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PENSION ACTUARIAL GAIN AND LOSS ANALYSIS

Teaching Session

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The gain and loss analysis is an essential part of the pension valuation process:

- It proves the correctness of the valuation;
- It answers the question as to why costs change;
- It helps establish the appropriateness of the actuarial assumptions (and is, therefore, valuable in supporting ERISA's requirements); and
- It provides valuable insight, useful in pension consulting.

ASSETS

Without loss of generality, assume assets are held at book value.

$$BV_0 + C + I + RG - B - E = BV_1 \quad \dots\dots (1)$$

$$BV_0(1+i) + {}^{i}C - {}^{i}B - {}^{i}E + IG = BV_1 \quad \dots\dots (2)$$

$$BV_0(1+i) + {}^{i}C - BV_1 = {}^{i}B + {}^{i}E - IG \quad \dots\dots (3)$$

Formula (1) can be adapted to fit any asset basis. It states that the book value at the start of the year, at time "0", plus contributions, plus interest and dividend income, plus net realized gains, less benefit payments, and less (any) expenses, equals the book value at the end of the year, at time "1".

Formula (2) introduces the valuation interest rate i and superscripts to indicate interest adjustments from the date of the event to time "1". The interest gain, IG, is the balancing element.

Formula (3) is the asset gain and loss. Since assets reduce accrued liabilities, the asset gain and loss is taken negatively in what follows.

IMMEDIATE GAIN--UNIT CREDIT

If we consider one active participant and one retired participant, a non-contributory pension plan providing a straight life annuity at age 65--where the accrued benefit at the beginning of the year plus the known future service benefit equals the accrued benefit at the end of the year--with interest and mortality assumptions only:

$$(AB)_{x-1} + {}_{x-1}(FSB)_x = (AB)_x \quad \dots\dots (4)$$

$$(AB)_{x-1} \cdot \frac{N_{65}^{(12)}}{D_{x-1}}(1+i) + {}_{x-1}(FSB)_x \cdot \frac{N_{65}^{(12)}}{D_{x-1}}(1+i) + q_{x-1} \cdot (AB)_x \cdot \frac{N_{65}^{(12)}}{D_x}$$

$$= (AB)_x \cdot \frac{N_{65}^{(12)}}{D_x} \quad \dots\dots (5)$$

$$(AB)_{65}\cdot\frac{N_{x-1}^{(12)}}{D_{x-1}}(1+i)-(AB)_{65}\cdot\left(1+\tfrac{13}{24}i\right)+q_{x-1}\cdot(AB)_{65}\left(\tfrac{11}{24}+\frac{N_x^{(12)}}{D_x}\right)$$
$$=(AB)_{65}\cdot\frac{N_x^{(12)}}{D_x}\ldots\ldots\ldots\ldots\ldots\ldots,\quad(6)$$

Formulas (4) and (5) refer to an active participant less than age 65 at the start of the year of age. Formula (6) refers to a retired participant age 65 or older. It is possible to generalize these results over the valuation year, for all participants:

$$(AL)_0(1+i)+(FSC)_0(1+i)+EM-{}^{i}EPP=(AL)_1\ \ldots\ldots\ldots\ (7)$$

or the accrued liability and the future service cost, at the start of the year, with a full year's interest, plus the expected mortality, less the expected pension payments with appropriate interest, equals the accrued liability at the end of the year.

The introduction of status codes, at time "1", permits $(AL)_1$ in formula (7) to be restated as

$$(AL)_1=(AL)_1^A+(AL)_1^T+(AL)_1^D+(AL)_1^R\ \ldots\ldots\ldots\ldots\ (8)$$

reflecting the accrued liabilities for active, terminated, deceased, and retired participants. Subtracting the accrued liability for new entrants from both sides of formula (7) and using formula (8), the liability gain and loss is:

$$(AL)_0(1+i)+(FSC)_0(1+i)-\left[(AL)_1^A+(AL)_1^{NE}+(AL)_1^R\right]$$
$$=\left[(AL)_1^D-EM\right]+(AL)_1^T-(AL)_1^{NE}+{}^{i}EPP\ \ldots\ldots\ (9)$$

