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CORPORATE MODELS

Teaching Session

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What can a corporate model do for you?
How do you work with it?
How do you vary or test your assumptions?
How can it help you solve your problems?

MR. ROBERT J. JOHANSEN: The purpose of this teaching session is to show you what you can do with a corporate model, how you can use it to solve your problems and hopefully not add to them. We will try to provide you with enough knowledge so that you will at least think seriously about using this technique.

A definition of a model appears in a paperback, The Use of Financial Models in Business, published 1975 by A. D. Little for the Financial Executives Research Foundation:

"... a symbolic and quantitative (i.e. mathematical) description of relationships among variables affecting, or reflecting, the results of business activity. Use of the model requires processing and provides for some degree of manipulation by the user It provides results intended for use by senior management (at the corporate, subsidiary or division level) in determining strategy for the firm or for a profit center within the firm The purpose of a financial model is to influence strategic decisions by revealing to the decision maker the implications of alternative values of these financial variables...."

According to a study published last June by Social Systems, Inc., over 2000 corporations in the United States, Canada and Europe are using, developing, or planning to develop a corporate model. Only a handful of companies in the survey had actually developed an integrated model including financial, marketing and production. Four out of five companies had modeled their financial structure, two out of five their marketing and production activities. About two-thirds of the models were prepared in-house. The average cost was \$80,000 and some eighteen man-months were required.

Most of the models were deterministic, that is, they did not include any random or stochastic values. Risk analysis models accounted for 6 percent; most models were the "what if" type designed to test for the effects of changes in management philosophy or changes in the external environment. Some were designed for optimization of items such as profit or cost.

The survey indicated that corporate models enjoyed high levels of support by management. In nearly half the cases, the president was involved; in nearly half the controller. The survey also showed that in most cases the planning department had development responsibility - the finance, operations research and management science units were frequently given the responsibility. In an insurance company, I daresay the actuarial department would have the job.

A recent limited survey of a few LOMA members identified 24 life insurance companies that are using a corporate model to project operational and financial results, and 4 other companies that are developing such a model. The definition of "corporate model" was one which would attempt to produce a balance sheet for the "whole company." Normally this would require the use and interconnection of at least the following submodels--liabilities, operations and surplus submodel; asset submodel; Federal income tax submodel; and expenses submodel.

The composition of these individual company models, their nature, how they were developed and the frequency of their making projections vary considerably among the 24 companies. Of surprise to some who have seen the results of the survey is the fact that half of the companies make only 5 or less alternative projections when they are testing proposed results.

The LOMA survey summary will be mailed to participating companies within the next 60 days, and will be available to all LOMA members at that time upon request.

A word about special simulation computer languages. These range from languages like GASP* which utilize FORTRAN or ALGOL together with subroutines to be called in by specific commands to produce a program, to those which use block diagrams like GPSS+, and to complete simulation languages like SIMSCRIPT which are specially designed for model construction.

Corporate models have many uses, and may be constructed to serve one or several purposes. For example, a corporate model could assist in the tests required under Proposed Recommendation 7, Statement of Actuarial Opinion for Life Insurance Companies annual statements - the section that reserves and other actuarial items make a good and sufficient provision for all un-matured obligations of the company guaranteed under the terms of its policies.

A word of caution, a corporate model should be used primarily to assess the direction and relative magnitude of the effects of changes in assumed future conditions such as interest, level of business, lapses, expenses, rather than to state that, on the basis of realistic assumptions, the gain from operations in 1982 after F.I.T. will be \$547,236.25.

The apparatus used for the teaching session consists of a computer terminal - it looks like an electric typewriter - and an acoustic coupler which converts electric impulses into audible signals to travel via telephone lines to the National CSS computer. We are using the National CSS computer because the SOFASIM model was designed on it and the program resides there. The TV monitor screens are here only to show you what is being typed on the computer terminal. I would like to thank National CSS for their assistance in this presentation.

We are indebted to a number of people for considerable effort expended to put on this session - to Dick Charnin and Bill Mulligan of National CSS, Hector Novell, Ron Hindman, Doug Weiss of IBM, who graciously supplied the 2741 terminal and also to Alice Goldstein of North American Re, Bill Hoop of American Bankers Life Assurance and Karen Edmondson and Mike Goldberg of Financial Data Planning.

*General Activity Simulation Program

+General Purpose Simulation System

Mr. John Woodydy, Chairman of the Joint Committee on Theory of Risk, which developed SOFASIM, will describe the model and tell why it was constructed. He will also cover some of the tasks for which it can be used. Dr. Barbara Markowitz, who has been deeply involved with SOFASIM from the beginning, will describe the model in detail. Her husband, Dr. Harry Markowitz, designed and programmed the model and was also responsible for SIMSCRIPT, the programming language intended for model building. Anthony Amodio will run the computer terminal and describe what he is doing. He will also act as the Recorder for this session.

Because of limitations of time and cost, we have taken some liberties for purposes of this demonstration. For some of the more time-consuming data entry items, we have already entered alternative items on the disk file and compiled a special executive routine to handle changes. An edit function has been built into the model. It can be used to substitute factors and is a very powerful tool. While we may not be able to handle in this session every change of assumption that you might think of, we can, for example, accept changes in one or two commission rates, or expense or lapse rates.

Before turning the teaching session over to Mr. Woodydy, I would like to call your attention to the fact that SOFASIM is not the only life company model available. For example, Insurance Systems of America has one and Scientific Time Sharing provides one on its time-sharing computer system. Use of an existing corporate model should be carefully considered. It permits you to have ready access for problem solving, avoids the necessity to provide the specialized expertise required to design your own, and, last but not least, avoids the considerable cost of development.

MR. JOHN WOODYDY: SOFASIM was built to answer questions with regard to adverse deviation in future mortality, interest, lapse and expense of stock life insurance companies. One way to model a company is to apply mortality and lapse rates to exposures, apply interest rates to assets, project expenses, calculate reserves, and move the company into the future on a sort of aggregate basis. However, the overall view of a life insurance company which we attempted to embody in the model was that of individual insureds taking out policies, paying premiums, dying and requiring death claims to be paid, surrendering and receiving cash values, with accompanying payment of commissions and other company expenses, and investment of company funds in bonds with specific coupon rates and terms to maturity. We did not succeed in getting all the way to individual policies, but SOFASIM does process each cohort separately. A cohort consists of all of the policies of a given type and size group issued at a given age in a given year. The numbers of deaths and lapses are determined by random numbers generated by the program. Random variation in the numbers of new policies sold was included to give a little more realism to the simulation.

