the asset values at each point during the ten-year period 1957–67. Basically, this test assumed that the asset value should not exceed market value, and, if it were below 80 per cent of market value, serious criticism might be expected from either the accountants, IRS, DCAA, or any other interested party (not the least of which are the current holders of the company's common stock, who might insist that inadequate recognition of unrealized appreciation merely increases pension costs for them which would ultimately result in a windfall to some subsequent generation of stockholders). It was felt that some criticism might arise if the asset value fell below 85 per cent of market and possibly even if asset value fell below 90 per cent of market. Accordingly, the method of testing levied three penalty points for each year-end asset value exceeding full market value and one penalty point for each year-end asset value below 90, 85, and 80 per cent, respectively. Since each year below 80 per cent is automatically a year below 85 and 90 per cent as well, this process effectively chalks up three bad

TABLE 2

PERCENTAGE OF EXCESS OF MARKET OVER BOOK VALUE OF STOCKS RECOGNIZED BY THE APPRECIATION ACCOUNT UNDER THE 3 PER CENT WRITE-UP METHOD

Year	Start at Book Value	Start at Market Value
1957	0.0%	100.0%
1958	3.3	61.4
1959	3.1	45.7
1960	5.4	55.4
1961	2.5	30.0
1962	10.8	68.7
1963	7.7	35.2
1964	6.8	26.9
1965	5.8	24.0
1966	7.2	35.9
1967	1.6	17.2

behavior points for asset value in excess of market or below 80 per cent of market, two points for asset value between 80 and 85 per cent of market, and one point for asset value between 85 and 90 per cent of market. Table 3 shows the number of penalty points accumulated by each of the methods over the ten-year period 1957–67.

The fewest points are, of course, accumulated by those asset values which are closest to market value. Under the adjusted market value methods the number of points generally increased as the amount of adjustment to market value increased. Book value showed up as the worst method, and the 3 per cent write-up methods, the 10 per cent offset method, and the perpetuity method also performed poorly. Over this particular ten-year period, the book value method, the 3 per cent write-up method starting at book value, the perpetuity method, and the 10 per cent offset method produced asset values that tended to get so far out of line with current market values as to make them impractical. This does not mean, for example, that the 3 per cent write-up method could not have been used satisfactorily, but a factor greater than the customary 3 per cent would have been necessary over this particular ten-year period in order to produce reasonable results.

FREQUENCY OF ADJUSTMENT TO MARKET VALUE FIGURES

In using either straight market value or the adjusted market value methods, the contribution requirement for a given plan year need not be based on the market value on only one particular day when, through unusual circumstances, the market might be unusually high or low. To

TABLE 3

SUMMARY OF TEST FOR FIT OF ASSET VALUE WITH MARKET VALUE

Method	Penalty Points	Rank
1. Book Value, I	24	15
2. Market Value, IV, *a*	0	1
3. Minimum Adjustment, V, *a*	0	2
4. Light Adjustment, V, *b*	0	3
5. Intermediate Adjustment, V, *c*	0	4
6. Heavy Adjustment, V, *d*	7	6
7. Full Adjustment, V, *e*	8	7
8. Variable Adjustment I, V, *f*	2	5
9. Variable Adjustment II, V, *g*	9	8
10. 10 Per Cent Offset, III, *c*	21	13
11. Perpetuity, VI, *a*	20	12
12. 3 Per Cent Write-up (Book), II, *a*	23	14
13. 3 Per Cent Write-up (Market), II, *a*	12	11
14. 7 Per Cent Yield (Book), II, *b*	12	10
15. 7 Per Cent Yield (Market), II, *b*	11	9

begin with, the trustee could be instructed to use an average market value over the last week or month of a plan year as the appropriate year-end value. Going further, it is also possible to adjust the contribution requirement either monthly or quarterly to reflect the changes in market value as they occur during the plan year. Table 4 shows the results of comparing the amortization portion of contribution requirements based on annual and quarterly market values with those determined on the basis of the average of the market values taken at the end of each of the twelve months during a plan year. Straight market value was used for this test without adjustment.

The underlying assumption in Table 4 is that a contribution based on the average of the twelve monthly market value figures during a plan year would provide the best asset value for pension cost purposes. Whether this is true or not is questionable at best, and the resulting differences appear too modest to dictate such refinement. Also, where adjusted market value is used, variations in the adjustment account could be expected to eliminate or reduce the larger deviations.

SIGNIFICANCE OF RESULTS

The results of any tests for smoothness of contribution and acceptability of asset value under the various methods that are carried out for a particular pension plan will vary somewhat, depending on the method of

TABLE 4

DEVIATION IN AMORTIZATION PAYMENTS FROM THOSE BASED ON TWELVE MONTHLY MARKET VALUES

Year	Using Market Value at Beginning of Plan Year		Using Market Value at the Beginning of the Four Quarters in Plan Year	
	Deviation	Deviation Squared	Deviation	Deviation Squared
1957	2.8%	7.8	0.7%	0.5
1958	6.7	44.9	1.0	0.1
1959	0.3	0.1	0.8	0.6
1960	9.9	98.0	1.7	2.9
1961	5.1	26.0	0.9	0.8
1962	10.3	106.1	1.9	3.6
1963	4.8	23.0	0.6	0.4
1964	0.4	0.2	1.1	1.2
1965	4.0	16.0	1.3	1.7
1966	10.8	116.6	2.3	5.3
Mean	5.5%	43.9	1.2%	1.8
Square root	....	6.6	....	1.3

funding, the actuarial assumptions, the degree of funding reached, and the investment results over the period of testing. For example, when the tests are applied to fairly new funds where the assets are relatively small and there are substantial unfunded liabilities, a given percentage variation in assets has a far smaller dollar effect on the contribution, and the contribution itself might be greater by reason of a relatively high payment for amortization of unfunded liabilities. Similarly, when the tests are applied to plans that are close to a fully funded condition, a given percentage change in the asset values has a larger relative effect on the contribution requirement.

Probably the test for acceptability of asset value was influenced more by the particular ten-year period chosen, and by the investment performance of the funds involved, than by all the differences in degree of funding, type of benefit, employment characteristics, and the like, put together. Over the ten-year period, high-grade bond yields rose from 4 per cent (perhaps a good long-term average value) to over 6 per cent, while the dividend yields on industrial common stocks fell from about 4 per cent (i.e., below the long-term average of about 5 per cent that prevailed from 1900–1955) to a twentieth-century low of only about 3 per cent. Clearly, asset valuation tests conducted over any different period could produce radically different results. The ten-year period 1957–67 did include several market downturns (1960, 1962, and 1966), and the total annual investment yields for the pension funds studied ranged from -10 to $+30$ per cent over this period. Even so, it is likely that future investment experience will differ significantly from that of the ten-year period studied, so that the specific results shown herein cannot necessarily be considered indicative of the results that would obtain in future years. About all that can be said is that some methods, particularly book value and modest variations therefrom, have already failed to perform well in certain cases.

The investment policy adopted by a particular pension fund will also have a bearing on the results. In some cases, significant amounts have already been invested in common stocks, and some trustees have recently shifted their investment strategy so as to improve their short-term investment performance. This can result in an increase in the turnover in equity investments with the result that substantial realized gains might develop from the equity portion of the portfolio. Where these realized gains approach 3 per cent annually, it is clear that a 3 per cent write-up method less realized gains for the equity portfolio would necessarily produce a final asset value close to book value.

The one firm conclusion that can be reached as a result of a testing process on various pension plans is that each plan must be considered by itself in the determination of the most appropriate asset valuation method, and in each case the choice will depend upon the relative weight assigned to smoothness of contribution, on the one hand, and to fit of asset value with market value, on the other.

CONCLUSION

In general, the matters set forth herein suggest rather strongly that no single measure of asset value is perfectly suited to every pension fund at every stage of its development. The choice of an appropriate measure for asset value will depend on the current investment climate, the particular strategy employed in the investment of a pension fund, the plan of benefits, the actuarial funding method used, the degree of funding actually achieved, and the actuarial assumptions, especially the interest rate and salary scale factors, that are used to determine the value of the liabilities.