Notice the bracket on the left-hand side of formula (9) is equal to the accrued liability shown in the valuation report. The sources of liability gain and loss are: mortality, terminations, new entrants, and expected pension payments with interest. Formula (9) less formula (3) produces the net actuarial gain and its sources in terms of the unfunded liability:

$$G=(UL)_0(1+i)+(FSC)_0(1+i)-{}^{i}C-(UL)_1$$
$$=\left[(AL)_1^D-EM\right]+(AL)_1^T-(AL)_1^{NE}+\left[{}^{i}EPP-{}^{i}B\right]-{}^{i}E+IG\ .(10)$$

If formula (4) is not satisfied, the usual case, it is necessary to introduce a corrective term on the left-hand side of formula (7) which becomes a separate source of loss in formulas (9) and (10),

$$-\left[(FSC)_0'-(FSC)_0\right](1+i)\ldots\ldots\ldots\ldots\ldots\ldots.(11)$$

or the excess of the actual future service cost over the expected future service cost with one year's interest.

IMMEDIATE GAIN--INDIVIDUAL ENTRY AGE NORMAL

If we introduce a normal cost expressed as a percentage of salary, salaries, and a compound interest salary scale (1+k), the last for convenience in these developments, the development similar to formula (5) is as follows:

$$\left[(PB)_{65}\cdot\frac{N_{65}^{(12)}}{D_{x-1}}-(NC)_{EA}^{\%}\cdot(AS)_{x-1}\cdot\frac{{}^{S}N_{x-1}-{}^{S}N_{65}}{{}^{S}D_{x-1}}\right](1+i)+(NC)_{EA}^{\%}\cdot(AS)_{x-1}\cdot(1+i)$$
$$+\,q_{x-1}\left[(PB)_{65}\cdot\frac{N_{65}^{(12)}}{D_x}-(NC)_{EA}^{\%}\cdot(AS)_{x-1}\cdot(1+K)\cdot\frac{{}^{S}N_x-{}^{S}N_{65}}{{}^{S}D_x}\right]$$
$$=(PB)_{65}\cdot\frac{N_{65}^{(12)}}{D_x}-(NC)_{EA}^{\%}\cdot(AS)_{x-1}\cdot(1+K)\cdot\frac{{}^{S}N_x-{}^{S}N_{65}}{{}^{S}D_x}\quad\ldots\ (12)$$

$(PB)_{65}$ is the projected benefit, at age 65, based on the salary scale applied to actual salary at age (x-1).

All of the previous results follow. Where actual salaries differ from expected salaries, there are two possible approaches. Under the first approach, accrued liabilities and normal cost are redetermined. Under the second approach, the change in projected benefit (actual over expected) is valued and added to the normal cost:

$$\frac{\left[(PB)'_{65}-(PB)_{65}\right]\dfrac{N_{65}^{(12)}}{D_x}}{\dfrac{{}^{S}N_x-{}^{S}N_{65}}{{}^{S}D_x}}\quad\ldots\ (13)$$

Formula (13) is important, since it is the first hint of a spread mechanism applied to an element of actuarial gain or loss.

SPREAD GAIN--AGGREGATE COST

In the Individual Entry Age Normal method, if we replace the individual normal cost percentages by a constant normal cost percentage, one which makes the unfunded liability zero, we find:

$$\left[TL_1-NC_1^{\%}\,({}^{s}PV_1)\right]-BV_1=0\quad\ldots\ (14)$$

Formula (14) states that the total liability for the projected benefit, less the constant normal cost percentage applied to the present value of future salaries, less assets, equals zero, so that:

$$NC_1^{\%}=\frac{TL_1-BV_1}{{}^{s}PV_1}\quad\ldots\ (15)$$

The normal cost, in dollars, is formula (15) applied to the covered payroll of active participants.

The actuarial gain and loss can be approached either as a special case of Individual Entry Age Normal or directly from first principles. As a special case of Individual Entry Age Normal, the constant normal cost percentage at the beginning of the year applies and if we assume the contribution was made timely and was equal to the normal cost:

$$G=-(UL)_1=-\left[TL_1-NC_0^{\%}\,({}^{s}PV_1)\right]+BV_1\quad\ldots\ (16)$$

using formula (10). If the contribution is not correct, in amount or timing, this becomes part of the gain or loss in this particular method. Combining formula (16) with formula (14), we have:

$$G = (NC_0^{\%} - NC_1^{\%})\,{}^{s}PV_1 \qquad (17)$$

From this, we have the following most useful relationship:

$$NC_1^{\%} = NC_0^{\%} - \frac{G}{{}^{s}PV_1} \qquad (18)$$

Under the alternative approach, $\Delta NC_0^{\%}$ is evaluated by separately considering each element in the numerator and denominator of $NC_0^{\%}$. (See Appendix.)