The handout you have received begins with a brief overall description of SOFASIM. The next item is a set of filled-out input forms followed by a listing of the input data for a company we put together and called BHBASE. This is a stock life insurance company which started business many years ago and still has policies in force from as far back as 1945. The sample set of input forms does not contain the complete input for BHBASE, since the data fill several copies of some forms. A vertical line was drawn in the right-hand margin of the input listing to identify the lines of data included in the sample set of forms.

The simulation begins on January 1, 1975, at which time the in-force is 260,000 policies with face amount of \$2.95 billion on four plans, ten-year term, twenty-year endowment, twenty-pay life, and ordinary whole life. Assets at this time totaled \$534 million, reserves \$464 million and surplus \$70 million. The handout includes the output of a simulation run of BHBASE data for 20 years to December 31, 1994, followed by a verbal description of each item of output. All of the input and output in the handout refer to BHBASE data, which is the starting point of today's demonstrations. The cases we will illustrate on the terminal are all variations of BHBASE.

The program will not, itself, generate variation in future expenses and interest rates. However, since the model permits these elements to be changed on the first day of any month during simulated time, any desired process for generating future expense rates and interest rates may be employed to provide the input which will conform with the user's decision as to the future or futures to be simulated. In today's demonstration, we have not changed future expense rates since the pretax effect of any such change seems fairly obvious. In order to achieve a dramatic demonstration of the effect of an interest rate change, we started out on January 1, 1975, with a money market in which bonds earned 6 percent net after investment expenses, which is the assumption used in calculating gross premiums; and we change, on January 1, 1977, to a market in which bonds earn only 3 percent net.

Here I should like to pause long enough to emphasize that in SOFASIM we do not change the interest rate earned by the company directly; we change the bond market. Thus all of the bonds in the portfolio on December 31, 1976, which are effectively earning 6 percent net, remain in the portfolio until they mature or are sold. Purchases and sales of bonds occurring on and after January 1, 1977, take place in a market three percentage points lower. Of course, we realize that a three percentage point drop overnight is not very realistic but a gradual, fluctuating change from month to month over a period of years, which is perfectly feasible for SOFASIM, might not have illustrated the interest rate change as forcefully.

The basic simulation uses Linton B lapse rates, which underlie the gross premiums. You will have noted that on the program of this meeting is a paper on the LIMRA lapse rates of 1971-1972. We thought it would be of some interest to do a simulation in which these lapse rates are substituted for Linton B. This involved more work than the usual switching of assumptions because the LIMRA rates are select and also we used their term rates for the term policies in our simulation and their permanent rates for the other three policy forms. Tony Amodeo spent a considerable amount of time using the editor, making changes in the input required by the substitution of the LIMRA rates for Linton B, and even so, we were only able to make the substitution for the first 20 policy years. In addition to the changes required to substitute a set of select rates for a set of rates varying only by duration and not by age at issue, since the lapse rates for a term policy issued at age x

differ from those for a permanent plan policy issued at age x, it was necessary to assign different age codes for term and permanent plans issued at the same true age. Also the information used by the program in making various edit checks had to be changed. At any rate, we will be able to show you the effect of the new lapse rates as compared with Linton B with all other elements held constant. We specified minimum cash values in the simulation, which corresponds to the gross premium assumption.

We could hardly carry out a demonstration of a Life Company model before an actuarial audience without showing the effect of a change in mortality. The basic simulation run was done with 1955-1960 select and ultimate mortality throughout, which conforms to the assumption in the gross premium rates. A second run then was done with mortality equal to 120 percent of 1955-1960 select and ultimate throughout. The general idea behind this comparison was to show that even as drastic and persistent an increase in mortality as 20 percent over twenty years does not drive a prosperous stock company to the wall. The comparison with the result of a change in interest rate is enlightening.

In describing the demonstration runs we have prepared for this session, I mentioned edit checks and also the necessity for using two different age codes for the same true age under certain circumstances. It seems appropriate to go into somewhat more detail on these points.

The SOFASIM user tells the computer what to simulate by means of punch cards or by direct entry from a terminal. Twelve different input forms have been designed to enable the user to convey his instructions to the computer. These instructions may require use of more than one of some of the forms. In order that the computer may test for inconsistencies in the input data, the user must specify such things as youngest and oldest issue ages, maximum number of years to maturity of the bonds in the company portfolio, number of policy types, etc. There are about sixty possible kinds of error in the input which the program has told the computer to recognize and announce to the user, meanwhile preventing the simulation from being executed with faulty data.

There are separate sections in the input for the mortality table controlling deaths of policyholders and the mortality table entering into the calculation of reserves and cash values. The death table is identified by an M, the valuation table by a V, and a similar section for lapses is identified by an L. All policies with a given age at issue code and a given duration will be governed by the same input entry for mortality, for valuation and cash value calculation, and for lapse determination. If older policies are to be valued on 1941 CSO, for instance, and more recent policies on 1958 CSO, it is necessary that different age at issue codes be assigned. For each age at issue code then, select mortality rates are entered from issue year to the final year of the simulation. Since this may result in the need for many more issue ages than the range of true issue ages, the input forms have been modified, subsequent to the filling in of the samples in the handout, to provide for up to three digits of issue age information, that is, from one to 999. Also, it is worth noting that there is some saving in using consecutive age at issue codes to reduce the storage otherwise required. For instance, in the demonstration today, data were used for true issue ages twenty, twenty-five, thirty, etc., but these ages were coded 1, 2, 3, etc.

Since for each policy type and issue age code, first-year and renewal valuation net premiums and initial deficits and adjusted premiums for cash value calculation purposes are entered on the input forms, it is possible to value some plans on a net level basis and others on a modified reserve basis without

resort to differentiation by age at issue code. If, however, different series of policies of the same plan are to have different gross premium rates and/or different valuation interest rates, they must be given different policy type codes. Thus, if gross premium rates vary by policy size, each size group simulated for a given age at issue code must have its own policy type. The gross premium rates used in the demonstration were calculated for JCOTOR by Bill Bailey with policy size assumptions of \$5,000, \$10,000, \$20,000, \$50,000, \$100,000. Again, the input forms have been modified to provide for up to 999 different policy types so that the user may have the desired degree of flexibility.