In each case, some special considerations may be present which take precedence over generalized analyses, and variances in these specific objectives from plan to plan clearly play as important a role in the selection of an appropriate measure for asset value as the various mathematical and financial considerations covered in this paper. In certain circumstances, the adjusted market value methods produce more meaningful asset values than the traditional methods based on book value and provide the actuary with a highly flexible set of tools with which to approach the actuarial valuation of the various pension funds he serves. Of course, the ultimate performance of a pension fund will not depend, in the final analysis, on the selection of a particular measure of asset value, or on a particular set of actuarial assumptions, or on a particular funding method. Far more important ingredients are the investment performance of the fund and the continued financial well-being of the sponsoring employer.

APPENDIX I

BERNSTEIN, LEONARD A.
Supervisor, Scientific Systems Unit
Operations Division
Merrill Lynch, Pierce, Fenner & Smith, Incorporated

HAMMER, GILBERT
Manager, Computer Applications Unit
Securities Research Division
Merrill Lynch, Pierce, Fenner & Smith, Incorporated

LORIE, JAMES H.
Professor of Business Administration
Director of the Center for Research in Security Prices
Graduate School of Business
University of Chicago

TREYNOR, JACK L.
Manager, Operations Research Department
Merrill Lynch, Pierce, Fenner & Smith, Incorporated

METHODS OF MEASURING PORTFOLIO PERFORMANCE

Made at a Series of Seminars Sponsored by Merrill Lynch, Pierce, Fenner & Smith, Incorporated

"All measurements of variability and rate of return should be based on the market value of assets; no distinction should be made between realized and unrealized capital gains. . . .

"Finally, it is not logical for an investor to focus on income to the exclusion of capital appreciation, or vice versa. Price appreciation leads to increased income and increased income when reinvested can lead to price appreciation. Low yield stocks are bought because the buyer believes that prospects for rapid price appreciation compensate for current low yields. Conversely, higher yields and stability compensate the investor for the prospect of lesser appreciation potential. It is always possible by concentrating on the appropriate kind of security to look good when judged on appreciation or income alone. Both factors must be measured to determine how well the fund is doing for its stockholders or its beneficiaries. Therefore, our measure is addressed to portfolio returns defined as the sum of appreciation and income."

* * *

CRAMER, JOSEPH J., JR., C.P.A.
Assistant Professor of Accounting
Pennsylvania State University

" 'UNREALIZED' APPRECIATION AND PRIVATE PENSION PLANS"
Business Horizons (Fall, 1965)

"Since a corporation's ability to increase contributions varies directly with its working capital position, it is not unreasonable to predict that recognition of unrealized appreciation in pension plan accounts as recommended above will be used as the most desirable alternative. Not only is such an approach to asset valuation theoretically superior to the cost method, but it is also more consistent with the actuarial valuation of pension plan liabilities."

* * *

DIETZ, PETER O.
Assistant Professor of Finance
Northwestern University

"PENSION FUNDS: MEASURING INVESTMENT PERFORMANCE"
The Graduate School of Business, Columbia University, and The Free Press, 1966

"For the purpose of measuring pension fund investment performance, income should be defined to include ordinary income plus realized and unrealized gains and losses. Unrealized appreciation is included because it reflects the total return during the period in question. When appreciation is experienced it is included because the trustee always has the option of realizing gains if he feels the value of his securities will decline in the future. When depreciation is shown, it is included in calculating return because it may have had to be realized if the trustee had to convert securities to cash for benefit payment purposes. The concept being used here is one of total performance. This concept of return is particularly pertinent for a nontaxable investment medium such as pension funds. Except for the factor of certainty of return, the trustee should have little preference for capital gains versus ordinary income."

* * *

FISHER, L.
Associate Professor of Finance
Associate Director of the Center for Research in Security Prices
Graduate School of Business
University of Chicago

LORIE, JAMES H.
Professor of Business Administration
Director of the Center for Research in Security Prices
Graduate School of Business
University of Chicago

"RATES OF RETURN ON INVESTMENTS IN COMMON STOCKS"

The Journal of Business, Vol. XXXVII, No. 1 (January, 1964)

[The methodology of "Cash-to-Cash" or "Cash-to-Portfolio" implies, without specifically stating since it is so obvious, that market value is the end value to use.]

"The method of calculation is analogous to that used to compute the yield-to-maturity of a bond. The cost of the stock at the beginning of a time period is analogous to the purchase price of the bond; dividends are analogous to interest payments; and the value of the stock at the end of the period is analogous to the sum received by the holder of a bond when it matures. The rate of return is the rate of discounting which makes the stream of after-tax cash flows have a present value of zero."

* * *

INVESTMENT PERFORMANCE MEASUREMENT COMMITTEE
(*Chairman*, William H. Gish, Jr.)
National Foundation of Health, Welfare, and Pension Plans

"MEASURING AND REPORTING INVESTMENT PERFORMANCE OF PENSION FUNDS"

Research Project No. 1, National Foundation of Health, Welfare, and Pension Plans

"Probably the most widely used—or rather, misused—method of measuring the investment return of a particular fund is simply to look at the latest annual or quarterly tabulation of the securities held in the fund, take the total indicated annual income figure, and divide this by the total cost, or 'book' value as reported in the tabulation. . . .

"There are two other major flaws in this method. The first is that it considers only income as constituting a recognizable increment to investment. . . .

". . . While it is a truism that 'paper profits' cannot be counted as 'money in the bank' it is true that, for any particular fund, a fairly computed market value is the best measure of its real, or realizable, value at any moment in time. Therefore, if the measurement technique is to quantify fully all past investment results of a fund up to a particular moment in time, it must take into account all elements making up the real value of the fund at that moment.

"The second flaw may be found in the reported figure for 'cost' value. When compared with the market value figure, the only thing it reveals as a matter of fact is that there is, at that moment, a certain amount of 'unrealized' market appreciation—the difference between reported 'cost' or 'book' value and market value. It does not tell us how much, if any, market appreciation or depreciation has been 'realized' prior to the statement date and thereby included in the 'cost' value figure.

"When reported 'cost' value is used as the investment base, we cannot tell whether we are using an original cost yardstick, or one that has been stretched to 50 inches by previously 'realized' gains, or shrunk to 30 inches by previously 'realized' losses."

* * *

LEVY, ROBERT A.
Computer Directions Advisors, Inc.

"MEASUREMENT OF INVESTMENT PERFORMANCE"

Journal of Financial and Quantitative Analysis,
Volume III, No. 1 (March, 1968)

"Third, should investment be cost-based or value-based for rate of return calculations? Although cost is utilized throughout the greater part of this paper, and utilized almost universally in practice, far wider attention should be given to the possibility of computing rate of return on a value-based investment. Only a value-based measure takes into account the implicit cost of capital (i.e., the return available on alternative investments at equal risk.)"

* * *

LORIE, JAMES H.
Professor of Business Administration
Director of the Center for Research in Security Prices
Graduate School of Business
University of Chicago

"NABAC STUDY ON PENSION FUNDS"

Financial Analysts Journal (March–April, 1968)

". . . . The single most important principle about which there was no disagreement was that the rate of return should be based essentially upon changes in the market value of the assets being managed after taking proper account of the contributions to and the distributions from the pension fund."

APPENDIX II

CLASSIFICATION OF ASSET VALUATION METHOD[15]

Class I: Initial Cost Methods

a) Value assets at initial cost. Realized gains, that is, the excess of proceeds at sale over the initial cost of the assets sold, would thus be included in asset value either as the initial cost of new purchases or as cash held by the fund. Usually called "book value."