SPREAD GAIN--FROZEN INITIAL LIABILITY

Entry Age Normal or Attained Age Normal, Frozen Initial Liability, follow directly from the previous results. If, in formula (14), we insist that the unfunded liability is equal to the Individual Entry Age Normal or Unit Credit unfunded liability, these methods follow. The effect is equivalent to introducing "assets" equal to the Individual Entry Age Normal or Unit Credit unfunded liability. These "assets" are treated as if they are debts to be amortized over a period of years.

Compared with the Aggregate Cost method, a distinct past service payment is introduced and the normal cost percentage is reduced. With this exception, all of the Aggregate Cost results hold.

SPREAD GAIN METHODS--SOME OBSERVATIONS

The gain and loss is handled exactly as if the method was Individual Entry Age Normal with the normal cost percentage, at the beginning of the year, used to establish the individual actuarial liabilities leading to the sources of gain and loss.

The salary scale gain or loss is established by using the projected benefit based upon actual salaries and then expected salaries. The normal cost percentage at the beginning of the year is used in both accrued liabilities.

The normal cost percentage remains unchanged over the year if actual experience equals expected experience and if the normal cost percentage, at the beginning of the year, is appropriate for new entrants (neither gain nor loss).

CONCLUSION

Projecting exact pension costs 30 to 40 or more years into the future is not possible. Less obvious, but equally true, it is not possible to choose realistic actuarial assumptions for much the same reasons. Realism itself is a short-term concept, not applicable in this connection (consider the choice of the valuation interest rate). The gain and loss analysis provides a sensible approach to the long-term problem: Understand the sources of gain and loss to date and, together with the plan sponsor, be ready to make periodic changes when appropriate.

Actuaries are not omniscient but move hesitatingly into the future, as well they should, one year at a time, adjusting the actuarial assumptions from time to time in an attempt to depict the unknown trend line of future pension costs. Beyond the assumptions, there is need for some conservatism to recognize that the pension benefit formula is variable, but increasing, over time. The objective should be to generate modest actuarial gains. The actuarial gain and loss analysis furnishes the basis for assessing such objective, as well as the basis for determining the corrective action necessary to achieve this objective.

APPENDIX

Assume interest and mortality assumptions only, actives only, and a contribution equal to the normal cost with interest.

$$NC_0 = \frac{TL_0 - BV_0}{TA_0} \quad \text{and} \quad NC_1 = \frac{TL_1 - BV_1}{TA_1}$$

where TL and BV are as before and TA is the sum of temporary annuities.

$$NC_0 + \Delta NC_0 = \frac{TL_1 - BV_1}{TA_1}$$

$$\Delta NC_0 \cdot TA_1 = TL_1 - BV_1 - NC_0 \cdot TA_1 - (TL_0 - BV_0 - NC_0 \cdot TA_0)$$

$$\Delta NC_0 \cdot TA_1 = (TL_1 - TL_0) - (BV_1 - BV_0) - NC_0 (TA_1 - TA_0)$$

$TL_1 = TL_0(1+i) + (q_0 - q_0')\, TL_1$, where prime means actual/experience.

$$BV_1 = BV_0(1+i) + NC_0(1+i) + IG$$

$$TA_1 = TA_0(1+i) - (1+i) + (q_0 - q_0')TA_1$$

$$\therefore\ \Delta NC_0 \cdot TA_1 = -(q_0' - q_0)(TL_1 - NC_0 \cdot TA_1) - IG$$

$\Delta NC_0 \cdot TA_1 = -MG - IG$, where MG = mortality gain.

$$\Delta NC_0 \cdot TA_1 = -G$$

$NC_1 = NC_0 - \dfrac{G}{TA_1}$, which is equivalent to formula (18).