From what I have said previously, it is probably clear that reserve factors and cash value factors are accumulated by use of the appropriate premiums with specified mortality and interest. The simple situation, of course, is one in which the appropriate premium rates are simply copied out of the tables published by the Society and a familiar statutory valuation mortality table used in the calculations. The model is not, however, necessarily limited to the simple concept. Any factors which can be calculated by means of first-year and renewal premiums and an interest rate or rates and a mortality table or tables, including select, are available. For a given policy type and age at issue the same interest and mortality must be used for both reserves and cash values. Perhaps this is the place to mention that SOFASIM is quite unaware of deficiency reserves.

Just incidentally, I should mention that the program calculates the cash values in force on each December 31 of simulated time by applying the cash value factor as of the next policy anniversary to the policies in force on December 31. The result is printed in the last section of year-by-year results. This figure is achieving some currency as an indicator of the rock-bottom asset requirement for a company approaching statutory insolvency.

I have referred above to twelve different kinds of input form and to changes in basic factors which the user causes to take place at predetermined times in the course of the simulation. Some discussion of these points seems called for. The first three input forms, A1, A2, and A3, specify the initial conditions at the beginning of the simulation and, to some extent, are controlling throughout the simulation. For instance, the Federal Income Tax rates on ordinary income and on capital gains may not be changed. All of the subsequent forms are used for entering data in what are called "event packets," each of which may be changed on the first day of any month during the course of the simulation independently of any of the other event packets. The first entry for each event packet is the event name, and the second entry is the effective date of the data specified in that event packet.

Input form B is for company parameters such as operating expenses, investment expenses, stockholder dividend decision ratios and investment decision parameters. These apply to the whole company and do not vary by, for example, policy type.

Input forms D1 and D2 specify the interest rate structure, that is, the yield curves in the bond market during the time period beginning on the effective date and applicable until another interest rate event packet with a later date changes the bond market. The D1 form gives the yields to maturity on bonds sold. The yield curve may be specified with as much precision as desired. In the demonstration, we have used the same set of yields to maturity varying by years to maturity but not by coupon rate. The model permits the user to have separate tables for each different

coupon rate or for different ranges of coupon rates. Additional variation as between callable and noncallable bonds is provided. The D1 form also calls for specification of net interest rates applicable to short-term borrowing and short-term lending. Input form D2 represents something of a simplification of bond purchases in that they must take place at par. However, the yield rates to maturity may vary by the number of years to maturity and by whether the bond is callable or not.

The E form requires the user to specify the desired percents invested by number of years to maturity by call category. Thus the user can test the results of a given investment maturity strategy under various assumptions as to actual market performance in the future.

The F1 and F2 forms are for entering the mortality and lapse rates, the F1 form for select rates and the F2 form for ultimate rates, where ultimate means by attained age for mortality and by duration for lapse. As mentioned above, three sets of these are required, one for valuation mortality, one for death mortality, and one for lapses.

Input form G is for commission rates; more precisely, it should be identified as specifying total percent-of-premium expenses, since this is the only source for such. (Input form B is where the user enters per-policy and per-thousand expenses for first year and all years.) The percent-of-premium rates on form G may be varied by policy type, since each set of "commission" rates is assigned a table number.

Input forms K1 and K2 describe the various policy types. Immediately following the card which identifies the event and the effective date of the packet comes a "41" card on which are specified the policy type, the valuation interest rate, and the applicable commission table number. This card is also used to tell the computer whether the policy is term, endowment or whole life and, if term or endowment, how many years the policy runs. For whole life it also permits specification of the number of years that premiums are paid (the term and endowment policies must be coterminous). The "42" cards are used to enter the valuation net premiums, the first-year deficit and adjusted premium for cash values, the gross premium rate, policy size, expected number of policies sold each year and standard deviation thereof for each age at issue. As mentioned earlier, a given policy type-age at issue combination may have only one gross premium. If the gross premium rates vary by policy size, and different sizes are to be simulated, the most convenient way of handling this is to assign different policy type codes to the different size categories. The "41" card and related "42" cards not only provide the necessary information for new issues during the course of the simulation but also permit calculation of reserves, cash values, etc. for the policies in force at the beginning of the simulation, which are specified on the K2 form.

DR. BARBARA MARKOWITZ*: My presentation will further explore three major aspects of SOFASIM: the inputs which describe the system to be simulated; the SOFASIM model within the computer; and the printouts which summarize the performance of the simulated company within the postulated environment. The system to be simulated is specified on the input forms, copies of which you have in your hands. We will look at these in detail later. On these forms you specify the parameters of the company you wish to simulate and the environment in which the company is to function.

*Dr. Markowitz, not a member of the Society, is currently a private consultant in the computer field.

These parameters are written on the 12 types of SOFASIM input forms. Usually these forms are given to a keypunch operator who punches their contents into a set of IBM cards. After you check a listing of the input cards to see if they are correct, and in the appropriate order for input to the computer, the cards are read into the computer. In the case of SOFASIM the cards are read into the National CSS computer and then read onto your own disk storage. Now you are ready to simulate the particular company and environment.

A SOFASIM simulation can be run in different ways. This afternoon we will operate online with immediate output on the terminal. You can also operate in an offline, or batch, mode where you execute the program during non-peak hours. You receive your output the next morning delivered by a messenger, or you can list the output on your terminal. The batch mode of operation costs approximately 60 percent less than the online mode.

Now let us look at some of the inputs required by SOFASIM. This will give you a feeling for the level of complexity represented in the model, as well as a feeling for the variables you can manipulate.

The input forms require that you specify the initial status of the company, which may be a company already in operation or a company just going into business. You must also provide a description of the initial and subsequent policies of the company. In addition, you must specify initial conditions of the environment, some of which will be in effect for the entire run and some of which you can change during the run. SOFASIM utilizes these values to simulate monthly operations of a stock life insurance company, and outputs an annual report detailing the performance of the company over the time frame specified on the input forms.

If you are simulating an ongoing company, the initial inputs that you must specify include the number of policies of each type, size, issue year and age at issue currently in effect at the beginning of the simulation; the sales rates and other characteristics of each policy type that the company currently issues; the number of bonds of various maturities, coupons and callability currently in the company's portfolio; and five years of tax-related tables, a stockholder dividend policy, and an investment policy must also be provided.

To describe the environment it is necessary to specify mortality tables, valuation tables, lapse tables, the interest rate structure, operating expense coefficients, and tax rates.