[15] For ease of description, reference herein is limited to "realized gains," "increases in value," "write-ups," etc. Each such reference should be considered as including the corresponding negative, such as "realized losses," "decreases in value," "write-downs," etc., wherever appropriate.

b) Use initial cost but apply judgment adjustments to the financial condition of the plan in facing new decisions, such as plan amendments or changes in funding methods or actuarial assumptions.

c) Use initial cost but increase the turnover of fund investments, even to the extent of using "wash sales,"[16] in order to maintain the desired aggregate difference between current market value and asset value.

d) Use initial cost but increase the actuarial interest assumption to anticipate (1) the future realized gains that can reasonably be anticipated with normal investment rollover or (2) the future increase in dividend income that can reasonably be anticipated for those equity securities that are held indefinitely.

e) In case of pooled funds use initial cost, but, where one security has been purchased on a number of different dates at different unit prices and only a portion of the holding is sold, value the securities sold either by using a first-in–first-out basis, a last-in-first-out basis, or the average unit cost of the security just prior to sale.

Class II: Initial Cost with Formula Modifications

a) Increase the asset value of equity securities each year by 3 per cent[17] of the previous year's asset value to recognize an assumed inherent long-run increase in the value of equity securities. Sometimes called the "3 per cent write-up method."

b) Increase the asset value of equity securities each year by 7 per cent[17] of the previous year's asset value reduced by any dividends received and by any realized gains. This method could also be applied to the entire portfolio with the reduction including interest as well. Sometimes called the "7 per cent yield method."

c) Increase the asset value of equity securities each year by the increase in net retained earnings per share as determined from the most recent corporate annual statement (security by security).

d) Value equity securities on the basis of initial cost times the ratio of current after-tax earnings per share to the after-tax earnings per share at the time of purchase after adjustment for stock splits, and so forth (security by security).

e) Value equity securities on the basis of initial cost times the ratio of current dividends per share to the dividends per share at the time of purchase after adjusting for stock splits and any effect on dividends of corporate taxes that are expected to apply only for a short time (security by security).

f) Value bonds on an amortized basis running from initial cost at purchase to par value at maturity, or earliest call date, using a uniform rate of interest throughout the period.

g) Value bonds at par value with the excess of par value over cost being treated as a realized gain at time of purchase.

[16] That is, selling a security with the intent of repurchasing the same security simultaneously in order to convert the excess of market value over initial cost into realized gain for the fund.

[17] The 3 per cent and 7 per cent figures shown are those commonly used. The choice of appropriate percentage will depend chiefly on an appraisal of the current and future investment outlook, but clearly the percentage used under II, *b*, will exceed that for II, *a*.

Class III: Modification Based on Both Initial Cost and Current Market Value

a) Value assets on the basis of the average of the market value on date of purchase (initial cost) and the market values on each plan valuation date up to the current date, possibly using a moving average after a security has been held for some maximum period, such as five years, to assure currency (security by security).

b) Write up asset values each year by some percentage of the excess of current market value over previous asset value in the aggregate, such as increasing the asset value by 25 per cent of the excess of current market value over 110 per cent of previous asset value (with new securities going in at initial cost), the 10 per cent being a margin for contingencies.

c) Increase asset values in the aggregate by some percentage (frequently 10 per cent is used) of the excess of current-year unrealized appreciation over the amount previously recognized, with such write up in value taken as a direct offset to current contribution requirements. Sometimes called the "10 per cent offset method."

d) Apply a series of percentages to spread the recognition of the increase in aggregate unrealized appreciation arising during each plan year over a period of future years until such appreciation is fully recognized with such write up taken as an offset to contributions.

e) Increase asset values in the aggregate by some percentage, such as 50 per cent, of the average dollar amount of unrealized appreciation at the last five or ten plan anniversaries.

f) Write up initial cost values in the aggregate to a new "judgment" level basis somewhere between initial cost and current market value at time of plan amendment to preserve a predetermined contingency margin or to freeze unfunded liabilities.

g) Use initial cost, or current market value if lower (but note IRS has specifically disapproved of this method in Revenue Ruling 63-11).

h) Apply some "credibility" factor to the excess of market value over book value (security by security or in the aggregate) at the end of each plan year based on the likelihood of actually realizing any such excess either at eventual sale or in future increased dividends.

i) Start at initial cost value and recognize initial unrealized appreciation over, say, ten years, with inclusion in asset value of 10 per cent of the increase in unrealized appreciation plus 10 per cent of the realized gains from each subsequent year.

j) Use book value or some percentage, say, 85 per cent, of market value if greater.

Class IV: Current Market Value Methods

a) Value assets at current market value. Frequently used for equity securities only, but occasionally used for the entire fund.

b) Value assets at current market value but use a lower actuarial interest assumption than otherwise in order to allow for possible future downward fluctuations in market value.

c) Value assets at current market value less, in the aggregate, the estimated selling expenses (broker's commissions, taxes, trustee charges, etc.) where such expenses are borne by the pension fund directly.

Class V: Adjusted Market Value Methods

a) Value assets at market value with a minimum downward adjustment. Adjustment account increased only by yields in excess of 200 per cent of mean yield and decreased only by negative yields.
b) Use light adjustment for yields beyond the range of 50–150 per cent of mean yield.
c) Use intermediate adjustment for yields beyond the range of 67–133 per cent of mean yield.
d) Use graded heavy adjustment. Adjustment account is reduced by full amount of actuarial interest loss. Grading formula operates over range from actuarial interest rate i to $i + 0.10$. When yield y is within the range, add $5(y - i)$ times gain from interest to the adjustment account. When yield y exceeds $i + 0.10$, add 50 per cent of the gain arising from the first 10 per cent of excess yield and 100 per cent of the remaining gain to the adjustment account.
e) Use full adjustment. All actuarial interest gains added to adjustment account subject to maximum adjustment account of 15 per cent of current market value. All actuarial interest losses deducted from account.
f) Use a variable adjustment reflecting the amount already held in the adjustment account. Adjustment account is reduced by full amount of actuarial interest loss. When adjustment account is zero, use formula in *d*, graded heavy adjustment. When adjustment account is n per cent of market value, use grading formula in *d* but operating over the range $(i + n)$ to $(i + n + 0.10)$.
g) Use a variable adjustment geared to the development of an optimum adjustment account of 15 per cent of market value. Adjustment account is reduced by full amount of actuarial interest loss. When adjustment account at beginning of year is n per cent of market value at year end, add $[1 - (n/15)^2]$ times gain from interest to the adjustment account.
h) Modify any of above formulas to include minimum asset value equal to current book value.

Class VI: Present Value Methods

a) Capitalize the current amount of aggregate interest and/or dividend income as a perpetuity, using the valuation rate of interest.
b) Capitalize bond interest income and principal repayment using the valuation rate of interest. (This method is closely related to the investment approach of purchasing specific bond issues in order to match up interest income and proceeds at maturity with estimated cash-flow requirements.)
c) Value equity securities by capitalizing future dividend income, assuming a uniform percentage increase each year in the dividend return from each security.
d) Value equity securities by capitalizing the most recent year's dividend income and the proceeds at sale on the assumption that the particular security will be sold at an arbitrarily determined future date and price.

APPENDIX III

Test of Smoothness of Contribution Requirements Based on Fifteen Asset Valuation Methods

Year	Book Value, I (1)	Market Value, IV, a (2)	Adjusted Market Value: Minimum Adjustment, V, a (3)	Light Adjustment, V, b (4)	Intermediate Adjustment, V, c (5)	Heavy Adjustment, V, d (6)	Full Adjustment, V, e (7)	Variable Adjustment I, V, f (8)	Variable Adjustment II, V, g (9)
	Ratio of Contribution for Year x to Contribution for Year $x-1$								
1958	0.980	0.921	0.921	0.923	0.936	0.960	0.976	0.960	0.978
1959	0.987	0.936	0.936	0.936	0.937	0.961	0.967	0.938	0.967
1960	1.032	1.039	1.039	1.036	1.019	1.031	1.044	1.032	1.029
1961	1.176	1.060	1.053	1.102	1.124	1.161	1.096	1.124	1.108
1962	0.967	1.085	1.089	1.040	1.019	0.952	0.966	0.970	0.951
1963	1.058	0.898	0.917	0.961	0.980	1.013	1.069	1.016	1.053
1964	0.995	1.267	1.239	1.177	1.166	1.133	1.015	1.119	1.069
1965	1.052	1.025	1.024	1.023	1.022	1.022	1.047	1.022	1.026
1966	1.031	1.128	1.118	1.096	1.079	1.030	1.048	1.069	1.030
	Deviation from Book Value Ratio as Percentage (Col. [X] − Col. [1] Times 100)								
1958	0	− 5.9	− 5.9	− 5.7	− 4.4	− 2.0	−0.4	− 2.0	−0.2
1959	0	− 5.1	− 5.1	− 5.1	− 5.0	− 2.6	−2.0	− 4.9	−2.0
1960	0	+ 0.7	+ 0.7	+ 0.4	− 1.3	− 0.1	+0.8	....	−0.3
1961	0	−11.6	−12.3	− 7.4	− 5.2	− 1.5	−8.0	− 5.2	−6.8
1962	0	+11.8	+12.2	+ 7.3	+ 5.2	− 1.5	−0.1	+ 0.3	−1.6
1963	0	−16.0	−14.1	− 9.7	− 7.8	− 4.5	−1.1	− 4.2	−0.5
1964	0	+27.2	+24.4	+18.2	+17.1	+13.8	+2.0	+12.4	+7.4
1965	0	− 2.7	− 2.8	− 2.9	− 3.0	− 3.0	−0.5	− 3.0	−2.6
1966	0	+ 9.7	+ 8.7	+ 6.5	+ 4.8	− 0.1	+1.7	+ 3.8	−0.1
Mean absolute deviation	0	10.1	9.6	7.0	6.0	3.2	1.8	4.0	2.4
Root mean square	0	12.6	11.7	8.4	7.1	5.1	2.8	5.2	3.5

APPENDIX III—*Continued*

Year	10 Per Cent Offset Method, III, *c* (10)	British Perpetuity Method, VI, *a* (11)	3 Per Cent Write-up Method on Stocks, II, *a* — 1957 Start at: Book Value (12)	Market Value (13)	7 Per Cent Yield Method on Total Fund, II, *b* — 1957 Start at: Book Value (14)	Market Value, (15)
	Ratio of Contribution for Year x to Contribution for Year $x-1$					
1958	0.889	0.967	0.976	0.975	0.962	0.959
1959	0.926	0.940	0.986	0.984	0.960	0.956
1960	1.073	1.072	1.030	1.032	1.007	1.005
1961	0.980	1.204	1.178	1.193	1.167	1.176
1962	1.241	0.933	0.961	0.957	0.926	0.920
1963	0.841	1.159	1.052	1.055	1.040	1.040
1964	1.246	1.089	1.011	1.065	1.139	1.193
1965	1.036	1.021	1.053	1.055	1.018	1.018
1966	1.209	0.995	1.033	1.034	1.016	1.016
	Deviation from Book Value Ratio as Percentage (Col. [X]-Col. [1] Times 100)					
1958	−9.1	−1.3	−0.4	−0.5	−1.8	−2.1
1959	−6.1	−4.7	−0.1	−0.3	−2.7	−3.1
1960	+4.1	+4.0	−0.2	....	−2.5	−2.7
1961	−19.6	+2.8	+0.2	+1.7	−0.9	....
1962	+27.4	−3.4	−0.6	−1.0	−4.1	−4.7
1963	−21.7	+10.1	−0.6	−0.3	−1.8	−1.8
1964	+25.1	+9.4	+1.6	+7.0	+14.4	+19.8
1965	−1.6	−3.1	+0.1	+0.3	−3.4	−3.4
1966	+17.8	−3.6	+0.2	+0.3	−1.5	−1.5
Mean absolute deviation	14.7	4.7	0.4	1.3	3.7	4.3
Root mean square	17.3	5.5	0.6	2.2	5.3	7.1

DISCUSSION OF PRECEDING PAPER

CHARLES L. TROWBRIDGE:

Mr. Jackson and Mr. Hamilton are to be congratulated for their very worthwhile contribution to pension literature. As they have well stated, the asset side of the pension balance sheet has too long been neglected by the actuarial profession.

One problem with which they concern themselves arises from the erratic nature of common stock market values. Investment yields on a common stock portfolio, calculated over any short period, will vary widely (even wildly) if end-of-period market values are the basis for asset valuation. The authors report over-all investment yields varying from -10 to $+30$ per cent in the sample of cases they have examined, and the extremes would surely have been even further apart had the dampening effect of fixed-income securities not been present. One horn of the dilemma has to do with the effect on contribution levels of the violently fluctuating actuarial gain or loss resulting from valuation of common stock at market.

The use of a book value based on cost for a time avoids this horn but eventually falls to the other—by postponing the effect of market appreciation (or depreciation) until such time as the particular security is sold, yields in early years are distorted, and the asset valuation slips further and further into unreality. Eventually some adjustment toward market value becomes necessary. Most sophisticated investment persons are coming to agree with the position taken by Mr. Binns and others quoted in the paper, that is, that there is no legitimate distinction (at least for a tax-exempt fund) between dividend income and capital gain or loss, or between the so-called realized and unrealized versions of the latter.

Jackson and Hamilton approach the problem by enumerating several methods of asset valuation, most of which, for common stocks, involve compromises between cost and market. As is to be expected, market valuation and its close companions show poorly on a smoothness test; cost and its derivatives get too many penalty points for lack of fit. Still others are complicated compromises which satisfy no one completely.

It is rather surprising that the authors have said so little about what appears to be the obvious way out with respect to the dilemma they so well describe. There is little reason why common stocks cannot be valued at market on the asset side of the pension balance sheet, if at the same time a powerful smoothing device is employed in the calculation of contributions to level out actuarial gain or loss. Several methods of "spread" adjustment for actuarial gain and loss already appear in pension literature. They smooth in the same mechanics other forms of actuarial gain or loss, some of which can be every bit as erratic as investment gain or loss and often exceed the latter in amplitude. I have always felt that the spreading device inherent in the frozen initial liability form of entry age normal (as just one example) will usually be strong enough to provide adequate smoothing of contributions even if a substantial portion of pension fund assets are in common stocks and even though straightforward market valuation be employed. I also feel that if stronger smoothing devices are needed they can be devised—and if necessary sold to the IRS and to the accounting profession. Stronger spreading methods may be needed, particularly for plans that have not used market values for asset purposes in the past but now intend to do so.

It is an injustice to the authors to imply that they are unaware of spread adjustment techniques, for I am reasonably sure that they employ some form of spread in their normal actuarial valuation procedures. They might tell us what techniques for actuarial gain or loss adjustment were used in calculating Table 1. I have the impression that they are letting actuarial gain or loss affect the unfunded supplemental liability and then are spreading the latter (and hence the actuarial gain or loss) over the period remaining for supplemental liability amortization. If so, the spreading technique becomes progressively weaker as the end of the amortization period approaches. In any event the details of the actuarial cost method used are needed for adequate interpretation of Table 1 and Appendix III. What is really being illustrated in these tables is the effect on contributions of the particular *combination* of asset valuation method and the gains adjusting features of the actuarial cost method.

Although it is apparent by now that I prefer for common stocks the combination of market valuation and a spread technique for adjustment of actuarial gain or loss, I look differently at valuation of bonds or other forms of indebtedness. The so-called amortized value represents the present value of future debt service discounted to the present at the rate of interest associated with the original investment. Market value is the present value of future debt service discounted at the rate of interest at which today's investor is willing to invest. The difference in the two discount rates can be associated with a change in the market's evaluation of the security behind the investment but is more likely to be largely a function of changes in the money market. If general interest levels have risen since the bond was bought, market value is likely to be lower than amortized value, but, if interest levels have declined, market value is likely to be above amortized value. The fact that bond (and mortgage) market values vary inversely with the interest rate may account in part for the tendency noted by the authors for bond values to vary inversely with common stock prices. This could possibly be demonstrated if common stock prices and long-term interest rates could be shown to move together. Unfortunately, investment forces are not that simple.