Now let us look at the actual forms and see what are some (but not all) of the actual values that you need to supply. You do not have to fill out all of these forms for each simulation, but can re-use information supplied previously. If you wish to change a few numbers describing the simulated environment or company, you need not go through a keypunch operator again, but can edit your file from a terminal.

Form A1 is called General System Description and can generate three IBM cards, a "10" card, an "11" card and, optionally, a "101" card. On the first card, the "10" card, you must specify information such as the number of different policy types that can exist anytime during the simulation, the youngest and oldest age at issue and the first and last year of the simulation. For example, if you write 1975 in columns 44 through 47 of the "10" card and 1994 in columns 50 through 53 -- and you have the forms keypunched and their content read into the computer as we described before -- then the simulated world will run from the first day of 1975 through the last day of 1994.

The "11" card expects federal income tax rates both for income and capital gains, and the maximum bond maturity as well as the range of bond coupons. The "101" card is used to make runs non-random as to deaths and lapses as well as to eliminate the federal income tax calculation, if those options are to be exercised.

On Form A2, the Initial Company Conditions Form, you specify on the "12" card the starting cash balance, on the "13" card, columns 15-35, the value of policyholders surplus account and shareholders surplus account. The next seven cards are for historical tax data such as ordinary gains and losses not yet carried back or forward, taxable investment income for gain years only and gain from operations after taxes which is used for dividend decisions.

It is not possible or appropriate here to describe how to fill out each field of each kind of form. Such instructions are part of the SOFASIM documentation which will be available soon. Our object now is to give you a general idea of the kind of information specified, and the way it is presented to SOFASIM.

The next form, A3, represents the investment portfolio of bonds already owned by the company when the simulation starts. On this form you specify the face value held for callable and noncallable bonds with various coupons and maturities.

Forms A1 through A3 are used only to specify initial conditions. They cannot be used to change values during the simulation.

In contrast, most of the remaining forms may be used both for initialization and to change policy or environmental parameters during the simulation. All of these forms require a time entry. For example, on Form B the time entry appears in columns 9 through 22 of the first card. The input deck is ordered according to this time of occurrence. All cards used for initialization have the time field as 01/01/start year/ 0 0.

On the first card of Form B, the CO.PRMS card, the user must specify the company number (SOFASIM can run more than one small company simultaneously, but we will generally ignore this feature). For the specified company, Form B asks for monthly operating expense coefficients such as the dollars of cost per policy for a new policy, and the dollars of cost per existing policy. On the "2" card on Form B you specify investment transaction costs and dividend parameters.

D1 and D2 forms are used for specifying interest rate structure. On these forms you specify the yield to maturity for all callable and noncallable bonds for every coupon category and for all maturities. These yields to maturity can change during the simulation, as they do change in the real world, and thus you can see the impact such changes would have on the condition of the simulated company.

Using Form E you specify the percentage of bonds you desire to hold in each maturity class. These numbers are used by the model when it determines what maturity bonds to buy and/or sell.

F1 and F2 provide the forms for inputting mortality, valuation and lapse tables. A "V" in Column 29 of the first card on F1 indicates that mortality rates used in reserve calculations are about to be described; an "M" in column 31 indicates that the expected rates of simulated deaths is about to be described. These "actual" mortality rates may or may not be the same as the valuation rates. An "L" in column 33 indicates that lapse rates follow.

Your valuation, mortality or lapse description can use either or both select or ultimate tables. Entries must be provided for all ages of issue from the youngest age at issue to the oldest possible survivor. As the simulation moves through time, random or nonrandom deaths and lapses are generated using the appropriate entries from the select or ultimate tables.

Form G is used to enter commission rates. Different tables can be used for different policy types.

Form K1 requires you to enter information about the different policy types in effect at the time the simulation begins or to be issued during the simulation. For example, on the "41" card you tell SOFASIM what type policy you are supplying data for, its contract type: term, endowment or whole life; the duration of the contract and what commission table to use. On the "42" card you enter values for valuation purposes such as the first year's net premium and the renewal year's net premium; for the calculation of surrender value you enter data for the first year's deficit and the adjusted premium.

On the K2 form you enter data which tells SOFASIM what cohorts were in the system when the simulation started: when these policies were sold, what type policy they were, what was their age of issue, the size of the policy, and how many policies remain for each month of issue as of the beginning of the simulation.

As you can see, these forms require extensive data and permit you to explore the effects of many important variables. Typically much of the data remains the same, and need not be reentered from one run to the next.

The internal structure of the SOFASIM model performs three major functions: initialization, the processing of events (including monthly, nonmonthly and external events), and output. During initialization, initial conditions are set up based on the inputs read in from the keypunched forms. Each input card is error edited. If an error is found a message is printed indicating the type of error that has been encountered and the card and card column in error. After all the initial inputs have been correctly entered, SOFASIM is ready to begin operation.

Each month SOFASIM processes policies, pays operating expenses, processes existing investments, and buys or sells investments -- in the order just listed. Other events are executed regularly, but not each month. These are the events which decide the stockholder dividend rate, pay dividends, calculate estimated and final income taxes, pay taxes or collect funds as appropriate, and report and reset various accounting totals.

Besides the monthly and other periodic events, external events are executed at the time specified on Forms B through K. Thus you can change company parameters, commission tables, investment profile, interest rates, mortality tables, or policy sales rates at any time during the simulation.

As the model runs, one line of output is created and printed at the end of each simulated year. Most of the output, however, is stored and printed at the completion of the entire run.

Each month the "PROCESS POLICIES" event draws deaths and lapses, either randomly or nonrandomly as you specified on the "101" card, using the appropriate select or ultimate mortality and lapse tables. It pays death benefits, maturity benefits on endowments, and surrender benefits for policies lapsing

with a positive cash value. Premiums are collected. Sales are generated randomly or nonrandomly for new cohorts. Reserves are computed for the company, and reserve factors, surrender benefit factors, cash and year-to-date totals are updated.

After policies are processed, the operating expense routine occurs. In this routine the current month's operating expenses are calculated, utilizing the factors you specified on Form B. Cash and year-to-date totals are updated.

Next, the existing investments are processed. This routine first collects or pays short-term interest on the current cash balance. Coupons are then clipped for 1/12 of the bonds held in each coupon category for each maturity. Next 1/12 of the bonds which mature this year are cashed. Then a calculation is made to see if any bonds should be called, and bonds are called if appropriate. The company's cash balance, capital gains and losses, if any, and the amounts invested in each call category, coupon category and maturity are updated.