Note that market valuation of bonds and other debt investment implies a "new money" approach to asset valuation—and might be viewed as calling for a similar approach to valuation of liabilities. If the interest rate assumed for valuation of pension liabilities is derived from interest rates now available for new investment, market valuation of assets is logical. If the valuation rate is derived from the average of interest rates available over a long-time span including the past, amortized value would seem to be more appropriate. As a practical matter pension actuaries do not change interest valuation assumptions very often and are today closer to an average figure than to a new-money rate.

Other factors as well lead to avoiding market value as the valuation measure for debt instruments. Many pension funds truly expect to hold bonds to maturity in more cases than not; so the long-run capital gain or loss is likely to be zero and the yield actually obtained equivalent to that on which amortized value is based. If so, there seems to be little point in introducing actuarial gain or loss, which will wash out eventually. A practical problem is that market value is not easily obtainable on mortgages or privately placed bonds or debentures, though there is a market for most types of publicly offered bonds.

It seems that both theory and practice lead to amortized value as the best measure of the value of debt instruments—except in the special situation where a new-money rate is being assumed in the valuation of pension liabilities. In a recent and as yet unpublished study undertaken by Frank Griffin and me, market values were chosen to value bonds (as well as stocks) because the pension assets were being compared to close-out liabilities calculated on a current interest basis.

DONOVAN T. BLANKLEY:

As noted by the authors, this subject has become of increasing importance in recent years, and I feel that we are fortunate in having this very thorough presentation of the different asset valuation procedures available. Naturally the variations and combinations of these procedures are innumerable, but one very simple type of variation which we have found useful is worthy of mention because of the flexibility and ease of its application as well as the satisfactory results produced.

The procedure classified by the authors as Class III, *a*, under which a moving average market value of each security is obtained, should be a satisfactory valuation basis for many situations. Anyone who has attempted to do a valuation of this type, however—with stock splits, stock dividends, spinoffs, and mergers each presenting separate problems—will hesitate to value a large fund in this way. On the other hand, the method of Class III, *e*, is applied to a portfolio in the aggregate, but there is no indication of how to adjust for fluctuations of the unrealized gains which can be produced by the incidence of taking realized gains and losses.

The moving average of market values has been spurned by other writers, but we believe that it can be applied in a way which exhibits the five desirable characteristics listed in this paper. Our procedure for this basis is to obtain an asset value which is equal to the aggregate book value plus an appropriate percentage of the average adjusted unrealized appreciation over each of the past four years. The adjusting merely consists of reducing the dollar amount of net unrealized appreciation in each previous year by the amount of net realized gains taken since that year. The adjusted amounts may be weighted in favor of the more recent years if deemed desirable. Also, in the case of a rapidly changing fund, one may wish to modify the adjusted amounts of previous years by the ratio of the current book value to the book value of each of such years. For the usual case, in which this latter correction is not made, an actuarial clerk can produce each succeeding year's total asset value for a fund in a few minutes, using a year-to-year worksheet form.

STUART J. KINGSTON:

The authors have created a milestone paper by presenting a strong case for the view that the valuation of assets is just as clearly in the province of the actuary as is the valuation of liabilities.

Furthermore, the authors have pointed out that the underlying concept in the valuation of assets is credibility theory and that all asset valuation methods in use are credibility methods, either refined or crude, as the case may be. Further development of the credibility approach will in time lead to asset valuations which are as reliable as liability valuations, and, unless this is done, the unfunded liability, the calculation of which is the primary goal of pension plan analysis, will necessarily be as unreliable, if not more unreliable, than the more unreliable of the twin valuations of assets and liabilities.

Actuaries have been reluctant to apply credibility theory to areas other than life contingencies and casualty contingencies.

A suggestion to apply credibility theory to the investment-year method of allocating income (in group pension business) appears on pages 342–44 of Volume XIII of the *Transactions of the Society of Actuaries* (1961). To the best of my knowledge, the suggestion has not yet been developed into workable form.

Perhaps the stimulation provided by Messrs. Jackson and Hamilton will pervade the many areas crying out for credibility theory applications.

RICHARD DASKAIS:

The authors have presented an excellent analysis of the methods of valuing assets and how they measure up to the five objectives listed in the paper. My discussion is directed to the desirability of the first objective (stability of values) and of the fifth objective (safety margins).

Is it necessarily desirable for the actuary to initiate the use of a conservative asset valuation method incorporating a margin of safety? The corporation's selling stockholders may not wish to pay for the charge to current earnings for conservatism. Management may be misled as to the cost of the benefit formula in effect, which may affect decisions to amend the plan or on other employee benefits. If a particular method—such as adjusted market—can be relied upon to produce, more often than not, a large adjustment account when there are temporary aberrations in market value, then this is a valuable investment tool that should be used by the fund's investment manager. A large adjustment account indicates that it is time to sell stocks and buy bonds; doing this seems preferable to adopting a safety margin in anticipation of the wrong decision (not switching from stocks to bonds) by the investment manager.

When we artificially stabilize asset values in order to stabilize contributions, we seem to be building stability upon stability. Under *APB Opinion 8*, the effect upon pension cost of gains and losses is to be spread or averaged. There seems to be little justification for providing a higher order of stability for a change in pension cost due to a change in asset values than for a change in pension cost due to large pay increases, bargaining a more liberal pension plan with the union, or the publication of Revenue Ruling 63-11. Perhaps there is a feeling that pension fund performance is not as much a part of an employer's operations as is, for example, granting steep pay increases. But is not the fact of the matter that the employer has made pension fund performance one of the business risks he takes when the fund is invested in common stocks? Smoothing the effects of changes in pension fund market value through an adjustment account does not seem much different to me from an air-conditioner manufacturer's establishing an adjustment account for a cool spring and early summer to which he would credit part of the income from sales in a hot June; if you have a pension fund, your earnings fluctuate with stock prices and, if you sell air conditioners, your earnings fluctuate with the weather.

Since most employers have some flexibility of contribution to their pension funds, use of market value need not result in sharp changes in contributions (as opposed to changes in pension expense). If employers remain reluctant to contribute different amounts than are charged, I am not sure that economists would generally agree with the paper's conclusion that "countercyclical changes in pension cost are more nearly in the best interest of all concerned." Heavy contributions in times of low stock prices (and low corporate earnings) would appear to reduce total corporation income taxes and to increase the supply of private capital with a resulting decrease in interest rates—and perhaps incidentally to produce better pension fund investment results through larger stock purchases when prices are low.

A possible disadvantage of insulating the employer's cost from fund investment results is that management may not pay as much attention to the quality of the fund investment management as it should.

If stability is desirable, is it not stability of contribution or charges rather than stability of asset value? Why not apply whatever dampening formula is necessary directly to the effect of all gains or losses on the plan's normal cost (as a percentage of pay or per employee, as appropriate) or on the unfunded prior-service cost? Adjustment accounts could indicate the amount and sign of the correction made through the dampening formula. Some ingenuity might be required to produce methods that would be satisfactory to the IRS, but the effect of the adjustment would be clear to the reader of the actuarial report. There should not be any difficulty in finding methods satisfactory to accountants, since the dampening would essentially consist of averaging or spreading large gains and losses over a longer period than small gains and losses.

An area in which asset stability may be desirable, to avoid misunderstandings, is the asset value used for disclosure under *APB Opinion 8* of the excess, if any, of the value of vested benefits over the sum of fund assets and balance sheet pension accruals.

The authors' conclusion that the choice of asset valuation method must be made for each plan should be emphasized. In most plans with which I am familiar (generally valued at cost), the difference between market and cost is not so great that there would be significantly different contributions and pension charges if market were used. In these cases, it may be easier for the actuary to use cost than to explain the less conventional use of market because it is theoretically justified. But, when significantly different contributions and pension charges would result from the use of market value, it seems market should be used, with smoothing by one of the methods suggested by the authors if required by the objectives of the employer.

CHARLES B. BAUGHMAN:

The inclusion of the quotations in Appendix I was helpful. One must conclude from these quotations that in qualified plans, which are exempt from taxes, there should be no distinction between income, realized gains, and unrealized gains and that the cost of securities should be ignored in measuring their value. I could not agree more, and I am sure that all good investment decisions are based on these principles.

There is a question, however, of whether these principles should be followed in actuarial valuations. I believe the answer is yes. Therefore, I disagree with the sentence in the paper which reads, "The use of straight market value with an inflexible funding method can easily result in pension contributions that increase with depression and decrease with prosperity, whereas countercyclical changes in pension cost appear to be more nearly in the best interest of all concerned." The first part of the sentence is correct, particularly when the contributions are calculated by an aggregate method in which the difference between unfunded liabilities and the funds is divided by a temporary annuity. However, it is the second part of the sentence which is bothersome. When the funds consist largely of common stocks, it seems more desirable that contributions increase with depression because that is when stocks are likely to be available at bargain prices. Valuing at market would probably reduce the employer's long-range cost, because the lower security prices are, the higher the contributions would be.

Incidentally, this same principle can be applied on the benefit side of the plan. When this is done, it indicates that a variable annuity plan is more desirable than a cost-of-living plan funded with common stocks. The variable annuity plan bases benefits on market price, but a cost-of-living plan bases benefits on a concept substantially equivalent to cost. The result is that in high markets there are high benefits with variable annuity plans and relatively less net money invested at high security prices than is true in a cost-of-living plan. Let me say that the point here is not that the price of a variable annuity plan will cost less; the point is that with a variable annuity plan the ratio of the value of employee benefits to employer cost is higher.

J. BRUCE MACDONALD:

I should like to compliment the authors on both this paper and the allied one presented to the International Congress in Munich. They contain a wealth of valuable information and should become classics. My comments are from the Canadian point of view.

Canadian actuaries have not tended to use such complicated techniques as many of those outlined. I believe that is because both the Department of National Revenue and the chartered accountants have been much more flexible in their approaches than their United States counterparts. Hence, less theoretically elegant methods have been acceptable.

There is no doubt that actuaries will be much more concerned with asset values in the future than in the past. With the advent of solvency standards in Canada, it will become necessary to pay closer attention to the valuation of assets. It should be remembered that Canadian pension funds usually have a smaller equity holding than American funds, so that the valuation of bonds assumes a more important role.

I cannot see how the valuation of assets can be divorced from the actuarial assumptions used in the valuation of the liabilities. It is logically indefensible to use a rate of, say, $4\frac{1}{2}$ per cent, in determining the liabilities and then to use a market value for bonds that effectively assumes a rate in excess of $6\frac{1}{2}$ per cent. Even using book value for the bond portfolio can imply a rate of $5\frac{1}{2}$ per cent or higher. This becomes particularly important if the valuation reveals an unfunded liability which provincial law requires to be amortized. It would thus seem to me that this problem can be solved by valuing the fixed income securities by the authors' Class VI, *b*, method.

As far as equities are concerned, I cannot completely separate their valuation from the salary scale used in determining the liabilities. For example, if one assumes that salaries increase by reason of inflation (i.e., by reasons other than promotion or seniority) by 7 per cent per year, it seems reasonable to make a corresponding assumption with respect to dividends and market value for equity portfolio—provided one accepts that salary levels and equity prices over the long run keep in line. (Of course, these comments have no application to career average plans.)

On final-average salary plans our office is coming to the conclusion that one solution is to value the liabilities at realistic rates, taking into account current conditions, such as interest of $8\frac{1}{2}$ per cent and an allowance for wage inflation of 6 per cent per annum in the salary scale, and to take equities at market value. This may well result in an irregular procession of asset values but may solve the unfunded liability problem.

In general I cannot accept market value as a proper valuation of the assets—primarily for the reasons alluded to in my comments on bonds. Market value has a meaning only if a plan is being liquidated. In that event the liabilities should be determined by using rates close to current ones or by using the cost of purchasing all the accrued benefits from an insurance company. As Canadian companies use an interest rate of around 7 per cent in their annuity rates, this highlights the absurdity of using market values for assets (especially bonds) but a conservative valuation rate of, say, $4\frac{1}{2}$ per cent for the liabilities.

JAMES L. CLARE:

Consider actuarial advice given purely for management purposes. The employer wants to know about such things as (1) the *real* surplus or deficit, (2) the best possible estimate of future cost, (3) how much the price to be paid or demanded for his company in the event of an acquisition or a merger should be modified by the pension funding situation, and (4) what to do about increasing benefits. (This last item is becoming of more importance as more employers are showing an interest in increasing benefits after retirement, through "cost-of-living pensions" in the United States or through "purchasing power pensions," as they are called in Canada.)

Consider the employer who has his plan valued simultaneously in four different offices of a firm providing actuarial advice. In each case the employee data will be absolutely identical. Assume also that the same funding methods and the same actuarial assumptions are used in all four cases. Assume further that in each case the asset portfolios at the valuation date are all absolutely identical. However, in spite of the identities, radically different answers could be given in each of the four cases, depending upon how the assets are treated.

Suppose that the first actuary is led to believe that the plan has been carried on a paid-as-you-go basis and has only just been funded by a lump-sum purchase of investments on the valuation date; then the asset value he uses would be the market value at that date in an amount, say, of $900,000. Suppose that the second actuary understood that the assets had been acquired over a ten-year period and were valued by the use of method *X*; then he might assume that their value was only $800,000.

Third, if the assets were valued by method X again but assumed to have been built up over a twenty-year period, their value might be taken as only $700,000. Fourth, if the assets were assumed to have been built up over the same twenty-year period in identically the same way as they were in the third method, but were valued by method Y, a value might be set on them of only $600,000.

I wonder whether actuaries should be put in the position of having to explain how they can get four different answers to the same problem, using the same employee data, the same funding methods, and the same actuarial assumptions?

Also, the methods set out by the authors in their paper seem to assume that the liabilities under any one plan will invariably be valued at the same interest rate, from valuation to valuation, with this valuation interest rate seldom changed.

I suggest that the problems under consideration in the paper result from our habit of first valuing liabilities and then moving to a consideration of the assets. I suggest that, at least in giving advice to a company management about the real surplus and the contribution picture under its pension plan, it would be better to proceed as follows. First consider assets, always taking these at their market value, and then move to a consideration of liabilities.

As I have noted several times in the *Reports of the Canadian Institute of Actuaries*,[1] when the market value of assets is high, the yield from dividends and from interest in relation to that market value tends to be lower, and vice versa. Thus, when asset values have fluctuated higher, a lower interest rate would be quite reasonable, not to say preferable, for valuing the liabilities.

Thus, always using the market value of assets when giving advice to managements would seem to me to have the following advantages: it is economical; it makes sense; it avoids the problem of the same calculations on the same data giving a variety of contradictory answers; it takes into account the fluctuating nature of investment conditions from time to time, both as to values and as to yields; it eliminates artificiality from at least the asset side of the valuation and concentrates such adjustments as may be required on the liability side.

Again, purely for giving advice to managements, if assets are taken at market value, substantial realism is needed throughout the work. It will be essential to use salary scales which really reflect past history and which reasonably estimate future rates of escalation, from all directions.

[1] September, 1965, pp. 22–23; September, 1966, p. 8; and February, 1967, p. 62.