The last regular monthly event to occur is Buy or Sell investments. In this routine a decision is made each month as to whether bonds need to be bought or sold depending on the ratio of cash to total reserves and on parameters you specified. After this decision is made it is necessary to decide which maturity of bonds to buy or sell. This depends in part on what the company already holds in each of these maturities.

After sales or purchases are made, investment expenses are calculated as a function of the face value of the bonds held and the amount of transactions made. Cash and year-to-date expenses are updated.

Among the non-monthly events, the Decide Dividend routine decides whether to change the current dividend for our simulated stock life insurance company. This dividend is paid quarterly for the next four quarters beginning on the date you specified on Form A2. The Pay Dividend routine updates the values of cash balance and dividends paid and, if necessary, makes an accounting transfer from the policyholders' surplus account to the shareholders' surplus account.

The routine which calculates estimated taxes four times a year and the actual tax on December 31 of each year is a complex program and I will not go into the details of it here. When the taxes are calculated it updates the appropriate values and then schedules a tax payment or refund event as appropriate. Alice Goldstein is the expert on this routine so if you have any questions as to how it operates I suggest that you ask her.

The last routine to occur each year is the report and reset routine. It stores the values required to print the yearly summaries at the end of the simulation. It also resets year-to-date totals.

Please look at the set of output that was given to you. This is the result from a case we refer to as BHBASE. The first table of output is not generated by the Report and Reset routine but is generated by a trace routine which prints one line as each year is completed. Thus if something happens to terminate the run before its completion, we can see how far the simulation ran and what had happened in the years completed. All of the rest of the output is printed after the run has been completed.

The second table of numbers shows year-end balance sheet information. It lists the face value of investments held, the cash on hand at the end of the year, total assets, total reserves, the tax due for the year, total liabilities, and the various surpluses (policyholders, shareholders and other), and lastly the year number and the date.

The third table shows annual profit-and-loss data such as the total premiums, investment income, investment expense, death benefits, matured endowments, surrender benefits, commissions, operating expenses, increase in reserves, gain from operations before taxes, federal tax excluding capital gain tax, and gain after taxes. The third set of data are tax related, the fourth miscellaneous. Finally, a historical summary is printed which lists the mean and standard deviation for gain after taxes and for stockholder dividends. Also the "present value" of dividends and of gain from operations after taxes is computed with three different discount rates.

Following Dr. Markowitz' presentation the terminal demonstration was held. The audience was solicited to supply changed company parameters for an actual computer run of SOFASIM. They felt that the first-year to maintenance expense ratio was too low, and that some maintenance expense would be better as first-year expense. The first-year factor was accordingly increased from \$40 to \$200, and the renewal factor was decreased from \$0.83 to \$0.40. The results of this run were printed at the terminal and on the television monitors. The summaries are included with those of the other cases mentioned by Mr. Woody. These summaries show Gain After Tax (G.A.T.) and Stockholder Dividends (S.D.) along with increases in surplus and in force. Following this page are sample pages from the handout that was distributed at the session.

MR. JOHANSEN: Now that you know what a corporate model can do, there is one more very important point. It is what is necessary to make a model worthwhile? The computer cannot help here. This requires people - you. Referring again to the Use of Financial Models in Business, let me paraphrase some of the worthwhile advice it may have for you. Someone must see a problem whose solution has value and which can be translated into mathematical terms. He must build and validate a model which behaves like its real world counterpart. Finally, he must bring about a decision to change the real world system along lines which the model indicates will effect beneficial results.

| | Average for period | Standard Deviation | Sum of discounted values as of January 1, 1975 at | | |
|--------|-----------------------|-----------------------|--|------------|------------|
| | | | 6% | 9% | 12% |
| Basic | | | | | |
| G.A.T. | 14,156.51 | 7,312.48 | 158,623.00 | 125,561.62 | 102,678.19 |
| S.D. | 3,264.92 | 802.53 | 41,086.82 | 34,143.25 | 29,092.75 |

Increase 1975-94

| | | | |
|------------|------------|------------|-----------|
| In Surplus | 217,660.00 | In Inforce | 1,669,313 |
|------------|------------|------------|-----------|

Interest Rate Changed from 6% to 3% 1/1/77

| | | | | | |
|--------|-----------|----------|------------|-----------|-----------|
| G.A.T. | 10,099.46 | 5,859.83 | 111,320.87 | 87,476.12 | 71,075.44 |
| S.D. | 2,603.81 | 846.81 | 34,190.52 | 28,964.52 | 25,115.70 |

Increase 1975-94

| | | | |
|------------|-----------|------------|-----------|
| In Surplus | 56,152.80 | In Inforce | 1,669,313 |
|------------|-----------|------------|-----------|

Mortality at 120% of 1955-60 Basic

| | | | | | |
|--------|----------|----------|-----------|-----------|-----------|
| G.A.T. | 4,773.52 | 7,509.79 | 73,560.69 | 66,047.44 | 60,013.33 |
| S.D. | 1,962.79 | 1,507.12 | 29,446.79 | 26,093.73 | 23,411.71 |

Increase 1975-94

| | | | |
|------------|------------|------------|-----------|
| In Surplus | 149,797.90 | In Inforce | 1,632,328 |
|------------|------------|------------|-----------|

LIMRA 1971-72 Lapses Instead of Linton B for First 20 Policy Years

| | | | | | |
|--------|-----------|----------|------------|------------|-----------|
| G.A.T. | 14,132.30 | 7,200.51 | 156,103.31 | 122,672.44 | 99,607.37 |
| S.D. | 3,477.12 | 460.55 | 43,037.28 | 35,491.71 | 30,036.19 |

Increase 1975-94

| | | | |
|------------|------------|------------|-----------|
| In Surplus | 212,746.60 | In Inforce | 2,736,148 |
|------------|------------|------------|-----------|

Changes in Expense Factors Suggested at Teaching Session

| | | | | | |
|--------|-----------|----------|------------|-----------|-----------|
| G.A.T. | 10,400.72 | 7,092.92 | 115,628.94 | 91,348.62 | 74,648.69 |
| S.D. | 2,652.56 | 1,038.15 | 34,938.18 | 29,618.21 | 25,686.26 |

Increase 1975-94

| | | | |
|------------|------------|------------|-----------|
| In Surplus | 154,851.30 | In Inforce | 1,669,313 |
|------------|------------|------------|-----------|

SOFASIM

General Description

The objective in building SOFASIM* was to develop a stock life insurance company model combining realism, flexibility, and simplicity to the greatest extent practicable. A brief outline of the characteristics of the model follows.

1. Plans of Insurance.

- (a) Co-terminous level term, any number of years
- (b) Co-terminous endowment insurance, any number of years
- (c) Whole Life, any number of premium payments

2. Gross premium rates are supplied by the user and must be level.

3. Insurance expenses.

- (a) Per policy: new and in force (monthly - beginning with month of issue)
- (b) Per \$1,000 of face value: new and in force (monthly - beginning with month of issue)
- (c) Per \$1 of premium: by policy year and plan
- (d) Constant monthly overhead

4. Investments, bonds only.

- (a) Existing portfolio, if any, is specified by callability, coupon and number of years to maturity measured from January 1 of first year of simulation.
- (b) Bond purchases are at par, hence the user of the model specifies, for each time interval over which the simulation is to run, the market (coupon) interest rate for callable and non-callable bonds separately for each number of years to maturity.
- (c) Callable bonds are called in accordance with a plausible fairly simple rule if future market interest rates are specified as sufficiently lower than then existing bond coupon rates.
- (d) When company operations require, bonds are sold at prices determined by user-specified tables of market yields to maturity varying by callability, coupon rate category and years to maturity at date of sale. The user may specify different market conditions for different time periods within one simulation.

* Society of Actuaries Simulation Model

- (e) Provision is made for a specified charge for each purchase and each sale and for a monthly maintenance expense per \$1,000 of invested assets.
5. Deaths are governed by select and ultimate mortality rates supplied by the user, who may change the rates from time to time. The number of deaths may be: (a) deterministic, that is, equal to the mortality rate multiplied by the in force; or, (b) stochastic, that is, determined by a random number drawn to correspond with the assumption that number of deaths is distributed about the deterministic mean in accordance with the Poisson distribution. The choice between (a) and (b) lies with the user.
 6. Lapses are handled in similar but not identical fashion.
 7. The user specifies the interest and mortality rates for reserve and cash value calculation.
 8. Federal Income Tax is calculated in accordance with the Society of Actuaries Study Note. Provision is made for input of required data for the five years preceding the start of the simulation to permit tax calculation to begin immediately. The tax calculation may be eliminated if desired.
 9. The expected amount of new business to be sold each year is specified by plan, age at issue, and policy size group. If it is desired to make actual new business always equal to expected, the standard deviation is specified as zero. The expected amount and/or the standard deviation may be changed from year to year.
 10. The business in force at the start of the simulation, if any, is specified by issue year, plan, age at issue, and policy size group.
 11. The amount of core storage, running time, and cost are largely a function of the number of cohorts, i.e., plan-issue age-size-issue year groups. In order to minimize this number, the concepts of background and foreground were developed.

- (a) The background consists of a few large cohorts whose size and other characteristics are to approximate, overall, the company being simulated.
- (b) The foreground consists of as many cohorts as necessary for the specific category of business under study.
12. There are three initial random number seeds for each run, one for deaths, one for lapses, and one for new business. Additional simulations of the same input data are obtained by changing the seeds. The number of simulations the user may run on the same data is constrained by cost considerations and the sheer difficulty of analysing an inordinate volume of output, not by the number of different possible seeds, which approaches 1,000,000,000 for each variable.
13. Most of the parameters of the model may be programmed in advance to change at specified points in time during the course of the simulation run.
14. The SOFASIM simulation model was programmed using the SIMSCRIPT programming language. A few words concerning the SIMSCRIPT view of a world to be simulated may be useful.
- 14.1. As of any moment in time a simulated world has a status. In the SIMSCRIPT view this status is described in terms of how many of each type of entity exist, what are the values of the attributes of each entity, what are the sets to which each entity belongs, and who are the members of the sets it owns.
- The principal entities in SOFASIM are COMPANY and COHORT. The attributes of COMPANY include its cash balance, its total reserves, its death benefits paid thus far this year, and so on. Each COMPANY owns a set of COHORTS.
- The attributes of COHORT include age at issue, policy type, policy size, etc. Each COHORT belongs to a set of COHORTS owned by some COMPANY. SOFASIM also refers to the attributes and set of relationships of other types of entities such as "gain memos" used to keep track of prior gains from operations as computed for tax purposes and not yet eliminated

by loss carry backs.

- 14.2. Status changes at points in time called events. Just as SOFASIM has different types of entities such as COMPANY, COHORT and "gain memo" it also has different types of events - different kinds of occurrences - such as those to process policies, buy or sell investments, calculate taxes, pay taxes, etc. Each type of event is described by an event routine. For example, there is a part of the SOFASIM programming which begins "EVENT BUY.SL" and then proceeds to describe the calculations, decisions and actions which are performed when this event is invoked to buy or sell investments.

A1

GENERAL SYSTEM DESCRIPTION

| CARD I.D. | N.R. OF COMPANIES | N.R. OF POLICY TYPES | YOUNGEST AGE AT ISSUE | OLDEST AGE AT ISSUE | OLDEST POSSIBLE SURVIVOR | FIRST YEAR OF SIMULATION | LAST YEAR OF SIMULATION | N.R. OF COMMISSION TABLES |
|--------------|-------------------|----------------------|-----------------------|---------------------|--------------------------|--------------------------|-------------------------|---------------------------|
| 1 0 01 02 | 1 08 07 | 009 11 13 | 01 19 20 | 10 26 27 | 060 34 36 | 1975 44 47 | 1994 50 53 | 04 59 60 |

| CARD I.D. | MAXIMUM YEARS BOND MATURITY | CALL PRICE CALLABLE BONDS | BOND COUPON RATES | | | FEDERAL INCOME TAX PERCENT | | CALL PROBABILITY COEFFICIENTS | |
|--------------|-----------------------------|---------------------------|-------------------|----------------|----------------|----------------------------|----------------|-------------------------------|-----------------|
| | | | FROM | THROUGH | BY | INCOME | CAPITAL GAIN | AA | BB |
| 1 1 01 02 | 15 06 07 | 105.00 11 16 | 6.200 18 23 | 7.000 25 30 | 0.010 32 36 | 48.00 38 42 | 30.00 44 48 | 1.000 50 55 | 0.0500 57 62 |

| CARD I.D. | MARK IF NO TAX CALCULATION | MARK IF NON-RANDOM | | INITIAL RANDOM SEEDS LEAVE BLANK FOR STANDARD SEEDS | | | | | |
|----------------|-----------------------------|-----------------------------|-----------------------------|--|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| | | DEATHS | LAPSES | DEATHS | LAPSES | SALES | | | |
| 1 0 1 01 03 | <input type="checkbox"/> 00 | <input type="checkbox"/> 10 | <input type="checkbox"/> 24 | <input type="checkbox"/> 35 | <input type="checkbox"/> 44 | <input type="checkbox"/> 46 | <input type="checkbox"/> 54 | <input type="checkbox"/> 56 | <input type="checkbox"/> 64 |