I am not suggesting the invariable valuation of assets at market value for other work, for the following reasons. The Internal Revenue Service does not permit realistic salary scales, and therefore valuing assets at market removes a "cushion" many employers like to have. In reporting under the Welfare Disclosure Legislation, any change in the interest rate requires substantial additional work to comply with that legislation, so that there is much to be said for continuing with the artificial type of valuation work which regulators seem to expect. Many of the requirements of accountants with respect to the valuation of pension plans, both in the United States and in Canada, would seem perhaps lacking in consistency and effectiveness, not least when considering assets; I wonder whether actuaries should not take the initiative and explain to accountants that they are missing their targets. In situations such as these (including, perhaps, some aspects of union bargaining), customary actuarial valuation practices, including all the various asset valuation methods set out by the authors in this paper, will no doubt continue to be employed.

With respect to giving advice to managements, however, I would agree most emphatically with the quotation presented by the authors that "for any particular fund, a fairly computed market value is the best measure of its real, or realizable, value at any moment of time."

BLACKBURN H. HAZLEHURST:

Perhaps the actuary should base his valuation on the market value of securities or some function of the market value of securities. Or perhaps the actuary should determine an actuarial value of assets based on a judgment determination of the dollar amount of future rights flowing from those assets, discounting those dollar amounts at the valuation interest rate. In any case, and particularly in view of the fact that there will be a diversity of opinion and bases for determining the actuarial value of assets, it seems important to disclose the framework of reference by which others may independently judge the assets and relate them to the value the actuary has used.

It is perhaps beyond the scope of the paper to discuss problems of determination of market and book values. It would appear important, however, to disclose these values, and, since a very large proportion of the pension assets in this country is held by insurance companies, it seems pertinent to the subject of the paper to inquire into the meaning of market value, for example, in the case of assets backed up by general insurance company portfolios (i.e., other than those supported by separate accounts which have a clearly determined market value).

The increasing attention being paid to the changes in asset values, and the growing recognition of various alternatives in investments, suggest that there may be an increasing amount of shifting of assets from one investment medium to another within an insurance company, among insurance companies, and among other investing agencies, as different modes of investment wax and wane in success from time to time. Accordingly, it seems important for the actuary to disclose in his report the realizable value of insurance assets, just as he should disclose the market value of noninsurance assets, whether or not he bases his determination of actuarial assets on these values.

In short, I encourage discussion of the determination of realizable value of insurance assets and the significance of such a value to the actuary in making his valuation, for example, in determining experience yield.

One approach to determining the realizable value of insurance assets which are able to be "sold," and so might be "sold" if the plan sponsor's investment adviser so suggests, would be to ask the insurance company to determine as of the valuation date the lump-sum withdrawable value. If such a determination is unavailable, the actuary can look to the contract and discount the contractual installment payments available in the event of withdrawal, at the valuation interest rate. Where the assets are expressed in terms of guaranteed benefits with no specific dollar values being available, the actuary might indicate the value of those benefits based on his valuation assumptions, perhaps footnoting the value of dividends discounted at the valuation rate on the presumption that dividends will continue in accordance with the past trend, if there is a trend.

HOWARD H. HENNINGTON:

In the conclusions in this interesting paper, the authors state that "the choice of an appropriate measure for asset value will depend on . . . the actuarial funding method used." The purpose of my discussion is to emphasize the validity of that point by calling attention to the fact that some actuarial funding methods naturally spread the effect of actuarial gains and losses and other actuarial funding methods do not spread such gains and losses. One of the tests used by the authors for acceptability of the different methods for determining asset value was a test of smoothness of the resulting annual contribution. The smoothness of the annual contribution is determined not only by the method for determining asset value but also by the actuarial funding method.

One of the actuarial cost methods most commonly used is the projected benefit method with supplemental liability, otherwise known as the entry age normal method. If this method is used with a frozen supplemental liability, all gains and losses are spread into future normal cost, and this spreading serves to increase the smoothness of the resulting annual costs. I believe that this aspect of smoothness has contributed significantly to the popularity of the projected benefit method. Under a typical calculation, the actuarial gain or loss in a particular year might flow into the next year's annual cost at the rate of approximately 8 per cent. The actual percentage would be the ratio of the total annual salary of the employees in the actuarial calculation to the present value of future salaries. Of course, there would continue to be further effects in subsequent years at reducing levels. This also means that gains and losses would to some extent be offsetting in their effects on future years' costs, and this, too, would contribute to the smoothness of the results. An actuarial cost method involving a spreading of actuarial gains and losses makes it easier to use some of the market value methods outlined by the authors.

The authors state that the smoothness tests presented in the paper were based on costs under the entry age normal cost method, but it was not clear to me whether or not a frozen supplemental liability was used. Since it has a bearing on the resulting tests, it would seem desirable for the authors to clarify this point.

THOMAS L. WILLS AND JAY C. RIPPS:

Messrs. Jackson and Hamilton have produced a paper which will be read by many actuaries confronted with the asset valuation problem. Their capsule descriptions of the various techniques will be especially important; therefore, we would like to suggest one further category of techniques which does not appear in their lists—the accumulated value methods. We have found them convenient for use in the valuation of the equity portion of pension funds.

An accumulated value method, in common with a present value method, does not depend on either initial cost or current market values in assigning fund values. The fund value is the accumulation at interest of the actual income to, and disbursements from, the pension fund, using some assumed rate of return.

This method is quite similar to the methods described by the authors as Class II, *b*, but it differs from them in that the asset value is computed without reference to either cost or market value. The income considered is all cash contributions to the fund since inception but only those directly from the employer (i.e., excluding dividends and interest); the disbursements are only those which are permanently removed from the fund (i.e., excluding those disbursements for investments which are retained as part of the fund). Depending on the action of the market or on the rate of investment turnover in the portfolio, this method may produce values which are above market value or below initial cost. We suspect, therefore, that the authors would assign it a large number of "penalty points." The theory is, of course, that both the market value and the cost value, at a point in time, may be irrelevant to the long-run value of a portfolio. However, if the fund value, according to the accumulated value technique, differs markedly from the long-term trend of market values, an adjustment is made to the fund value or to the assumed rate of return, or to both. As the authors suggest, the actuarial interest assumption should probably be the same for the valuation of both assets and liabilities. It does not follow, however, that the only practical adjustment to fund values is the one-shot type; if the assumed rate of return proves to be inappropriate, it should probably be changed along with any other adjustment. The effect on contribution rates could, of course, be substantial, but, in such a situation, a substantial change in contributions will generally be in order.

Any asset valuation technique, viewed in retrospect, is merely a device for assigning portions of income and appreciation to various periods; as such, it is somewhat similar to an actuarial cost method, which assigns portions of pension liabilities to various periods. The accumulated value methods divide the growth of the fund into level percentage increments, and we would suggest, therefore, that they parallel, and are suitable for use with, those actuarial cost methods which spread liabilities in a level manner.

To move on to more general comments on the paper, in testing for smoothness, the authors have chosen the book value method as the "smoothest" method. It is admittedly difficult to devise a test for smoothness, but, since the book value method builds in the effect of deviations in dividend income, realized gains, and so on, it may not be a particularly appropriate index. A better, but more difficult, test might be performed by considering the progression of unit costs (percentage of payroll, cost per hour, etc.) from year to year, after removing the effects of plan amendments, changes in assumptions, and actuarial gains and losses other than those based on assets. The smoothest method, but not necessarily the best method, would then be the method with the smoothest set of unit costs.

Finally, we are somewhat puzzled by the section headed "Acceptability of Asset Measure." The authors assign penalty points if the asset value under a given method was significantly less than or exceeded, even by small amounts, the market value during the ten-year test period. On the other hand, in footnote 8, they state, "For example, special asset valuation methods were imposed by state insurance commissioners during the early 1930's to prevent the temporary apparent insolvency of some insurance companies, which, taking a realistic long-run view, were in sound condition." It seems to us that the asset valuation technique used in a pension funding program should take "a realistic long-run view" and that, so long as the pension plan is not likely to terminate imminently, market value may well be unrelated to such a long-run view. Therefore, a test of acceptability which depends on the degree of closeness to market value and on never exceeding market value is questionable.