A2

INITIAL COMPANY CONDITIONS

| CARD NO. | CO. NR. | STARTING CASH BALANCE (\$1000) | FEDERAL INCOME TAX, OR REFUND IF (-), PAYABLE 3/15 (IN \$1000) | ANNUAL STOCKHOLDER DIVIDEND | | | | |
|----------|---------|--------------------------------|--|-----------------------------|-----|------------|-----|--------------------------------------|
| | | | | DECIDED | | FIRST PAID | | CURRENT AGGREGATE AMOUNT (IN \$1000) |
| | | | | MO. | DAY | MO. | DAY | |
| 12 | 01 | 4.00 | 0.0 | 02 | 15 | 03 | 15 | 390.0 |

| CARD NO. | LAST YEAR'S ORDINARY FEDERAL INCOME TAX BEFORE ESTIMATED PAYMENTS (\$1000) | VALUE OF POLICYHOLDERS SURPLUS ACCOUNT (\$1000) | VALUE OF SHAREHOLDERS SURPLUS ACCOUNT (\$1000) | TOTAL RESERVES IN 1958 FOR TAX CALCULATIONS (\$1000) |
|----------|--|---|--|--|
| | | | | |

| CARD NO. | DESCRIPTION | HISTORICAL EARNINGS AND TAX DATA (IN \$1000 EXCEPT EARNINGS IN %) | | | | |
|----------|-------------|---|----------------|----------------|----------------|----------------|
| | | YEAR JUST ENDED | 2ND PRIOR YEAR | 3RD PRIOR YEAR | 4TH PRIOR YEAR | 5TH PRIOR YEAR |
| 14 | ORDI | 1200.0 | 1100.0 | 1000.0 | 90.0 | 80.0 |
| 15 | T I I. | 1100.0 | 1060.0 | 1010.0 | 96.0 | 91.0 |
| 16 | T X X W | 554.0 | 518.4 | 480.0 | 432.0 | 384.0 |
| 17 | S D N P | 0. | 0. | 0. | 0. | 0. |
| 18 | C . G . | 0. | 0. | 0. | 0. | 0. |
| 19 | E R N R | 6.000 | 6.000 | 6.000 | 6.000 | 6.000 |
| 20 | G . A T | 6456.0 | 5816.0 | 5200.0 | 4680.0 | 4160.0 |

LEGEND: 14 ORDI ORDINARY GAIN OR LOSS (-) NOT YET CARRIED BACK OR FORWARD IN FEDERAL TAX CALCULATION.
 15 T I I. TAXABLE INVESTMENT INCOME.
 16 T X X W ORDINARY TAX (EXCLUDING POLICYHOLDERS SURPLUS ACCOUNT WITHDRAWALS) NOT CARRIED.
 17 S D N P SPECIAL DEDUCTION FOR NON-PARTICIPATING CONTRACTS. NEEDED FOR LOSS YEARS.
 18 C . G . CAPITAL GAIN OR LOSS (-) NOT YET CARRIED BACK OR FORWARD.
 19 E R N R EARNINGS RATES—TO COMPUTE ADJUSTED RESERVES RATE AS A PERCENT.
 20 G . A T GAIN FROM OPERATIONS AFTER TAXES (FOR DIVIDEND POLICY).
 } NEEDED FOR GAIN YEARS

Output of Simulation runs.

1. Summary output from TRACE routine (all amounts to nearest \$1,000.).

| | |
|--|---|
| YR. NO. | First, second, third, ... year of simulation run. |
| AMOUNT TERMINATED BY SURRE. BY MATUR. BY DEATH | The first of these three columns shows, for each year of the simulation, the amount of insurance terminated by application of lapse rates to policies in force. The second column shows the reduction in amount in force by reason of endowment policies reaching maturity. The amounts in this column are also the respective year's maturity benefit payments. The third column shows amounts of insurance terminated by death; these are also the death benefit payments for the respective years. Terminations by expiry of term policies are not shown explicitly. |
| END OF YEAR IN FORCE ASSETS | These two columns show, respectively, amount of insurance in force and amount of assets at the end of each simulated year. |
| TOTAL PRE- MIUMS | This is the total of the annual Gross Premiums, first year and renewal, at the rates given on the K1 forms, collected on anniversaries during the calendar year. |
| INSUR. EXPEN. PCT. OF PREMIUM OTHER | These two columns give the annual expenses which are expressed as percentages of Gross Premiums in accordance with the G forms and the annual sum of the monthly per-policy, per thousand, and overhead expenses based on the entries on the CO.PRMS card as specified on the B form. |
| INVESTMENT GROSS INCOME EX- PEN | The first of these columns shows gross annual investment income derived from collected bond coupons whose amounts are determined by the complex interplay of many elements of the input. The second column gives the annual amount of the investment transactions expense and the monthly investment maintenance expense, as specified on the "2" card of the B form. |

SURR.
BENS. Total cash values paid during the calendar year based on the lapses called forth by the rates on the (L) F1 and/or F2 forms and the unit cash values derived from information on the K1 form and the valuation mortality given by the (V) F1 and/or F2 forms.

INCR.
IN
RESRV Terminal reserve factors are calculated from the first year and renewal valuation premiums and interest rate of the K1 forms and the valuation mortality of the (V) F1 and/or F2 forms. Mean reserve factors are derived therefrom and applied to December 31 in force by issue age, issue year and policy type. The increase in aggregate statutory reserve is shown in this column.

FR. OPERATIONS The first of these columns is the net excess, positive or negative, for the year, of income over outgo before Federal Income Tax. The second column is the Federal Income Tax on ordinary income calculated at the rate given on the "11" card of the A1 form in accordance with the Society of Actuaries Study Note on Federal Income Taxation of Life Insurance Companies.

GN.BE-
FORE TX.
FED.
IN.TAX The first column gives the year's capital gains, if any, on bond sales. Such gains would arise from differences between the rates on the D1 and D2 forms and reflect any changes in these forms over time. Bond sales, as distinguished from maturities or exercise of call provisions, take place only in response to requirements of the insurance operations. There is no programmed bond trading per se, nor switching of maturities. No default events are provided for explicitly, but could be made part of the input by, e.g., specifying a particular simulated year or years in which investment expenses would be high enough to wipe out the desired fraction of the portfolio, with subsequent return to regular expense rates. Other ingenious devices might be conceived. The second column gives the Federal Income Tax on the amount in the first column, with allowance for loss carry forwards and carry backs, at the rate given on the "11" card of the A1 form.

CAPITAL AND SURPLUS The first of these columns gives the total capital and surplus of the simulated company as it would appear in a statutory statement, except that it includes all items of allocated surplus such as the Mandatory Security Valuation Reserve. The second and third columns respectively show Policyholder and Stockholder Surplus as defined in the Federal Income Tax law. The tax routine does deal with Phase 3 situations.

2. Balance sheet output (all amounts to nearest \$100.).

| | |
|----------------------|--|
| INVEST- MENTS | Total bonds held by company at year end. |
| CASH | Cash held by company at year end. |
| TOTAL ASSETS | Sum of INVESTMENTS plus CASH. |
| TOTAL RESERVES | Aggregate of mean reserves at year end calculated as described in the INCR. IN RESRV item under 6.5.1. |
| TAX DUE | Final Federal Income Tax for the year on both ordinary income and capital gains, calculated as described under TAXCALC in 6.3.3, minus payments of estimated tax during the calendar year. |
| TOTAL LIABILITIES | Sum of TOTAL RESERVES plus TAX DUE. |
| PLCYHLDRS SURPLUS | Policyholders surplus at year end as defined in the Federal Income Tax Law. |
| SHRHLDRS SURPLUS | Shareholders surplus at year end as defined in the Federal Income Tax Law. |
| OTHER SURPLUS | That portion of total surplus at year end not classified as Policyholders or Shareholders surplus. |
| TOTAL SURPLUS | The sum of the preceding three columns. |
| YR NR | The line designated 0 in YR NR gives certain opening information. For the other lines the number in this column is the serial number of the year just ending. |

YEAR The number in this column is that of the calendar year corresponding to YR NR on the basis of the specification on the "10" card of the A1 form as to the first year to be simulated.

3. Income statement output (all amounts to nearest \$100.).

Every amount column on this section of the printout except GAIN AFTR TAXES corresponds to some column described in 6.5.1., but here the amounts are shown to one more significant digit. Note that: DEATH BENEFITS is the same as AMOUNT TERMINATED BY DEATH; COMMISSIONS is the same as INSUR.EXPEN.-PCT. OF PREMIUM; OPERATING EXPENSES is the same as INSUR.EXPEN.-OTHER; FED TAX (X CG TX) is the same as PR.OPERATIONS - FED.IN.TAX; and GAIN AFTR TAXES is after taxes on ordinary income.

4. Federal Income Tax calculation output (percentages to tenths of a percent; amounts to nearest \$100.).

| | |
|--------------------------|--|
| MEAN INTR ASSUMPTN | Average valuation interest rate calculated in accordance with the tax law. |
| CUR.EARN RATE | Average investment yield rate for the year calculated in accordance with the tax law. |
| AVG EARN RATE | Five-year moving average of rates in preceding column. |
| ADJ RSRVS RATE | Adjusted valuation interest rate for calculating required interest in accordance with tax law. |
| TXL INVST INCOME | Taxable Investment Income calculated in accordance with tax law. |
| GFO FOR TAX CALC | Gain from Operations calculated in accordance with tax law. |
| LOSS C.F. OR C.B. (-) | A positive figure in this column represents the amount of prior year ordinary loss or losses used to reduce current year Gain from Operations for tax purposes. A negative figure in this column represents the amount of current year loss carried back against prior Gains from Operations to produce an ordinary tax refund for the current year. |

TEACHING SESSION

C.LOSS CB OR CF (-) A positive figure in this column represents the amount of capital loss in the current year carried back against prior capital gains to produce a capital gains tax refund for the current year. A negative figure in this column represents the amount of prior year capital loss or losses used to reduce current year capital gains.

CAP GAIN OFFSET Amount of loss from operations applied against capital gains.

EST. TAX PAID Amount of estimated tax already paid on account of the current year as of December 31.

EST. TAX PAY 1/15 Amount of estimated tax on account of current year not yet paid as of December 31 and due on the coming January 15.

TAX PYMNT 4/15 Excess of final tax calculated for current year over sum of preceding two columns.

5. Miscellaneous output (amounts to nearest \$100.).

CAPITAL GAIN Positive figures are capital gains on sale of bonds; negative figures are capital losses.

CAP GAIN TAX Federal capital gains tax if positive; recovery of prior year tax paid if negative.

DIVIDENDS Stockholder dividends disbursed in accordance with instructions on A2 form and B form.

PHSA TRNS FOR DIVS Policyholder surplus transferred in order to pay stockholder dividends.

POLICY FACE VAL Amount of life insurance in force at year end.

5 YR AVG GFO AFT TX Five year moving average of GAIN AFTR TAXES column of Income statement output described in 6.5.3.

QTR DIV RATE Amount of quarterly dividend paid on last dividend date prior to year end.

CSV'S IN FORCE Aggregate amount of cash values which would be paid in coming year if all policies in force on December 31 should surrender on their anniversaries in the coming year.

NR COHS Number of cohorts in being at end of year.

NR These four columns do not have any significance at
GNS the present time.

NR
LSS

NR
CGNS

NR
CLSS

6. Summary output (amounts to nearest \$10.).

The following summary figures for all simulated years are calculated for gain after taxes and for stockholder dividends..

MEAN Algebraic sum of all gains after tax and of all
 stockholder dividends respectively for entire period
simulated, divided by number of years in period.

STD.DEV Standard deviation of gains after tax and stockholder
 dividends respectively, about the mean previously
described.

PV AT 6PCT Present value at beginning of simulation, calculated
 at 6%, of all gains after tax and stockholder dividends
respectively generated during course of simulation.

AT 9PCT Same present values but calculated at 9%.

AT 12PCT Same present values but calculated at 12%.