(AUTHORS' REVIEW OF DISCUSSION)

PAUL H. JACKSON AND JAMES A. HAMILTON:

The authors wish to express their appreciation to each of the Society members who has taken the time to prepare a formal discussion. The number of such discussions seems to indicate a considerable amount of current interest in this topic, and the discussions themselves, representing so many different points of view, serve to add significantly to the value of the paper.

Mr. Trowbridge suggests that one obvious way of avoiding substantial variation in pension contributions as a result of market value changes is to employ a powerful smoothing device in the calculation of contributions. If such a smoothing device were powerful enough, for example, to minimize the effect of a 10 per cent swing in asset value on current contributions, the end result would be that a 10 per cent variation in the funding achieved under the plan would have a similarly minimal effect on contributions. For the plans actually valued and tested in the paper, the actuarial cost methods probably produce a spreading comparable to that under the frozen initial liability form of entry age normal funding approach. For example, the average amortization factors used in adjusting contribution requirements for asset changes ranged from about 5 to 8 per cent over the ten-year period studied, so that gains and losses were effectively spread over a period comparable to that under the frozen initial liability method.

Mr. Trowbridge indicates a preference for market value of common stocks and amortized value for bonds. In fairness to Mr. Trowbridge, he is no doubt addressing himself to the problems faced by an insurance company holding assets that back pension liabilities. While insurance companies have followed the practice of using amortized value for bonds, among noninsured trusteed pension plans amortized value is generally not available, since the trustee would normally report initial cost for the securities held, along with their current market value. For the pension funds studied, the use of market value for common stocks and amortized value for bonds would have produced a total asset value in excess of market value in every single one of the ten years valued, with the asset value ranging from 101 to 109 per cent of market value. Where the total assets of a pension fund stand in back of all the benefit expectations, it seems unreasonable to deliberately assign a value to the assets that is consistently greater than the aggregate market value. As to the interest rates being used, here again the insured approach is likely to differ from that used for a noninsured trust fund, since a single, over-all rate of interest would normally be used to discount all potential future benefits without regard to those liabilities backed by bond investments or common stock investments, as might be the case with an insured arrangement.

Mr. Blankley and others have found that satisfactory results can be produced by using an average of the ratios of market value to book value over a three- or five-year period. The authors agree that in many cases such a method can produce reasonable asset values, and the method is certainly simple and understandable in application. On the other hand, to the extent that book value is irrelevant, any formula based on the ratio of market value to book value or an average of such ratios must contain some degree of irrelevancy, and values actually produced by the method described by Mr. Blankley can vary considerably among various funds, depending upon whether the trustee is turning over common stocks at a rapid pace and realizing substantial capital gains on a book value basis or, alternatively, shifting over from bonds into stocks and taking substantial capital losses. In short, we believe the method requires the most careful attention by the actuary as to the reason for the increases and decreases in the book value itself should any unusual increases or decreases arise.

Mr. Kingston suggests that the credibility theory may well be the most satisfying approach to asset valuation, at least from a theoretical standpoint. While this approach seems to hold considerable promise, at the moment so little has been done in the asset valuation area that credibility theory simply is not currently productive of truly practical formulas.

Mr. Daskais and others have observed that smoothness in pension contributions is not the only desirable objective. The authors generally agree with Mr. Daskais' comments, although we believe that methods for stabilizing asset value constitute another useful tool that an actuary may employ in achieving a desired goal. Mr. Daskais observes that heavy pension contributions during bad times and light contributions during good times may well produce better pension fund investment results over a long period of time. To the extent, however, that companies in the past have found it useful to contribute, and to charge off as pension cost on their financial statements, maximum amounts in good business years and then to pass up pension contributions entirely in poor business years, it seems clear that countercyclical changes in pension costs have been viewed, in the past at least, as being generally desirable from the standpoint of the employer and his financial statement. In any case, as Mr. Daskais points out, the choice of asset valuation methods must be tailor-made for each plan so as to meet the employer's objectives and, furthermore, the details of the method should be carefully explained to the employer so that he can understand just what is involved.

Mr. Baughman, too, questions the desirability of countercyclical changes in pension costs from the standpoint of the long-range pension values accumulated under a variable annuity plan. To the authors, however, while investment considerations suggest rather strongly that an increase in contributions during a depression may well result in far greater investment return, from a practical standpoint the individuals and corporations paying the cost find themselves in the poorest financial condition at that very time, and an artificial increase in the required contribution might actually lead to the termination of many plans rather than to the investment advantages pointed out.

Messrs. MacDonald and Clare addressed their comments to the Canadian scene. It is interesting to note that Mr. MacDonald believes that market value is not a proper value for assets but that a method comparable to the British perpetuity method is to be preferred, whereas Mr. Clare believes that market value is clearly the preferred method since it eliminates artificiality from the asset side of the valuation and concentrates any adjustments on the liability side. Both Mr. MacDonald and Mr. Clare, by implication at least, suggest the desirability of running several actuarial valuations on differing assumptions at one and the same time; for larger funds such an approach is desirable if management is to appreciate the true financial condition of its pension fund.

Mr. Hazlehurst points to the application of the asset valuation methods, considered appropriate for noninsured trusteed pension funds, to the assets held by an insurance company covering a portion of the liabilities under a split-funded program. This is, in reality, another application of the suggestion that double valuations be conducted—in this case one by the insurance company on its regular assumptions and the other by the consulting actuary on the basis of realistic, nonconservative assumptions—in order to get some estimate of the potential future dividends that might arise from the insurance portion.

Mr. Hennington also raises the question of the effect of the actuarial cost method on the relative smoothness of the annual contribution. The paper indicated that contribution requirements for the pension plans tested had been based on the entry age normal cost method. For the particular funds studied, the amortization period ranged from a minimum of fourteen years to a maximum of thirty years, with the average amortization factors falling in the 5–8 per cent range, as compared with the rough 8 per cent figure mentioned by Mr. Hennington. Clearly, had shorter amortization periods been involved, the amortization factors could have been significantly greater, although in practice such programs are generally converted over to an aggregate cost method or to a frozen initial liability method in order to avoid the wide variations in cost that might otherwise occur under the entry age normal method with fixed amortization as the end of the amortization period approaches.

Messrs. Wills and Ripps suggest an accumulated value method, which is independent of both initial cost and current market value and which would operate in much the same manner as the 7 per cent yield method. To the extent that such a method produces asset values completely independent of the current market value or current book value, the general concept is not likely to meet with the approval of the financial executives responsible for the management of the funds. Messrs. Ripps and Wills appropriately point out that the choice of book value as the smoothest method may not be appropriate in all circumstances, since book value itself may vary considerably with deviations in dividend income, realized gains and losses, and the like. For the funds under consideration, however, the pattern of such gains and losses, along with the dividend and interest income received by the funds, was quite stable in the aggregate over the period studied, so that a determination of unit costs as a percentage of payroll or costs in cents per hour would not have produced measurably different results. Finally, Messrs. Ripps and Wills point to the apparent inconsistency in our stating that asset value in depression periods might well exceed market value while assessing penalty points in our test of acceptability of asset value if the asset value rose above the market value. Over the ten-year test period there were no unusual investment considerations such as those which applied during the early 1930's, and it was felt by the authors that an asset valuation method producing a value in excess of market value during a prolonged ten-year period of favorable investment conditions would not be considered to produce reasonable results. In fact, the authors have not taken the position that the asset value for actuarial purposes ought to be close to market value and should never exceed market value. That test of acceptability was introduced in order to take into account the current attitude of the accountants and other interested parties in the sense that, if total asset value were within a reasonable percentage of market value, they would not be likely to raise undue criticism; while, at the other end of the scale, if asset value exceeded market value, the financial executives responsible for the management of the fund might well indicate their displeasure with an unrealistic asset value.

The authors are delighted that the paper has evoked thoughtful discussion from so many Society members. It is indeed one of the best compliments which can be offered under the circumstances, and we are deeply appreciative.